Casimir scaling versus Abelian dominance in QCD string formation

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We show that the hypothesis of Abelian dominance in the maximal Abelian gauge, which is known to work for Wilson loops in the fundamental representation, fails for Wilson loops in higher group representations. Monte Carlo simulations are performed on lattice SU(2) gauge theory, in $D=3$ dimensions, in the maximal Abelian gauge, in the confined phase. It is well known that Creutz ratios extracted from loops in various group representations are proportional to the quadratic Casimir invariant of each representation, in a distance interval from the confinement scale to the point where color screening sets in. In contrast, we find numerically, in the same interval, that string tensions extracted from loops built from Abelian projected configurations are the same for the fundamental and $j=3/2$ representations, and vanish for the adjoint representation. In addition, we perform a lattice Monte Carlo simulation of the Georgi-Glashow model in $D=3$ dimensions. We find that the representation dependence of string tensions is that of pure Yang-Mills theory in the symmetric phase, but changes abruptly to equal tensions for the $j=1/2$, 3/2 representations, and zero tension for $j=1$, at the transition to the Higgs phase. Our results indicate that an effective Abelian theory at the confinement scale, invoking *only* degrees of freedom (monopoles and photons) associated with a particular Cartan subalgebra, is inadequate to describe the actual interquark potential in an unbroken non-Abelian gauge theory. [S0556-2821(96)05910-3]

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I. INTRODUCTION

Many years ago, in a very influential paper [1], Polyakov demonstrated quark confinement in the Higgs phase of the $D=3$ Georgi-Glashow model, the mechanism being condensation of monopoles in the unbroken U(1) subgroup. It is natural to suppose that such a mechanism might also explain quark confinement in the symmetric phase of non-Abelian gauge theories, in $D=4$ as well as $D=3$ dimensions, and that the effective theory at the confinement scale and beyond is essentially Abelian, i.e., compact QED. The most explicit version of this idea is the Abelian projection theory due to 't Hooft [2], where a special gauge-fixing condition on the gauge fields, rather than the Higgs field, is used to single out an Abelian subgroup of the full gauge group. For an SU(N) theory, 't Hooft's Abelian projection gauge-fixing leaves an unbroken $U(1)^{N-1}$ subgroup; condensation of the magnetic monopoles associated with this subgroup is the conjectured confinement mechanism. This picture is one possible realization of the idea of dual superconductivity in non-Abelian gauge theories, as originally proposed by 't Hooft [3] and Mandelstam [4].

In the $D=3$ Georgi-Glashow (GG_3) model in the Higgs phase, Polyakov computed the area law contribution to Wilson loops in terms of an effective Abelian theory, invoking only the monopoles and ''photons'' associated with the unbroken U(1) gauge group. The Abelian gauge field (A^3_μ, say) is singled out by a unitary gauge choice, and for the calculation of the string tension (in this theory) it is a reasonable approximation to ignore the contribution of the other color components, i.e.,

$$\langle W(C)\rangle=\left\langle \mathrm{Tr}\exp\left(i\oint dx^\mu A^a_\mu \tau_a\right)\right\rangle \sim\left\langle \mathrm{Tr}\exp\left(i\oint dx^\mu A^3_\mu \tau_3\right)\right\rangle, \tag{1}$$

where $\tau_a=(1/2)\sigma_a$. The same approximation, in the context of 't Hooft's theory, has come to be known as ''Abelian dominance'' [5].

In this work we address the question of whether Abelian dominance, which implies the existence of an effective Abelian theory of monopoles and photons at large scales, is adequate to describe the infrared dynamics of $D=3$ Yang-Mills theory, in the maximal Abelian gauge. Our tool for studying this question will be Wilson loops in higher group representations. It should be noted, at the outset, that we are *not* addressing the possible relevance or irrelevance of monopoles, or the validity of dual-superconductor pictures in general. Our investigation is limited to one issue only, namely, are vacuum fluctuations, at the confinement scale and beyond, dominated by fluctuations in the gauge field associated with a Cartan subalgebra of the gauge group, as is the case for GG_3 in the Higgs phase? In particular, the question here

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is not whether magnetic monopoles, defined with respect to an Abelian projection gauge, are condensed (evidence for condensation of such monopoles is found in [6]; condensation of these and perhaps other types of field configurations is not unexpected in a magnetic-disordered vacuum state). Rather, the issue we address is whether the fluctuations of the corresponding (Cartan subalgebra) gauge field dominate the large-scale vacuum fluctuations, justifying the use of Eq. (1).

There are a number of reasons to believe in Abelian dominance for pure Yang-Mills theory in maximal Abelian gauge. There are, for example, several kinematical similarities between that theory, and GG_3 in the Higgs phase. First, in both cases, the underlying SU(2) symmetry is reduced to a U(1) symmetry by a gauge choice: the unitary gauge in GG_3, and the maximal-Abelian gauge [7] for pure Yang-Mills theory. Second, magnetic monopoles can be identified in both theories, associated with the remaining U(1) symmetry. Third, on the lattice, one finds in both cases that most of the quantum fluctuations of the link variables are in the A^3_μ degrees of freedom. Apart from these kinematical similarities, it is reasonable to suppose that if Abelian monopoles are the crucial confining configurations, then a truncation to the associated A^3_μ degrees of freedom (Abelian dominance) would retain the essential features of magnetic disorder and flux-tube formation. In support of this supposition, Monte Carlo simulations have found that the Abelian dominance approximation, i.e., Eq. (1), accurately reproduces the string tension for Wilson loops in the fundamental representation of the gauge group [5].

However, the fundamental representation is not the only group representation, and Wilson loops in higher group representations may also have a tale to tell. In particular let us recall the suggestion, made many years ago, that the string tension of planar Wilson loops in $D=3$ and $D=4$ dimensions could be computed from an effective two-dimensional gauge theory. This suggestion, known as ''dimensional reduction,'' was put forward independently (and for quite different reasons) in [8,9], and some numerical evidence for the idea, based on a Monte Carlo evaluation of loop spectral densities, was presented by Belova *et al.* in [10]. It was Ambjørn, Olesen, and Peterson, in [11], who noticed that dimensional reduction implies that the ratio of string tensions between quarks in different group representations should equal the ratio of the corresponding quadratic Casimir invariants, since this can be shown to be true in two dimensions. In particular, for SU(2) lattice gauge theory at weak couplings, the prediction is

$$\frac{\chi_j[I,J]}{\chi_{1/2}[I,J]}=\frac{4}{3}\,j(j+1), \tag{2}$$

where $\chi_j[I,J]$ is the Creutz ratio for Wilson loops in the $j=0,1/2,1,3/2,\ldots$ representations. These authors tested the above prediction numerically, in both $D=3$ and $D=4$ dimensions, and found it to be accurate to within 10%. Their results have since been confirmed, for larger loops and with better statistics, by a number of other studies in both three and four dimensions [12]. Similar results have also been obtained in SU(3) gauge theory [13]. Of course, this ''Casimir scaling'' of string tensions cannot hold at arbitrarily large distances, since at some distance the screening of heavy quark charges by gluons will become energetically favorable, reducing the effective charge. Numerical simulations indicate, however, that there is a large-distance interval, between the onset of confinement and the onset of charge screening, where Casimir scaling of string tensions holds quite accurately.[1] It is reasonable to demand that any theory of quark confinement, which purports to explain the behavior of gauge fields beyond the confinement scale, should account for the observed Casimir scaling of interquark forces in this interval.

Does the hypothesis of Abelian dominance allow for the existence of Casimir scaling? According to a simple heuristic argument, found in [14], the answer is probably no. Instead, beginning at the onset of confinement, one expects

$$\begin{aligned}\chi_j&=\chi_{1/2}, \quad j=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\ldots,\\ \chi_j&=0, \qquad j=1,2,3,\ldots,\end{aligned} \tag{3}$$

for an SU(2) gauge theory. We refer to the expectations of Eq. (3) as the ''Abelian monopole prediction.'' We then test this prediction numerically in two cases where one may be fairly sure that Abelian monopole configurations give the crucial contributions: (1a) The calculation of Creutz ratios in lattice $D=3$ Yang-Mills theory, using ''Abelian-projected'' lattice configurations obtained in maximal Abelian gauge; and (2a) the calculation of Creutz ratios in the lattice $D=3$ Georgi-Glashow model in the Higgs phase.

The results for these two cases are compared with (1b) the actual Creutz ratios in lattice $D=3$ Yang-Mills theory, obtained from the full, unprojected lattice configurations; and (2b) Creutz ratios in the lattice $D=3$ Georgi-Glashow model in the symmetric phase.

It will be found that cases (1a) and (2a) agree quite well with the Abelian monopole prediction, and utterly disagree with the corresponding cases (1b) and (2b), which instead follow the predictions of dimensional reduction. This has two consequences. First, it means that in the case of GG_3 in the Higgs phase, where it is known that an effective U(1) theory describes the infrared dynamics, the monopole prediction is verified. Second, in the case of pure Yang-Mills theory, where Casimir scaling is observed, the Abelian dominance approximation has failed entirely.

Before proceeding to discuss the simulations, let us first recall the heuristic argument leading to the Abelian monopole prediction (3). Suppose that, in an SU(2) gauge theory, the area law for Wilson loops is due to fluctuations of the gauge field A^3_μ, associated with a remaining U(1) symmetry. This U(1) symmetry is assumed to be singled out either by an Abelian-projection gauge choice (as in 't Hooft's theory), or by a unitary gauge choice ($D=3$ Georgi-Glashow model). In that case, we would have

[1] In fact, it is not even clear that color screening has been seen yet, in lattice Monte Carlo simulations of $D=3$ Yang-Mills theory, inside the scaling region (cf. Poulis and Trottier in [12]).

$$\langle W_j(C)\rangle = \left\langle \mathrm{Tr}\,\exp\left(i \oint dx^\mu A^a_\mu T^j_a \right) \right\rangle$$
$$\sim \left\langle \mathrm{Tr}\,\exp\left(i \oint dx^\mu A^3_\mu T^j_3 \right) \right\rangle$$
$$\sim \sum_{m=-j}^{j} \left\langle \exp\left(im \oint dx^\mu A^3_\mu \right) \right\rangle, \tag{4}$$

where the T^j_a are the SU(2) group generators in the j representation. If an area law is obtained from Abelian configurations, this is presumably due to monopole effects. Following Polyakov's analysis [1], one then expects

$$\langle W_j(C)\rangle \sim \sum_{m=-j}^{j} \exp[-\mu_m \mathrm{Area}(C)]. \tag{5}$$

The μ_m will increase with the magnitude of the U(1) charge, which is given by $|m|$ (for $m=0$, $\mu_0=0$). The above sum would then be dominated by those terms which are falling most slowly with increasing area, i.e., $m=\pm 1/2$ for $j=$half-integer, and $m=0$ for $j=$integer. In this way, we arrive at the monopole prediction (3).

Now the behavior (3) is, in fact, what one expects asymptotically, due to charge screening. The problem, however, is that according to the argument above this behavior actually begins right at the confinement scale, and has nothing whatever to do with the physics of charge screening.[2] The fact that adjoint loops are unconfined, in the Abelian projection theory, is simply due to the fact that the $m=0$ component of an adjoint charge is neutral (and thereby unconfined) with respect to the remaining U(1) symmetry. The $m=0$ contribution therefore dominates the sum in (5).[3] A flux tube between adjoint quarks does not form and then break due to charge screening; in this picture the tube does not form at all. As already mentioned, this conclusion appears to be contradicted by the numerical evidence presented in [11–13], which find a force between adjoint quarks which is about 8/3 that of the fundamental quarks, over a fairly large-distance interval in the confinement regime.

The Abelian monopole prediction, however, is based on a heuristic argument; it could be that there is some subtlety of monopole dynamics that we have missed. Let us turn, then, to the numerical simulations.

II. BREAKDOWN OF ABELIAN DOMINANCE

We perform Monte Carlo simulations of $D=3$ lattice SU(2) gauge theory, at lattice coupling $\beta=5$, which is just inside the scaling regime. Maximal Abelian gauge fixing, which maximizes the quantity

$$Q_{\mathrm{sum}} = \sum_{x,\mu} \mathrm{Tr}[U_\mu(x)\sigma^3 U^\dagger_\mu(x)\sigma^3] \tag{6}$$

is implemented. Wilson loops in the fundamental ($j=1/2$), adjoint ($j=1$), and $j=3/2$ representations, normalized to a maximum value of one, are given by

$$W_{1/2}(C) = \tfrac{1}{2}\mathrm{Tr}[UUU\cdots U],$$
$$W_1(C) = \tfrac{1}{3}[4W^2_{1/2}(C) - 1],$$
$$W_{3/2}(C) = \tfrac{1}{4}[8W^3_{1/2}(C) - 4W_{1/2}(C)]. \tag{7}$$

We calculate the expectation values of these loops using both the full link configurations (for which the gauge fixing is irrelevant), and also using the Abelian-projected link configurations (or ''Abelian links''). For a full SU(2) link matrix, represented by

$$U = a_0 I + i\sum_{k=1}^{3} a_k \sigma^k, \tag{8}$$

the corresponding Abelian link U' is given by a truncation to the diagonal component, followed by a rescaling to restore unitarity, i.e.,

$$U \rightarrow U' = \frac{a_0 I + i a_3 \sigma^3}{\sqrt{a_0^2 + a_3^2}}. \tag{9}$$

Wilson loops of the Abelian-projected configurations are obtained by inserting the Abelian links (9) into Eq. (7), and the corresponding Creutz ratios are computed in the usual way.

Our simulation involved 100 000 sweeps of a 12^3 lattice at $\beta=5$, comprising 10 000 thermalization sweeps, with data taken every tenth of the remaining sweeps. Figure 1 shows the ratios of Creutz ratios

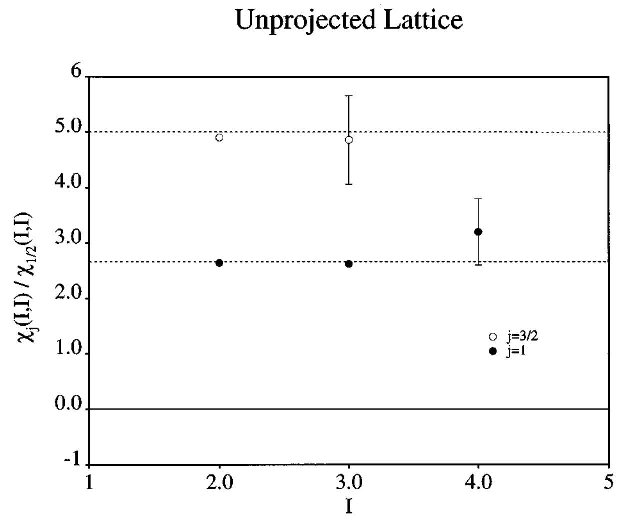

FIG. 1. The ratio of Creutz ratios $\chi_j[I,I]/\chi_{1/2}[I,I]$, for $j=1$ (solid circles) and $j=3/2$ (open circles), in $D=3$ lattice SU(2) gauge theory at $\beta=5$. Dashed lines show the corresponding ratio of quadratic Casimir invariants (8/3 for $j=1$, and 5 for $j=3/2$).

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[2] We have emphasized this lack of connection to charge screening in the Abelian projection theory in a previous publication, which was mainly concerned with large-N behavior [15]. In the present work, we turn our attention to $N=2$.

[3] Of course, if one would simply toss out the $m=0$ contribution, then $W_j(C)$ *would* decay exponentially with the area. But we can see no justification for such a procedure.

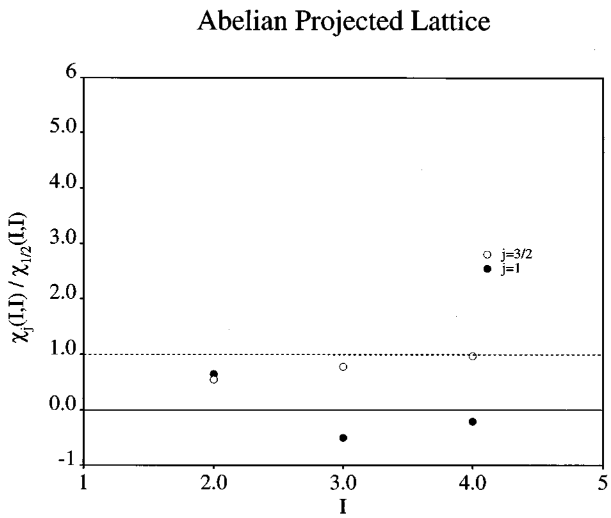

FIG. 2. Same as Fig. 1, except that Creutz ratios have been computed using the Abelian-projected lattice configurations in maximal Abelian gauge.

$$\frac{\chi_1[I,I]}{\chi_{1/2}[I,I]} \quad \text{and} \quad \frac{\chi_{3/2}[I,I]}{\chi_{1/2}[I,I]} \tag{10}$$

for $I=2,3,4$. The agreement with Casimir scaling (8/3 and 5, respectively) is fairly good, as found in previous studies [11,12]. Figure 2 shows the same ratio of Creutz ratios, for the same loop sizes, but this time computed with Abelian-projected configurations. It is clear that Figs. 1 and 2 display completely different behavior. In the Abelian projection, the adjoint Creutz ratio actually goes negative at $I=3$; the adjoint tension is consistent with zero at $I=4$, as predicted by (3). Likewise, $\chi_{3/2}[I,I]$ appears to converge to $\chi_{1/2}[I,I]$, again as expected from the Abelian monopole prediction. However, this behavior of the Abelian-projected loops is clearly inconsistent with the corresponding behavior of the full Wilson loops. Evidently, for higher-representation Wilson loops, Abelian dominance has failed entirely.

III. *D*=3 LATTICE GEORGI-GLASHOW MODEL

Polyakov's seminal work [1] was concerned with the Higgs phase of the Georgi-Glashow model in $D=3$ dimensions. Because of this work, we may be confident that the confinement mechanism in the Higgs phase is due to monopole condensation. In that case one may ask: is the monopole prediction (3) for higher representations confirmed? And does this prediction also hold in the symmetric phase?

There have been a number of lattice Monte Carlo simulations of this model, both in $D=3$ [16,17] and $D=4$ [7,18] dimensions, and these have been mainly concerned with finding the phase diagram of the theory. To our knowledge, there has been no study of the behavior of Wilson loops, as one goes across the symmetry-breaking transition. We have therefore carried out such a calculation. However, as there is a three-dimensional coupling constant space for the lattice Georgi-Glashow model, we have not attempted to compute the Wilson loop behavior throughout the phase diagram. Instead, we have only computed loops along a particular line of the coupling constant space, which crosses from the symmetric to the Higgs phase. We believe the behavior that we find for Creutz ratios is typical, as the system goes across the Higgs transition, but of course this will have to be verified by a more extensive study.

The lattice action of the Georgi-Glashow model is

$$\begin{aligned} S=&\frac{1}{2}\beta_G\sum_{plaq}\mathrm{Tr}[UUU^\dagger U^\dagger]\\ &+\frac{1}{2}\beta_H\sum_{n,\mu}\mathrm{Tr}[U_\mu(n)\phi(n)U_\mu^\dagger(n)\phi^\dagger(n+\mu)]\\ &-\sum_n\left\{\frac{1}{2}\mathrm{Tr}[\phi\phi^\dagger]+\beta_R\left(\frac{1}{2}\mathrm{Tr}[\phi\phi^\dagger]-1\right)^2\right\}, \end{aligned} \tag{11}$$

where the adjoint Higgs field $\phi(n)$ has three degrees of freedom per lattice site

$$\phi(n)=i\sum_{a=1}^{3}\phi^a(n)\sigma_a. \tag{12}$$

In performing the Monte Carlo simulations it is useful to go to a unitary gauge where $\phi(n)=i\rho(n)\sigma_3$, reducing the degrees of freedom of the Higgs field from three to one per site. The details may be found in [16].

To map out the phase structure of the theory in $D=3$ dimensions, we compute the following observables: (1) the rms value of the Higgs field

$$R=\langle\mathrm{Tr}[\phi\phi^\dagger]\rangle^{1/2}, \tag{13}$$

(2) the value

$$Q=\tfrac{1}{2}\langle\mathrm{Tr}[U_\mu(n)\sigma^3U_\mu^\dagger(n)\sigma^3]\rangle \tag{14}$$

in unitary gauge. A jump in these two quantities is an indication of a transition from the symmetric phase to the Higgs phase.

We begin by looking for a region of couplings where it is possible to see a (fundamental) string tension in both the symmetric and Higgs phases. The strategy we have chosen is to keep β_G and β_R fixed, and vary β_H. One would like to use a value of β_G where the pure gauge theory is in the scaling regime, i.e., $\beta_G\geq5$. In practice, however, we have not been able to detect a string tension in the Higgs phase at such large values of β_G. Since presumably there *must* be a string tension in the Higgs phase in $D=3$ dimensions, we interpret this result as meaning that the monopole is quite heavy, in lattice units, at the larger values of β_G, and therefore the confinement scale (in the Higgs phase) probably lies beyond the size of our lattice.[4] So we have been forced to go to a rather small value of β_G, using $\beta_G=2$ throughout. A fixed (and rather arbitrary) value of $\beta_R=0.01$ was also chosen; this was mainly in order to compare our values for the location of the phase transition with those in [16]. Simulations in the region of the transition were run on a 12^3 lattice with a total

[4]A related observation has been made by Laursen and Müller-Preussker in [17], who noted that monopoles in the Higgs phase, at $\beta_G=5$, are very dilute.

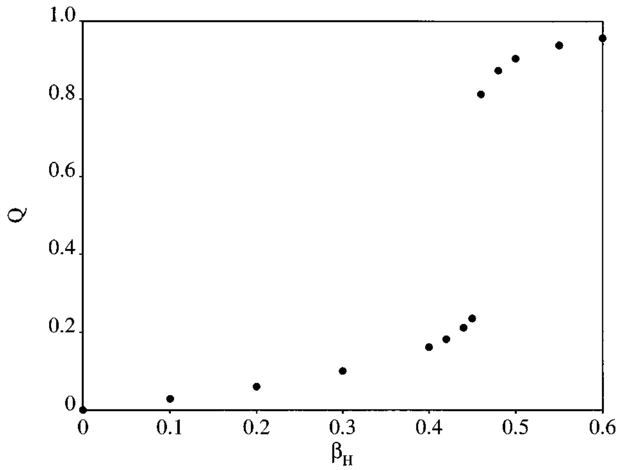

FIG. 3. Variation of the Q parameter with β_H in the 3D Georgi-Glashow model, at $\beta_G=2$ and $\beta_R=0.01$,

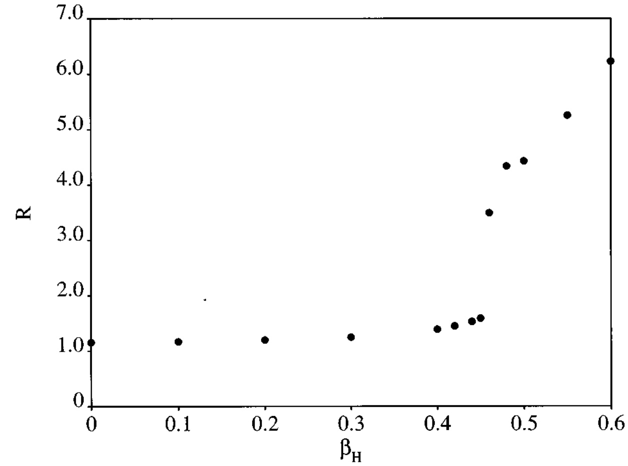

FIG. 4. Variation of the rms Higgs field R with β_H, same model and parameters as in Fig. 3.

of 35 000 sweeps; of which 5000 were thermalizing sweeps, with data taken every tenth of the remaining sweeps.

Figure 3 shows the variation of the Q parameter with β_H, at fixed $\beta_G=2$, $\beta_R=0.01$. There is clear evidence of a first-order transition between the symmetric and Higgs phases at $0.45<\beta_H<0.46$; which is supported by the behavior of the rms value of the Higgs field, shown in Fig. 4, showing a similar jump at the same value of β_H.

Having located the transition to the Higgs phase, we then study the behavior of Creutz ratios. Figure 5 shows the $\chi_j[2,2]$ Creutz ratios for fundamental and adjoint loops. Up to the Higgs transition, we are in the strong-coupling regime and the Creutz ratios do not appear to be strongly dependent on β_H (for comparison, to lowest-order in the strong-coupling expansion at $\beta_H=0$, we have string tensions $\mu_{1/2}=0.84$ and $\mu_1=2.01$).[5,6] At the Higgs transition the fundamental string tension drops, but remains finite, while the adjoint string tension appears to be consistent with zero.

In Fig. 6 we display the $\chi_j[I,I]$ Creutz ratios in the Higgs phase, for the fundamental ($j=1/2$), adjoint ($j=1$), and $j=3/2$ representations, at $I=2,3,4$. The coupling is $\beta_H=0.46$, which is just past the transition (once again, $\beta_G=2$ and $\beta_R=0.01$). Note that the adjoint ratio actually goes *negative* at $I=3$, and is consistent with zero at $I=4$. Since the signal for the $j=3/2$ loops is quite small, we have not obtained good data for the $j=3/2$ Creutz ratio beyond $I=3$. Nevertheless, from the data at $I=2$ and $I=3$, it does appear that the $j=3/2$ string tension is converging to the $j=1/2$ value.

In short, up to the Higgs transition, our Creutz ratios essentially follow the strong-coupling expansion, which (at lowest order) is in agreement with the notion of dimensional reduction. At the Higgs transition, both the absolute and relative values of the string tensions change abruptly, and all indications are that the Abelian monopole prediction (3) is fulfilled.

IV. CONCLUSIONS

At a minimum, our results cast considerable doubt on the hypothesis of Abelian dominance in maximal Abelian gauge. If the ''photon'' gauge field associated with the remaining U(1) symmetry is mainly responsible for forces between heavy fundamental quarks beyond the confinement scale, that same gauge field should also explain the forces between heavy quarks in higher group representations. Given that the projection to Abelian lattice configurations is found to reproduce the fundamental string tension, then according to these ideas the string tensions for higher representations should also be reproduced, at any distance beyond the confinement scale. We have found, however, that this is not at all the case.

There have been previous indications of trouble for the

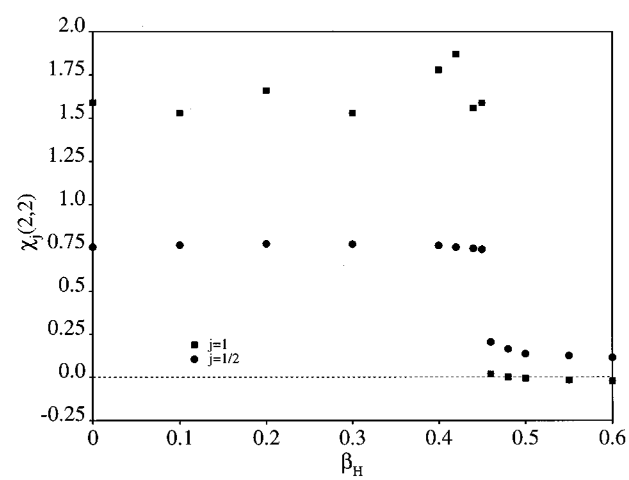

FIG. 5. Creutz ratios $\chi_j[2,2]$ vs β_H, for $j=1/2$ (solid circles) and $j=1$ (grey squares), same model and parameters as in Fig. 3.

[5]The lowest-order strong-coupling result in three-dimensions is the same as that in two dimensions, consistent with the idea of dimensional reduction, and the string tension is given by a ratio of Bessel functions. This ratio becomes, in the limit of weak couplings, a ratio of quadratic Casimirs, which is the origin of the Casimir scaling prediction of [9].

[6]Creutz ratios for the $j=3/2$ representation are not shown, since the statistical errors are quite large for 2×2 loops in the symmetric phase. However, we have found that the smaller 1×1 and 1×2 loops are quite close to their strong-coupling values in the symmetric phase, right up to the transition.

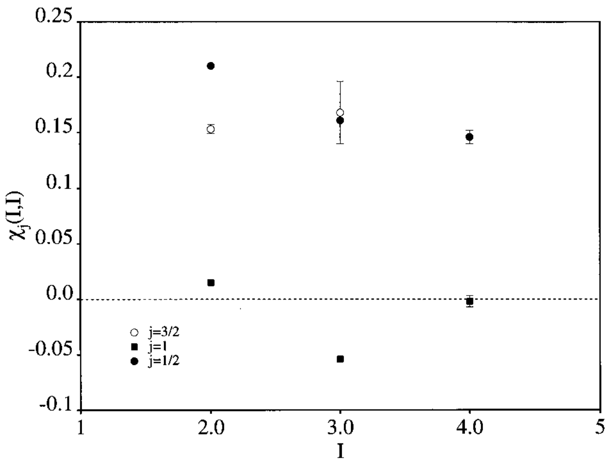

FIG. 6. Creutz ratios $\chi_j[I,I]$ vs I, in the Higgs phase of the 3D Georgi-Glashow model at $\beta=0.46$, just past the transition. Again $\beta_G=2$, $\beta_R=0.01$; representations $j=1/2$ (solid circles), $j=1$ (grey squares), and $j=3/2$ (open circles) are shown.

Abelian projection theory. As three of us have pointed out in a previous publication [15], for SU(N) theories there is a significant difference in the coefficients of subleading (perimeter-law) contributions to adjoint Wilson loops, as predicted, respectively, by large-N counting arguments, and by the Abelian-projection theory. The origin of this difference is that according to the large-N picture, the perimeter-law term is due to the binding of gluons to the adjoint quarks (a $1/N^2$ suppressed process), while perimeter law behavior in the Abelian projection theory is just due to the fact that $N-1$ of the N^2-1 adjoint quark charges are neutral with respect to the Abelian subgroup, and this leads only to a $1/N$ suppression factor. The different powers of N reflect the fact that there are different mechanisms involved; only one of these can be the right explanation of the perimeter law. We refer the reader to [15] for a more extensive discussion of this point. Some other types of numerical evidence against the abelian projection theory are found in [19].

Not everyone finds large-N arguments persuasive, so in this work we have considered the opposite limit, namely $N=2$. For such a small value of N, it is hard to understand, in the context of the Abelian projection theory, why the Abelian neutral ($m=0$) adjoint quark component should not completely dominate the value of the adjoint loop, at and beyond the confinement scale. In fact we find, in Abelian-projected lattice configurations, that this is exactly what happens, and the corresponding adjoint loop has no discernable string tension at any of the distances studied. However, such behavior is in complete contrast to adjoint Creutz ratios, measured at the same distances, constructed from the full lattice configuration. The latter follow Casimir scaling (2). The breakdown of Abelian dominance in pure SU(2) lattice gauge theory, not only for the adjoint but also for the $j=3/2$ representations, seems to be quite evident from comparing Figs. 1 and 2. Conversely, in the $D=3$ Georgi-Glashow model in the Higgs phase, where the infrared dynamics is essentially that of compact QED, it is the monopole prediction, rather than Casimir scaling, which agrees with the data.

A breakdown of Abelian dominance implies that large-scale vacuum fluctuations are *not* adequately represented by the fluctuations of only those degrees of freedom associated with a particular Cartan subalgebra (A^3_μ, in the Yang-Mills case considered here), not even in the maximal Abelian gauge. Large-scale fluctuations in the ''off-diagonal'' degrees of freedom (A^1_μ and A^2_μ) have been found to be important; were it not for these fluctuations, Wilson loops would follow the Abelian monopole prediction found for Abelian-projected configurations. It may be, of course, that there exists a simple effective theory, perhaps even an Abelian gauge theory involving some sort of composite fields, which does capture the essential dynamics of confinement in Yang-Mills theory. It may also be that the Yang-Mills vacuum does, in some way, exhibit the properties of a dual superconductor. Concerning these possibilities, we have nothing to say here. What can be asserted, however, is that an effective theory of the long-wavelength dynamics cannot be based on the A^3_μ degrees of freedom alone. The validity of a theory of that sort would imply the validity of the Abelian dominance approximation, and this simply conflicts with our data.

Some caveats about the data, however, are in order. We have looked only at rather small loops (up to 4×4 lattice spacings) at $\beta=5$ in $D=3$ pure Yang-Mills, and only along a single line (varying one coupling) in the three-dimensional phase diagram of the $D=3$ Georgi-Glashow model. Certainly much more numerical work is needed to extend and solidify our results. This work is in progress, and will be reported in due course.

Finally, in view of the observed Casimir scaling of Creutz ratios, we believe that a certain scepticism regarding proposed monopole confinement mechanisms, at least in their most naive forms, may be appropriate. Whatever may be the importance of monopoles, it appears doubtful that the effective infrared dynamics of Yang-Mills theory is essentially that of compact QED. It may also be that there is an element of truth in some of the old ideas regarding dimensional reduction. In any event, Casimir scaling of heavy interquark forces is a striking result of many numerical simulations, and any satisfactory theory of quark confinement must eventually take this scaling into account.

Note added: After submitting the present paper for publication, a paper appeared by Poulis [20] which also addresses the problem of Abelian dominance for higher representation sources. Poulis modifies the usual Abelian dominance approximation, in an attempt to allow for some of the effects of the off-diagonal degrees of freedom, and finds that in this modified approximation the $m=0$ adjoint loop component still has no area law falloff in any distance range. In our opinion his results support our conclusions, although he chooses to interpret those results in a different way. We will return to this issue in a future publication.

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