Solar neutrino problem within the left-right model

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Limiting ourselves to the two flavor approximation within the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge model the motion of the neutrino flux in solar matter is considered. For the neutrino system described by the eight-component wave function $\Psi^T=(\nu_{eL},\nu_{XL},N_{eR},N_{XR},\overline{\nu}_{eR},\overline{\nu}_{XL},\overline{N}_{eR},\overline{N}_{XR})$, where $X=\mu,\tau$, an evolution equation in a Schrödinger-like form is found. It is shown that the number of possible resonance transitions for the flux of solar ν_{eL} is defined both by the mass hierarchy within the system $\nu_{eL},\nu_{XL},N_{eR},N_{XL}$, and by the choice of mixing scheme. Factors influencing the neutrino flux crossing a region of solar flares are defined.

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I. INTRODUCTION

Two of the most fundamental unanswered questions in particle physics, cosmology, and astrophysics concern the issues of whether neutrinos have nonzero masses and a Major-
ana or Dirac nature. The existence of a neutrino rest mass leads to the picture of oscillations which can be tested by two independent kinds of experiments: (1) experiments with the solar neutrino; (2) experiments with so-called terrestrial neutrino (neutrino, born in the atmosphere, in accelerators and nuclear reactors). Notwithstanding the negative result for accelerator and nuclear neutrino an oscillation scheme can be considered as the possible solution of solar and atmospheric neutrino puzzle. A flux of solar ν_{eL} measured in four various experiments (Hometstake, Kamiokande, SAGE, and GALLEX) turns out to be greatly suppressed in comparison with predictions of existing solar models. Thus, for example, the Homestake experiment, which uses for neutrino detection the reaction

$$\nu_{eL}+{}^{37}\mathrm{Cl}\rightarrow{}^{37}\mathrm{Ar}+e^-$$

gives for ν_{eL} capture rate the result

$$2.1\pm0.3 \ \text{solar neutrino units (SNU)} \ \ (1 \ \ \mathrm{SNU}=10^{-36} \ \text{capture/atom s}).$$

Though theoretical predictions are

$$7.9 \ \ \mathrm{SNU}$$

for the Bahcall *et al.* model (standard solar model) [1] and

$$6.4 \ \ \mathrm{SNU}$$

for the Turck-Chieze model [2]. Let us note, that there are two approaches which predict neutrino oscillations. First, the Mikheyev-Smirnov-Wolfenstein (MSW) [3] mechanism is based on the resonant angle enhancement in matter. Second, motivated by observations in the Homestake experiment of anti-correlations between neutrino flux and solar activity was suggested in Ref. [4] by Voloshin, Vysotsky, and Okun (VVO). In this approach the neutrino, having as large a magnetic moment as $\sim10^{-11}\mu_B$ (μ_B-Bohr magneton) while moving in the solar magnetic field undergoes both flavor oscillations and spin precession. Thus only a combination of MSW and VVO effects gives a whole picture while studying various oscillation schemes (for review see [5]).

Analogously, a theoretical ratio of atmospheric fluxes of muon and electron neutrinos without involving an oscillation picture contradicts those measured in Kamiokande ($\eta=R^{\mathrm{expt}}_{\mu/e}/R^{\mathrm{theor}}_{\mu/e}\approx0.6$) [6], IMB ($\eta\approx0.54$) [7], and Soudan II ($\eta\approx0.69$) [8] experiments. As the problem both of solar and atmospheric neutrinos finds its solution within one and the same hypothesis, one should expect that their oscillation parameters regions (squared mass difference δm^2 and mixing angle in vacuum θ) should coincide. However, if one is to operate with the standard model (SM) of electroweak interaction, in which due to any mechanism (an extension of either fermion or Higgs sectors, radiative corrections, etc.), neutrino acquired its mass, then in the two flavor basis it does not take place. Thus, for example, the usage of MSW mechanism gives

$$\delta m^2\sim6(9)\times10^{-6} \ \ \mathrm{eV}^2,$$

$$\sin^2 2\theta\sim7\times10^{-3} \ \ (0.6)$$

for the solar neutrino in adiabatic (nonadiabatic) case and

$$\delta m^2\approx10^{-3}-10^{-2} \ \ \mathrm{eV}^2,$$

$$\sin^2 2\theta\approx0.5$$

for the atmospheric neutrino. The solution of this problem demands either use of a three flavor approximation, or exit beyond the SM.

As up to now the most part of neutrino physics aspects was discussed within the SM, we take a natural interest in what really new information an exit beyond the SM can give. The aim of this work is to consider the solar neutrino problem within the left-right model (LRM) based on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group [9]. The oscillation picture in this model considerably differs from the SM scenario. New resonance transitions appear, probability expressions are changed, and places of resonance localization are shifted. It is obvious that existing experiments for solar neutrino observations due to their low statistics cannot choose between different models of electroweak interactions. However, it is possible to be sure that this task will be solved in the next generation of solar neutrino experiments, where statistics of neutrino events will increase on two orders of magnitude. Thus, for example, for Superkamiokande 23 events per day is planned, HELLAZ 5×10^3 events per year, BOREXINO 50 events per day, and ICARUS 3×10^3 events per year.

In the second chapter we give the necessary information about the Sun structure. There we obtain an evolution equation for a neutrino system both for Dirac and Majorana neutrinos. In the third chapter we investigate the possible resonance transitions for a solar left-handed electron neutrino flux. The fourth chapter will be dedicated to the study of a neutrino flux motion through a solar flare (SF) region. In the fifth section we discuss the obtained results.

II. OSCILLATIONS SCHEMES

Let us consider an evolution of the neutrino system in two flavor approximation. In doing this we shall take into consideration effects of neutrino interaction both with the Sun matter and with its magnetic field. Electron neutrinos are mostly born in the central core of the Sun. High energetic neutrinos ^{7}Be and ^8B are produced in the hottest part of the core, where magnetic field should not exceed the value $B_c = 0.5\times10^8$ G [10] (at $B > B_c$ due to a buoyancy effect this magnetic field would have been lost by the Sun during its time of existence). In the radiative zone ($0.1 < r/R_\odot < 0.7$) the magnetic field value can be as strong as 10^4–10^5 G. Both in the center core and in the radiative zone the field does not display the time dependence. In the convective zone ($0.7 < r/R_\odot < 1$) the magnetic field module has an 11.2-yr cycle. During the years of the active Sun, in the bottom of the convective zone in the region of 10^3 km the field has a value of 10^5 G. With the increasing of r a field decreases and its value on the surface totally depends on the existence on the surface of the so-called active region (AR). AR first appears as a developing magnetic field, preferably within or close to an old expanding magnetic region whose field has fallen to about 1 G or less. Its characteristic measures on the surface is $R_\odot$ in diameter, and its height reaches corona level. The number of these regions and their location on the disk changes within the solar cycle. In the period of the highest maximum (1957–58), the activity involved practically the whole solar disk. In those places of AR where the field value reaches 500 G the process of sunspot formation begins. The field strength of a developed sunspot is maximal in the center (B_1) and directed up the solar radius, near periphery it decreases (B_2), and force lines are more strongly inclined to the surface. Fields $B_1 \sim 5\times10^3$ G and $B_2 \sim 2\times10^3$ G are characteristic for big spots ($d \sim 2\times10^5$ km). According to modern view points, the geometrical depth of a spot is approximately 300 km. Magnetic field above a spot slowly decreases with the growing of height. Thus, for example, it may reach the values of 10^3 G in upper levels of the chromosphere, while beyond a spot region a field value is only 1 G. Usually sunspots form a group. Such a group exists for about a month, and most big spots live up to one hundred days.

A field in a convective zone is characterized by geometrical phase, $\Phi(z)$, defined by

$$B_x \pm iB_y = B_\perp e^{i\Phi(z)}$$

and its first derivative $\dot{\Phi}(z)$ (we have chosen a coordinate system with the z axis along the solar radius). Nonzero values $\Phi(z)$ and $\dot{\Phi}(z)$ also exist both in photosphere and chromosphere in regions above sunspots. A magnetic field above and under a spot has the nonpotential character [11]

$$(\mathrm{rot}\vec{B})_z = 4\pi j_z \neq 0.$$

The data concerning centimeter radiation above a spot testify of a gas heating up to the temperatures of a coronal order. Thus, for example, at the height $\sim 2\times10^2$ km the temperature reaches the values of the order of 10^6 k, which results in a great value of solar plasma conductivity ($\sigma \sim T^{3/2}$). That allows us to suppose that the density of longitudinal electric current might be large enough in a region above a spot.

Now we may study an evolution of solar neutrinos for various oscillation schemes. At first we shall consider a situation where only mixing angles between neutrinos having the same helicity but belonging to different generations are not equal to zero (OS1): i.e.,

$$\nu_{eL} = \nu_1 \cos\theta_\nu + \nu_2 \sin\theta_\nu, \quad \nu_{XL} = -\nu_1 \sin\theta_\nu + \nu_2 \cos\theta_\nu,$$

$$N_{eR} = \nu_3 \cos\theta_N + \nu_4 \sin\theta_N, \quad N_{XR} = -\nu_3 \sin\theta_N + \nu_4 \cos\theta_N,$$

takes place, where ν_i ($i = 1,2,3,4$) are mass eigenstates, θ_ν and θ_N are mixing angles in vacuum, and $X = \mu, \tau$. As we are limited only by two generations, we should consider a neutrino system consisting of $\nu_{eL}, \nu_{XL}, N_{eR}, N_{XR}$ and their antiparticles $(\nu_{eL})^c, (\nu_{XL})^c, (N_{eR})^c, (N_{XR})^c$, where c means an operation of charge conjugation. We stress that Majorana neutrino is also not a charge conjugation operator eigenstate due to a switching on of weak interaction. As $(\nu_L)^c$ and $(N_R)^c$ are right- and left-handed ν and N neutrinos, respectively, further on we shall use for them both in Majorana and Dirac cases following the notions, $\overline{\nu_L}$ and $\overline{N_R}$. To obtain an evolution equation in a Schrödinger-like form let us make standard assumptions: (1) the solar matter is electroneutral; (2) the speed of solar matter particles is negligibly small. Then a required equation for the Majorana neutrino case will be

$$i\frac{d}{dz}\Psi = \mathscr{H}\Psi, \tag{2.1}$$

where $\Psi^T = (\nu_{eL}, \nu_{XL}, N_{eR}, N_{XR}, \overline{\nu_{eL}}, \overline{\nu_{XL}}, \overline{N_{eR}}, \overline{N_{XR}})$,

$$\mathcal{H}=\begin{pmatrix} \mathcal{H}_{pp} & \mathcal{H}_{pa} \\ \mathcal{H}^{\dagger}_{pa} & \mathcal{H}_{aa} \end{pmatrix}, \quad \mathcal{H}_{pp}=\begin{pmatrix} \mathcal{H}_{\nu\nu} & \mathcal{H}_{\nu N}B_{\perp}e^{i\Phi} \\ \mathcal{H}^{\dagger}_{\nu N}B_{\perp}e^{-i\Phi} & \mathcal{H}_{NN} \end{pmatrix},$$

$$\mathcal{H}_{\nu\nu}=\begin{pmatrix} \delta_c^{12}+V_{eL}+4\pi a_{\nu_e\nu_e}j_z+\Delta & -\delta_s^{12}+4\pi a_{\nu_e\nu_X}j_z \\ -\delta_s^{12}+4\pi a_{\nu_X\nu_e}j_z & -\delta_c^{12}+V_{XL}+4\pi a_{\nu_X\nu_X}j_z+\Delta \end{pmatrix},$$

$$\mathcal{H}_{NN}=\mathcal{H}_{\nu\nu}(\{m_1,m_2,L,j_z\}\rightarrow\{m_3,m_4,R,-j_z\}),$$

$$\mathcal{H}_{\nu N}=\begin{pmatrix} \mu_{\nu_e N_e} & \mu_{\nu_e N_X} \\ \mu_{\nu_X N_e} & \mu_{\nu_X N_X} \end{pmatrix}, \quad \delta^{12}_{c(s)}=\frac{m_1^2-m_2^2}{4E}\cos 2\theta_\nu(\sin 2\theta_\nu),$$

$$\delta^{34}_{c(s)}=\frac{m_3^2-m_4^2}{4E}\cos 2\theta_N(\sin 2\theta_N), \quad \Delta=\frac{m_1^2+m_2^2-m_3^2-m_4^2}{8E},$$

$$V_{eL}=\sqrt{2}G_F n_e+V^{\rm NC}_{eL}, \quad V_{eR}=\frac{n_e g_R^2}{4}\left(\frac{\sin^2\xi}{m^2_{W_1}}+\frac{\cos^2\xi}{m^2_{W_2}}\right)+V^{\rm NC}_{eR},$$

$$V_{XL,R}=V^{\rm NC}_{eL,R}=n_n\sum_{i=1}^{2}\frac{g_i^{\nu_{L,R}}g_i^n}{4m^2_{Z_i}},$$

$$g_1^f=ec_W^{-1}[s_W^{-1}c_\phi(T^f_{3L}-2Q_f s_W^2)+s_\phi\alpha^{-1}(T^f_{3L}+T^f_{3R}c_W^2e^{-2}g_R^2-2Q_f)],$$

$$g_2^f=g_1^f\left(\phi\rightarrow\phi+\frac{\pi}{2}\right),$$

$$c_W=\cos\theta_W, \quad s_W=\sin\theta_W, \quad c_\phi=\cos\phi, \quad s_\phi=\sin\phi, \quad \alpha=\sqrt{c_W^2e^{-2}g_R^2-1},$$

$$\mathcal{H}_{pa}=\begin{pmatrix} M_{\nu\nu}B_{\perp}e^{i\Phi} & 4\pi A_{\nu N}j_z \\ -4\pi A_{N\nu}j_z & M_{NN}B_{\perp}e^{i\Phi} \end{pmatrix},$$

$$M_{\nu\nu}=\begin{pmatrix} 0 & \mu_{\nu_e\overline{\nu_X}} \\ -\mu_{\nu_e\overline{\nu_X}} & 0 \end{pmatrix}, \quad M_{NN}=M_{\nu\nu}(\{\nu_e,\nu_X\}\rightarrow\{N_e,N_X\}),$$

$$A_{\nu N}=\begin{pmatrix} a_{\nu_e\overline{N_e}} & a_{\nu_e\overline{N_X}} \\ a_{\nu_X\overline{N_e}} & a_{\nu_X\overline{N_X}} \end{pmatrix}, \quad A_{N\nu}=A_{\nu N}(\nu_l\leftrightarrow N_l),$$

$$\mathcal{H}_{aa}=\mathcal{H}_{pp}(\{V_{lL,R},j_z\}\rightarrow\{-V_{lL,R},-j_z\}), \quad l=e,X.$$

μ_{ik} and a_{ik} are the magnetic and anapole moments between i- and k-neutrino states, and n_e and n_n are electron and neutron densities, respectively. Hereafter we shall use notations of Ref. [12]. For the sake of simplicity we consider all the particles to be ultrarelativistic. As we have for solar neutrinos energies,

$$0.14<E<14 \ \text{MeV},$$

this in its turn limits masses m_{N_l} to the keV range. Generalization in case of $m_{N_l}\approx$MeV does not present us with any kind of principal difficulties, but greatly complicates an algebraic essence of the problem.

For Dirac neutrinos we should take a matrix $M_{\nu\nu}$ in the form

$$M_{\nu\nu}=\begin{pmatrix} \mu_{\nu_e\overline{\nu_e}} & \mu_{\nu_e\overline{\nu_X}} \\ \mu_{\nu_e\overline{\nu_X}} & \mu_{\nu_X\overline{\nu_X}} \end{pmatrix}$$

and assume $V_{lL,R}$ equal to zero in the expression for $\mathcal{H}_{aa}$.

To further analyze the equation (2.1) one should get rid of an imaginary part in a Hamiltonian. That can be achieved by transformation into reference frame (RF), rotating at the same angle speed as a magnetic field. A matrix of transition to the new RF has a form

$$S=\mathrm{diag}(\lambda,\lambda,\lambda^{-1},\lambda^{-1},\lambda^{-1},\lambda^{-1},\lambda,\lambda),$$

where $\lambda=\exp(i\Phi/2)$.

A Hamiltonian in a rotating RF follows from the initial one by a replacement

$$e^{\pm i\Phi}\rightarrow 1,\quad V_{lL,R}\rightarrow V_{lL,R}\mp\frac{\dot{\Phi}}{2}.$$

Let us go to the analysis of the terms constituting the Hamiltonian. Magnetic dipole moments entering a whole Hamiltonian can be divided in two classes. First let us discuss them on the example of moments, induced by charged gauge bosons exchange $W^{\pm}_{L,R}$. Let us notice that these moments do not depend on the assumed neutrino nature. The first class appears from diagrams with one photon and two neutrino external lines with the helicity flipping on an external neutrino line. Its value is defined by the expression [13]

$$\mu^{W}_{\nu_i\overline{\nu_j}}=-\frac{3e(m_{\nu_i}+m_{\nu_j})}{256\pi^2}\sum_{\alpha=1}^{3}m^2_{l_\alpha}\left[g_L^2\left(\frac{\cos^2\xi}{m^4_{W_1}}+\frac{\sin^2\xi}{m^4_{W_2}}\right)\times\mathscr{U}^{\dagger}_{j\alpha}\mathscr{U}_{\alpha i}+g_R^2\left(\frac{\sin^2\xi}{m^4_{W_1}}+\frac{\cos^2\xi}{m^4_{W_2}}\right)V^{\dagger}_{j\alpha}V_{\alpha i}\right],\qquad(2.2)$$

where $m_{\nu_\alpha}=\mathscr{U}_{\alpha i}m_{\nu_i}$ and $m_{N_\alpha}=V_{\alpha i}m_{N_i}$.

The second class is produced by diagrams with the helicity flipping on the internal lepton line. The magnetic moment of this class is much bigger than in the first case and its expression is [13]

$$\mu^{W}_{\nu_i N_j}=\frac{eg_Lg_R}{32\pi^2}\sin2\xi\sum_{\alpha=1}^{3}m_{l_\alpha}(\mathscr{U}^{\dagger}_{j\alpha}V_{\alpha i}+V^{\dagger}_{j\alpha}U_{\alpha i})\frac{\times\left(m^2_{W_2}-m^2_{W_1}\right)}{m^2_{W_1}m^2_{W_2}}.\qquad(2.3)$$

Now let us discuss moments induced by the exchange of the physical Higgs bosons. In the LRM there are two possible choices of Higgs multiplets [in brackets quantum numbers $T_L,T_R,(B-L)/2$ are given]: (a) bidoublet $\Phi(\frac{1}{2},\frac{1}{2},0)$ and two triplets $\Delta_L(1,0,1),\Delta_R(0,1,1)$; (b) bidoublet $\Phi(\frac{1}{2},\frac{1}{2},0)$ and two doublets $\mathscr{X}_L(\frac{1}{2},0,-\frac{1}{2})$, $\mathscr{X}_R(0,\frac{1}{2},-\frac{1}{2})$. At (a) neutrinos are Majorana particles and at (b) they are Dirac ones. First we consider the Majorana case. A convenient representation of the multiplets is given by the 2×2 matrices

$$\Phi=\begin{pmatrix}\Phi_1^0 & \Phi_1^+\\ \Phi_2^- & \Phi_2^0\end{pmatrix},\quad \Delta_{L,R}=\begin{pmatrix}\delta^+_{L,R}/\sqrt{2} & -\delta^{++}_{L,R}\\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2}\end{pmatrix}.$$

The most general Yukawa couplings involving the leptons is given by

$$-L_Y=\sum_{a,b}\{h_{ab}\overline{\Psi}_{aL}\Phi\Psi_{bR}+\overline{h_{ab}}\overline{\Psi}_{aL}\overline{\Phi}\Psi_{bR}+if_{ab}[\Psi^T_{aL}C\tau_2(\vec{\tau}\cdot\vec{\Delta}_L)\Psi_{bL}+(L\rightarrow R)]+\mathrm{conj}\},\qquad(2.4)$$

where $\tau_{1,2,3}$ are Pauli matrices, $\overline{\Phi}=\tau_2\Phi^*\tau_2$, and $a,\underline{b}=e,\mu,\tau$. Next for the sake of simplicity we shall set $h\ \ _{ab}=h_{ab}$ and ignore mixing between generations. After spontaneous symmetry violation

$$\langle\Phi\rangle=\begin{pmatrix}k & 0\\ 0 & k'\end{pmatrix},\quad\langle\Delta_L\rangle=\begin{pmatrix}0\\0\\v_L\end{pmatrix},\quad\langle\Delta_R\rangle=\begin{pmatrix}0\\0\\v_R\end{pmatrix},$$

where $v_R\gg\max(k,k')\gg v_L$, we are left with four doubly charged, four singly charged, and six neutral physical Higgs bosons. Among them only singly charged ones $\tilde{\delta}^{\pm}_L$ and $H^{\pm}$ can contribute to the multipole moments (MM). These Higgs bosons correspond to the quanta of the fields [14]

$$\tilde{\delta}^{\pm}_L=\frac{\delta^{\pm}_L+\frac{\sqrt{2}v_L}{k}\Phi_2^{\pm}}{\sqrt{1+\frac{2v_L^2}{k^2}}},\quad H^{\pm}=\frac{\Phi_1^{\pm}+\frac{k}{\sqrt{2}v_R}\delta^{\pm}_R}{\sqrt{1+\frac{k^2}{2v_R^2}}}.$$

Neglecting the terms which are proportional to k/v_R and v_L we obtain the following Lagrangians describing the interaction between leptons and singly charged Higgs bosons

$$L_{\tilde{\delta}_L}=g_{\tilde{\delta}_L}(\overline{\nu^c_{lL}}l_L+\overline{l^c_L}\nu_{lL})\tilde{\delta}^+_L+\mathrm{conj},\qquad(2.5)$$

$$L_H=g_H(\overline{\nu_{lL}}l_R-\overline{N_{lR}}l_L)H^++\mathrm{conj},\qquad(2.6)$$

where $g_H\sim g_Lm_l/\sqrt{2}m_{W_1}$ and $g_{\tilde{\delta}_L}\sim g_Lm_{N_l}/m_{W_2}$ [15].

Diagrams with $\tilde{\delta}^{\pm}_L$ exchange give magnetic moments corresponding to the first class and their value is defined by a relation

$$\mu^{\tilde{\delta}_L}_{\nu_i\overline{\nu_j}}\sim\sum_{\alpha}\frac{eg_L^2m^2_{N_e}(m_{\nu_i}+m_{\nu_j})m_{\nu_i}m_{\nu_j}}{\sqrt{2}\pi^2m^2_{\tilde{\delta}_L}m^2_{W_2}}\mathscr{U}^{\dagger}_{i\alpha}\mathscr{U}_{j\alpha}.\qquad(2.7)$$

Diagrams with $H^{\pm}$ in an intermediate state lead to magnetic moments of the second class

$$\mu^{H}_{\nu_iN_j}\sim\frac{eG_F}{2\sqrt{2}\pi^2m_H^2}\sum_{\alpha=1}^{3}m^3_{l_\alpha}(\mathscr{U}^{\dagger}_{j\alpha}V_{\alpha i}+V^{\dagger}_{j\alpha}\mathscr{U}_{\alpha i}).\qquad(2.8)$$

For Dirac neutrino the contribution to a magnetic dipole moment will produce only diagrams with an exchange of the $H^{\pm}$ boson.

Having taken values $\xi \approx 3\times 10^{-2}$, $g_L = g_R$, $m_H = 80$ GeV, and $m_{\tilde{\delta}_L} = 40$ GeV and assuming that an electron neutrino is mixed with a τ-lepton one, we obtain

$$\mu^{W}_{\nu_i \overline{\nu_j}} \sim 10^{-22}\left(\frac{m_{\nu_i}+m_{\nu_j}}{\mathrm{eV}}\right)\mu_B,$$

$$\mu^{W}_{N_i \overline{N_j}} \sim 10^{-22}\left(\frac{g_R m_{W_1}}{g_L m_{W_2}}\right)^2\left(\frac{m_{N_i}+m_{N_j}}{\mathrm{eV}}\right)\mu_B,$$

$$\mu^{W}_{\nu_i \overline{\nu_j}} \gg \mu^{\tilde{\delta}_L}_{\nu_i \overline{\nu_j}}, \quad \mu^{W}_{\nu_i N_j} \sim 10^{-11}\mu_B, \quad \mu^{H}_{\nu_i N_j} \sim 3.7\times 10^{-13}\mu_B. \tag{2.9}$$

Thus we see that diagrams with $W^{\pm}_{L,R}$ exchange make the main contribution to magnetic moments. At the introduction of a singly charged singlet $S_1(0,0,-2)$ or doubly charged one $S_2(0,0,4)$, values $\mu_{\nu_i(N_i)\overline{\nu_j(N_j)}}$ could also have an order of $10^{-11}\mu_B$. Remembering this we shall not ignore them in our further consideration. Let us note that an account of influence of polarization of dispersive solar plasma results in a value of an induced magnetic moment, which proves to be comparable and even substantially larger than a vacuum one [16].

Let us recall existing on a vacuum level problem of inevitable connection of a big magnetic moment with an unacceptably large neutrino mass. As diagrams defining a magnetic moment follow from diagrams of self-neutrino energy by means of the addition of an external photon line, there is a connection

$$\mu_{\nu_e} \approx \frac{2 m_e m_{\nu_e}}{M^2}\mu_B,$$

where M is a typical mass in a loop. Then for $M \approx 100$ GeV and $\mu_{\nu_e} \sim 10^{-11}\mu_B$ we have $m_{\nu_e} \sim 1$ keV. The mass suppression down to phenomenologically acceptable values (~ 1 eV) is satisfied by introducing either additional gauge degrees of freedom [17] or discrete horizontal symmetry between lepton generations [18].

In the present work we shall not do any calculations of anapole moment (AM) and limit ourselves only to display of results for the SM. At neutrino mass neglecting AM is connected with a radius of an electric charge (REC) by a relation

$$a^{l}_{L,R} = \frac{1}{6}\langle r^2\rangle^{l}_{L,R}. \tag{2.10}$$

Let us stress that this connection exists only for nondiagonal AM moments. REC of neutrino was calculated in Ref. [19]. Since an analytical expression for it is rather cumbersome, we shall limit ourselves only to numerical values of REC for all the three kinds of neutrino found at $m_t = 180$ GeV, $m_H = 100$ GeV, and $m_{W_1} = 80.2$ GeV. They have the form

$$\langle r^2\rangle^{e}_{L} = (49.8\pm 3.6)\times 10^{-34}\ \mathrm{cm}^2,$$

$$\langle r^2\rangle^{\mu}_{L} = (82.4\pm 3.6)\times 10^{-34}\ \mathrm{cm}^2,$$

$$\langle r^2\rangle^{\tau}_{L} = (99.6\pm 3.6)\times 10^{-34}\ \mathrm{cm}^2. \tag{2.11}$$

Induced AM has nonzero value only for anisotropic media. Magnetized plasma above a sunspot could serve such an example.

It is also instructive to consider the example of LRM in which due to certain reasons multipole moments of neutrino are negligibly small. Let us assume that in this case the following mixing scheme (OS2) takes place:

$$\begin{pmatrix} \nu_{lL} \\ \nu_{XL} \\ N_{eR} \\ N_{XR} \end{pmatrix} = (U^g)^{-1}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix},$$

where

$$(U^g)^{-1} = \begin{pmatrix} c_{\theta_\nu} & s_{\theta_\nu} & 0 & 0 \\ -s_{\theta_\nu} & c_{\theta_\nu} & 0 & \\ 0 & 0 & c_{\theta_N} & s_{\theta_N} \\ 0 & 0 & -s_{\theta_N} & c_{\theta_N} \end{pmatrix}\begin{pmatrix} c_{\varphi_e} & 0 & s_{\varphi_e} & 0 \\ 0 & c_{\varphi_X} & 0 & s_{\varphi_X} \\ -s_{\varphi_e} & 0 & c_{\varphi_e} & 0 \\ 0 & -s_{\varphi_X} & 0 & c_{\varphi_X} \end{pmatrix},$$

$c_{\theta_\nu} = \cos\theta_\nu$, $s_{\theta_\nu} = \sin\theta_\nu$, etc.

As a result, an evolution equation in such a scheme decouples into two independent equations: one is for neutrino, the other is for antineutrino systems. By means of standard method for neutrino flux, described by wave function $\Psi^T = (\nu_{eL}, \nu_{XL}, N_{eR}, N_{XR})$ we obtain the following equation for both Dirac and Majorana neutrinos:

$$i\frac{d}{dz}\Psi = \begin{pmatrix} \mathcal{H}_{\nu\nu} & \mathcal{H}_{\nu N} \\ \mathcal{H}^{\dagger}_{\nu N} & \mathcal{H}_{NN} \end{pmatrix}\Psi, \tag{2.12}$$

where

$$\mathscr{H}_{\nu\nu}=\begin{pmatrix} \delta_c^{12}c_{\varphi_e}^2+\delta_c^{34}s_{\varphi_e}^2+\Delta c_{2\varphi_e}+V_{eL} & \delta_s^{12}c_{\varphi_e}c_{\varphi_X}+\delta_s^{34}s_{\varphi_e}s_{\varphi_X} \\ \delta_s^{12}c_{\varphi_e}c_{\varphi_X}+\delta_s^{34}s_{\varphi_e}s_{\varphi_X} & -\delta_c^{12}c_{\varphi_X}^2-\delta_c^{34}s_{\varphi_X}^2+\Delta c_{2\varphi_X}+V_{XL} \end{pmatrix},$$

$$\mathscr{H}_{\nu N}=\begin{pmatrix} 2^{-1}s_{2\varphi_e}(\delta_c^{34}-\delta_c^{12}-2\Delta) & \delta_s^{34}s_{\varphi_e}c_{\varphi_X}-\delta_s^{12}c_{\varphi_e}s_{\varphi_X} \\ \delta_s^{34}c_{\varphi_e}s_{\varphi_N}-\delta_s^{12}s_{\varphi_e}c_{\varphi_X} & 2^{-1}s_{2\varphi_X}(\delta_c^{12}-\delta_c^{34}-2\Delta) \end{pmatrix},$$

$$\mathscr{H}_{NN}=\mathscr{H}_{\nu\nu}\left(\{\varphi_e,\varphi_X\}\rightarrow\left\{\varphi_e+\frac{\pi}{2},\varphi_X+\frac{\pi}{2}\right\}\right).$$

To conclude this section let us discuss experimental limits on neutrino parameters. Bounds on multipole moments values follow from laboratory experiments and astrophysical estimations for the following processes: (1) cooling of young white dwarves and red giant stars, (2) an observation of an explosion of supernova, and (3) primordial nucleosynthesis. The upper boundary on the magnetic moment varies from $0.5\times10^{-12}\mu_B$ up to $7\times10^{-10}\mu_B$ (see [20] and references therein). However, most parts of the estimations refer to the Dirac neutrino. Upper bounds on magnetic moments transitions for Majorana neutrino, obtained by studies of red giant luminosity before and after helium flash, are given by expressions [21]

$$\mu_{\nu_e}\leqslant3\times10^{-12}\mu_B.$$

Limitations on neutrino REC can be obtained by studies of elastic scattering reactions $\nu_{lL}(l=e,\mu)$ on electrons. The usage of the CHARM II Collaboration data for muon neutrino gives the result [22]

$$\langle r^2\rangle_L^{\mu}<(2.0-2.5)\times10^{-32}\ \ \mathrm{cm}^2,$$

while for electron neutrino an analysis of the LAMPF experiment leads to the inequality [23]

$$\langle r^2\rangle_L^{e}<(9.6-10.2)\times10^{-32}\ \ \mathrm{cm}^2.$$

The upper limit on right-handed electron neutrino REC is obtained by observation of SN 1987A and has the form [24]

$$\langle r^2\rangle_R^{e}\leqslant2\times10^{-33}\ \ \mathrm{cm}^2.$$

For masses of left-handed neutrino existing experiments give the bounds

$$m_{\nu_e}\leqslant0.9\ \ \mathrm{eV},\quad m_{\nu_\mu}\leqslant160\ \ \mathrm{keV},\quad m_{\nu_\tau}\leqslant29\ \ \mathrm{MeV}. \tag{2.13}$$

As concerns the mass of the right-handed neutrino, the situation is less definite because the obtained experimental constraints depend on the other parameters of the LRM. (We recall that in the seesaw picture, the right-handed neutrino masses are in the tens of GeV range. However, when one gives up the seesaw picture, their masses could be arbitrary.) The lower bound on the mass of Majorana neutrinos follows from the lack of neutrinoless double β decay ($\beta\beta_{0\nu}$) for ^{76}Ge [25]:

$$m_{N_e}>(377-413)\left(\frac{1\ \ \mathrm{TeV}}{m_{W_R}}\right)^4\ \ \mathrm{GeV}, \tag{2.14}$$

where the different values in brackets are connected with the way in which a nuclear matrix element is calculated.

Bounds on both Dirac and Majorana neutrino masses could be obtained by analysis of experiments on Z_1 bozon decays at the CERN e^+e^- collider LEP I. Assuming OS2, we have, for the width of decay,

$$Z_1\rightarrow\nu_k^f\overline{\nu_l^f},$$

where $\nu_k^f=\nu_{eL},\nu_{XL},N_{eR},N_{XR}$ obtain the expression in the case of Majorana neutrino

$$\Gamma_{Z_1\rightarrow\nu_k^f\overline{\nu_l^f}}=\frac{m_{Z_1}g_1^{\nu_k^f}g_1^{\nu_l^f}}{12\pi}\sum_{i=1}^{4}U_{ik}^gU_{il}^g\left(1-\frac{4m_km_l}{m_{Z_1}^2}\right)\sqrt{\left(1-\frac{m_l^2+m_k^2}{m_{Z_1}^2}\right)^2-\frac{4m_k^2m_l^2}{m_{Z_1}^4}}, \tag{2.15}$$

where m_k stands for $m_{\nu^f_k}$. The use of (2.15) and the results of L3 Collaboration for the limit of the branching ratio $Z_1 \to \nu \overline{N}$ [26],

$$B_{Z_1 \to \nu \overline{N}} < 3 \times 10^{-5},$$

will give us bounds only for the regions of the allowed values of N_{lR} masses and oscillation angles.

III. RESONANCES AND TRANSITIONS PROBABILITIES

Now we discuss the possible resonances within oscillation schemes OS1 and OS2. In addition, conventional for the SM, here we will have resonance transitions including neutrinos from an additional $SU(2)_R$ sector. Further on we will label them as exotic resonance transitions. In our analysis we will limit ourselves to consideration of resonance conversions only for left-handed electron neutrinos. Then for OS1 in the case of the Majorana neutrino we have six resonance transitions.

(1) $\nu_{eL} \to \nu_{XL}$ is an MSW resonance, which is realized if the condition is satisfied

$$\Sigma_{\nu_e \nu_X} = 2\delta_c^{12} + V_{eL} - V_{XL} + 4\pi(a_{\nu_e \nu_e} - a_{\nu_X \nu_X}) j_z = 0, \tag{3.1}$$

with the transition width

$$\delta n_e(\nu_e \nu_X) \sim [n_e(\nu_e \nu_X) - 4\pi\beta^{-1}(a_{\nu_e \nu_e} - a_{\nu_X \nu_X}) j_z] \tan 2\theta, \tag{3.2}$$

where $\beta = \sqrt{2} G_F - n_e^{-1}(\nu_e \nu_X)(V_{eL}^{\rm NC} - V_{XL}^{\rm NC})$, and $n_e(\nu_e \nu_X)$ is electron density at which the resonance takes place. A value $V_{eL}^{\rm NC} - V_{XL}^{\rm NC}$ is different from zero only if radiation corrections (RC), connected with REC calculations, are taken into account. According to Ref. [19] these RC lead to nonequal values of the Weinberg angle for various fermions. In other words, $\sin^2\theta_W$ acquires a flavor index and is defined by the relation

$$(\sin^2\theta_W)^l = \sin^2\theta_W \left(1 + \frac{m_{W_1}^2}{2} \langle r^2 \rangle_L^l \right). \tag{3.3}$$

Calculations show that at $X = \tau$ and $n_n \approx \frac{1}{6} n_e$ (these very densities are realized in the upper radiative and convective zones of the Sun) $n_e^{-1}(\nu_e \nu_X)(V_{eL}^{\rm NC} - V_{XL}^{\rm NC}) \sim 10^{-3} G_F$. However in neutron stars, where $n_n > n_e$ the influence of this quantity will increase.

Let us show that the last addendum in (3.1) does not exert any influence on the resonance under consideration. According to the existing assumptions about solar matter density distribution, $V_{eL} - V_{XL}$ for the low bound of convective zone is equal to 4×10^{-16} eV. If one assumes that diagonal AM elements are of the same order as nondiagonal ones, then for anapole interaction to have the same order one requires anomalously large electric current density ($\sim 10^{14}$ A cm^{-2}). On the other hand, an analysis of existing experiments with solar neutrinos, which uses two-flavor approximation and the standard solar model, gives for the first addendum in (3.1) the following interval ($E = 10$ MeV):

$$2\delta_e^{12} \approx (0.8 \cdot 10^{-12} - 0.4 \times 10^{-14}) \ \text{eV}. \tag{3.4}$$

Therefore the MSW resonance may only occur before the convective zone where even at the existence of the nonzero value of j_z the anapole interaction would have the order of magnitude much less than the rest of the terms in (3.1).

(2) $\nu_{eL} \to N_{eR}$ is exotic resonance with spin flipping. It takes place at the condition

$$\Sigma_{\nu_e N_e} = \delta_c^- + 2\Delta + V_{eL} - V_{eR} + 4\pi(a_{\nu_e \nu_e} + a_{N_e N_e}) j_z + \dot{\Phi} = 0, \tag{3.5}$$

where $\delta_c^{\pm} = \delta_c^{12} \pm \delta_c^{34}$. The width of this transition is given by the expression

$$\delta n_e(\nu_e N_e) \sim \frac{2\mu_{\nu_e N_e} B_\perp n_e(\nu_e N_e)}{\delta_c^- + 2\Delta + \dot{\Phi} + 4\pi(a_{\nu_e \nu_e} + a_{N_e N_e}) j_z}. \tag{3.6}$$

Let us discuss the mass terms $\delta_c^{\pm} + 2\Delta$. Their values and signs are entirely defined by the mass hierarchy (MH) in the system $\nu_{eL}, \nu_{XL}, N_{eR}, N_{XR}$. In the LRM there are the five possible versions of this hierarchy.

(a) The choice $m_1 \approx m_2 \approx m_3 \approx m_4 \approx$ eV (MH1) leads to the following. The terms $\delta_c^+ + 2\Delta$ and $\delta_c^- + 2\Delta$ could have any signs with the values being arbitrarily small.

(b) When $m_1 \approx m_2 \ll m_3 \approx m_4$ (MH2) the terms $\delta_c^{\pm} + 2\Delta$ exceed the matter potential by several orders of magnitude even at the central core of the Sun (V_c^s).

(c) The scheme with

$$m_1 \ll m_2, \quad m_3 \ll m_4$$

and m_2 which surpasses m_3 so that $\delta^- + 2\Delta$ could turn into zero at the definite values of the angles θ_ν and θ_N (MH3). MH3 is very attractive for the reason that unlike MH1 and MH2 it does not contradict the following quadratic mass formula for neutrinos:

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_u^2 : m_c^2 : m_t^2, \tag{3.7}$$

which is predicted by the grand unified theories (GUT's).

When

$$m_2^2 \sin^2\theta_\nu < (>) [m_4^2 \sin^2\theta_N + m_3^2 \cos^2\theta_N - m_1^2 \cos^2\theta_\nu] \tag{3.8}$$

is satisfied the term $\delta_c^- + 2\Delta$ has the positive (negative) sign. As for $\delta_c^+ + 2\Delta$ its values are much larger than V_c^s.

(d) The hierarchy with $m_1 \ll m_2 \ll m_3 \ll m_4$ (MH4) just as the previous one is in agreement with the neutrino mass spectrum given by GUT's. In this case we have

$$|\delta_c^{\pm} + 2\Delta| \gg |V_c^s|. \tag{3.9}$$

(e) The choice $m_1 \approx m_3 \ll m_2 \approx m_4$ (MH5) once again does not contradict (3.7) and could lead to $\delta_c^- + 2\Delta$ with positive and negative sign and with the values of the same order of magnitude as the solar matter potential. In this case $\delta_c^+ + 2\Delta$ is so large that in the conditions of the Sun none of the resonances with its participation could be realized.

As it is easy to see, $\nu_{eL}\rightarrow N_{eR}$ resonance will take place only if MH1 or MH3 is realized, while MSW resonance will take place at MH1 and MH2.

Now the situation when anapole interaction will play a considerable role is possible. To achieve this one may assume, for example, that $\dot{\Phi}$ in the region above sunspot satisfies the relation

$$\delta_c^- + 2\Delta + \dot{\Phi} = 0. \tag{3.10}$$

Then resonance, i.e.,

$$V_{eL} - V_{eR} + 4\pi(a_{\nu_e\nu_e} + a_{N_eN_e})j_z = 0, \tag{3.11}$$

may take place in the chromosphere above sunspot when $j_z \sim 10^4$ A cm^{-2} [to estimate AM formulas (2.11) were used].

(3) $\nu_{eL}\rightarrow N_{XR}$ is an exotic resonance with spin and flavor flipping. The corresponding condition for its observation has the form

$$\Sigma_{\nu_eN_X} = \delta_c^+ + 2\Delta + V_{eL} - V_{XR} + 4\pi(a_{\nu_e\nu_e} + a_{N_XN_X})j_z + \dot{\Phi} = 0. \tag{3.12}$$

For the width of this transition we get the expression

$$\delta n_e(\nu_e N_X) \sim \frac{2\mu_{\nu_eN_X}B_\perp n_e(\nu_e N_X)}{\delta_c^+ + 2\Delta + \dot{\Phi} + 4\pi(a_{\nu_e\nu_e} + a_{N_XN_X})j_z}. \tag{3.13}$$

This resonance may take place only at MH1. Then, once again, we may imagine a situation in which an anapole interaction will turn out to be decisive for the existence of $\nu_{eL}\rightarrow N_{XR}$ resonance conversion.

(4) $\nu_{eL}\rightarrow_{XL}$ resonance with flavor and spin flipping occurs at the condition

$$\Sigma_{\nu_e\overline{\nu_X}} = 2\delta_c^{12} + V_{eL} + V_{XL} + 4\pi(a_{\nu_e\nu_e} + a_{\nu_X\nu_X})j_z + \dot{\Phi} = 0, \tag{3.14}$$

which may be satisfied only for MH1 and MH2. The resonance transition width is defined as

$$\delta n_e(\nu_e\overline{\nu_X}) \sim \frac{2\mu_{\nu_e\overline{\nu_X}}B_\perp n_e(\nu_e\overline{\nu_X})}{2\delta_c^{12} + 4\pi(a_{\nu_e\nu_e} + a_{\nu_X\nu_X})j_z + \dot{\Phi}}. \tag{3.15}$$

The given resonance, as well as the MSW resonance, is predicted also within the SM. However, while for the MSW resonance the condition for its fulfillment has one and the same form in both models, for $\nu_{eL}\rightarrow\overline{\nu_{XL}}$ resonance it is not true. Now by means of the $V_{eL}+V_{XL}$ term there are contributions, connected with an exchange of additional Z_2 boson. For the convective zone they change matter potential on the value of 1%. With the growth of neutron concentration these contrbutions are growing and will become considerable for such objects as neutron stars.

(5) $\nu_{eL}\rightarrow\overline{N_{eR}}$ is an exotic resonance without spin and flavor flipping. It takes place if the following condition is satisfied:

$$\Sigma_{\nu_e\overline{N_e}} = \delta_c^- + 2\Delta + V_{eL} + V_{eR} + 4\pi(a_{\nu_e\nu_e} - a_{N_eN_e})j_z = 0, \tag{3.16}$$

which might occur at MH1 and MH3. The resonance transition width is defined as the following:

$$\delta n_e(\nu_e\overline{N_e}) \sim \frac{8\pi a_{\nu_e\overline{N_e}}j_z n_e(\nu_e\overline{N_e})}{\delta_c^- + 2\Delta + 4\pi(a_{\nu_e\nu_e} - a_{N_eN_e})j_z}. \tag{3.17}$$

(6) $\nu_{eL}\rightarrow\overline{N_{XR}}$ is an exotic resonance with the flavor flipping. It takes place at the condition

$$\Sigma_{\nu_e\overline{N_X}} = \delta_c^+ + 2\Delta + V_{eL} + V_{XR} + 4\pi(a_{\nu_e\nu_e} - a_{\overline{N_X}\overline{N_X}})j_z = 0, \tag{3.18}$$

which is realized only at MH1.

For the resonance transition width we have the expression

$$\delta n_e(\nu_e\overline{N_X}) \sim \frac{8\pi a_{\nu_e\overline{N_X}}j_z n_e(\nu_e\overline{N_X})}{\delta_c^+ + 2\Delta + 4\pi(a_{\nu_e\nu_e} - a_{\overline{N_X}\overline{N_X}})j_z}. \tag{3.19}$$

A common property for $\nu_{eL}\rightarrow\overline{N_{eR}}$, $\overline{N_{XR}}$ resonances is the fact that if the anapole interaction is absent, they would have been possible only in the second order of the perturbation theory by means of two-step resonance conversions

$$\nu_{eL}\rightarrow I\rightarrow\overline{N_{eR}}, \overline{N_{XR}},$$

where $I\neq\overline{\nu_{eL}}$, N_{eR} for (5) and $I\neq\overline{\nu_{eL}}$, N_{XR} for (6) resonances, respectively.

The analysis of the picture in the case of the Dirac neutrino is analogous. Let us only notice the fact that now the maximally possible number of resonance conversions for ν_{eL} is 7.

Now let us discuss OS1 predictions from the point of view of experiment. If lifetimes of new relative to the SM neutrinos N_{lR} turn out to be longer than the time of the motion from the birth place to the detecting place, N_{lR} could be detected in the "appearance" experiments. Let us consider, for example, perspectives for the $\overline{N_{eR}}$ observation. At $m_{N_e} < 14$ MeV the following decay modes dominate:

$$\overline{N_{eR}}\rightarrow\overline{\nu_{lL}}e^+e^-, \overline{\nu_{lL}}\gamma, 3\overline{\nu_{lL}}.$$

Formulas defining the widths of these decays could be found in [25]. The choice of parameters LRM in the form

$$m_{N_e} = 9 \text{ MeV}, \quad m_{\nu_e} = 0.9 \text{ eV}, \quad \xi = \theta = 10^{-2},$$

$$E = 10 \text{ MeV}$$

leads to the domination of $\overline{\nu_{\tau L}}\gamma$ decay mode with the width of

$$\Gamma_{\overline{N_e}\rightarrow\overline{\nu_\tau}\gamma} \approx 8\times10^{-22}\xi^2\theta^2\left(\frac{m_{N_e}}{1 \text{ MeV}}\right)^2 \text{ MeV}.$$

It gives for the lifetime $\overline{N_{eR}}$ the value $\sim 5\times10^5$ s, while the time of the motion from the Sun to a terrestrial detector has an order of $\sim 10^3$ s. So, if in the solar conditions $\nu_{eL}\rightarrow\overline{N_{eR}}$ resonance is realized and a degree of deviation from adiabaticity is small, then falling down on the Earth N_{eR} flux is rather intensive. The following process could serve as a candidate for an observation of such neutrinos:

$$\overline{N_{eR}}+e_R^-\rightarrow\mu^-+\overline{\nu_{\mu L}}.$$

It should be noted that in the SM the reaction of the $\mu^-\overline{\nu_{\mu L}}$-pair production takes place only at the collision of the left-handed electron antineutrino on left-polarized electrons.

In ''disappearance'' experiments they measure the so-called survival probability of left-handed electron neutrino $\mathcal{P}(\nu_{eL}\rightarrow\nu_{eL})$, which could be found by means of solving the (2.1). It is obvious that we cannot always hope to obtain a precise solution in the analytical form. Even for a two-component neutrino system $\Psi^T=(\nu_{eL},\nu_{XL})$ at $\dot{\Phi}=j_z=0$ this task was solved only in the following case of matter density distribution: (1) $n_e(z)\sim z$ [27]; (2) $n_e(z)\sim\exp(\text{const}\times z)$ [28]; (3) $n_e(z)\sim\tanh z$ [29]; (4) $n_e(z)\sim z^{-1}$ [30]. To simplify the case let us assume that the resonance localization places are situated rather far from one another, i.e., the conditions are satisfied

$$n_e(k)+\delta n_e(k)<n_e(i)-\delta n_e(i), \tag{3.20}$$

where $i,k=\nu_e\nu_X,\nu_eN_e,\ \nu_eN_X,\nu_e\overline{\nu_X},\nu_e\overline{N_e},\nu_e\overline{N_X}$. That allows us to consider them as independent ones. Then transition probabilities on resonances are defined by the expression

$$\mathcal{D}_i=\exp\{-\gamma^i(z_i)F_i\}, \tag{3.21}$$

where $\gamma^i(z)$ is the adiabaticity parameter of i resonance, z_i is the z coordinate of i resonance, the F_i value depends on a kind of a resonance, and in the most general case, on the behavior of such quantities as $\dot{\Phi}(z),V_{lL,R}$, and j_z near resonance. Assuming that all three above-stated quantities are linear functions on z, we have $F_i=\pi/4$. Adiabaticity parameters are defined according to the relation

$$\gamma^i(z)=\frac{8(\mathcal{H}_i)^2}{\sin^3 2\theta_i\left|\dfrac{d}{dz}\Sigma_i\right|}, \tag{3.22}$$

where $\sin^2 2\theta_i=2\mathcal{H}_i^2/(\Sigma_i^2+2\mathcal{H}_i^2)$, and $\mathcal{H}_i$ is a nondiagonal element of the Hamiltonian in Eq. (2.1), corresponding to an i-resonance transition.

In our further analysis we shall not take into consideration an anapole interaction and for the sake of definiteness shall consider the neutrino to be a Majorana particle. Then assuming that MH1 is realized in the nature, we have four possible resonance transitions for ν_{eL}:

$$\nu_{eL}\rightarrow\nu_{XL},N_{XR},N_{eR},\overline{\nu_{XL}}.$$

To such a sequence of resonances corresponds the choice of model parameters in the form

$$\delta_c^{12}<\delta_c^+ +2\Delta<\delta_c^- +2\Delta<\delta_c^{12}+\dot{\Phi}<0.$$

In the presence of several resonances we should average not only over ν_{eL} birth and detection regions, but also over the regions separating these resonances. That results in the following expression for the ν_{eL} survival probability:

$$\langle\mathcal{P}(\nu_{eL}\rightarrow\nu_{eL})\rangle=(\cos^2\theta_\nu,\sin^2\theta_\nu,0,0,0)B^{\nu_X}B^{N_X}B^{N_e}\times B^{\overline{\nu_X}}\begin{pmatrix}(c_{\nu_X}c_{N_X}c_{N_e}c_{\overline{\nu_X}})^2\\(s_{\nu_X}c_{N_X}c_{N_e}c_{\overline{\nu_X}})^2\\(s_{N_X}c_{N_e}c_{\overline{\nu_X}})^2\\(s_{N_e}c_{\overline{\nu_X}})^2\\s^2_{\overline{\nu_X}}\end{pmatrix}, \tag{3.23}$$

where k is a resonance number; $(k+1)\times(k+1)$ matrices B^d $(d=\nu_X,N_X,N_e,\overline{\nu_X})$, describing transitions between eigenstates, have the nonzero elements

$$B^d_{\nu_e\nu_e}=B^d_{dd}=1-\overline{D}_{\nu_ed},\quad B^d_{\nu_ed}=B^d_{d\nu_e}=\overline{D}_{\nu_ed};$$

$B^d_{ii}=1$ at $i\neq\nu_e,d$, $c_d=\cos\overline{\theta}_{\nu_ed}$, $s_d=\sin\overline{\theta}_{\nu_ed}$, $\overline{D}_{\nu_e\nu_X}=\Theta(\Sigma_{\nu_e\nu_X})D_{\nu_e\nu_X}$, the function $\Theta(x)$ [$\Theta(x)=0$ for $x<0$ and $\Theta(x)=1$ at $x>0$] allows the transition probability to come into play only after reaching a resonance point and a line above an effective angle means that it is defined in the ν_{eL} birth point.

Let us switch over to an MH2 analysis. In this case LRM, as well as the SM, predicts for ν_{eL} two resonance transitions

$$\nu_{eL}\rightarrow\nu_{XL},\overline{\nu_{XL}}.$$

The expressions for a survival probability have the form

$$\langle\mathcal{P}(\nu_{eL}\rightarrow\nu_{eL})\rangle=(\cos^2\theta_\nu,\sin^2\theta_\nu,0)B^{\nu_X}B^{\overline{\nu_X}}\begin{pmatrix}c^2_{\nu_X}c^2_{\overline{\nu_X}}\\s^2_{\nu_X}c^2_{\overline{\nu_X}}\\s^2_{\nu_X}\end{pmatrix}. \tag{3.24}$$

The corresponding expression for the SM follows from (3.24) by a replacement

$$\xi=\phi=g_R=0,\quad m_{Z_2}\rightarrow\infty.$$

Consider now a situation when a MH3 is true. Provided that

$$\delta_c^- +2\Delta+\dot{\Phi}<0,$$

there is only one resonance transition for ν_{eL}

$$\nu_{eL}\rightarrow N_{eR}.$$

Having averaged, we obtain for the sought probability

$$\langle\mathcal{P}(\nu_{eL}\rightarrow\nu_{eL})\rangle=(\cos^2\theta_\nu,0)B^{N_e}\begin{pmatrix}c^2_{N_e}\\s^2_{N_e}\end{pmatrix}. \tag{3.25}$$

At MH4 and MH5 the oscillation scheme under consideration does not predict any resonance transitions for a ν_{eL} flux.

In the conclusion of this chapter let us briefly discuss OS2 predictions. First of all, let us notice the absence of transitions to an antiparticle sector even in the highest orders of the perturbation theory. All the possible resonance conversions of ν_{eL} flux are already going in the first order of interaction. The resonance conditions have the form

$$\Sigma^{\mathrm{OS2}}_{\nu_e\nu_X}=\delta_c^{12}(c^2_{\varphi_e}+c^2_{\varphi_X})+\delta_c^{34}(s^2_{\varphi_e}+s^2_{\varphi_X})+\Delta(c_{2\varphi_e}-c_{2\varphi_X})+V_{eL}-V_{XL}=0 \quad (\nu_{eL}\to\nu_{XL}), \tag{3.26}$$

$$\Sigma^{\mathrm{OS2}}_{\nu_eN_e}=c_{2\varphi_e}(\delta_c^{12}-\delta_c^{34}+2\Delta)+V_{eL}-V_{eR}=0 \quad (\nu_{eL}\to N_{eR}), \tag{3.27}$$

$$\Sigma^{\mathrm{OS2}}_{\nu_eN_X}=\delta_c^{12}(c^2_{\varphi_e}+s^2_{\varphi_X})+\delta_c^{34}(s^2_{\varphi_e}+c^2_{\varphi_e})+\Delta(c_{2\varphi_e}+c_{2\varphi_X})+V_{eL}-V_{XR}=0 \quad (\nu_{eL}\to N_{XR}). \tag{3.28}$$

It is not difficult to show that by corresponding choice of oscillation parameters $\nu_{eL}\to\nu_{XL}(N_{eR})$ resonance takes place at any mass hierarchy except for MH5 (MH2) while $\nu_{eL}\to N_{XR}$ resonance can occur in the conditions of the Sun only at MH1. It is important to notice that even when $m_{N_l}>14$ MeV the mass values of the right-handed neutrinos influence the MSW resonance in the case of the small angles $\varphi_{e,X}$.

Once again, as for OS1, assuming the resonances to be well separated, we can write down analytical expressions for the survival probability of ν_{eL} flux for all the five possible mass hierarchies. Thus, for example, in case of MH1 realization we have

$$\langle\mathcal{P}(\nu_{eL}\to\nu_{eL})\rangle=(c^2_{\theta_\nu}c^2_{\varphi_e},s^2_{\theta_\nu},s^2_{\varphi_e},0)B^{\nu_X}_{\mathrm{OS2}}B^{N_e}_{\mathrm{OS2}}B^{N_X}_{\mathrm{OS2}}\times\begin{pmatrix} c^2_{\nu_X}c^2_{N_e}c^2_{N_X}\\ s^2_{\nu_X}c^2_{N_e}c^2_{N_X}\\ s^2_{N_e}c^2_{N_X}\\ s^2_{N_X}\end{pmatrix}.$$

IV. SOLAR FLARES AND THEIR INFLUENCE ON THE NEUTRINO FLUX

At certain conditions an AR evolution may lead to an appearance of SF which represents as itself the most powerful of all the solar activity events. Magnetic energy of sunspots transforms into kinetic energy of matter emission (at a speed of 10^6 m/s), into energies of hard electromagnetic radiation, and into fluxes of so-called solar cosmic rays (SCR). In general SCR consist of protons $\mathscr{E}_k\geqslant10^6$ eV, of nuclei with charges $2\leqslant Z\leqslant28$ and energy within an interval from 0.1 to 100 eV/nucleon and of electrons with $\mathscr{E}_k\geqslant30$ MeV. Relative content of nuclei with $Z\geqslant2$ reflects in general solar atmosphere composition, while proton portion depends on flare power (for big SF power is about of $\sim10^{29}$ erg/s). Pretty popular mechanism of flare appearance is based on breaking and reconnection of magnetic field strength lines of neighboring spots. This mechanism suggested in Ref. [31] further on was developed in detail in Ref. [32]. According to this model a change of magnetic field configuration in a sunspots group of fairly opposite polarity might lead to the appearance of a limiting strength line common for the whole group. Throughout the limiting line the redistribution of magnetic fluxes takes place, which is necessary for magnetic field to have the minimum energy. The limiting strength line rises from photosphere to the corona. From the moment of this line appearance an electric field induced by magnetic field variations causes current along the line, which due to the interaction with a magnetic field takes a form of a current layer. As the current layer prevents the magnetic fluxes redistribution, the process of magnetic energy storage of the current layer begins. Duration of appearance and formation period of the current layer (initial SF phase) varies from several to dozens of hours. The second stage (an explosion phase of SF) has a time interval of 1–3 minutes. It begins from the appearance in some part of the current layer of a high resistance region, which leads to a current dissipation. Then due to penetration of the magnetic field through the current layer a strong magnetic field appears perpendicular to it; this process of magnetic force breaks the current layer and throws out plasma at a great speed. Observations and theoretical analysis show that main mechanisms of accelerating hot plasma particles from a breaking region are the following: (1) particles are accelerated by a quasiregular electric field, appearing at a current layer breaking; (2) betatron mechanism, at which particles are accelerated by the influence of a fluctuating magnetic field; (3) Fermi mechanism, when particles are accelerated by the collisions with magnetic nonhomogeneities. The height where particles acceleration takes place is not the same for different flares. Acceleration regions may be located either in chromosphere where plasma particles concentrations is $n\sim10^{13}$ cm^{-3}, or in the corona at $n\sim10^{11}$ cm^{-3}. The particle distribution according to energies and charges while in motion through the interplanetary medium is defined by a mechanism of their acceleration at SF and by pecularities of an exit from an acceleration region. For high energy particles with $\mathscr{E}_k\geqslant10^8$ eV time dependence of flux intensity near Earth represents itself as a nonsymmetrical bell-like curve with a very quick increase (minutes to dozens of minutes) and a slow (from several hours to one day) decrease. An increase amplitude on the Earth's surface for the most powerful SF may reach $\sim$4500% in comparison to background flux of cosmic particles. The concluding stage (hot phase of SF) is characterized by the existence of a high temperature coronal region and can continue for several hours. The heating of dense atmospheric layers leads to an evaporation of a large amount of gas, which favors a long-continued existence of a dense hot plasma cloud.

One of the characteristic flare features is its isomorphism, i.e., the repetition in one and the same place with the same field configuration. A small flare may repeat up to 10 times per day, while a large one may take place the next day and even several times during active region lifetime. Besides, the stronger a flare is, the larger magnetic field gradient precedes its appearance. After the flare the general decrease of magnetic field gradients comes, but by the time of the next flare the gradients return to their previous values.

Now let us discuss the factors, which may influence on a neutrino flux, crossing an SF region. Let us start from a case when a mixing according to OS1 takes place.

(1) A change in a twisting rate of magnetic field $\dot{\Phi}$ and electric current density j_z in an active zone, beginning from

a preflare period, may result in disappearance or appearance $\nu_{eL}\rightarrow N_{eR}$, $\nu_{eL}\rightarrow N_{XR}$, and $\nu_{eL}\rightarrow\overline{\nu}_{XL}$ of resonance transition.

(2) At motion of neutrino flux along a current layer the resonance conditions for all the resonances under study will change due to abrupt increase of an anapole interaction.

(3) Adiabaticity conditions may be violated on all the three SF phases which leads to an increase of ν_{eL} flux observed by terrestrial detectors.

(4) While a neutrino flux crosses a dense plasma cloud evaporated during an SF in the upper atmospheric layers, the resonance conditions for $\nu_{eL}\rightarrow N_{XR},\overline{N}_{XR}$ transitions could be satisfied.

(5) A flux of particles accelerated up to gigantic energies will also influence the neutrino moving along it. In the simplified form the flux motion could be imagined as a motion with the different speed of three regions, each separately consisting of electrons, nuclei $2\leqslant Z\leqslant 28$, and protons, respectively. Further on for the sake of simplicity we shall limit ourselves to consideration of electron and proton regions only. The terms, describing multipole neutrino interactions with an electromagnetic field, should be changed in the following manner:

$$4\pi a_{ik}j_z\rightarrow 4\pi a_{ik}\left(\rho_f+\frac{1}{4\pi}\frac{\partial E_z}{\partial z}\right), \tag{4.1}$$

$$\mu_{ik}B_{\perp}\rightarrow d_{ik}E_z, \tag{4.2}$$

where ρ_f is a volume charge density in a region consisting of $f(f=e,p)$ particles, and d_{ik} is a dipole electric transition moment between neutrino states i and k. If for the AM a_{ik} one uses values (2.11) then terms $4\pi\rho_f$ and V_{eL} have one and the same order of magnitude, provided that a flare occurred in the chromosphere. To estimate a contribution of a dipole electric interaction let us use for d its upper experimental bound [33] $d<1{,}3\times10^{-40}$ C cm. Then it has the same order of magnitude as a matter potential in the chromosphere at $E_z\sim10^5$ V cm. Now matter potential for the neutrinos, found in one of the two regions, are defined by the expressions

$$V_{eL}=\frac{1}{2}(1-Q_f)\sqrt{2}(1-v_z^f)G_Fn_f+V_{XL}, \tag{4.3}$$

$$V_{eR}=\frac{g_R^2}{8}(1-Q_f)\left(\frac{\sin^2\xi}{m_{W_1}^2}+\frac{\cos^2\xi}{m_{W_2}^2}\right)(1-v_z^f)n_f+V_{XR}, \tag{4.4}$$

$$V_{XL,R}=n_f(1-v_z^f)\sum_{i=1}^{2}\frac{g_i^{\nu_{L,R}}g_i^f}{4m_{Z_i}^2}, \tag{4.5}$$

where v_z^f is an averaged longitudinal velocity of f region, $Q_p=-Q_e=1$, and n_f is defined as a density of f particles in a flux. Now it is possible to imagine a situation when a flux ν_{eL} undergoes one and the same conversion both in the solar matter and in a flux of accelerated particles. Let us consider, for example, $\nu_{eL}\rightarrow N_{eR}$ resonance. We assume that $\delta_c^-+2\Delta$ is negative and its module is $\sim V_{eL}^{\mathrm{chr}}$, where a superscript chr means that a value in brackets is taken for the chromosphere. Let the twisting rate $\dot{\Phi}$ be such that a resonance condition (3.5) is satisfied in a convective zone. Then a flux ν_{eL} crossing a region consisting of accelerated protons may undergo $\nu_{eL}\rightarrow N_{eR}$ resonance for the second time provided that

$$\delta_c^-+2\Delta+4\pi(a_{\nu_e\nu_e}+a_{N_eN_e})\rho^p=0. \tag{4.6}$$

To satisfy (4.6) it is enough for the anapole interaction to be positive and SF to take place in the chromosphere.

At OS2 realization the number of factors influencing the ν_{eL} flux is reduced to 2.

(1) Resonance conversions $\nu_{eL}\rightarrow\nu_{XL},N_{eR},N_{XR}$ may occur during crossing the plasma cloud evaporated after an SF.

(2) A resonance appearance is also possible during a motion in a flux of particles accelerated by an SF.

V. CONCLUSION

The analysis of the two flavor system of Majorana neutrino from the point of view of the LRM shows that, depending on the neutrino mass correlation in lepton family and on the chosen mixing scheme, the oscillation picture may considerably differ relative to the SM. In the case when all the masses are close enough to one another, the number of possible resonance transitions for Majorana (Dirac) ν_{eL} may reach 6 (7). The appearance of new resonances is caused by N_{lR} neutrinos belonging to an $SU(2)_R$ sector. At $m_1\approx m_2\leqslant m_3\approx m_4$ and mixing of neutrino with the same helicity but different flavor, the resonance transition numbers of ν_{eL} equal 2 and the difference from the SM is only reduced to matter potential values. At the existing limitations on LRM parameters the difference in potentials should not exceed 1,2% [34]. The problem of solar neutrinos in the LRM may as well be explained by the choice

$$m_1\leqslant m_2,\quad m_3\leqslant m_4,$$

which is dictated by mass hierarchy in grand unification theories.

In this work we considered two possible mixing schemes OS1 and OS2. The obtained expressions for the ν_{eL} survival probability may be used for obtaining allowed regions of oscillation parameters in both schemes. Now, however, even in assumption

$$\frac{d\dot{\Phi}}{dz}=\frac{dj_z}{dz}=0,$$

while fitting such parameters as δm_{34}^2, θ_ν, and θ_N in OS1 it is necessary to know values of neutrino multipole moments.

Neutrinos, crossing sunspots practically all the time on their motion through the Sun and its atmosphere, are located in the region of an intensive magnetic field. According to the existing viewpoints the structure of this field is rather complicated ($\dot{\Phi}\neq0$, rot$\mathbf{B}\neq0$), and a value can vary from $\sim10^8$ G in the central region to $\sim10^3$ G in the coronal part of the Sun. On the condition that OS1 is realized, a resonance picture for such a neutrino might prove to be richer than for the neutrinos which on their way out do not face sunspots. Thus, for example, resonances $\nu_{eL}\rightarrow N_{eR}$, $\nu_{eL}\rightarrow N_{XR}$, $\nu_{eL}\rightarrow\overline{\nu}_{XL}$

due to a change $\dot{\Phi}$ and j_z may occur both in the convective zone and in the atmosphere. Consequently, a comparison of neutrino fluxes crossing and escaping sunspot regions will serve as a source of information on the following: (a) a structure of the solar electromagnetic field; (b) values of neutrino multipole moments; (c) low bounds on neutrino masses additional in respect to the SM.

It is well known that when a magnetic dipole moment of neutrino is of about $10^{-11}\mu_B$, the change of a magnetic field in the convective zone will lead to an anticorrelation of a neutrino flux with the solar activity [35]. This time correlation is not the only one from the ones predicted nowadays for a solar neutrino flux. Experimentalists may hope to observe some more variations with a smaller time scale: (1) seasonal variations with the maximum in July and December connected with the inclination of the Earth orbit plane to the Sun equator plane on the angle of 7° 15 ft [4]; (2) half-year variation with a September maximum caused by a change in the twisting rate of the magnetic field in the convective zone [36]; (3) day-night variation, caused by a neutrino interaction with a field of the Earth matter [37]. We also discussed one more correlation: the correlation of a neutrino flux with solar flares whose possibilities for the first time were noted in Ref. [34]. We have shown that this correlation occurs both at OS1 and OS2. Let us notice that detection of the neutrino flux correlation with solar flares is already possible on neutrino telescopes of the next generation, where events statistics increase on two orders of magnitude.

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