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Barbero's Hamiltonian derived from a generalized Hilbert-Palatini action

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Barbero recently suggested a modification of Ashtekar's choice of canonical variables for general relativity. Although leading to a more complicated Hamiltonian constraint this modified version, in which the configuration variable still is a connection, has the advantage of being real. In this article we derive Barbero's Hamiltonian formulation from an action, which can be considered as a generalization of the ordinary Hilbert-Palatini action. [S0556-2821(96)04310-X]

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In 1986 Ashtekar presented a new pair of canonical variables for the phase space of general relativity [1]. These variables led to a much simpler Hamiltonian constraint than that in the Arnowitt-Deser-Misner (ADM) formulation [2], but had the drawback of introducing complex variables in the phase-space action—something that leads to difficulties with reality conditions which then must be imposed. A couple of years later the Lagrangian density corresponding to Ashtekar's Hamiltonian was given independently by Samuel, and by Jacobson and Smolin [3]. That was seen simply to be the Hilbert-Palatini (HP) Lagrangian with the curvature tensor replaced by its self-dual part only.

Recently Barbero pointed out that it is possible to choose a pair of canonical variables that is closely related to Ashtekar's, but in contrast to the latter, they are real [4]. The price paid is that the simplicity of Ashtekar's Hamiltonian constraint is destroyed. However, some advantages are still present with Barbero's choice of variables. For example, they provide a real theory of gravity with a connection as configuration variable, and with the usual Gauss and vector constraint, thus fitting into the class of diffeomorphism invariant theories considered in [5] in the context of quantization. In this paper we derive Barbero's result from an action, and since his formulation includes also that of ADM and Ashtekar via a parameter, the Lagrangian density used as starting point in this paper, also includes these cases. Hence we have found, in a sense, a generalized HP action.

The notation for indices adopted below is as follows. $\alpha,\beta,\gamma,...$ are used as spacetime indices whereas $a,b,c,...$ denote spatial components. t denotes the time component. $I,J,K,...$ are used as Lorentz indices and $i,j,k,...$ as spatial such. The time component of a Lorentz vector is denoted by 0.

To begin with, let us list Barbero's formulation so that we know what we are heading for. The variables are ${}^{\mathrm{Bar}}A_{ai}$ and E_{ai}, and they satisfy the fundamental Poisson brackets

$$\{{}^{\mathrm{Bar}}A_a^i(x),{}^{\mathrm{Bar}}A_b^j(y)\}_{\mathrm{PB}}=0,$$

$$\{{}^{\mathrm{Bar}}A_a^i(x),E_j^b(y)\}_{\mathrm{PB}}=-\beta\delta_j^i\delta_a^b\delta^3(x,y),$$

$$\{E_i^a(x),E_b^j(y)\}_{\mathrm{PB}}=0,$$

where β is a complex parameter. $E_i^{\,a}$ is the densitized triad and $A_i^{\,a}=\Gamma_i^{\,a}+\beta K_i^{\,a}$, where $\Gamma_i^{\,a}$ is the spin connection [see (2) below], and $K_i^{\,a}$ the extrinsic curvature. For the choice $\beta^2=1$ these variables are real and lead to the constraints

$$\partial_a E^{ai}+\epsilon^{ijk}\,{}^{\mathrm{Bar}}A_{aj}E_k^a=0,$$

$$E^{bi}F_{abi}=0,$$

$$\epsilon^{ijk}E_i^aE_j^b(F_{abk}-2R_{abk})=0, \tag{1}$$

where

$$F_{abi}=2\partial_{[a}\,{}^{\mathrm{Bar}}A_{b]i}+\epsilon_{ijk}\,{}^{\mathrm{Bar}}A_a^j\,{}^{\mathrm{Bar}}A_b^k,$$

$$R_{abi}=2\partial_{[a}\Gamma_{b]i}+\epsilon_{ijk}\Gamma_a^j\Gamma_b^k$$

$$\text{with}\quad \Gamma_{ai}=-\frac{1}{2}\,\epsilon_{ijk}e_b^j\nabla_a e^{bk} \tag{2}$$

corresponding to Gauss' law, the vector and the scalar constraints, respectively.

We now write down the action that will be the starting point in this paper. Thereafter we will motivate that it is a good canditate for an action leading to Barbero's formulation, which then will be explicitly derived from it:

$$S=\frac{1}{2}\int ee^{\alpha}{}_{I}e^{\beta}{}_{J}(F_{\alpha\beta}{}^{IJ}-\alpha{*}F_{\alpha\beta}{}^{IJ})$$

$$\equiv\frac{1}{2}\int ee^{\alpha}{}_{I}e^{\beta}{}_{J}\left(F_{\alpha\beta}{}^{IJ}-\frac{\alpha}{2}\,\epsilon^{IJ}{}_{KL}F_{\alpha\beta}{}^{KL}\right). \tag{3}$$

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Here $e_{\alpha I}$ is the tetrad, e its determinant, $F_{\alpha\beta}{}^{IJ}$ is the curvature considered as a function of the connection $A_{\alpha IJ}$, and α is a (complex) parameter which will allow us to account for all the cases mentioned above. The star (*) denotes, as is seen, the usual duality operator.

Note that if $\alpha=0$ (3) is simply the HP action, which, when 3+1 decomposes, leads to the usual ADM formulation. On the other hand, when $\alpha=i$ the integrand is $ee^{\alpha}{}_{I}e^{\beta}{}_{J}{}^{+}F_{\alpha\beta}{}^{IJ}$, where ${}^{+}F_{\alpha\beta}{}^{IJ}(A)=F_{\alpha\beta}{}^{IJ}({}^{+}A)$ denotes the self-dual part of the curvature, thus yielding Ashtekar's formulation. Our claim here is that $\alpha=1$ leads to Barbero's Hamiltonian with *his* parameter $\beta=1$, that is, formula (1). (However our α is not identical to his β, rather $\alpha=\beta^{-1}$, as we will see.)

As a first check let us study the variation of (3) with respect to $A_{\beta}{}^{IJ}$. Using

$$\delta F_{\alpha\beta}{}^{IJ}=2\mathscr{D}_{[\alpha}\delta A_{\beta]}{}^{IJ},$$

where $\mathscr{D}$ denotes the covariant derivative acting on both spacetime and Lorentz indices, we get

$$\delta S=\frac{1}{2}\int ee^{\alpha}{}_{I}e^{\beta}{}_{J}\left(\delta F_{\alpha\beta}{}^{IJ}-\frac{\alpha}{2}\epsilon^{IJ}{}_{KL}\delta F_{\alpha\beta}{}^{KL}\right)$$
$$=\int \delta B_{\beta}{}^{IJ}\mathscr{D}_{\alpha}(ee^{\alpha}{}_{I}e^{\beta}{}_{J}), \qquad (4)$$

where a partial integration was performed, and where

$$\delta B_{\beta}{}^{IJ}\equiv\delta A_{\beta}{}^{IJ}-\alpha*\delta A_{\beta}{}^{IJ}.$$

If $\alpha\neq\pm i$ one easily finds $\delta A=\delta A(\delta B)$ from this, and hence one can choose an arbitrary variation δB in (4). For $\alpha=\pm i$, that is, Ashtekar's case, $\delta B_{\beta}{}^{IJ}$ clearly is the self-dual (anti-self-dual) part of $\delta A_{\beta}{}^{IJ}$ and the action contains only ${}^{\pm}F_{\alpha\beta}{}^{IJ}(A)=F_{\alpha\beta}{}^{IJ}({}^{\pm}A)$. Then we can choose to vary S with respect to ${}^{\pm}A_{\beta}{}^{IJ}$ instead. So in either case (4) implies

$$\mathscr{D}_{\alpha}(ee^{\alpha}{}_{[I}e^{\beta}{}_{J]})=0,$$

which gives (see, for example, [6])

$$A_{\alpha IJ}=e_{\beta I}\nabla_{\alpha}e^{\beta}{}_{J}. \qquad (5)$$

Hence, if (3) is considered as a first-order action it implies, exactly as the ordinary HP action does, that $A_{\alpha IJ}$ is the Levi-Civita spin connection.

Before we perform the 3+1 decomposition of (3) we convince ourselves that it gives the right theory, that is, general relativity, for all complex values of α, and not only for $\alpha=0$ or $\alpha=i$ when it is known to do that. Study the second term in the action:

$$ee^{\alpha}{}_{I}e^{\beta}{}_{J}\epsilon^{IJKL}F_{\alpha\beta KL}=e_{\gamma}{}^{K}e_{\delta}{}^{L}\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta KL}=\epsilon^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}. \qquad (6)$$

Here $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor, and (5) was used in the last step, meaning that this is only true when the evolution equations are used. But this is clearly equal to zero, since $R_{[\alpha\beta\gamma]}{}^{\delta}=0$.

Thus (3) differ from the case $\alpha=0$, that is ADM, by at most a canonical transformation, so we are indeed working with the right theory. Of course, for this conclusion to be valid it would suffice if (6) was a total derivative—it does not have to be zero. But in our case it should be, since the canonical transformation that we are heading for is of the form

$$A_a^i\rightarrow A_a'^i+\gamma\,\frac{\delta f[E]}{\delta E_i^a},$$
$$E_i^a\rightarrow E_i'^a=E_i^a, \qquad (7)$$

where γ is some parameter, and

$$f[E]=\int\Gamma_a^i[E]E_i^a \quad \text{with} \quad \Gamma_{ai}=-\frac{1}{2}\epsilon_{ijk}e_b^j\nabla_a e^{bk}, \qquad (8)$$

Γ thus being the (spatial) Levi-Civita spin connection. Then the functional derivative of $f[E]$ equals Γ, and a simple calculation shows that

$$\int \dot{A}_a'^i E_i'^a=\int \dot{A}_a^i E_i^a.$$

Thus all transformations of the form (7) correspond to no change in the Lagrangian density.

Now that we have collected enough confidence that (3) is the action we are looking for, let us do the 3+1 decomposition to verify that this indeed is the case. We write the time component of the tetrad as

$$e_{0I}=Nn_I+N^a e_{aI}, \qquad (9)$$

where n_I is the normalized gradient to the time coordinate function defined on the spacetime, and hence orthogonal to surfaces with $t=$const. More precisely $n^I e_{aI}=0$ and $n^I n_I=-1$. N and N^a are the usual lapse and shift, respectively. Now we choose the so-called "time gauge:" We choose the tetrad in such a way that $n_I=(1,0,0,0)$. This simply means that the spatial vectors of the tetrad $e_{\alpha i}$ now span the tangent space to a $t=$const surface, and that $e_{a0}=0$. Of course, this gauge choice does not put any restrictions on the ADM metric itself.

With the time gauge imposed, the 3+1 decomposition of (3) looks like

$$S=\frac{1}{2}\int\epsilon^{abc}\epsilon^{ijk}e_{ai}(e_{bj}\hat{F}_{ctk0}+N^d e_{dj}\hat{F}_{bck0}+\tfrac{1}{2}N\hat{F}_{bcjk}), \qquad (10)$$

where

$$\hat{F}_{ctk0}=2\partial_{[c}A_{t]k0}+2A_{[c|k|}{}^{m}A_{t]m0}-\alpha\epsilon_{kmn}(\partial_{[c}A_{t]}{}^{mn}+A_{[c}{}^{mp}A_{t]p}{}^{n}+A_{[c}{}^{m0}A_{t]0}{}^{n}),$$

$$\hat{F}_{bck0}=2\partial_{[b}A_{c]k0}+2A_{[b|k|}{}^{m}A_{c]m0}-\alpha\epsilon_{kmn}(\partial_{[b}A_{c]}{}^{mn}+A_{[b}{}^{mp}A_{c]p}{}^{n}+A_{[b}{}^{m0}A_{c]0}{}^{n}),$$

$$\hat{F}_{bcjk}=2\partial_{[b}A_{c]jk}+2A_{[b|j|}{}^{m}A_{c]mk}+2A_{[b|j|}{}^{0}A_{c]0k}-2\alpha\epsilon_{jkm}(\partial_{[b}A_{c]}{}^{m0}+A_{[b}{}^{mp}A_{c]p}{}^{0}), \qquad (11)$$

and where $\epsilon^{abc}\equiv\epsilon^{tabc}$, $\epsilon^{ijk}\equiv\epsilon^{0ijk}$, and $\epsilon_{ijk}\equiv\epsilon_{0ijk}$.

The only terms in (10) containing time derivatives are

$$-E^{ck}\partial_t\left(A_{ck0}-\frac{\alpha}{2}\,\epsilon_{kmn}A_c^{mn}\right),$$

where the useful identity

$$\frac{1}{2}\,\epsilon^{abc}\epsilon^{ijk}e_{ai}e_{bj}=ee^{ck}\equiv E^{ck}$$

was used. This motivates the introduction of new variables:

$${}^{+}\mathcal{A}_{ck}\equiv A_{ck0}+\frac{\alpha}{2}\,\epsilon_{kmn}A_c^{mn},$$

$${}^{-}\mathcal{A}_{ck}\equiv A_{ck0}-\frac{\alpha}{2}\,\epsilon_{kmn}A_c^{mn}, \tag{12}$$

with the inverse

$$A_{ck0}=\frac{1}{2}\,({}^{+}\mathcal{A}_{ck}+{}^{-}\mathcal{A}_{ck}),$$

$$A_c^{ij}=\frac{1}{2\alpha}\,\epsilon^{kij}({}^{+}\mathcal{A}_{ck}-{}^{-}\mathcal{A}_{ck}). \tag{13}$$

Substituting $(A_{ck0},A_c^{\ mn})$ for $({}^{+}\mathcal{A}_{ck},{}^{-}\mathcal{A}_{ck})$ in (11) gives

$$\hat{F}_{ctk0}=-\partial_t\,{}^{-}\mathcal{A}_{ck}+\partial_c\left(A_{tk0}-\frac{\alpha}{2}\,\epsilon_{kmn}A_t^{mn}\right)$$
$$+\epsilon_{klm}A_{t0}{}^{m}\left(\frac{\alpha^2+1}{2\alpha}\,{}^{+}\mathcal{A}_c^{l}+\frac{\alpha^2-1}{2\alpha}\,{}^{-}\mathcal{A}_c^{l}\right)$$
$$-A_{tkl}\,{}^{-}\mathcal{A}_c^{l},$$

$$\hat{F}_{bck0}=2\partial_{[b}{}^{-}\mathcal{A}_{c]k}-\frac{\alpha^2+1}{4\alpha}\,\epsilon_{klm}{}^{+}\mathcal{A}_{[b}{}^{l}\,{}^{+}\mathcal{A}_{c]}{}^{m}$$
$$+\frac{-\alpha^2+3}{4\alpha}\,\epsilon_{klm}{}^{-}\mathcal{A}_{[b}{}^{l}\,{}^{-}\mathcal{A}_{c]}{}^{m}$$
$$-\frac{\alpha^2+1}{2\alpha}\,\epsilon_{klm}{}^{+}\mathcal{A}_{[b}{}^{l}\,{}^{-}\mathcal{A}_{c]}{}^{m},$$

$$\hat{F}_{bc}{}^{jk}=\epsilon^{ljk}\partial_{[b}\left(\frac{\alpha^2+1}{\alpha}\,{}^{+}\mathcal{A}_{c]l}+\frac{\alpha^2-1}{\alpha}\,{}^{-}\mathcal{A}_{c]l}\right)$$
$$-\frac{\alpha^2+1}{2\alpha^2}\,{}^{+}\mathcal{A}_{[b}{}^{j}\,{}^{+}\mathcal{A}_{c]}{}^{k}+\frac{3\alpha^2-1}{2\alpha^2}\,{}^{-}\mathcal{A}_{[b}{}^{j}\,{}^{-}\mathcal{A}_{c]}{}^{k}$$
$$+\frac{\alpha^2+1}{\alpha^2}\,{}^{+}\mathcal{A}_{[b}{}^{[j}\,{}^{-}\mathcal{A}_{c]}{}^{k]}.$$

From this we see that ${}^{-}\mathcal{A}_{ai}$ is the dynamical variable and E^{ai} its conjugated momentum. Nondynamical variables, apart from the lapse N and the shift N^d, are A_{ti0}, A_{tij}, and ${}^{+}\mathcal{A}_{ai}$. Note that if $\alpha=\pm i$ all terms involving ${}^{+}\mathcal{A}_{ai}$ disappear, and the constraint below implied by variation of ${}^{+}\mathcal{A}_{ai}$ does not exist. This simplifies things a lot, and, in fact, Ashtekar's Hamiltonian follows almost immediately. However, for general α we get the following constraints when varying the Lagrangian density with respect to A_{ti0}, A_{tij}, and ${}^{+}\mathcal{A}_{ai}$, respectively:

$$\frac{\partial\mathcal{L}}{\partial A_{tk0}}=-\partial_cE^{ck}-\epsilon^{ilk}E_i^c\left(\frac{\alpha^2+1}{2\alpha}\,{}^{+}\mathcal{A}_{cl}+\frac{\alpha^2-1}{2\alpha}\,{}^{-}\mathcal{A}_{cl}\right)=0,$$

$$\frac{\partial\mathcal{L}}{\partial A_{tmn}}=\frac{\alpha}{2}\,\epsilon^{kmn}\partial_cE^c{}_k-E^{c[m}\,{}^{-}\mathcal{A}_c{}^{n]}=0,$$

$$\frac{\partial\mathcal{L}}{\partial\,{}^{+}\mathcal{A}_c^{l}}=\frac{\alpha^2+1}{2\alpha}\,\epsilon_{klm}E^{ck}A_{t0}{}^{m}+\frac{\alpha^2+1}{4\alpha}\,\epsilon^{abc}\epsilon^{ijk}\epsilon_{klm}e_{ai}N^de_{dj}({}^{+}\mathcal{A}_b^{m}+{}^{-}\mathcal{A}_b^{m})-\frac{\alpha^2+1}{4\alpha}\,\epsilon^{abc}\epsilon^{ijk}\epsilon_{ljk}\partial_b(Ne_{ai})$$
$$+\frac{\alpha^2+1}{4\alpha^2}\,\epsilon^{abc}\epsilon_{ilk}Ne_a^i({}^{+}\mathcal{A}_b^{k}-{}^{-}\mathcal{A}_b^{k})=0.$$

A lengthy manipulation of these yields

$${}^{+}\mathcal{A}_{ai}={}^{-}\mathcal{A}_{ai}-2\alpha\Gamma_{ai}, \tag{14}$$

where Γ_{ai} is defined as in (8), and they also determine $A_{ti0}=A_{ti0}({}^{-}\mathcal{A},\Gamma,N,N_a)$ which will show up as (part of) a Lagrange multiplier. Equation (14) is naturally taken as a second-class constraint in Dirac's notation, since trivially we have for the conjugated momenta ${}^{+}\Psi_{ai}$ to ${}^{+}\mathcal{A}_{ai}$ that ${}^{+}\Psi_{ai}=0$, and then $\{{}^{+}\mathcal{A}_{ai}-{}^{-}\mathcal{A}_{ai}+2\alpha\Gamma_{ai},{}^{+}\Psi_{ai}\}_{\rm PB}=1$ using the naive canonical Poisson brackets. Hence (14) should be inserted into the action. This gives, after some algebraic work,

$$S=\int[-{}^{-}\dot{\mathcal{A}}_{ck}E^{ck}+\alpha\Lambda_k(\partial_cE^{ck}+\alpha^{-1}\epsilon^{kli}\,{}^{-}\mathcal{A}_{cl}E_i^c)$$
$$+N^dE^{ck}\widetilde{F}_{dck}+N\,\tfrac{1}{2}\,\epsilon^{abc}e_{ai}[\alpha\widetilde{F}_{bc}{}^{i}-(1+\alpha^2)R_{bc}{}^{i}]], \tag{15}$$

where R_{abi} is given by (2) and

$$^{-}\mathcal{A}_{ai}=A_{ai0}-\frac{\alpha}{2}\,\epsilon_{ijk}A_a^{jk},$$

$$\Lambda_k=\alpha A_{tk0}+\frac{1}{2}\,\epsilon_{kmn}A_t^{mn},$$

$$\widetilde{F}_{abi}=2\partial_{[a}\,{}^{-}\mathcal{A}_{b]i}+\alpha^{-1}\epsilon_{ijk}\,{}^{-}\mathcal{A}_a^j\,{}^{-}\mathcal{A}_b^k.$$

Note that if one uses the definition (12) of $^{\pm}\mathcal{A}_{cl}$ in (14) one obtains

$$A_{aij}=-\epsilon_{ijk}\Gamma^k_{\ a},$$

that is, A_{aij} is simply the spatial Levi-Civita spin connection. Hence, what we effectively have done is to solve for the rotational (spatial) part of the connection from the evolution equations and to reinsert it into the action.

Before we comment on the terms in (15) we should say something about how our dynamical variable $^{-}\mathcal{A}_{ai}$ is connected with the dynamical variables used by Ashtekar and Barbero. From the fact that $A_{\alpha IJ}$ is the Levi-Civita spin connection, that is (5), we have

$$A_{ai0}=e_i^{\beta}\nabla_a e_{\beta 0}\equiv K_{ai},$$

which in fact is nothing but the extrinsic curvature of a t=const surface. Hence, our dynamical variable $^{-}\mathcal{A}_{ai}$ can be written

$$^{-}\mathcal{A}_{ai}=A_{ai0}-\frac{\alpha}{2}\,\epsilon_{ijk}A_a^{jk}=K_{ai}+\alpha\Gamma_{ai},$$

which should be compared with Barbero's dynamical variable

$$^{\mathrm{Bar}}A_{ai}=\Gamma_{ai}+\beta K_{ai}.$$

From this we see that

$$\alpha=\beta^{-1}\quad\text{and}\quad {}^{-}\mathcal{A}_{ai}=\beta^{-1}\,{}^{\mathrm{Bar}}A_{ai}=\alpha\,{}^{\mathrm{Bar}}A_{ai}. \qquad (16)$$

Note that for $\alpha=i$, $^{-}\mathcal{A}_{ai}$ and $^{\mathrm{Bar}}A_{ai}$ equals Ashtekar's dynamical variable up to a factor

$$^{-}\mathcal{A}_{ai}|_{\alpha=i}=A_{\alpha i0}-\tfrac{1}{2}\epsilon_{ijk}A_a^{jk}=2\,{}^{+}A_{ai0}=-{}^{\mathrm{Ash}}A_{ai},$$

and hence

$$^{\mathrm{Bar}}A_{ai}|_{\alpha=i}=-i\,{}^{-}\mathcal{A}_{ai}=i\,{}^{\mathrm{Ash}}A_{ai}.$$

Putting $\alpha=1$ we recognize the four terms in (15) as the kinetical term "$\dot{q}p$," Gauss' law constraint, the vector constraint and the scalar constraint in Barbero's formulation (1). Furthermore, one easily finds the ADM or the Ashtekar formulation by putting $\alpha=0$ or $\alpha=i$, respectively, in (15).

To summarize, we have shown that the action (3) with $\alpha=\beta^{-1}$ is the one corresponding to Barbero's formulation. This was effectively done by solving for the rotational part of the spin connection $A_{\alpha IJ}$ from the evolution equations, and by inserting this result into the action. For $\alpha=0$ the action (3) is the HP action, leading to ADM when 3+1 decomposed, and for $\alpha=i$ it is the self-dual part of the HP action, leading to Ashtekar's Hamiltonian. Hence it could be looked upon as a nice generalization of the HP action containing Ashtekar's formulation as a special case.

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