Decay $Z^0 \rightarrow \pi^0 \gamma$ reexamined

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The decay $Z^0 \rightarrow \pi^0 \gamma$ is treated using a relativistic nonperturbative quark model for the pion. The model assumes that the pion is made of a $q\overline{q}$ valence pair and of an effective neutral component representing the nonelementary excitations of the quark-gluonic field responsible for the confinement. The decay width is expressed in terms of the pion decay constant F_π and of current quark masses. Its value calculated for 2 MeV $\leqslant m_u \leqslant$ 8 MeV, 5 MeV$\leqslant m_d \leqslant$15 MeV is at least four orders of magnitude under the experimental upper bound but in agreement with other theoretical models.

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It was recently shown [1,2] that the soft pion techniques [3] are not applicable on the $Z^0 \rightarrow \pi^0 \gamma$ decay. First theoretical estimation was then given by resorting to the standard theory of the electroweak interaction supplemented by an effective $\pi q\overline{q}$ coupling which led to a quark triangle diagram [1,2] of the kind already known from the neutral pion decay [4].

A careful analysis of the elementary processes implied by this model shows that the existence of a $\pi q\overline{q}$ Hamiltonian introduces, in addition to other quantum fluctuations, the spontaneous creation and absorption from the vacuum of a pion *and* of a quark-antiquark pair. This is questionable, since it is as if there were in the same time elementary quarks and pions. This feature is hidden in the Lorentz covariant form of perturbation theory, but it can be easily seen in the noncovariant form, where the structure of the quantum fluctuations participating to a certain process appears more clearly. For this reason we consider that in such processes involving quarks and hadrons, the effective quark-hadron coupling should be avoided and hadrons have to be treated as systems of quarks.

Starting from this observation we proposed recently a relativistic quark model [5] where the hadrons are systems made of valence quarks and some other, more complex excitations of the quark gluonic field, which cannot be represented in terms of a few elementary excitations. The fundamental conjecture of the model is that a system of this kind is in a stable equilibrium state as long as it is not subjected to an external interaction. Conversely, any interaction of a hadron with some other external field can be viewed as a perturbation of the equilibrium state. This conjecture will allow a separate treatment of the structure and interaction of the hadrons. In this respect our model recalls the Furry representation in QED [6]. There, as in our model, the interaction of a bound electron with the electrostatic field of the nucleus is introduced by means of the stationary internal wave function and is treated separately from the interaction with the external radiation field. As it will be soon clear, in some cases, as for instance the present one, this separation introduces substantial simplifications.

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In accordance with the above assumptions, the π meson state was written [5]

$$|\pi_i(P)\rangle = \int d^3p\frac{m}{e}d^3q\frac{m'}{\epsilon}d^4Q\,\varphi(p,q;Q)\overline{u(\mathbf{p})}\gamma_5 v(\mathbf{q}) \times \delta^{(4)}(p+q+Q-P)\chi^\dagger\lambda_i\psi\Phi^\dagger(Q)a^\dagger(\mathbf{p})b^\dagger(\mathbf{q})|0\rangle, \tag{1}$$

where $a^\dagger$, $b^\dagger$ are the creation operators of the valence quark and antiquark (supposed to be free bare particles) and u,v are Dirac spinors ensuring the Lorentz-covariant coupling of the quark spins; χ and ψ are vectors in the space of flavors and λ_i is the Gell-Mann matrix; $\Phi^\dagger(Q)$ represents globally all the other internal excitations of the quark gluonic field which are far from being elementary. Their total momentum is Q. $\varphi(p,q;Q)$ is the time-independent internal distribution of momenta corresponding to the equilibrium state.

Assuming, as usually, the independence of the strong, weak, and electromagnetic interaction, the $\mathscr{S}$ matrix element of the gauge boson decay in the lowest order of perturbation with respect to the weak and electromagnetic interaction is

$$\langle\pi^0\gamma|\mathscr{S}|Z^0\rangle = \int d^4x d^4y \langle\pi\gamma|T[H^{\text{weak}}(x)H^{\text{em}}(y) \times U_e(+\infty,-\infty)]|Z^0\rangle, \tag{2}$$

where the weak and electromagnetic Hamiltonians involving the fields of interest for our problem are [7]

$$H^{\text{weak}}(x) = \frac{g}{2\,\cos\theta_W}\sum_{i=u,d}\overline{q_i}(x)\gamma^\mu(g_V^i - g_A^i\gamma_5)q_i(x)Z_\mu(x), \tag{3}$$

$$H^{\text{em}}(x) = e\sum_{i=u,d} g_i\overline{q_i}(x)\gamma_\mu q_i A^\mu(x), \tag{4}$$

with $q_u(x), q_d(x)$ the up and down quark fields, eg_i the quark electric charges, $g_V^i = t_{3L}(i) - 2g_i\sin^2\theta_W$, $g_A = t_{3L}(i)$, and t_{3L} is the weak isospin of the quarks.

In Eq. (2), $U_e(t,t')$ is the time evolution operator which, according to our dynamical conjecture, describes the evolution of a quark system perturbed by an external interaction.

Introducing the explicit chronological order, the matrix element (2) becomes

$$\int d^4xd^4y\langle\pi^0\gamma|T[H^{\rm weak}(y)H^{\rm em}(x)U_e(+\infty,-\infty)]|Z^0\rangle$$
$$=\int d^4xd^4ye^{iKx-iky}\varepsilon^{\mu}_{Z^0}\varepsilon^{\nu}_{\gamma}[\langle\pi|U_e(+\infty,y_0)j^{\rm em}_{\nu}(y)$$
$$\times U_e(y_0,x_0)j^{\rm weak}_{\mu}(x)U_e(x_0,-\infty)|0\rangle\theta(y_0-x_0)$$
$$+\langle\pi|U_e(+\infty,x_0)j^{\rm weak}_{\mu}(x)U_e(x_0,y_0)j^{\rm em}_{\nu}(y)$$
$$\times U_e(y_0,-\infty)|0\rangle\theta(x_0-y_0)]. \quad (5)$$

Matrix elements such as (5) have been evaluated by using either a perturbative expansion of the time evolution operator $U(t,t')$ or some nonperturbative methods, such as, for instance, the operator product expansion which considers the modifications produced on the nonlocal product of currents by gluon exchange [8,9]. Instead of this, we suggest preserving the free field form of the currents and looking at the action of the time evolution operator on the quark systems originating from the initial and final states. Then, because of the specific dynamical conjecture of our model, the expression (5) can be essentially simplified in some particular cases. Indeed, observing that the initial state contains no hadrons and that the pion is the single one in the final state, one concludes that there is no external interaction other than those already treated perturbatively which can disturb the initial and final state before and after, respectively, the interaction of the quarks with a gauge boson. It follows therefore that the time evolution operators $U_e(t,-\infty)$ and $U_e(+\infty,t')$ where t and t' are the moments of the first and second interaction with a gauge boson can be replaced by unity.

It remains then to analyze only the effects of the time evolution operator $U_e(t,t')$ on the quark system originating from the first quantum fluctuation involving a gauge boson and transforming into a pion as a result of a second quantum fluctuation implying a gauge boson. In fact, we have to describe the modifications produced by the quark gluon interaction in a free bare $q\bar{q}$ pair in the time interval between two interactions with gauge bosons. Globally, this can be seen like a momentum transfer process from the quark pair to the surrounding quark gluonic field.

Starting from this conjecture and using the analogy with the bremsstrahlung, we shall analyze further the effects of the time translation operator $U_e(t,t')$ and finally conclude that, in the given kinematical situation, it can also be replaced by unity. For transparency of the physical arguments, we shall work in the following with the explicit chronological order of the quantum fluctuations, treating separately the two terms appearing in Eq. (5).

The first term in the right-hand side of Eq. (5) describes the decay of the gauge boson Z^0 into a $q\bar{q}$ pair, followed by the emission of the photon by one of the quarks and the formation of a pion [Fig. 1(a)]. In the rest frame of Z^0, the first process gives rise to a quark and an antiquark moving in opposite directions with equal and very large energies. A "QCD-inspired" argument says that the confinement of color will force them to slow down by transferring a large part of their energy to the surrounding quark-gluonic field. After a while, one of the quarks changes suddenly its direction of motion by emitting a photon and gives rise, together with the other quark, to the final pion.

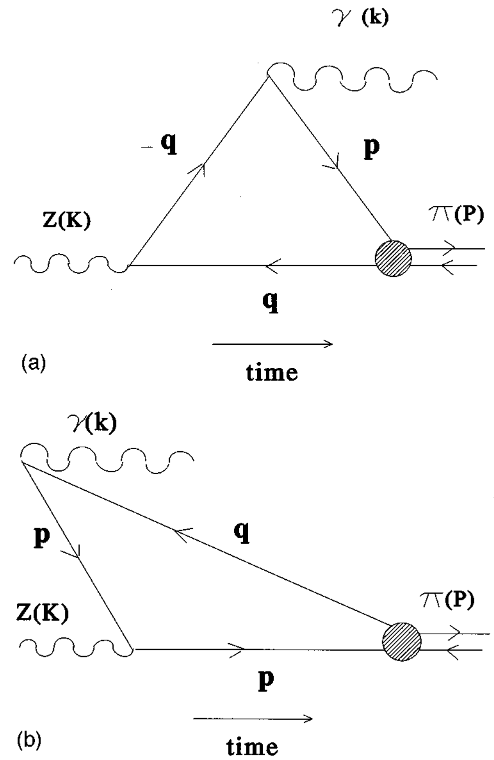

FIG. 1. Quark diagrams for the matrix elements in Eq. (5). The internal lines represent on-mass-shell quarks and are labeled by the quark momenta. The vertices denoting the quark interactions with the gauge fields are drawn in chronological order.

For the calculation of the decay width, it is important to know what happens with the excitations of the quark-gluonic field appearing in the time interval between the creation moment of the $q\bar{q}$ pair and the moment of the photon emission.

Observing that the pion is the single particle in the final state which could take in the excitations of the quark-gluonic field, we conclude that these excitations must be found in the effective component Φ of the pion and their total momentum has to be equal with that carried by Φ. On the other side, since Φ is a component of a pion moving with very large velocity, one can expect for its momentum to be large in the pion direction.

However, in the kinematical situation of interest, because of the symmetry in the quark-movement, the total momentum transferred to the quark-gluonic field, during the slowing down must have a rather small spatial component. One is then forced to conclude that the excitations of the quark gluonic field, created by slowing down the quarks, cannot be absorbed into the final pion. This means that in this case there cannot exist any momentum transfer from the valence quarks to the surrounding quark-gluonic field and hence the quark momenta do not change since the moment of their creation until the emission of the photon. Consequently, the time evolution operator $U_e(y_0,x_0)$, which is supposed to introduce the modifications produced in the initial quark system by the interaction with the quark-gluon surrounding field, can also be replaced by unity in the first term of Eq. (5).

This term can be easily evaluated by introducing the expression of the θ function

$$\theta(x_0)=\frac{1}{2\pi i}\int d\tau\frac{e^{i\tau x_0}}{\tau-i\epsilon},$$

the plane wave decomposition of the quark fields

$$\psi(x)=\frac{1}{(2\pi)^3}\int d^3p\frac{m}{e}[e^{-ip\cdot x}a(\mathbf{p})u(\mathbf{p})+e^{ip\cdot x}b^+(\mathbf{p})v(\mathbf{p})] \tag{6}$$

and the free field canonical commutation relations

$$[a_i^+(\mathbf{p}),a_j(\mathbf{p}')]_+=[b_i^+(\mathbf{p}),b_j(\mathbf{p}')]_+ =(2\pi)^3\frac{e(\mathbf{p})}{m}\delta_{ij}\delta^{(3)}(p-p'). \tag{7}$$

A straightforward calculation then gives

$$\begin{aligned}\mathscr{S}^{(1)}(Z^0\to\pi^0\gamma)=&-i(2\pi)^4\delta^{(3)}(K-P-k)\frac{e^2}{\sqrt{2}}\varepsilon^{\mu}_{Z^0}\varepsilon^{\nu}_{(\gamma)}\sum_{j=u,d}\int d^3p\frac{m_j}{e(\mathbf{p})}d^3q\frac{m_j}{e(\mathbf{q})}d^4Q\,\delta^{(3)}(\mathbf{p}+\mathbf{q}+\mathbf{Q}-\mathbf{P})\\&\times\delta(M_{Z^0}-e(\mathbf{p})-e(\mathbf{q})-k_0)3\varphi(p,q;Q)\langle 0|\Phi(Q)|0\rangle\Bigg[\mathrm{Tr}\left(\gamma_5\frac{(\hat{p}+m_j)}{2m_j}\gamma_\nu\frac{e(\mathbf{q})\gamma_0+\gamma\cdot\mathbf{q}+m_j}{2m_j}\gamma_\mu\frac{(\hat{q}-m_j)}{2m_j}\right)\\&\times\frac{m_j}{e(\mathbf{q})}\frac{1}{2e(\mathbf{q})-M_{Z^0}}-\mathrm{Tr}\left(\gamma_5\frac{(\hat{p}+m_j)}{2m_j}\gamma_\mu\frac{e(\mathbf{p})\gamma_0+\gamma\cdot\mathbf{p}-m_j}{2m_j}\gamma_\nu\frac{(\hat{q}-m_j)}{2m_j}\right)\frac{m_j}{e(\mathbf{p})}\frac{1}{2e(\mathbf{p})-M_{Z^0}}\Bigg],\end{aligned} \tag{8}$$

where $\hat{k}=e(\mathbf{k})\gamma^0-\gamma\cdot\mathbf{k}$,

$$S_u=-\frac{2}{3}\left(\frac{1}{4}-\frac{2}{3}\sin^2\theta_W\right)/(\sin\theta_W\cos\theta_W),$$

$$S_d=\frac{1}{3}\left(\frac{1}{4}-\frac{1}{3}\sin^2\theta_W\right)/(\sin\theta_W\cos\theta_W).$$

Using a relation obtained in the case of the leptonic decay of the pion [5],

$$3\varphi(p,q;Q)\langle 0|\Phi(Q)|0\rangle=i\frac{F_{\pi^0}M_\pi}{\pi(p_um_u+p_dm_d)}\delta^{(4)}(Q), \tag{9}$$

where F_{π^0} is the pion decay constant and $p_j=\frac{1}{2}M_\pi\sqrt{1-4m_j^2/M_\pi^2}$, performing the trace and the integration over the quark momenta, and introducing a covariant notation, one finally has

$$\mathscr{S}^{(1)}(Z^0\to\pi^0\gamma)=-i(2\pi)^4\delta^{(4)}(K-P-k)\times\varepsilon^{\mu}_{(Z^0)}\varepsilon^{\nu}_{(\gamma)}\epsilon_{\mu\nu\alpha\beta}P^\alpha K^\beta\mathscr{T}^{(1)}(Z^0\to\pi^0\gamma), \tag{10}$$

where

$$\mathscr{T}^{(1)}(Z^0\to\pi^0\gamma)=\frac{16\pi\alpha\sqrt{2}F_{\pi^0}M_\pi}{(m_up_u+m_dp_d)M_Z^2}\times\sum_{j=u,d}S_jm_j\ln\frac{M_\pi+2p_j}{2m_j} \tag{11}$$

with the fine structure constant $\alpha\approx 1/137$.

We pass now to the second term in Eq. (5). The first fluctuation here is the creation from the vacuum of a photon and of a $q\bar{q}$ pair. The second fluctuation is the absorption of the gauge boson Z^0 at rest by one of the quarks [Fig. 1(b)]. Noticing that in this case the amplitude of Z^0 absorption is proportional with $\varepsilon^\mu\overline{u(v)}(\mathbf{p})\gamma_\mu u(v)(\mathbf{p})=\varepsilon^\mu\cdot p_\mu\overline{u(v)}(\mathbf{p})u(v)(\mathbf{p})$, it follows that a pseudoscalar, such as that in Eq. (10), required by CP invariance, must be formed from the polarization vector of the photon and *three* momenta. This leads to a vanishing final result since there are only *two* independent external momenta. Therefore the amplitude (11) is the total amplitude of the decay process and the decay width is

$$\Gamma(Z^0\to\pi^0\gamma)=\frac{32\pi\alpha^2F_{\pi^0}^2M_\pi^2}{M_{Z^0}(m_up_u+m_dp_d)^2}\left(\sum_{j=u,d}S_jm_j\ln\frac{M_\pi+2p_j}{2m_j}\right)^2. \tag{12}$$

For a comparison of the decay width (12) with the experimental data and other theoretical results, we represent it in function of m_d with m_u parameter in the domain of current quark masses [7], 2 MeV$\leqslant m_u\leqslant$8 MeV, 5 MeV$\leqslant m_d\leqslant$15 MeV, for $F_{\pi^0}\approx$93 MeV, $\sin^2\theta_W\approx$0.23 (Fig. 2).

One sees in Fig. 2 that the partial decay width obtained in our model is at least four orders of magnitude under the experimental upper bound which is 1.37×10^{-2} MeV. Our results are of the same order of magnitude with other theoretical estimations [1,2,9,10], giving a partial width of about 10^{-8} MeV. Comparing their methods with ours, it can be seen that the difference comes mainly from the treatment of the $\pi q\bar{q}$ coupling. Indeed, the introduction of the $q\bar{q}$ pair by means of an effective $\pi q\bar{q}$ interaction Lagrangian leads through Wick's theorem to the Lorentz-invariant quark tri-

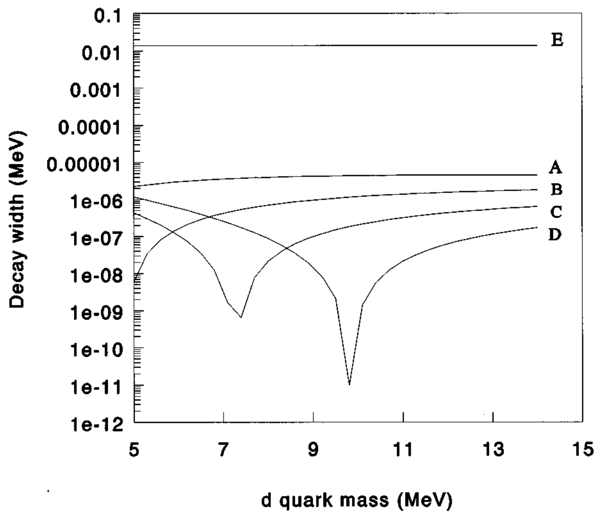

FIG. 2. The decay width (12) represented as function of the d quark mass with m_u-like parameter. The graphs A,B,C,D correspond to $m_u=2,4,6,8$ MeV, respectively. The graph E represents the experimental upper bound.

angle diagram with propagators for the internal quark lines. One must recall, however, that this diagram is nothing else but a sum of six quark diagrams corresponding to the six different chronological arrangements of the vertices, where the internal quark lines represent projectors on the positive or negative energy states. Remarking now that the quark diagrams in Fig. 1 are just two of these six diagrams, one concludes that our model rejects the quark diagrams where the quark and/or the antiquark in the pion coexists with the pion itself. This is explicitly the result of the definition (1) of the pion state.

The remarkable fact that various approaches, such as operator product expansion [9], constituent quark models [1,2], current algebra, and axial anomaly [10] give small values of the branching ratio can be explained by the overall presence of the factor $1/M_Z^2$ in the decay amplitude coming from a far-off shell quark. In our model this factor is because of a quantum fluctuation whose energy is far under the energy of the initial state [Eq. (8)], which is mainly the same thing as above.

One could finally ask if it is possible to immediately transpose the result (12) to the $W^{\pm}\rightarrow\pi^{\pm}\gamma$ and $Z^0\rightarrow W^{\pm}\pi^{\mp}$ decays. The answer is negative, because in these cases one has additional contributions from a parity-violating term in the decay amplitude, analogous of the axial form factor in $\pi^{\pm}\rightarrow l^{\pm}\nu\gamma$ decay. The first term in Eq. (5) can be calculated by taking into account the additional contributions and by following the same arguments as above but the second term cannot be so easily rejected because the argument used in $Z^0\rightarrow\pi^0\gamma$ does not hold in these cases.

The evaluation of the $W^{\pm}\rightarrow\pi^{\pm}$ and $Z^0\rightarrow W\pi$ decay widths would require more detailed information about the internal wave function of the pion and the mechanism of momentum transfer between the bare quarks and the surrounding quark-gluonic field.

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