Soldering chiralities

R. Amorim,* A. Das, and C. Wotzasek†
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
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We study how to solder two Siegel chiral bosons into one scalar field in a gravitational background. [S0556-2821(96)02810-X]

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I. INTRODUCTION

The research of chiral scalars in two space-time dimensions has attracted much attention [1,2]. These objects can be obtained from the restriction of a scalar field to move in one direction only, as done by Siegel [3], or by a first-order Lagrangian theory, as proposed by Floreanini and Jackiw [4]. The equivalence of these two independent descriptions for chiral scalars has been established both in the context of the Senjanovic [5] formalism of a constrained path integral in [6], and later, in the operatorial canonical approach in [7], by gauge fixing an existing symmetry in Siegel's model. This procedure will leave behind only one degree of freedom in phase space, corresponding to the chiral excitations, just as in the Floreanini-Jackiw model.

Scalar fields in two dimensions (2D) can be viewed as bosonized versions of Dirac fermions and chiral bosons can be seen to correspond to two-dimensional versions of Weyl fermions. In a more formal level, it has been shown by Sonnenschein [8] and by Tseytlin and West [9] that, in some sense, the sum of the flat space-time actions of two chiral scalars of opposite chirality does correspond to the action of a single 2D scalar field. This seems correct since the number of degrees of freedom adds up correctly, and a 2D, conformally invariant field theory is known to possess two independent current algebras associated with each of the chiral components.

In a more detailed study, Stone [10] has shown that one needs more than the direct sum of two fermionic representations of the Kac-Moody group to describe a Dirac fermion. Stated differently, the action of a bosonized Dirac fermion is not simply the sum of the actions of two bosonized Weyl fermions, or chiral bosons. Physically, this is connected with the necessity to abandon the separated right and left symmetries, and accept that vector gauge symmetry should be preserved at all times. This restriction will force the two independent chiral scalars to belong to the same *multiplet*, effectively soldering them together. The basic idea in [10] to sew the two dequantized left and right Dirac seas of each Weyl component, was to introduce a gauge field to remove the obstruction to vector gauge invariance. This gauge field, being just an auxiliary field, without any dynamics, can be eliminated in favor of the physically relevant quantities.

In this work we follow the basic physical principles of [10] to solder together two Siegel chiral bosons of opposite chiralities to establish the formal equivalence with a single scalar field in a gravitational background. In Sec. II we present the gauge procedure necessary for the soldering of the chiral scalars, obtaining a Lagrangian that is invariant under vector gauge transformations. In Sec. III we study the symmetry group of the quoted Lagrangian, showing that it can be written in a way where the full diffeomorphism invariance becomes manifest. In Sec. IV some considerations regarding truncations, generators of gauge transformations, and a hidden duality symmetry are discussed. Further geometrical considerations are done in Sec. V. The last section is reserved for some final comments and perspectives.

II. THE GAUGING PROCEDURE

To begin with, let us review a few known facts about Siegel's theory.[1] First, one can see this model as the result of gauging the semilocal affine symmetry [11]

$$\delta\varphi=\epsilon^{-}\partial_{-}\varphi,\quad \partial_{-}\epsilon^{-}=0 \tag{1}$$

possessed by the action of a free scalar field. This can be done with the introduction of a gauge field λ_{++}, as long as it transforms as

$$\delta\lambda_{++}=-\partial_{+}\epsilon^{-}+\epsilon^{-}\partial_{-}\lambda_{++}-\lambda_{++}\partial_{-}\epsilon^{-}. \tag{2}$$

The result of this procedure is a Siegel action for a left-moving chiral boson

$$\mathscr{L}_0^{(+)}=\partial_{+}\varphi\partial_{-}\varphi+\lambda_{++}\partial_{-}\varphi\partial_{-}\varphi. \tag{3}$$

One can also interpret this theory as describing the action for the coupling of scalar field to a chiral $\mathscr{W}_2$ gravity [11]. The gauge field is just a Lagrange multiplier imposing the constraint

$$T_{--}=\partial_{-}\varphi\partial_{-}\varphi\approx 0\,, \tag{4}$$

known to satisfy the conformal algebra. Similarly, one can gauge the semilocal affine symmetry

$$\delta\varphi=\epsilon^{+}\partial_{+}\varphi,\quad \partial_{+}\epsilon^{+}=0 \tag{5}$$

*Permanent address: Instituto de Física, Universidade Federal do Rio de Janeiro, Brazil. Electronic address: amorim@urhep.pas.rochester.edu

†Permanent address: Instituto de Física, Universidade Federal do Rio de Janeiro, Brazil. Electronic address: clovis@urhep.pas.rochester.edu

[1]We are using standard light-front variables: $x^{\pm}=(1/\sqrt{2})(x^0\pm x^1)$.

to obtain the right-moving Siegel chiral boson, by introducing the gauge field λ_{--} such that

$$\delta\lambda_{--}=-\partial_{-}\epsilon^{+}+\epsilon^{+}\partial_{+}\lambda_{--}-\lambda_{--}\partial_{+}\epsilon^{+}. \tag{6}$$

In fact, if we write the right and left chiral boson actions as

$$\mathcal{L}_0^{(\pm)}=\frac{1}{2}J_{\pm}(\varphi)\partial_{\mp}\varphi \tag{7}$$

with

$$J_{\pm}(\varphi)=2(\partial_{\pm}\varphi+\lambda_{\pm\pm}\partial_{\mp}\varphi), \tag{8}$$

it is easy to verify that these models are indeed invariant under Siegel's transformations (1,2) and (5,6), using that

$$\delta J_{\pm}=\epsilon_{\pm}\partial_{\mp}J_{\pm}. \tag{9}$$

It is worth mentioning at this point that Siegel's actions for left and right chiral bosons can be seen as the action for a scalar field immersed in a gravitational background whose metric is appropriately truncated. In this sense, Siegel symmetry for each chirality can be seen as a truncation of the reparametrization symmetry existing for the scalar field action. We should mention that the Noether current J_{+}, defined above, is in fact the nonvanishing component of the left chiral current $J_{+}=J_{(L)}^{-}$, while J_{-} is the nonvanishing component of the right chiral current $J_{-}=J_{(R)}^{+}$, with the left and right currents being defined in terms of the axial vector and vector currents as

$$J_{\mu}^{(L)}=J_{\mu}^{(A)}+J_{\mu}^{(V)},$$

$$J_{\mu}^{(R)}=J_{\mu}^{(A)}-J_{\mu}^{(V)}. \tag{10}$$

Let us next consider the question of the vector gauge symmetry. We can use an iterative Noether procedure to gauge the global U(1) symmetry,

$$\delta\varphi=\alpha,$$

$$\delta\lambda_{++}=0, \tag{11}$$

possessed by Siegel's model (3). Under the action of the group of transformations (11), written now with a local parameter, the action (3) changes as

$$\delta\mathcal{L}_0^{(+)}=\partial_{-}\alpha J_{+} \tag{12}$$

with the Noether current $J_{+}=J_{+}(\varphi)$ being given as in (8). To cancel this piece, we introduce a gauge field A_{-} coupled to the Noether current, redefining the original Siegel's Lagrangian density as

$$\mathcal{L}_0^{(+)}\rightarrow\mathcal{L}_1^{(+)}=\mathcal{L}_0^{(+)}+A_{-}J_{+}, \tag{13}$$

where the variation of the gauge field is defined as

$$\delta A_{-}=-\partial_{-}\alpha. \tag{14}$$

As the variation of $\mathcal{L}_1^{(+)}$ does not vanish modulo total derivatives, we introduce a further modification as

$$\mathcal{L}_1^{(+)}\rightarrow\mathcal{L}_2^{(+)}=\mathcal{L}_1^{(+)}+\lambda_{++}A_{-}^{2} \tag{15}$$

whose variation gives

$$\delta\mathcal{L}_2^{(+)}=2A_{-}\partial_{+}\alpha. \tag{16}$$

This left-over piece cannot be canceled by a Noether counterterm, so that a gauge-invariant action for φ and A_{-} does not exist, at least with the introduction of only one gauge field. We observe, however, that this action has the virtue of having a variation dependent only on A_{-} and not on φ. Expression (16) is a reflection of the standard anomaly that is intimately connected with the chiral properties of φ.

Now, if the same gauging procedure is followed for an opposite chirality Siegel boson, say

$$\mathcal{L}_0^{(-)}=\partial_{+}\rho\partial_{-}\rho+\lambda_{--}\partial_{+}\rho\partial_{+}\rho \tag{17}$$

subject to

$$\delta\rho=\alpha,$$

$$\delta\lambda_{--}=0,$$

$$\delta A_{+}=-\partial_{+}\alpha, \tag{18}$$

then one finds that the sum of the right- and left-gauged actions $\mathcal{L}_2^{(+)}+\mathcal{L}_2^{(-)}$ can be made gauge invariant if a contact term of the form

$$\mathcal{L}_C=2A_{+}A_{-} \tag{19}$$

is introduced. One can check that indeed the complete gauged Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm tot}=&\partial_{+}\varphi\partial_{-}\varphi+\lambda_{++}\partial_{-}\varphi\partial_{-}\varphi+\partial_{+}\rho\partial_{-}\rho+\lambda_{--}\partial_{+}\rho\partial_{+}\rho\\&+A_{+}J_{-}(\rho)+A_{-}J_{+}(\varphi)+\lambda_{--}A_{+}^{2}+\lambda_{++}A_{-}^{2}\\&+2A_{-}A_{+}\end{aligned} \tag{20}$$

with $J_{\pm}$ defined in Eq. (8) above, is invariant under the set of transformations (11), (14), and (18). For completeness, we note that Lagrangian (20) can also be written in the form

$$\begin{aligned}\mathcal{L}_{\rm tot}=&D_{+}\varphi D_{-}\varphi+\lambda_{++}D_{-}\varphi D_{-}\varphi+D_{+}\rho D_{-}\rho\\&+\lambda_{--}D_{+}\rho D_{+}\rho+(\varphi-\rho)E,\end{aligned} \tag{21}$$

modulo total derivatives. In the above expression, we have introduced the covariant derivatives $D_{\pm}\varphi=\partial_{\pm}\varphi+A_{\pm}$, with a similar expression for $D_{\pm}\rho$, and $E\equiv\partial_{+}A_{-}-\partial_{-}A_{+}$. In form (21), $\mathcal{L}_{\rm tot}$ is manifestly gauge invariant.

III. DIFFEOMORPHISM

What may be the most remarkable consequence of the gauging procedure we have presented in the previous section is that the two decoupled Siegel's symmetries, associated with each sector originally described by the pairs φ,λ_{++} and ρ,λ_{--}, have now been enlarged to a complete diffeomorphism, while these quantities have become effectively coupled in a highly nontrivial way to have full diffeomorphism invariance. To see how these features occur, we will first redefine the fields $A_{\pm}$ by a shift that would diagonalize the Lagrangian in Eq. (20). Let

$$\overline{A_\pm}=A_\pm-\frac{1}{\Delta}(J_\pm-\lambda_{\pm\pm}J_\mp), \tag{22}$$

where $J_+=J_+(\varphi)$, $J_-=J_-(\rho)$, and $\Delta=2(\lambda_{++}\lambda_{--}-1)$. Under these redefinitions of the fields, the Lagrangian becomes

$$\mathcal{L}_{\text{tot}}=\mathcal{L}_g+\mathcal{L}_{\overline{A}}, \tag{23}$$

where

$$\mathcal{L}_g=\frac{1}{2}\sqrt{-g}g^{\alpha\beta}\partial_\alpha\Phi\,\partial_\beta\Phi,$$

$$\mathcal{L}_{\overline{A}}=\lambda_{--}\overline{A}_+^2+\lambda_{++}\overline{A}_-^2+2\overline{A_-}\,\overline{A_+}\,. \tag{24}$$

In the above expressions we have introduced the metric tensor density

$$G^{--}=\sqrt{-g}g^{--}=-4\frac{\lambda_{++}}{\Delta},$$

$$G^{++}=\sqrt{-g}g^{++}=-4\frac{\lambda_{--}}{\Delta},$$

$$G^{+-}=\sqrt{-g}g^{+-}=-\frac{2}{\Delta}(1+\lambda_{++}\lambda_{--}), \tag{25}$$

as well as the field

$$\Phi=\frac{1}{\sqrt{2}}(\rho-\varphi). \tag{26}$$

We observe that in two dimensions, $\sqrt{-g}g^{\alpha\beta}$ needs only two parameters to be defined in a proper way. As it should be, $\det(\sqrt{-g}g^{\alpha\beta})=-1$. We also note that because of conformal invariance, we cannot determine $g_{\alpha\beta}$ itself.

Before studying the symmetries of the model given by $\mathcal{L}_{\text{tot}}$, we note that in the path-integral approach, the fields $\overline{A_\pm}$ can be integrated out, contributing in a trivial way to the vacuum functional. We could, therefore, think of $\mathcal{L}_g$ as an effective theory, which represents a scalar boson Φ in a gravitational background. Later, we will come back to this question.

To study the symmetries associated with $\mathcal{L}_{\text{tot}}$, in (23) and (24), let us first note that the original vectorial symmetry given by (11) and (18) is now hidden. Actually, since the metric is a function only of the λ's, it does not transform at all. The field Φ is also invariant, which can be seen from (11), (18), and (26). Finally, from (22), we see that $\delta\overline{A_\pm}=0$. Collecting all these facts, we see that (23) is trivially invariant under the local vectorial symmetry.

Under a diffeomorphism

$$\delta x^\alpha=-\epsilon^\alpha, \tag{27}$$

a symmetric tensorial density $G^{\alpha\beta}$ transforms as

$$\delta G^{\alpha\beta}=\partial_\gamma(G^{\alpha\beta}\epsilon^\gamma)-G^{\gamma\alpha}\partial_\gamma\epsilon^\beta-G^{\gamma\beta}\partial_\gamma\epsilon^\alpha. \tag{28}$$

From (25) and (28), we derive after some algebraic calculations that under a diffeomorphism

$$\begin{aligned}\delta\lambda_{++}&=-\partial_+\epsilon^-+\lambda_{++}^2\partial_-\epsilon^+\\&\quad+(\partial_+\epsilon^+-\partial_-\epsilon^-+\epsilon^+\partial_++\epsilon^-\partial_-)\lambda_{++},\\ \delta\lambda_{--}&=-\partial_-\epsilon^++\lambda_{--}^2\partial_+\epsilon^-\\&\quad+(\partial_-\epsilon^--\partial_+\epsilon^++\epsilon^+\partial_++\epsilon^-\partial_-)\lambda_{--}.\end{aligned} \tag{29}$$

Since Φ transforms as a scalar under diffeomorphism, i.e., $\delta\Phi=\epsilon^\alpha\partial_\alpha\Phi$, $\mathcal{L}_g$ in (24) can be seen to be invariant modulo total derivatives, while $\mathcal{L}_{\overline{A}}$ can be made invariant once we choose

$$\begin{aligned}\delta\overline{A_+}&=\epsilon^-\partial_-\overline{A_+}+\partial_+(\epsilon^+\overline{A_+})+\overline{A_-}\partial_+\epsilon^--\frac{1}{4}\partial_+\epsilon^-\frac{\delta S}{\delta A_+},\\ \delta\overline{A_-}&=\epsilon^+\partial_+\overline{A_-}+\partial_-(\epsilon^-\overline{A_-})+\overline{A_+}\partial_-\epsilon^+-\frac{1}{4}\partial_-\epsilon^+\frac{\delta S}{\delta A_-},\end{aligned} \tag{30}$$

where

$$\frac{\delta S}{\delta A_\pm}=2(\overline{A_\mp}+\lambda_{\pm\pm}\overline{A^\pm}). \tag{31}$$

We can see that the transformations for $\overline{A_\pm}$ are just those of the vectors plus terms that vanish on shell.

After some lengthy algebraic calculations we can show that the algebra of diffeomorphism closes on the field Φ as well as on the λ's as

$$\begin{aligned}[\delta_1,\delta_2]\Phi&=\delta_3\Phi,\\ [\delta_1,\delta_2]\lambda_{\pm\pm}&=\delta_3\lambda_{\pm\pm}\end{aligned} \tag{32}$$

where

$$\epsilon_3^\alpha=-\epsilon_1^\beta\partial_\beta\epsilon_2^\alpha+\epsilon_2^\beta\partial_\beta\epsilon_1^\alpha, \tag{33}$$

while variations on $\overline{A_\pm}$ satisfy an open algebra [12] of the form

$$\begin{aligned}[\delta_1,\delta_2]\overline{A_+}&=\delta_3\overline{A_+}+V_{+-}\frac{\delta S}{\delta A_-},\\ [\delta_1,\delta_2]\overline{A_-}&=\delta_3\overline{A_-}+V_{-+}\frac{\delta S}{\delta A_+},\end{aligned} \tag{34}$$

where

$$V_{+-}=-V_{-+}=\frac{1}{4}(\partial_+\epsilon_1^-\partial_-\epsilon_2^+-\partial_+\epsilon_2^-\partial_-\epsilon_1^+). \tag{35}$$

In other words, the algebra closes on shell for the $\overline{A_\pm}$ fields. This, in turn, implies that one can introduce auxiliary fields to close the algebra off shell.

IV. TRUNCATIONS, DUALITIES, AND GENERATORS

So far we have shown that we can consider $\mathscr{L}_g$ as an effective Lagrangian after integrating out the $A_\pm$ fields from the complete theory, which solders two initially decoupled Siegel bosons of opposite chiralities.

We would like to comment that if we restrict the diffeomorphism to just one sector, say by requiring $\epsilon^+=0$, we reproduce the original Siegel symmetry for the sector described by the pair Φ,λ_{++} in the same way as it appears in (1) and (2). However, under this restriction, λ_{--} transforms in a nontrivial way as

$$\delta\lambda_{--}=\lambda_{--}^2\partial_+\epsilon^-+(\partial_-\epsilon^-+\epsilon^-\partial_-)\lambda_{--}. \tag{36}$$

The original Siegel model, therefore, is not a subgroup of the original diffeomorphism group but it is only recovered if we also make a further truncation, by imposing that $\lambda_{--}=0$. The existence of the residual symmetry (36) seems to be related to a duality symmetry satisfied by $\mathscr{L}_g$ when the metric is parametrized as in (25). Under the transformation

$$\lambda_{\pm\pm}\rightarrow\frac{1}{\lambda_{\mp\mp}}, \tag{37}$$

we see that (i) the set of variations (29) is invariant and (ii) $\mathscr{L}_g$ goes into $-\mathscr{L}_g$, so that the equations of motion are invariant. The duality present in the equations of motion of the theory can also be seen to work in a first-order formalism. By introducing

$$\Pi=\frac{\partial\mathscr{L}_g}{\partial(\partial_+\Phi)} \tag{38}$$

as the momentum canonically conjugated to Φ, we can rewrite $\mathscr{L}_g$ as

$$\mathscr{L}_{\mathrm{FO}}=\Pi\partial_+\Phi+\frac{1}{2}\lambda_{++}T_{--}-\frac{1}{2}\frac{1}{\lambda_{--}}T_{++}, \tag{39}$$

where

$$T_{\pm\pm}=\frac{1}{2}(\Pi\mp\partial_-\Phi)^2 \tag{40}$$

are the diffeomorphism generators and satisfy the Virasoro algebra. We see from the above equations that the duality invariance of (39) is achieved not only with (37) but also with the change $T_{\pm\pm}\rightarrow T_{\mp\mp}$, which is the same as having $x^-\rightarrow -x^-$, but keeping x^+ unchanged. This is obviously related to the symmetry under the interchange of the right- and the left-moving sectors of the theory. In the next section we will come back to this point by writing in a geometric manner the original left- and right-moving Siegel models.

V. FURTHER GEOMETRICAL CONSIDERATIONS

The Siegel model as well as the soldering can be described in a geometrical manner also. Let us look at the Siegel model in one sector only and note that we can write it also as

$$\mathscr{L}_0^{(+)}=\partial_+\varphi\partial_-\varphi+\lambda_{++}(\partial_-\varphi)^2=\frac{1}{2}\sqrt{-g}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi, \tag{41}$$

where

$$\sqrt{-g}g^{\alpha\beta}\equiv G_{(+)}^{\alpha\beta}=\begin{pmatrix}0 & 1\\ 1 & 2\lambda_{++}\end{pmatrix}. \tag{42}$$

From (41) and (42), we note that we can think of the Siegel particle as propagating in a background gravitational field in a light cone gauge for which the invariant length has the form

$$ds^2=2g_{+-}dx^+dx^-+g_{++}(dx^+)^2. \tag{43}$$

The Siegel invariance of Eq. (41) can be understood as the residual one-parameter coordinate invariance in this gauge, defined by

$$x^-\rightarrow x^--\epsilon^-(x^+,x^-),$$

$$\delta\varphi=\epsilon^\alpha\partial_\alpha\varphi, \tag{44}$$

where $\delta G_{(+)}^{\alpha\beta}$ is given by Eq. (28) and we assume

$$\epsilon^+=0. \tag{45}$$

Similarly, the Siegel Lagrangian in the other sector can also be written as

$$\mathscr{L}_0^{(-)}=\partial_+\rho\partial_-\rho+\lambda_{--}(\partial_-\rho)^2=\frac{1}{2}\sqrt{-\tilde{g}}\tilde{g}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi, \tag{46}$$

where

$$\sqrt{-\tilde{g}}\tilde{g}^{\alpha\beta}\equiv G_{(-)}^{\alpha\beta}=\begin{pmatrix}2\lambda_{--} & 1\\ 1 & 0\end{pmatrix}. \tag{47}$$

In other words, the Siegel particle in the other sector can also be thought of as moving in a background gravitational field with a light cone metric of the form

$$ds^2=2\tilde{g}_{+-}dx^+dx^-+\tilde{g}_{--}(dx^-)^2. \tag{48}$$

The Siegel invariance, in this sector, can again be thought of as a one-parameter residual coordinate invariance of the form

$$x^+\rightarrow x^+-\epsilon^+(x^-,x^+). \tag{49}$$

The gauged Lagrangian in one sector, Eq. (15), cannot be written in a diffeomorphism-invariant manner. Therefore, gauging in one of the sectors breaks Siegel invariance. However, let us note the following from Eq. (15):

$$\mathcal{L}_2^{(+)}=(\partial_-\varphi)(\partial_+\varphi+\lambda_{++}\partial_-\varphi)+2(\partial_+\varphi+\lambda_{++}\partial_-\varphi)+\lambda_{++}A_-^2=-(\partial_+\varphi)\left(\partial_-\varphi+\frac{1}{\lambda_{++}}\partial_+\varphi\right)+\lambda_{++}\left(A_-+\partial_-\varphi+\frac{1}{\lambda_{++}}\partial_+\phi\right)^2. \quad (50)$$

This shows that if we integrate out the A_- field, the Siegel theory changes chirality with the identification

$$\lambda_{--}=\frac{1}{\lambda_{++}}, \quad (51)$$

that again, is related to the duality symmetry discussed in the last section.

Finally, we note that the complete Lagrangian of Eq. (20) can also be written in form (23), but we can rewrite $\mathcal{L}_A$ as

$$\mathcal{L}_A=\frac{1}{2}M^{\alpha\beta}\overline{A_\alpha}\overline{A_\beta}, \quad (52)$$

where

$$M=(G_{(+)}+G_{(-)})=2\begin{pmatrix}\lambda_{--} & 1\\ 1 & \lambda_{++}\end{pmatrix}=-\frac{\Delta}{2}(G-\sigma_1),$$

$$\overline{A}=\begin{pmatrix}A_++\frac{1}{2}\partial_+(\rho+\varphi)\\ A_-+\frac{1}{2}\partial_-(\rho+\varphi)\end{pmatrix}+i\sqrt{2}\sigma_2 G\begin{pmatrix}\partial_+\Phi\\ \partial_-\Phi\end{pmatrix}. \quad (53)$$

Here, σ_1 and σ_2 represent the usual Pauli matrices. We also note that the Lagrangian in Eq. (52), with the identifications in Eq. (53), can also be written as

$$\mathcal{L}_{\rm tot}=\frac{1}{2}\sqrt{-g}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi-\frac{1}{4}\sqrt{-g}g^{\alpha\beta}\Delta\overline{A_\alpha}\overline{A_\beta}+\frac{\Delta}{4}\overline{A_+}\overline{A_-}=\frac{1}{2}\sqrt{-g}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi-\sqrt{-g}g_{\alpha\beta}\hat{A}_\alpha\hat{A}_\beta+\hat{A}_+\hat{A}_-, \quad (54)$$

where we have defined $\hat{A}_\alpha=(\sqrt{\Delta}/2)\overline{A_\alpha}$. There are several things to note here. First, the combination of the scalar fields, $\varphi+\rho$, has gone into the redefinition of the vector field. From the structure of the Lagrangian, it is obvious that the original gauge transformations have disappeared completely, as we saw in the beginning of Sec. III. The diffeomorphism invariance of the theory is almost obvious. The presence of the last term in Eq. (54) suggests that $\hat{A}$ cannot transform the same as a vector under a coordinate invariance. In fact, $\mathcal{L}_{\rm tot}$ can be checked to be invariant under the set of transformations (28), the already quoted diffeomorphism transformation for Φ, as well as

$$\delta\hat{A}_\alpha=\hat{A}_\beta\partial_\alpha\epsilon^\beta+\epsilon^\beta\partial_\beta\hat{A}_\alpha+\frac{1}{2}M^{-1}_{\alpha\beta}\begin{pmatrix}\hat{A}_+\partial_-\epsilon^+\\ \hat{A}_-\partial_+\epsilon^-\end{pmatrix}^\beta. \quad (55)$$

The algebra, of course, trivially closes on the Φ and $G^{\alpha\beta}$ variables. For $\hat{A}_\alpha$, however, we can show that

$$[\delta_1,\delta_2]\hat{A}_\alpha=\delta_3\hat{A}_\alpha+\frac{1}{2\Delta^2}[(i\sigma_2)M\hat{A}]_\alpha\times(\partial_+\epsilon_2^-\partial_-\epsilon_1^+-\partial_+\epsilon_1^-\partial_-\epsilon_2^+). \quad (56)$$

As expected, the algebra of the variations closes on shell for the gauge fields.

VI. CONCLUSIONS

In this work we have shown how to solder two initially decoupled Siegel bosons of different chiralities. This has been done through the implementation of a vector gauge symmetry, which has forced the two bosons to belong to the same multiplet. The complete theory so obtained presents full diffeomorphism invariance and can be represented in a geometric manner. We have shown, as expected, that the diffeomorphism algebra closes on the fields appearing in the theory. The way we have parametrized the metric has made explicit that the naive sum of two Siegel metrics is not the metric of a full diffeomorphism-invariant theory. In this sense, we could reveal the relationship between Siegel symmetry and diffeomorphism. We have also discovered a surprising duality in the model, which is related to the symmetry under the exchange between the left and right movers.

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