Supersymmetric SO(10) model with inflation and cosmic strings

Rachel Jeannerot*
Department of Applied Mathematics and Theoretical Physics, Cambridge University, Silver Street, Cambridge, CB3 9EW, United Kingdom
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We have built a supersymmetric SO(10) model consistent with cosmological observations. The model gives rise to a false vacuum hybrid inflationary scenario which solves the monopole problem. We argue that this type of inflationary scenario arises naturally in supersymmetric SO(10) models. No external field or external symmetry has to be added. It can just be a consequence of the theory. In our specific model, at the end of inflation, cosmic strings form. The properties of the strings are presented. The cosmic background radiation anisotropies induced by the inflationary perturbations and the cosmic strings are estimated. The model produces a stable lightest superparticle and a very light left-handed neutrino which may serve as the cold and hot dark matter. The properties of a mixed scenario of cosmic string and inflationary large-scale structure formation are discussed. [S0556-2821(96)06510-1]

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I. INTRODUCTION

Supersymmetric SO(10) models have received much interest in the past ten years. SO(10) is the minimal grand unified gauge group which unifies all kinds of matter, thanks to its 16-dimensional spinorial representation to which all fermions belonging to a single family can be assigned. The running of the gauge coupling constants measured at LEP in the minimal supersymmetric standard model with supersymmetry broken at 10^3 GeV merge in a single point at 2×10^{16} GeV [1], hence strongly favoring supersymmetric versions of grand unified theories (GUT's). The doublet-triplet splitting can be easily achieved in supersymmetric SO(10), through the Dimopoulos-Wilczek mechanism [2]. The fermions masses can be beautifully derived [3]. The gauge hierarchy problem can be solved [4]. A Z_2 symmetry subgroup of the Z_4 center of SO(10) can be left unbroken down to low energies, provided only ''safe'' representations [5] are used to implement the symmetry breaking from SO(10) down to the standard model gauge group. We should point out here that, when we write SO(10), we really mean its universal covering group, spin(10). The Z_2 symmetry can suppress rapid proton decay and provide a cold dark matter candidate, stabilizing the lightest superparticle (LSP). Finally, introducing a pair of Higgs fields in the $\mathbf{126}+\overline{\mathbf{126}}$ representations can give a superheavy Majorana mass to the right-handed neutrino, thus providing a hot dark-matter candidate and solving the solar neutrino problem through the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [6]. All these features make supersymmetric SO(10) models very attractive.

In a recent paper [7], we have constrained supersymmetric SO(10) models using cosmological arguments. We have in particular studied the formation of topological defects in all possible symmetry breaking patterns from supersymmetric SO(10) down to the standard model, considering no more than one intermediate symmetry breaking scale. Domain walls and monopoles lead to a cosmological catastrophe, while cosmic strings can explain large scale structures, part of the baryon asymmetry of the universe, and thermal fluctuations in the cosmic background radiation (CBR). Since SO(10) is simply connected and the standard model gauge group involves an unbroken U(1) symmetry, which remains unbroken down to low energy, all symmetry breaking patterns from supersymmetric SO(10) down to the standard model automatically lead to the formation of topologically stable monopoles via the Kibble mechanism [8]. All supersymmetric SO(10) models are therefore cosmologically irrelevant without invoking some mechanism for the removal of the monopoles, such as an inflationary scenario. The conclusions in [7], is that there are only two possibilities for breaking SO(10) down to the standard model which are consistent with observations. SO(10) can break via $SU(3)_c\times SU(2)_L \times SU(2)_R\times U(1)_{B-L}$; here SO(10) must be broken with a combination of a 45-dimensional Higgs representation and a 54-dimensional one, and via $SU(3)_c\times SU(2)_L\times U(1)_R \times U(1)_{B-L}$. In these models, the intermediate symmetry group must be broken down to the standard model gauge group with unbroken matter parity, $SU(3)_c\times SU(2)_L \times U(1)_Y\times Z_2$. In supergravity SO(10) models, the breaking of SO(10) via flipped SU(5) is also possible.

In this paper, we study a supersymmetric SO(10) model involving an intermediate $SU(3)_c\times SU(2)_L\times U(1)_R\times U(1)_{B-L}$ symmetry. The resultant cosmological model is compatible with observations.

In Sec. II, we describe a hybrid inflationary scenario first introduced by [9], and we argue that this type of inflationary scenario occurs naturally in global supersymmetric SO(10) models. Neither any external field nor any external symmetry has to be imposed. Inflation is driven by a scalar field singlet under SO(10).

In the next sections, we construct a specific supersymmetric SO(10) model, as mentioned above. The latter aims to be consistent with observations. In Sec. III we study the symmetry breaking pattern. We conclude on the proton lifetime and on a hot dark-matter candidate provided by the model.

*Electronic address: R.Jeannerot@damtp.cam.ac.uk

Using homotopy theory, we find topological defects which form according to the Kibble mechanism [8].

In Sec. IV, we explain how to implement the symmetry breaking pattern which solves the doublet-triplet splitting and includes the inflationary scenario described in Sec. II. We write down the superpotential and find its global minimum with corresponding Higgs VEV's.

In Sec. V, we evaluate the dynamics of the symmetry breaking and inflationary scenario, studying the scalar potential. It is shown that the monopole problem may be solved and that cosmic strings form at the end of inflation.

In Sec. VI, we give general properties of the strings formed at the end of inflation. In particular, we study the possibility that the strings may be superconducting.

In Sec. VII, we estimate the observational consequences. The temperature fluctuations in the CBR due to the mixed inflation-cosmic strings scenario are evaluated. Using the temperature fluctuations measured by COBE we find values for the scalar coupling constant, the scale at which the strings formed and the strings mass per unit length. We specify the dark-matter present in the model and give a qualitative discussion of the large-scale structure formation scenario in this model.

We finally conclude in Sec. VIII.

In order to simplify the notation, we shall make use of the following:

$$\text{(a)}\quad 4_c 2_L 2_R \equiv \mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R,$$

$$\text{(b)}\quad 3_c 2_L 2_R 1_{B-L} \equiv \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L},$$

$$\text{(c)}\quad 3_2 2_L 1_R 1_{B-L} \equiv \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L},$$

$$\text{(d)}\quad 3_c 2_L 1_Y Z_2 \equiv \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times Z_2,$$

$$\text{(e)}\quad 3_c 1_Q Z_2 \equiv \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q \times Z_2.$$

II. INFLATION IN SUPERSYMMETRIC SO(10) MODELS

In this section, we argue that false vacuum hybrid inflation, with a superpotential in the inflaton sector similar to that studied in [11,12], is a natural mechanism for inflation in global supersymmetric SO(10) models. Neither any external field nor any external symmetry has to be imposed, it can just be a consequence of the theory.

The first thing to note in SO(10) models, is that the rank of SO(10) is greater than one unit from the rank of the standard model gauge group. The rank of SO(10) is five, whereas the rank of the standard model gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y (\times Z_2)$ is 4. In other words, SO(10) has an additional U(1) symmetry, named $U(1)_{B-L}$, compared to the standard model gauge group. Therefore the rank of the group must be lowered by one unit at some stage of the symmetry breaking pattern, i.e., $\mathrm{U}(1)_{B-L}$ must be broken. This can be done using a pair of $\mathbf{16}+\overline{\mathbf{16}}$ Higgs representations or by a pair of $\mathbf{126}+\overline{\mathbf{126}}$ representations. If a $\mathbf{16}+\overline{\mathbf{16}}$ pair of Higgs fields are used, then the Z_2 symmetry, subgroup of both the Z_4 center of SO(10) and of $\mathrm{U}(1)_{B-L}$, playing the role of matter parity, is broken. On the other hand, if a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields are used, then the Z_2 symmetry can be kept unbroken down to low energy if only safe representations [5] are used to implement the full symmetry breaking pattern, such as the 10, the 45, the 54 or the 210-dimensional representations. If a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields are used, the right-handed neutrino can get a superheavy Majorana mass, and the solar neutrino problem can be solved via the MSW mechanism [6].

In order to force the VEV's of the $\mathbf{16}+\overline{\mathbf{16}}$ or $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields, needed to lower the rank of the group, to be the order of the GUT scale, we can use a scalar field $\mathscr{S}$ singlet under SO(10). The superpotential can be written as follows:

$$W_1 = \alpha \mathscr{S} \bar{\Phi} \Phi - \mu^2 \mathscr{S}, \tag{1}$$

where $\Phi + \bar{\Phi}$ stand for a $\mathbf{16}+\overline{\mathbf{16}}$ or a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields, and the field $\mathscr{S}$ is a scalar field singlet under SO(10). The constants α and μ are assumed to be both positive and must satisfy $\mu/\sqrt{\alpha} \sim (10^{15} - 10^{16})$ GeV. The superpotential W_1 is natural in the strong sense [12]. It is of the most general form consistent with R-symmetry under which $W \to e^{i\gamma} W$, $\mathscr{S} \to e^{i\gamma} \mathscr{S}$ and the product $\bar{\Phi}\Phi$ is invariant.

It is easy to see that the superpotential given in equation (1), used to break the rank of the group by one unit, is the same superpotential used by Dvali *et al.* [11,13] to implement a false vacuum hybrid inflationary scenario, identifying the scalar field $\mathscr{S}$ with the inflaton field. Hence, as shown below, in supersymmeytric SO(10) models, the superpotential used to break $\mathrm{U}(1)_{B-L}$ can also lead to a period of inflation. Inflation is then just a consequence of the theory. In order to understand the symmetry breaking and the inflationary dynamics, we can study the scalar potential. The latter is given by (keeping the same notation for the bosonic component of the superfields as for the superfields):

$$V_1 = |F_S|^2 + |F_\Phi|^2 + |F_{\bar{\Phi}}|^2, \tag{2}$$

where the F terms are such that $F_{\Psi_i} = |\partial W / \partial \Psi_i|$, for $\Psi_i = \mathscr{S}$, Φ, and $\bar{\Phi}$. Thus

$$V_1 = \alpha^2 |\mathscr{S} \bar{\Phi}|^2 + \alpha^2 |\mathscr{S} \Phi|^2 + |\alpha \bar{\Phi} \Phi - \mu^2|^2. \tag{3}$$

The potential is minimized for $\arg(\Phi) + \arg(\bar{\Phi}) = 0$, $(\alpha > 0)$, and it is independent of $\arg(\mathscr{S}) + \arg(\Phi)$ and $\arg(\mathscr{S}) + \arg(\bar{\Phi})$. From the vanishing condition of the D terms, we have $\langle|\Phi|\rangle = \langle|\bar{\Phi}|\rangle$. Thus we can rewrite the scalar potential with the new fields which minimize the potential, keeping the same notation for the old and new fields,

$$V_1 = 4\alpha^2 |\mathscr{S}|^2 |\Phi|^2 + (\alpha |\Phi|^2 - \mu^2)^2. \tag{4}$$

The potential has a unique supersymmetric minimum corresponding to $\langle|\Phi|\rangle = \langle|\bar{\Phi}|\rangle = \mu/\sqrt{\alpha}$ and $\mathscr{S} = 0$. The potential has also a local minimum corresponding $S > \mu/\sqrt{\alpha}$ and $\langle|\Phi|\rangle = \langle|\bar{\Phi}|\rangle = 0$. We identify the scalar field $\mathscr{S}$ with the inflaton field and we assume chaotic initial conditions. All the fields are thus supposed to take initial values the order of the Planck scale, and hence the initial value of the inflaton field $S \gg \mu/\sqrt{\alpha}$. Since the potential is flat in the $\mathscr{S}$ direction, we can minimize it at a fixed value of $\mathscr{S}$. The Φ and $\bar{\Phi}$ fields roll down their local minimum corresponding to

$\langle|\Phi|\rangle = \langle|\overline{\Phi}|\rangle = 0$. The vacuum energy density is then dominated by a nonvanishing F_S term, $|F_S| = \mu^2$. The inflationary epoch takes place as the inflaton field slowly rolls down the potential. $F_S \neq 0$ implies that supersymmetry is broken. Quantum corrections to the effective potential will help the fields to slowly roll down their global minimum [11]. At the end of inflation, the phase transition mediated by the Φ and $\overline{\Phi}$ fields takes place.

Now, in order to break SO(10) down to the standard model gauge group, we need more than a $\mathbf{16}+\overline{\mathbf{16}}$ or a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields. We need Higgs in other representations, like the 45, 54 or 210-dimensional representations if the Z_2 parity is to be kept unbroken down to low energy, as required from proton lifetime measurements. Thus the full superpotential needed to break SO(10) down to the standard model must, apart of Eq. (1), contains terms involving the other Higgs needed to implement the symmetry breaking. Due to the nonrenormalization theorem in supersymmetric theories, we can write down the full superpotential which can implement the desired symmetry breaking pattern, just adding to Eq. (1) terms mixing the other Higgs needed to implement the symmetry breaking pattern. There can be no mixing between the latter Higgs and the pair of Higgs used to break $U(1)_{B-L}$ (see Sec. IV for example) and the superpotential can be written as follows:

$$W = W_1(\mathcal{S}, \Phi, \overline{\Phi}) + W_2(H_1, H_2, \ldots), \tag{5}$$

where $\mathcal{S}$ is a scalar field singlet under SO(10) identified with the inflaton field, the Φ and $\overline{\Phi}$ fields are the Higgs fields used to break $U(1)_{B-L}$ and the H_i fields, $i=1, \ldots, m$, are the m other Higgs fields needed to implement the full symmetry breaking pattern from SO(10) down to the standard model gauge group. W_1 is given by Eq. (1) and W_1+W_2 has a global supersymmetric minimum such that the SO(10) symmetry group is broken down to the standard model gauge group. The scalar potential is then given by

$$V = V_1(\mathcal{S}, \Phi, \overline{\Phi}) + V_2(H_i). \tag{6}$$

V_1 is given by Eq. (4) and V_1+V_2 has a global minimum such that the SO(10) symmetry is broken down to the standard model gauge group. The evolution of the fields is then as follows. The fields take random initial values, just subject to the constraint that the energy density is at the Planck scale. The inflaton field is distinguished from the other fields from the fact that the GUT potential is flat in its direction; the potential can be minimized for fixed $\mathcal{S}$. Chaotic initial conditions imply that the initial value of the inflaton field is greater than $\mu/\sqrt{\alpha}$. Therefore, the noninflaton fields will roll very quickly down to their global (or local) minimum, at approximatively a fixed value for the inflaton, $\langle|H_i|\rangle \neq 0$, for $i=1, \ldots, n$, $\langle|H_j|\rangle = 0$, for $j=n+1, \ldots, m$, and $\langle|\Phi|\rangle = \langle|\overline{\Phi}|\rangle = 0$; a first symmetry breaking, implemented by the n Higgs fields H acquiring VEV, takes place, SO(10) breaks down to an intermediate symmetry group G. Then inflation occurs as the inflaton rolls slowly down the potential. The symmetry breaking implemented with the $\Phi+\overline{\Phi}$ fields occurs at the end of inflation, and the the imtermediate symmetry group G breaks down to the standard model gauge group.

In the scenario described above, the rank of the intermediate symmetry group G is equal to the rank of SO(10), which is 5, and hence involves an unbroken $U(1)_{B-L}$ symmetry. If the rank of the intermediate symmetry group were that of the standard model gauge group, that is if $U(1)_{B-L}$ were broken at the first stage of the symmetry breaking, the inflationary scenario would unable to solve the monopole problem, since the later would form at the end of inflation or once inflation completed. Finally, in models where supersymmetric SO(10) is broken directly down to the standard model gauge group, such hybrid inflationary scenarios cannot cure the monopole problem.

We conclude that if inflation has to occur during the evolution of the universe described by a spontaneous symmetric breaking pattern from the supersymmetric grand unified gauge group SO(10) down to the minimal supersymmetric standard model, it can thus just be a consequence of the theory. No external field and no external symmetry has to be imposed. One can use the superpotential given in Eq. (1) to lower the rank of the group by one unit and then identify the scalar field $\mathcal{S}$, singlet under SO(10), with the inflaton field. A false vacuum hybrid inflationary scenario will be implemented. It emerges from the theory.

III. THE SUPERSYMMETRIC SO(10) MODEL AND THE STANDARD COSMOLOGY

We now construct a supersymmetric SO(10) model which aims to agree with observations. SO(10) is broken down to the standard model gauge group with unbroken matter parity $3_c 2_L 1_Y Z_2$, via the intermediate symmetry group $3_c 2_L 1_R 1_{B-L}$. We study the symmetry breaking pattern of the model and deduce general impacts of the model on observations. We look for topological defects formation.

The model initially assumes that the symmetries between particles, forces and particles, are described by a supersymmetric SO(10) theory. The SO(10) symmetry is then broken down to the standard model gauge group via $3_c 2_L 1_R 1_{B-L}$,

$$\mathrm{SO}(10) \xrightarrow{M_{\mathrm{GUT}}} 3_c 2_L 1_R 1_{B-L} \xrightarrow{M_G} 3_c 2_L 1_Y Z_2 \xrightarrow{M_Z} 3_c 1_Q Z_2, \tag{7}$$

$M_{\mathrm{GUT}} \sim 10^{16}$ GeV, $M_G \sim M_{\mathrm{GUT}}$ with $M_G < M_{\mathrm{GUT}}$ and $M_Z \simeq 100$ GeV, and supersymmetry is broken at $M_s \simeq 10^3$ GeV. The Z_2 symmetry, which appears at the second stage of the symmetry breaking in (7), is the discrete $\{1, -1\}$ symmetry, subgroup of both the Z_4 center of SO(10) and of $U(1)_{B-L}$ subgroup of SO(10). Recall that when we write SO(10) we really mean its universal covering group spin(10). The Z_2 symmetry acts as matter parity. It preserves large values for the proton lifetime and stabilizes the LSP; it is thus necessary that this Z_2 symmetry be kept unbroken down to low energies.

We can look for topological defect formation in the symmetry breaking pattern using topological arguments [8]. Since spin(10), the universal covering group of SO(10), is simply connected, the second homotopy group $\pi_2(\mathrm{spin}(10)/3_c 2_L 1_R 1_{B-L}) = \pi_1(3_c 2_L 1_R 1_{B-L}) = Z \oplus Z$ is nontrivial. Therefore, according to the Kibble mechanism [8], topological monopoles form during the first phase transition in equation (7) when SO(10) breaks down to $3_c 2_L 1_R 1_{B-L}$. They have a mass $M_m 10^{17}$ GeV. Furthermore, the second homotopy groups $\pi_2(\mathrm{spin}(10)/3_c 2_L 1_Y Z_2) = \pi_1(3_c 2_L 1_Y Z_2) = Z$ and $\pi_2(\mathrm{spin}(10)/3_c 1_Q Z_2) = \pi_1(3_c 1_Q Z_2) = Z$ are nontrivial so that the monopoles are topologically stable down to low energies. These monopoles if present today would dominate the energy density of the universe, and are thus in conflict with cosmological observations.

Now the first homotopy group $\pi_1(3_c 2_L 1_R 1_{B-L}/3_c 2_L 1_Q Z_2)$ is nontrivial and therefore topological cosmic strings form according to the Kibble mechanism during the second phase transition in equation (7), when the $3_c 2_L 1_R 1_{B-L}$ symmetry group breaks down to $3_c 2_L 1_Y Z_2$. The strings connect half of the monopole-antimonopole pairs formed earlier [7]. Some closed strings can also form. The strings can break with monopole-antimonopole pair nucleation. The monopoles get attracted to each other and the whole system of strings disappear [14]. Nevertheless, the other half of the monopoles remain topologically stable. If present today, these monopoles would lead to a cosmological catastrophe.

Now the rank of $3_c 2_L 1_R 1_{B-L}$ is equal to five, as the rank of SO(10), and is therefore greater than the rank of $3_c 2_L 1_Y Z_2$ from one unit. Thus we can couple the inflaton field with the Higgs field mediating the breaking of $3_c 2_L 1_R 1_{B-L}$ down to $3_c 2_L 1_Y Z_2$, see Sec. II, and the monopole problem can be cured. If the monopoles are pushed away before the phase transition leading to the strings formation takes place, then the evolution of the string network is quite different than previously said. It is that of topologically stable cosmic strings.

IV. MODEL BUILDING

A. Ingredients

In this section, we explain how to implement the symmetry breaking pattern given in Eq. (7). The model solves the doublet-triplet splitting and includes an inflationary scenario as described in Sec. II.

In order to implement the symmetry breaking pattern given in Eq. (7), and in order to preserve the Z_2 symmetry unbroken down to low energy, see Eq. (7), we must only use Higgs fields in ''safe'' representations [5], such as the adjoint 45, the 54, the 126 or the 210-dimensional representations.

In order to implement the first stage of the symmetry breaking, we could use only one Higgs in the 210-dimensional representation; unfortunately the model would then not solve the doublet-triplet splitting problem. The latter can be easily solved using the Dimopoulos-Wilczek mechanism [2], using two Higgs, one in the adjoint 45-dimensional representation and one in the 54-dimensional one. The VEV of the adjoint 45, which we call A_{45}, which implements the Dimopoulos-Wilczek mechanism is in the *B-L* direction, and breaks SO(10) down to $3_c 2_L 2_R 1_{B-L}$. The Higgs in the 54 dimensional representation, which we call S_{54}, breaks SO(10) down to $4_c 2_L 2_R$. Altogether the SO(10) symmetry is broken down to $3_c 2_L 2_R 1_{B-L}$.

We want to break SO(10) directly down to $3_c 2_L 1_R 1_{B-L}$, we therefore need more Higgs. We use another 54, which we call S'_{54}, and another 45, which we call A'_{45}, in the T_{3R} direction. The latter breaks SO(10) down to $4_c 2_L 1_R$. S'_{54} and A'_{45} break together SO(10) down to $4_c 2_L 1_R$.

The role of S_{54} and S'_{54} is to force A_{45} and A'_{45} into $B-L$ and T_{3R} directions. SO(10) breaks down to $3_c 2_L 1_R 1_{B-L}$ with A_{45}, S_{54}, A'_{45}, and S'_{54} acquiring VEV's, and as mentioned in Sec. III, topologically stable monopoles form.

During the second stage of symmetry breaking, see Eq. (7), the rank of the group is lowered by one unit. Indeed the rank of $3_c 2_L 1_R 1_{B-L}$ is equal to the rank of SO(10) which is 5 whereas the rank of $3_c 2_L 1_Y Z_2$ is 4. We can therefore implement a false vacuum hybrid inflationary scenario as described in Sec. II, if we couple the inflaton field to the Higgs field used to break the intermediate symmetry gauge group $3_c 2_L 1_R 1_{B-L}$. The monopole problem can be solved and cosmic strings can form at the end of inflation when the $3_c 2_L 1_R 1_{B-L}$ symmetry group breaks down to the standard model gauge group with unbroken matter parity, $3_c 2_L 1_Y Z_2$.

To break $3_c 2_L 1_R 1_{B-L}$, we use a $126+\overline{126}$ pair of Higgs fields, which we call Φ_{126} and $\overline{\Phi}_{126}$. The latter are safe representations [5] and therefore keeps the Z_2 symmetry unbroken. A $\mathbf{16}+\overline{\mathbf{16}}$ pair of Higgs fields usually used for the same purpose would break the Z_2 symmetry. The VEV of the **126** and $\overline{\mathbf{126}}$ are in the *X* direction, the U(1) symmetry of SO(10) which commutes with SU(5). They break SO(10) down to $SU(5)\times Z_2$. All together, i.e., with A_{45}, S_{54}, A'_{45}, S'_{54}, Φ_{126}, and $\overline{\Phi}_{126}$ acquiring VEV's, the SO(10) symmetry group is broken down to $3_c 2_L 1_Y Z_2$.

The symmetry breaking of the standard model is then achieved using two Higgs in the 10-dimensional representation of SO(10), H_{10} and H'_{10}.

To summarize, the symmetry breaking is implemented as follows:

$$\mathrm{SO}(10) \xrightarrow{\langle A_{45}\rangle\langle S_{54}\rangle\langle A'_{45}\rangle\langle S'_{54}\rangle} 3_c 2_L 1_R 1_{B-L} \xrightarrow{\langle\Phi_{126}\rangle\langle\overline{\Phi_{126}}\rangle} \times 3_c 2_L 1_Y Z_2 \xrightarrow{\langle H'_{10}\rangle\langle H'_{10}\rangle} 3_c 1_Q Z_2. \tag{8}$$

B. The superpotential

We now write down the superpotential involving the above mentioned fields. A consequence of the superpotential is the symmetry breaking pattern given in Eq. (7), which involves an inflationary sector.

As discussed above, our model involves four sectors. The first sector implements the doublet-triplet splitting and involves A_{45}, with VEV in the $U(1)_{B-L}$ direction. It also involves S_{54} and two Higgs 10-plets, H and H'. The superpotential in the first sector is given by W_1+W_2, with, dropping the subscripts,

$$W_1 = m_A A^2 + m_S S^2 + \lambda_S S^3 + \lambda_A A^2 S \tag{9}$$

and

$$W_2 = HAH' + m_{H'}H'^2. \tag{10}$$

W_1 has a global minimum such that the SO(10) symmetry group is broken down to $3_c 2_L 2_R 1_{B-L}$, with A_{45} and S_{54} acquiring VEV's. W_2 implements the doublet-triplet splitting; H and H' break $SU(2)_L \times U(1)_Y$ down to $U(1)_Q$.

The second sector involves A'_{45}, with VEV in the T_{3R} direction, and S'_{54}. The superpotential in the second sector is given by

$$W_3 = m_{A'}A'^2 + m_{S'}S'^2 + \lambda_{S'}S'^3 + \lambda_{A'}A'^2S'. \tag{11}$$

W_3 has a global minimum such that the SO(10) symmetry group is broken down to $3_c 2_L 2_R 1_{B-L}$, with A'_{45} and S'_{54} acquiring VEV's.

The superpotential $W_1 + W_2 + W_3$ has a global minimum such that the SO(10) symmetry is broken down to $3_c 2_L 1_R 1_{B-L}$, with A_{45}, S_{54}, A'_{45}, and S'_{54} acquiring VEV's.

The third sector involves Φ_{126} and $\overline{\Phi}_{126}$, and breaks SO(10) down to SU(5) $\times Z_2$. In order to force the Φ_{126} and $\overline{\Phi}_{126}$ fields to get their VEV's the order of the GUT scale, we use a scalar field $\mathscr{S}$ singlet under SO(10). The superpotential is of the form, dropping the subscript

$$W_4 = \alpha \mathscr{S} \overline{\Phi} \Phi - \mu^2 \mathscr{S}. \tag{12}$$

α and μ are both positive and we must have $\mu/\sqrt{\alpha} = M_G$, with $M_G \simeq 10^{15} - 10^{16}$ GeV for the unification of the gauge coupling constants. Identifying the scalar field $\mathscr{S}$ with the inflaton field, W_4 leads to a false vacuum hybrid inflationary scenario, as described in Sec. II.

The superpotential $W_1 + W_2 + W_3 + W_4$ has a global minimum such that the $3_c 2_L 1_R 1_{B-L}$ symmetry group is broken down to the standard model gauge group with unbroken matter parity, $3_c 2_L 1_Y Z_2$, with A_{45}, S_{54}, A'_{45}, S'_{54}, Φ_{126}, and $\overline{\Phi}_{126}$ acquiring VEV's.

The full superpotential $W = W_1 + W_2 + W_3 + W_4$ does not involve terms mixing A'_{45} and S_{54}, S'_{54} and A_{45} etc . . . . In other words the three sectors are independent. Thanks to the nonrenormalizable theorem, we are not obliged to write down these terms, and it is not compatible with any extra discrete symmetry [15], therefore we do not have to fear any domain wall formation when the symmetry breaks. Nevertheless, in order to avoid any undesirable massless Goldstone Bosons, the three sectors have to be related. The two first sectors, which involve the adjoints A_{45} and A'_{45}, can be related introducing a third adjoint A''_{45} whose VEV does not have the Dimopoulos-Wilczek form, and adding a term of the form $\mathrm{Tr}(AA'A'')$ to the superpotential [15]. The latter does neither affect the symmetry breaking pattern, nor the inflationary scenario discussed below. The light states resulting from the separation of the Φ and $\overline{\Phi}$ sector from the rest are complete multiplets of SU(5) [since the VEVs of Φ and $\overline{\Phi}$ are SU(5) invariant] and therefore do not affect the cosmological scenario, and they do not affect the running of the gauge couplings either. The full superpotential of the model is therefore

$$\begin{aligned} W = {} & m_A A^2 + m_S S^2 + \lambda_S S^3 + \lambda_A A^2 S + HAH' + m_{H'}H'^2 \\ & + m_{A'}A'^2 + m_{S'}S'^2 + \lambda_{S'}S'^3 + \lambda_{A'}A'^2S' + \alpha \mathscr{S} \overline{\Phi} \Phi \\ & - \mu^2 \mathscr{S} + AA'A''. \end{aligned} \tag{13}$$

In Eq. (13), A^2 really means $\mathrm{Tr}(A^2)$, A^2S really means Tr (A^2S), etc. The superpotential given in Eq. (13) leads to the desired pattern of symmetry breaking and the VEV's of A_{45}, S_{54}, A'_{45}, S'_{54}, Φ_{126}, and $\overline{\Phi}_{126}$ are given as follows (see Appendix). The adjoint $\langle A_{45} \rangle$ is in the B-L direction,

$$\langle A_{45} \rangle = \eta \otimes \mathrm{diag}(a,a,a,0,0) \tag{14}$$

where $\eta = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$ and $a \sim M_{\mathrm{GUT}}$. $\langle S_{54} \rangle$ is a traceless symmetric tensor given by

$$\langle S_{54} \rangle = I \otimes \mathrm{diag}\left(x,x,x,-\frac{3}{2}x,-\frac{3}{2}x \right), \tag{15}$$

where I is the unitary 2×2 matrix and $x = -m_A/2\lambda_A$. $\langle A'_{45} \rangle$ is in the T_{3R} direction,

$$\langle A'_{45} \rangle = \eta \otimes \mathrm{diag}(0,0,0,a',a'), \tag{16}$$

where $a' \sim M_{\mathrm{GUT}}$. S'_{54} is a traceless antisymmetric tensor,

$$\langle S'_{54} \rangle = I \otimes \mathrm{diag}\left(x,'x',x',-\frac{3}{2}x',-\frac{3}{2}x' \right), \tag{17}$$

where $x' = 2m_{A'}/3\lambda_{A'}$.

$$\langle |\Phi_{126}| \rangle_{\nu^c\nu^c} = \langle |\overline{\Phi}_{126}| \rangle_{\overline{\nu^c\nu^c}} = d. \tag{18}$$

With the VEV's chosen above, if $S=0$, $d = \mu/\sqrt{\alpha}$, and the superpotential has a global minimum such that the SO(10) symmetry is broken down to the standard model gauge group with unbroken matter parity $SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$, and supersymmetry is unbroken (see the Appendix).

V. THE INFLATIONARY EPOCH

In this section we evaluate the details of the symmetry breaking pattern and of the inflationary scenario. We write down the scalar potential and find values of the scalar coupling constant and the mass scales M_G and M_{GUT} for which the inflationary scenario is successful.

We are interested in the dynamics of the symmetry breaking pattern and how the inflationary scenario fits in the symmetry breaking pattern. We therefore need to study the scalar potential. In order fully to understand the dynamics of the model, one would need to use finite temperature field theory. Nevertheless, study of the scalar potential derived from the superpotential given in Eq. (13) leads a good understanding of the field evolution. The scalar potential is given by

$$\begin{aligned} V = {} & (2m_A A + 2\lambda_A AS)^2 + (2m_S S + 3\lambda_S S^2 + \lambda_A A^2)^2 \\ & + (2m_{A'}A' + 2\lambda_{A'}A'S')^2 + (2m_{S'}S' + 3\lambda_{S'}S'^2 \\ & + \lambda_{A'}A'^2)^2 + \alpha^2 |\mathscr{S}\overline{\Phi}|^2 + \alpha^2 |\mathscr{S}\Phi|^2 + |\alpha \overline{\Phi}\Phi - \mu^2|^2. \end{aligned} \tag{19}$$

We remind the reader that A and A' are two Higgs fields in the 45-dimensional representation of SO(10) with VEV in the $B-L$ and T_{3R} directions, respectively. S and S' are two Higgs fields in the 54-dimensional representation of SO(10). Φ and $\bar{\Phi}$ are two Higgs in the **126** and $\overline{\mathbf{126}}$ representations, with VEVs in the right-handed neutrino direction. The scalar field $\mathcal{S}$ is a scalar field singlet under SO(10). It forces Φ and $\bar{\Phi}$ to get VEV of the order of the GUT scale. The scalar field $\mathcal{S}$ is identified with the inflaton field. α and μ are both positive constants which must satisfy the relation $\mu/\sqrt{\alpha}=M_G$. The potential is minimized for $\arg(\Phi)+\arg(\bar{\Phi})=0$, $(\alpha>0)$, and it is independent of $\arg(\mathcal{S})+\arg(\Phi)$ and $\arg(\mathcal{S})+\arg(\bar{\Phi})$. We rewrite the potential with the new fields which minimize the potential, keeping the same notation for the old and new fields. The scalar potential becomes

$$V=(2m_A A+2\lambda_A AS)^2+(2m_S S+3\lambda_S S^2+\lambda_A A^2)^2 +(2m_{A'}A'+2\lambda_{A'}A'S')^2+(2m_{S'}S'+3\lambda_{S'}S'^2 +\lambda_{A'}A'^2)^2+4\alpha^2|\mathcal{S}|^2|\Phi|^2+(\alpha|\Phi|^2-\mu^2)^2 +\frac{1}{2}m^2|\mathcal{S}|^2, \tag{20}$$

where we have also introduced a soft supersymmetry breaking term for S, and $m\sim10^3$ GeV.

The scalar potential is flat in the $\mathcal{S}$ direction; we thus identify the scalar field $\mathcal{S}$ with the inflaton field. We suppose chaotic initial conditions; that is we suppose that all the fields have initial values of the order of the Planck scale. We then minimize the superpotential for fixed $\mathcal{S}$. We easily find that for $\mathcal{S}>\mu/\sqrt{\alpha}=s_c$, (recall that $\mu,\alpha>0$), there is a local minimum corresponding to $|\Phi|=|\bar{\Phi}|=0$, and A, A', S and S' taking values as given above in Eqs. (14), (15), (16), and (17). Since all the fields are assumed to take initial values of the order of the Planck scale, the inflaton field has an initial value greater than $\mu/\sqrt{\alpha}$. Then, because the potential is flat in the inflaton direction, the fields Φ, $\bar{\Phi}$, A, A', S and S' settle quickly to the local minimum corresponding to $\langle S\rangle$, $\langle A\rangle$, $\langle S'\rangle$, and $\langle A'\rangle$ as in Eqs. (14), (15), (16), and (17) respectively, and $\langle|\Phi|\rangle=\langle|\bar{\Phi}|\rangle=0$. The first phase transition takes place and the SO(10) symmetry group breaks down to the $3_c2_L1_R1_{B-L}$ symmetry group. As shown in Sec. III, topologically stable monopoles form according to the Kibble mechanism [8] during this first phase transition.

Once the fields A, S, A', and S' have settled down to their minimum, since the first derivatives $\partial V/\partial A$, $\partial V/\partial S$, $\partial V/\partial A'$, and $\partial V/\partial S'$ are independent of Φ and $\mathcal{S}$, the fields A, A', S, and S' will stay in their minimum independently of what the fields Φ and $\mathcal{S}$ do. When the VEV of the inflaton field is greater than $\mu/\sqrt{\alpha}=s_c$, $|\Phi|=|\bar{\Phi}|=0$, $F_{\mathcal{S}}$ term has a nonvanishing VEV, which means that supersymmetry is broken in the $\mathcal{S}$ direction, by an amount measured by the VEV of the $\mathcal{S}$ superfield. There is a nonvanishing vacuum energy density, $V=\mu^4$. An inflationary epoch (an exponentially extending universe) can start.

As has been pointed out recently [12], the fact that supersymmetry is broken for $\langle\mathcal{S}\rangle>\langle\mathcal{S}\rangle_c$ implies that the one loop corrections to the effective potential are nonvanishing. They are given by [11]

$$\Delta V(\mathcal{S})=\Sigma_i\frac{(-1)^F}{64\pi^2}M_i(\mathcal{S})^4\ln\left(\frac{M_i(\mathcal{S})}{\Lambda^2}\right), \tag{21}$$

where the summation is over all helicity states for both fermions and bosons. Λ denotes a renormalization mass and $(-1)^F$ indicates that the bosons and fermions make opposite sign contributions to the sum; (-1) stand for the fermions. Therefore the one loop effective potential obtained from Eqs. (20) and (21) is given by [11]

$$V_{\rm eff}=\mu^4 b\left(1+\frac{\alpha^2}{32\pi^2}\left[2\ \ln\left(\frac{\alpha^2 s^2}{\Lambda^2}\right)+\left(\frac{\alpha s^2}{\mu^2}-1\right)^2\ln\left(1-\frac{\mu^2}{\alpha s^2}\right) +\left(\frac{\alpha s^2}{\mu^2}+1\right)^2\ln\left(1+\frac{\mu^2}{\alpha s^2}\right)\right]+\frac{m^2}{2\mu^4}s^2 b\right), \tag{22}$$

where $s=\langle\mathcal{S}\rangle\gg$. Now $m\simeq10^3$ GeV and $\mu/\sqrt{\alpha}\sim10^{15-16}$ GeV, hence unless $\alpha\ll1$, the soft supersymmetry breaking term can be neglected. Its contribution to the scalar potential is negligible. For $s>s_c$, the quantum corrections to the effective potential help $\mathcal{S}$ to roll down its minimum. Below s_c, the $\mathcal{S}$ field is driven to zero by the positive term $\alpha^2|\mathcal{S}|^2|\Phi|^2$ which becomes larger with increasing $|\Phi|$. Rapidly the Φ, $\bar{\Phi}$, and $\mathcal{S}$ fields settle down the global minimum of the potential, corresponding to $\langle\Phi\rangle_{\nu^c\nu^c}=\bar{\Phi}_{\overline{\nu^c\nu^c}}=\mu/\sqrt{\alpha}$ and $s=0$. This does not affect the VEV's of the S, A, S', and A' fields which remain unchanged. The $3_c2_L1_R1_{B-L}$ symmetry group breaks down to $3_c2_L1_YZ_2$. As shown in Sec. III, topological cosmic strings form during this phase transition. If inflation ends after the phase transition, the strings may be inflated away.

Inflation ends when the ''slow roll'' condition is violated. The slow roll condition is characterized by [10],

$$\epsilon\ll1, \quad \eta\ll1, \tag{23}$$

where

$$\epsilon=\frac{M_p^2}{16\pi}\left(\frac{V'}{V}\right)^2, \quad \eta=\frac{M_p^2}{8\pi}\left(\frac{V''}{V}\right) \tag{24}$$

and the prime refers to derivatives with respect to s. As pointed out by Copeland *et al.* [10], the slow-roll condition is a poor approximation. But as shown in [10], the number of e-foldings which occur between the time when η and ϵ reach unity and the actual end of inflation is a tiny fraction of unity. It is therefore sensible to identify the end of inflation with ϵ and η becoming of order unity.

From the effective potential (22) and the slow-roll parameters (24) we have [11]

$$\epsilon=\left(\frac{\alpha^2 M_p}{8\pi^2 M_G}\right)^2\frac{x^2}{16\pi}\left[(x^2-1)\ln\left(1-\frac{1}{x^2}\right) +(x^2+1)\ln\left(1+\frac{1}{x}\right)\right]^2, \tag{25}$$

$$\eta=\left(\frac{\alpha M_p}{2\pi M_G}\right)^2\frac{1}{16\pi}\left[(3x^2-1)\ln\left(1-\frac{1}{x^2}\right)+(3x^2+1)\ln\left(1+\frac{1}{x}\right)\right]^2, \quad (26)$$

where x is such that $s=xs_c$. The phase transition down to the standard model occurs when $x=1$. The results are as follows. We find the values of the scalar coupling α, the scale $M_{\rm GUT}$ and the scale M_G which lead to successful inflation. For $\alpha\geqslant 35-43$, $M_G\sim 10^{15}-10^{16}$ GeV, ϵ is always greater than unity, and the slow roll condition is never satisfied. The scale $M_{\rm GUT}$ at which the monopoles form satisfies $M_{\rm pl}\geqslant M_{\rm GUT}\geqslant 10^{16}-10^{17}$ GeV. For $\alpha\leqslant 0.02-0.002$ and $M_G\sim 10^{15}-10^{16}$ GeV, neither η nor ϵ ever reaches unity. $\mathcal{S}$ reaches $\mathcal{S}_c$ during inflation. Inflation must end by the instability of the Φ and $\bar{\Phi}$ fields. In that case, inflation ends in less than a Hubble time [10] once $\mathcal{S}$ reaches $\mathcal{S}_c$. Cosmic strings, which form when $x=1$, are not inflated away. The scale $M_{\rm GUT}$ at which the monopoles form must satisfy $M_{\rm pl}\geqslant M_{\rm GUT}\geqslant 10^{16}-10^{17}$ GeV. For the intermediates values of α, inflation occurs, and ends when either ϵ or η reaches unity; the string forming phase transition takes places once inflation completed.

VI. FORMATION OF COSMIC STRINGS

In this section we give general properties of the strings which form at the end of inflation when the $3_c2_L1_R1_{B-L}$ symmetry group breaks down to $3_c2_L1_YZ_2$. We find their width and their mass, give a general approach for their interactions with fermions and study their superconductivity.

A. General properties

Recall that, since the first homotopy group $\pi_1(3_c2_L1_R1_{B-L}/3_c2_L1_YZ_2)$ is nontrivial, cosmic strings form during the second phase transition [see Eq. (7)] when the $3_c2_L1_R1_{B-L}$ symmetry breaks down to $3_c2_L1_YZ_2$. We note that the subspace spanned by R and $B-L$ is also spanned by X and Y. The generator of the string corresponds to the U(1) of SO(10) which commutes with SU(5), and the gauge field forming the string is the corresponding gauge field, which we call X. The strings are Abelian and physically viable. The model does not give rise to Alice strings, like most of the non-Abelian GUT phase transitions where Abelian and non-Abelian strings form at the same time. This is a good point of the model, since Alice strings give rise to quantum number nonconservation, and are therefore in conflict with the standard cosmology. The strings arising in our model can be related to the Abelian strings arising in the symmetry breaking pattern of SO(10) down to to the standard model with SU(5) $\times Z_2$ as intermediate scale, since they have the same generator; the latter have been widely studied in the nonsupersymmetric case [16,17]. Nevertheless, in our model, inside the core of the string, we do not have an SO(10) symmetry restoration, but a $3_c2_L1_R1_{B-L}$ symmetry restoration. We therefore don't have any gauge fields mediating baryon number violation inside the core of the strings, but one of the fields violates $B-L$. We also expect the supersymmetric strings to have different properties and different interaction with matter due to the supersymmetry restoration inside the core of the string. These special properties will be studied elsewhere.

The two main characteristics of the strings, their width and their mass, are determined through the Compton wavelength of the Higgs and gauge bosons forming the strings. The Compton wavelength of the Higgs and gauge bosons are respectively

$$\delta_{\Phi_{126}}\sim m_{\Phi_{126}}^{-1}=(2\alpha M_G)^{-1} \quad (27)$$

and

$$\delta_X\sim m_X^{-1}=(\sqrt{2}eM_G)^{-1}, \quad (28)$$

where e is the gauge coupling constant in supersymmetric SO(10) and it is given by $e^2/4\pi=1/25$ and M_G is the scale at which the strings form.

As mentioned above, the strings formed in our model can be related to those formed during the symmetry breaking pattern SO(10) $\rightarrow$SU(5)$\times$U(1)$\rightarrow$SU(5)$\times Z_2$. These strings have been studied by Aryal and Everett [16] in the nonsupersymmetric case. Using their results, with appropriate changes in the gauge coupling constant and in parameters of the Higgs potential, we find that the string mass per unit length of the string is given by

$$\mu\simeq(2.5-3)(M_G)^2, \quad (29)$$

for the scalar coupling α ranging from 5×10^{-2} to 2×10^{-1}. Recall that the mass per unit- ength characterizes the entire properties of a network of cosmic strings.

B. No superconducting strings

One of the most interesting feature of GUT strings is their superconductivity. Indeed, if they become superconducting at the GUT scale, then vortons can form and dominate the energy density of the universe; the model loses all its interest. The strings arising in our model are not superconducting in Witten's sense [18]. They nevertheless can become current carrying with spontaneous current generation at the electroweak scale through Peter's mechanism [19]. But it is believed that this does not have any disastrous impact on the standard cosmology. It has been shown in the nonsupersymmetric case that the Abelian strings arising when SO(10) breaks down to SU(5)$\times Z_2$ have right-handed neutrino zero modes [20]. Since the Higgs field forming the string is a Higgs boson in the **126** representation which gives mass to the right-handed neutrino and winds around the string, we expect the same zero modes on our strings. Since supersymmetry is restored in the core of the string, we also expect bosonic zero modes of the superpartner of the right-handed neutrino. Now, the question of whether or not the string will be current carrying will depend on the presence of a primordial magnetic field, and the quantum charges of the right-handed neutrino with respect to this magnetic field. If there is a primordial magnetic field under which the right-handed neutrino has a nonvanishing charge, then the current will be able to charge up. On the other hand, if such magnetic field does not exist, or if the right-handed neutrino is neutral, then there will be nothing to generate the current of the string.

Although it is possible to produce a primordial magnetic field in a phase transition [21], we do not expect the fields produced through the mechanism of Ref. [21] to be able to charge up the current on the string, since the latter are correlated on too large scales. Nevertheless, the aim of this section is to show that the strings will not be superconducting at the GUT scale in any case. We can therefore assume a worse situation, that is, suppose that the magnetic fields are correlated on smaller scales, due to any mechanism for primordial magnetic field production any time after the Planck scale. In our model, cosmic strings form when $3_c 2_L 1_R 1_{B-L}$ breaks down to $3_c 2_L 1_Y Z_2$. Therefore the symmetric phase $3_c 2_L 1_Y Z_2$ will be associated with color, weak and hypercharge magnetic fields. The color and weak magnetic fields formed when SO(10) broke down to $3_c 2_L 1_R 1_{B-L}$, and and the hypercharge magnetic field formed at the following phase transition, formed from the R and $B-L$ magnetic fields. Since the charges of the right-handed neutrino with respect to the color, weak and hypercharge magnetic fields are all vanishing, no current will be generated.

We conclude that the strings will not be superconducting at the GUT scale. They might become superconducting at the electroweak scale, but this does not seem to affect the standard big-bang cosmology in any essential way.

If the strings formed at the end of inflation are still present today, they would affect temperature fluctuations in the CBR and have affected large scale structure formation.

VII. OBSERVATIONAL CONSEQUENCES

We show here that the strings formed at the end of inflation may be present today. We find the scale M_G at which cosmic strings form and the scalar coupling of the inflaton field which are consistent with the temperature fluctuations observed by COBE. We then examine the dark-matter content of the model and make a qualitative discussion regarding large scale structure formation.

A. Temperature fluctuations in the CBR

If both inflation and cosmic strings are part of the scenario, temperature fluctuations in the CBR are the result of the quadratic sum of the temperature fluctuations from inflationary perturbations and cosmic strings.

The scalar density perturbations produced by the inflationary epoch induce temperature fluctuations in the CBR which are given by [11]

$$\left(\frac{\delta T}{T}\right)_{\rm inf} \simeq \sqrt{\frac{32\pi}{45}\frac{V^{3/2}}{V' M_{\rm pl}^3}}\Bigg|_{x_q} \tag{30}$$

$$\approx (8\pi N_q)^{1/2}\left(\frac{M_G}{M_{\rm pl}}\right)^2, \tag{31}$$

where the subscript indicates the value of $\mathscr{S}$ as the scale (which evolved to the present horizon size) crossed outside the Hubble horizon during inflation, and N_q ($\sim 50-60$) denotes the appropriate number of e-foldings. The contribution to the CBR anisotropy due to gravitational waves produced by inflation in this model is negligible.

The cosmic strings density perturbations also induce CBR anisotropies given by [22]

$$\left(\frac{\delta T}{T}\right)_{\rm c.s.} \approx 9G\mu, \tag{32}$$

where μ is the strings mass per unit length, which is given by Eq. (29). It depends on the scalar coupling α. Since the later is undetermined, we can use the order of magnitude

$$\mu \sim \eta^2, \tag{33}$$

which holds for a wide range of the parameter α; see Eq. (29) and Ref. [16]. In Eq. (33), η is the symmetry breaking scale associated with the strings formation, here $\eta = M_G$.

Hence, from Eqs. (31) and (32) the temperature fluctuations in the CBR are given by

$$\left(\frac{\delta T}{T}\right)_{\rm tot} \approx \sqrt{\left(\frac{\delta T}{T}\right)_{\rm inf}^2 + \left(\frac{\delta T}{T}\right)_{\rm c.s.}^2} \tag{34}$$

$$\approx \sqrt{8\pi N_q + 81}\left(\frac{M_G}{M_{\rm pl}}\right)^2. \tag{35}$$

The temperature fluctuations from both inflation and cosmic strings add quadratically. Since they are both proportional to M_G/M_{pl} their computation is quite easy.

An estimate of the coupling α is obtained from the relation [11]

$$\frac{\alpha}{x_q} \sim \frac{8\pi^{3/2}}{\sqrt{N_q}}\frac{M_G}{M_{\rm pl}}. \tag{36}$$

With $x_q \sim 10$, using Eqs. (35) and (36) and using the density fluctuations measured by COBE $\simeq 1.13 \times 10^{(-5)}$ [23] we get

$$\alpha \simeq 0.03, \tag{37}$$

$$M_G \simeq 6.710^{15}\ \text{GeV}. \tag{38}$$

With these values, we find that η reaches unity when $x \simeq 1.4$ and the scale $M_{\rm GUT}$ at which the monopoles form must satisfy

$$M_{\rm pl} \geqslant M_{\rm GUT} \geqslant 6.7\ 10^{16}\ \text{GeV}, \tag{39}$$

where $M_{\rm pl}$ is the Planck mass $\simeq 1.2210^{19}$ GeV.

From the above results, we can be confident that the strings forming at the end of inflation should still be around today.

Now that we have got values for the scalar coupling α and the scale M_G at which the strings form, the Compton wavelength of the Higgs and gauge bosons forming the strings given by Eqs. (27) and (28) can be computed. We find

$$\delta_{\Phi_{126}} \sim m_{\Phi_{126}}^{-1} \sim 0.4210^{-28}\ \text{cm} \tag{40}$$

for the Compton wavelength of the Higgs field forming the string and

TABLE I. The table shows the values obtained for the scale M_G at which the strings form, the scalar coupling α, the Higgs and gauge boson Compton wavelengths δ_Φ and δ_X of the strings, and $G\mu$, where μ is the strings mass per unit length and G is the Newton's constant, for different values of the number of e-foldings N_q and for different initial values used for their computation for the string mass-per-unit length.

N_q	50	50	60	60
μ_{init}	η^2	$2.5\eta^2$	η^2	$2.5\eta^2$
M_G	6.8×10^{15}	6.3×10^{15}	6.5×10^{15}	6.1×10^{15}
α	0.03	0.03	0.03	0.29
δ_Φ	0.42×10^{-28}	0.44×10^{-28}	0.43×10^{-28}	0.46×10^{-28}
δ_X	0.29×10^{-29}	0.31×10^{-29}	0.30×10^{-29}	0.32×10^{-29}
$G\mu$	7.7×10^{-7}	6.7×10^{-7}	7.1×10^{-7}	6.3×10^{-7}

$$\delta_X \sim m_X^{-1} \sim 0.2910^{-29} \ \text{cm} \tag{41}$$

for the Compton wavelength of the strings gauge boson. $\delta_{\Phi_{126}} > \delta_X$ thus the strings possess an inner core of false vacuum of radius $\delta_{\Phi_{126}}$ and a magnetic flux tube with a smaller radius δ_X. The string energy per unit length is given by Eq. (29), thus, using above results, we have

$$G\mu \sim 7.7\times10^{-7}, \tag{42}$$

where G is Newton's constant. The results are slightly affected by the number of e-folding and by the order of magnitude (33) used to compute the temperature fluctuations in the CBR due to cosmic strings in Eqs. (34) and (35). Once we have found the value for the scalar coupling α for successful inflation, we can redo the calculations with a better initial value for the string mass per unit length; see Eq. (29); the scalar coupling α is unchanged. The results are summarized in Table I.

B. Dark matter

We specify here the nature of dark matter generated by the model.

If we go back to the symmetry breaking pattern of the model given by Eq. (7), we see that a discrete Z_2 symmetry remains unbroken down to low energy. This Z_2 symmetry is a subgroup of both the Z_4 center of SO(10) and of $U(1)_{B-L}$ subgroup of SO(10). This Z_2 symmetry acts as matter parity. It preserves large values for the proton lifetime and stabilizes the lightest superparticle. The LSP is a good cold dark matter candidate.

The second stage of symmetry breaking in Eq. (7) is implemented with the use of a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs multiplets, with VEV's in the direction of the $U(1)_X$ of SO(10) which commutes with SU(5). The $\overline{\mathbf{126}}$ multiplet can couple with fermions and give superheavy Majorana mass to the right-handed neutrino, solving the solar neutrino problem via the MSW mechanism [6] and providing a good hot dark matter candidate. This can be done if all fermions are assigned to the 16-dimensional spinorial representation of SO(10). In that case, couplings of the form $f\overline{\Psi}\Psi\overline{\mathbf{126}}$, where Ψ denotes a 16-dimensional spinor to which all fermions belonging to a single family are assigned, provide right-handed neutrinos masses of order $m_R \simeq 10^{12}$ GeV, if $f\sim10^{-4}$ GeV. Neutrinos also get Dirac masses which are typically of the order of the mass of the up-type quark of the corresponding family; for instance $m_D^{\nu^e} \simeq m_u$. After diagonalizing the neutrino mass matrix, one finds that the right-handed neutrino mass $m_{\nu_R} \simeq m_R$ and the left-handed neutrino mass $m_{\nu_L} \simeq m_D^2/m_R$. With the above values we get

$$m_{\nu^e} \sim 10^{-7} \ \text{eV}, \tag{43}$$

$$m_{\nu^\mu} \sim 10^{-3} \ \text{eV}, \tag{44}$$

$$m_{\nu^\tau} \sim 10 \ \text{eV}. \tag{45}$$

The tau neutrino is a good hot dark matter candidate.

Our model thus provides both CDM and HDM and is consistent with mixed cold and hot DM scenarios.

It is interesting to note that CDM and HDM are, in this model, related to each other. Indeed, the Z_2 symmetry in Eq. (7), which stabilizes the LSP, is kept unbroken because a $\mathbf{126}+\overline{\mathbf{126}}$ and not $\mathbf{16}+\overline{\mathbf{16}}$ pair of Higgs fields are used to break $U(1)_{B-L}$. If a 16 dimensional Higgs representation were used, the right-handed neutrino could not get a superheavy Majorana mass and thus no HDM could be provided, also the Z_2 symmetry would have been broken, and thus the LSP destabilized. The $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields provide superheavy Majorana to the right-handed neutrino and keeps the Z_2-parity unbroken. It leads to both HDM and CDM. We conclude that, in this model, CDM and HDM are intimately related. Either the model provides both cold and hot dark matter, or it does not provide any. Our model provides both CDM and HDM.

C. Large scale structure

We give here only a qualitative discussion of the consistency of the model with large scale structure. We do not make any calculations which would require a full study on their own. We can nevertheless use various results on large scale structure with inflation or cosmic strings. Since we determined the nature of dark matter provided by the model, we may make sensible estimations about the the consistency of the model with large scale structure.

Presently there are two candidates for large scale structure formation, the inflationary scenario and the topological defects scenario with cosmic strings. Both scenarios are always considered separately. Indeed, due to the difference in the nature of the density perturbations in each of the models, density perturbation calculations due to a mixed strings and inflation scenario are not straightforward. Indeed in the inflation-based models density perturbations are Gaussian adiabatic whereas in models based on topological defects inhomogeneities are created in an initially homogeneous background [24].

In the attempt to explain large scale structure, inflation-seeded cold dark matter models or strings models with HDM are the most capable [24]. In adiabatic perturbations with hot dark matter small scale perturbations are erased by free streaming whereas seeds like cosmic strings survive free streaming and therefore smaller scale fluctuations in models with seeds + HDM are not erased, but their growth is only delayed by free streaming.

Our model involves both hot and cold dark matter, and both inflation and cosmic strings. It is therefore sensible to suggest that our model will be consistent with large scale structure formation, with the large scale fluctuations resulting from the inflationary scenario and small scale fluctuations being due to cosmic strings.

VIII. CONCLUSIONS

We have successfully implemented a false vacuum hybrid inflationary scenario in a supersymmetric SO(10) model. We first argued that this type of inflationary scenario is a natural way for inflation to occur in global supersymmetric SO(10) models. It is natural, in the sense that the inflaton field emerges naturally from the theory, no external field and no external symmetry has to be added. The scenario does not require any fine tuning. In our specific model, the SO(10) symmetry is broken via the intermediate $3_c 2_L 1_R 1_{B-L}$ symmetry down to the standard model with unbroken matter parity $3_c 2_L 1_Y Z_2$. The model gives a solution for the doublet-triplet splitting via the Dimopoulos-Wilczek mechanism. It also suppresses rapid proton decay.

The inflaton, a scalar field singlet under SO(10), couples to the Higgs mediating the phase transition associated with the breaking of $3_c 2_l 1_R 1_{B-L}$ down to the standard model. The scenario starts with chaotic initial conditions. The SO(10) symmetry breaks at $M_{\rm GUT}$ down to $3_c 2_L 1_R 1_{B-L}$ and topologically stable monopoles form. There is a nonvanishing vacuum energy density, supersymmetry is broken, and an exponentially extending epoch starts. Supersymmetry is broken, and therefore quantum corrections to the scalar potential can not be neglected. The latter help the inflaton field to roll down its minimum. At the end of inflation the $3_c 2_L 1_R 1_{B-L}$ breaks down to $3_c 2_L 1_Y Z_2$, at a scale M_G, and cosmic strings form. They are not superconducting.

Comparing the CBR temperature anisotropies measured by COBE with that predicted by the mixed inflation-cosmic strings scenario, we find values for the scalar coupling α and for the scale M_G at which the strings form. $M_{\rm GUT}$ is calculated such that we get enough e-foldings to push the monopoles beyond the horizon. The results are summarized in Table I. The evolution of the strings is that of topologically stable cosmic strings. The model is consistent with a mixed HCDM scenario. Left-handed neutrinos get very small masses and the tau neutrino may serve as a good HDM candidate. They could also explain the solar neutrino problem via the MSW mechanism. The unbroken matter parity stabilizes the LSP, thus providing a good CDM candidate. A qualitative discussion leads to the conclusion that the model is consistent with large scale structures, very large scale structures being explained by inflation and cosmic strings explaining structures on smaller scales. An algebraic investigation for this purpose would be useful, but will require further research.

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APPENDIX: MINIMIZING THE SUPERPOTENTIAL

In this Appendix, we find the true minimum of the superpotential of the model. We calculate the F terms and find the VEV's of the Higgs fields which correspond to the global minimum.

The full superpotential of the model is given by Eq. (13),

$$\begin{aligned}W = {}& m_A A^2 + m_S S^2 + \lambda_S S^3 + \lambda_A A^2 S + HAH' + m_{H'} H'^2 \\ & + m_{A'} A'^2 + m_{S'} S'^2 + \lambda_{S'} S'^3 + \lambda_{A'} A'^2 S' + \alpha \mathcal{S} \overline{\Phi} \Phi \\ & - \mu^2 \mathcal{S} + \mathrm{Tr}(AA'A''). \end{aligned} \tag{A1}$$

where A and A' are 54-dimensional Higgs representations therefore traceless second rank antisymmetric tensors. Thus in the 10-dimensional representation of SO(10) they are of the form, with appropriate subscripts,

$$\langle S_{54} \rangle = I \otimes \mathrm{diag}\left(x, x, x, -\frac{3}{2}x, -\frac{3}{2}x \right) \tag{A2}$$

and

$$\langle S'_{54} \rangle = I \otimes \mathrm{diag}\left(x,'x', x', -\frac{3}{2}x', -\frac{3}{2}x' \right), \tag{A3}$$

where x and x' are the order of $M_{\rm GUT}$ and are determined by the vanishing condition of the F terms. The Higgs bosons A_{45} and A'_{45} are 45-dimensional representations and must be in the $B-L$ and T_{3R} directions respectively (see Sec. IV). Therefore in the 10-dimensional representation of SO(10) A_{45} and A'_{45} are given by

$$\langle A_{45} \rangle = \eta \otimes \mathrm{diag}(a, a, a, 0, 0), \tag{A4}$$

where $a \sim M_{\rm GUT}$ and

$$\langle A'_{45} \rangle = \eta \otimes \mathrm{diag}(0, 0, 0, a', a'), \tag{A5}$$

where $a' \sim M_{\rm GUT}$. The Φ and $\overline{\Phi}$ fields are Higgs bosons in the **126** and $\overline{\Phi}$-dimensional representations. The Φ and $\overline{\Phi}$ fields must break the $\mathrm{U}(1)_X$ symmetry which commutes with SU(5), and thus acquire VEV's in the right-handed neutrino direction. From the vanishing condition for the D terms, $\langle \Phi \rangle = \langle \overline{\Phi} \rangle$ and thus, with appropriate subscripts,

$$\langle \Phi_{126} \rangle_{\nu^c \nu^c} = \langle \overline{\Phi}_{126} \rangle_{\overline{\nu^c \nu^c}} = d, \tag{A6}$$

where $d \sim M_G$.

The true vacuum corresponds to F terms vanishing. Supersymmetry is unbroken. Using the same notation for the scalar component than for the superfield, the F terms are given by

$$F_A = 2 m_A A + 2 \lambda_A A S, \tag{A7}$$

$$F_S = 2 m_S S + 3 \lambda_S S^2 + \lambda_A A^2, \tag{A8}$$

$$F_{A'} = 2 m_{A'} A' + 2 \lambda_{A'} A' S', \tag{A9}$$

$$F_{S'} = 2m_{S'}S' + 3\lambda_{S'}S'^2 + \lambda_{A'}A'^2, \quad (A10)$$

$$F_{\Phi} = \alpha\mathscr{S}\overline{\Phi}, \quad (A11)$$

$$F_{\overline{\Phi}} = \alpha\mathscr{S}\Phi, \quad (A12)$$

$$F_{\mathscr{S}} = \alpha\overline{\Phi}\Phi - \mu^2. \quad (A13)$$

Using the VEV's of the Higgs fields given above, we easily get the VEV's of the F terms. The vanishing condition for the latter leads to the following relations, for each term respectively:

$$m_A a + 2\lambda_A a x = 0, \quad (A14)$$

$$-m_S x + \frac{3}{4}\lambda_S x^2 + \frac{1}{5}\lambda_A a^2 = 0, \quad (A15)$$

$$2m_{A'}a' - 3\lambda_{A'}a'x' = 0, \quad (A16)$$

$$-m_{S'}x' + \frac{3}{4}\lambda_{S'}x'^2 - \lambda_{A'}a'^2 = 0, \quad (A17)$$

$$\alpha s d = 0, \quad (A18)$$

$$\alpha d^2 - \mu^2 = 0, \quad (A19)$$

where s is the VEV of the scalar field $\mathscr{S}$. We note that the roles of the 54 dimensional representations S_{54} and S'_{54} are to force the adjoint A_{45} and A'_{45} into $B-L$ and T_{3R} directions. With the VEV's chosen above, see Eqs. (A2), (A4), (A3), (A5), and (A6), if $s=0$ and $d=\mu/\sqrt{\alpha}$ the potential has a global minimum, such that the SO(10) symmetry is broken down to $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ and supersymmetry is unbroken and we have $x = 2m_A/3\lambda_A$ and $x' = 2m_{A'}/3\lambda_{A'}$. $a \sim M_{GUT}$, $a' \sim M_{GUT}$, and $\mu/\sqrt{\alpha} \sim M_G$, where $M_G \sim 10^{15-16}$ GeV and $M_G \leqslant M_{GUT} \leqslant M_{pl}$ and M_{pl} is the Planck mass $\sim 10^{19}$ GeV.

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