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Sum rules for charmed baryon masses

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The measured masses of the three charge states of the charmed Σ_c baryon are found to be in disagreement with a sum rule based on the quark model, but relying on no detailed assumptions about the form of the interaction. This poses a significant problem for the charmed baryon sector of the quark model. Other relations among charmed baryon masses are also discussed.

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In recent years, measurements have been made [1] of the masses of the three charge states of the charmed Σ_c baryon. These measurements can be applied to sum rules [2] that were derived some time ago using fairly minimal assumptions within the quark model. The sum rules depend on standard quark model assumptions, and the additional assumption that the interaction energy of a pair of quarks in a particular spin state does not depend on which baryon the pair of quarks is in. No assumptions are made about the type of potential, and no internal symmetry is assumed. More detail about the derivation of the sum rules is given in Sec. II of Ref. [2].

The Σ sum rule relates electromagnetic mass differences of the Σ_c baryon with corresponding mass differences of the Σ and Σ^* [2, 3]

$$D_{uu} + D_{dd} - 2D_{ud} = \Sigma^+ + \Sigma^- - 2\Sigma^0 = 1.7 \pm 0.2 \tag{1}$$

$$= \Sigma^{*+} + \Sigma^{*-} - 2\Sigma^{*0} = 2.6 \pm 2.1 \tag{2}$$

$$= \Sigma_c^{++} + \Sigma_c^0 - 2\Sigma_c^+ = -2.1 \pm 1.3. \tag{3}$$

The baryon symbol has been used as its mass, and the D_{ij} represent the two-body interaction energies between pairs of quarks in states of spin one. Experimental values in MeV are given for the baryon combination in each equation. The experimental value in Eq. (3) uses the measured mass differences $\Sigma_c^{++} - \Sigma_c^0 = 0.7 \pm 0.4$ and $\Sigma_c^+ - \Sigma_c^0 = 1.4 \pm 0.6$.

The sum rule relating the Σ_c and the Σ, which is among the most rigorous in I, is violated by three standard deviations. This sum rule relates baryons with the same quark spin states, and, because they are stable baryons, their mass differences can be measured more accurately than resonance masses. Each baryon in the sum rule consists of two light nucleon quarks with a heavier spectator quark. The only difference is that, for Eq. (3), the spectator strange quark is replaced by a heavier charmed quark. The equality represented by the sum rule follows because the two-body interaction energies given by the D_{ij} are the same for each combination of baryons. This is because they all have the same spin-one state for corresponding pairs of quarks. A number of two-body interaction energies (also involving other spin states) and the spectator quark cancel in the linear combinations formed in the sum rule. Although no assumption has been made about the form of the interaction energies, this sum rule is probably purely electromagnetic because the QCD mass corrections to the combination $D_{uu} + D_{dd} - 2D_{ud}$ cancel to first order in the ratio $\delta = (m_d - m_u)/m$ (m is the average of the nucleon quark masses) and the second-order correction is negligible.

It has been suggested that there should be some dependence of the two-body interaction energy on the third quark in the baryon [4, 5]. We have estimated this effect, following the procedure suggested in Ref. [5] using their parameters. The net change in the Σ_c sum is only 0.1 MeV so that the sum rule seems to be quite robust with respect to this type of correction. One reason for this is that all cancellations of interaction terms take place between pairs of quarks that are in corresponding positions in the baryons. Of the nine original interaction terms in each combination of Σ baryons, the six that cancel are essentially unaffected by this type of mass correction because of the cancellation of mass effects to first order.

In looking more deeply at the Σ-Σ_c sum rule, the +1.7 MeV for the uncharmed Σ combination seems to be sensible, but the −2.1 MeV for the Σ_c is difficult to understand. If it is purely electromagnetic, the mass difference for the Σ's is given by [6]

$$D_{uu} + D_{dd} - 2D_{ud} = \alpha_{\rm em}\langle 1/r\rangle - D_m, \tag{4}$$

where r is the distance between the two nucleon quarks. The magnetic contribution is given by

$$D_m = \frac{2\pi\alpha_{\rm em}}{3m^2}|\psi(0)|^2. \tag{5}$$

It is shown in Ref. [6] [in the discussion following Eq. (32)] that the +1.7 MeV for the uncharmed Σ combination leads to reasonable values for $\langle 1/r\rangle$ and $|\psi(0)|^2$. On the other hand, even the sign of the Σ_c sum is hard to understand. There is no reasonable potential for which the magnetic term in Eq. (4) could be large enough compared to the electric term to give a large negative overall result. This is especially true for the case of the Σ baryons which have the two nucleon quarks in a pure spin one state, for which the short range QCD spin-spin interaction is repulsive. It is difficult to think of any quark wave function and masses that could lead to a negative sign for Eq. (4). If future experiments do not result in a different value for the combination $\Sigma_c^{++} + \Sigma_c^0 - 2\Sigma_c^+$, the quark model for charmed baryons would require considerable revision. That is the main conclusion of this paper [7].

Other sum rules given in I can be applied to measurements of the masses of the Ω_c^0 and the two charge states of the Ξ_c. We present these here, but with the caveat that they would not apply if the above violation of the more rigorous electromagnetic sum rule for the charmed Σ_c baryons cannot be resolved. The first of these is [2]

$$D_{uu} + D_{ss} - 2D_{us} = \Delta^{++} + \Xi^{*0} - 2\Sigma^{*+} = -3 \pm 1 \quad (6)$$

$$= \Sigma_c^{++} + \Omega_c^0 - 2\Xi_c'^{+}. \quad (7)$$

We use the prime on $\Xi_c'^{+}$ to signify that its u and s quarks are in a spin-one state. The unprimed Ξ_c^+ has the u and s quarks in a spin-0 state. Note that this convention is opposite to the notation in I.

We can use this sum rule to predict the mass of the $\Xi_c'^{+}$ to be [8]

$$\Xi_c'^{+} = 2583 \pm 3. \quad (8)$$

This is consistent with the prediction $\Xi_c'^{+} = 2580 \pm 20$ in Ref. [9]. If we modify this sum rule by the mass corrections of Ref. [5], we find that individual terms (there are 18 in the sum rule) can be changed by as much as 5 MeV in substituting a c quark for a spectator u quark. However, these changes tend to cancel out in taking the mass differences, and the net contribution of these effects on the sum rule would be to lower the predicted mass of the $\Xi_c'^{+}$ by only 1 MeV. Incidentally, the sum rule makes it clear that the observed charmed Ξ is the Ξ_c^+, since the $\Xi_c'^{+}$ would violate the sum rule by a large amount if it had the mass of the observed $\Xi_c^+(2465)$.

A combination of sum rules from I can be used to predict the isospin-breaking mass difference of the Ξ_c' baryon:

$$\Xi_c'^{0} - \Xi_c'^{+} = (\Xi^{*-} - \Xi^{*0}) - (\Sigma^{*0} - \Sigma^{*+}) + (\Sigma_c^+ - \Sigma_c^{++})$$
$$= 3.0 \pm 1.4. \quad (9)$$

The interaction energy difference in Eq. (9) comes from the QCD $1/m_i m_j$ interaction as well as electric Coulomb and magnetic dipole-dipole interactions, similar to those in Eq. (4).

However, this prediction is made ambiguous by the experimental failure of the Σ_c sum rule. There are theoretically equivalent expressions for the Ξ_c' mass difference given by

$$\Xi_c'^{0} - \Xi_c'^{+} = (\Xi^{*-} - \Xi^{*0}) - (\Sigma^{*-} - \Sigma^{*0}) + (\Sigma_c^0 - \Sigma_c^+)$$
$$= -1.7 \pm 1.0, \quad (10)$$

$$\Xi_c'^{0} - \Xi_c'^{+} = (\Xi^{*-} - \Xi^{*0}) - \frac{1}{2}[(\Sigma^{*0} - \Sigma^{*++})$$
$$+ (\Sigma_c^0 - \Sigma_c^{++})]$$
$$= 0.6 \pm 0.8. \quad (11)$$

The inconsistency of these theoretically equivalent predictions highlights the failure of the Σ_c sum rule.

[1] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994). All baryon masses (in MeV) are taken from this reference.

[2] J. Franklin, Phys. Rev. D **12**, 2077 (1975). We refer to this paper as I.

[3] The same combinations of Σ_c^* and Σ_{bottom} baryons also enter the sum rule, but these have not yet been measured.

[4] Y. Wong and D. B. Lichtenberg, Phys. Rev. D **42**, 2404 (1990).

[5] R. Roncaglia, A. Dzierba, D. B. Lichtenberg, and E. Predazzi, Phys. Rev. D **51**, 1248 (1995).

[6] J. Franklin, Phys. Rev. **172**, 1807 (1968).

[7] R. E. Cutkosky and P. Geiger, Phys. Rev. D **48**, 1315 (1993), reach a similar conclusion in a more detailed model calculation. They predict +1.64 for the right-hand side of Eq. (3), consistent with our sum rule.

[8] We use the experimental values $\Sigma_c^{++} = 2453.1 \pm 0.6$ and $\Omega_c^0 = 2710 \pm 5$.

[9] R. Roncaglia, D. B. Lichtenberg, and E. Predazzi, Phys. Rev. D **52**, 1722 (1995).