New fermions and a vectorlike third generation in $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ models

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We study two 3-3-1 models with (i) five (four) charge 2/3 (−1/3) quarks and (ii) four (five) charge 2/3 (−1/3) quarks and a vectorlike third generation. Possibilities beyond these models are also briefly considered.

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I. INTRODUCTION

Nowadays it seems that in some sense the third generation may be different from the other ones: although a heavy *top* quark [1] can still be barely accommodated in the standard model, it can bring some unexpected features to the mass spectrum problem, also, possibly the *bottom* quark couples to the Z^0 with a strength which is different from the strength of the d and s quarks [2] and, finally, the properties of the τ lepton and its neutrino can still bring up surprises [3]. On the other hand, if the cross section $\sigma(p\bar{p} \to t\bar{t} + X)$ obtained by the CDF Collaboration [1] is in fact higher than the prediction of quantum chromodynamics, this may be a signature of new quarks.

In the model based on the gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ of Ref. [4], the effective $SU(2) \otimes U(1)$ model coincides with the usual electroweak one. The three families belong to left-handed doublets and right-handed singlets of SU(2). Hence, all of them have, at leading order, the same couplings to the $W^\pm$ and Z^0 bosons. Also the extra quarks in that model have exotic 5/3 and −4/3 charges. The lepton sector is exactly the same as in the standard model (SM). Although this model coincides at low energies with the usual electroweak model, it explains the fundamental questions of (i) the family number and (ii) why $\sin^2\theta_W < 1/4$ is observed. Therefore, it is possible from the last constraint to compute an upper limit to the mass scale of the SU(3) breaking of about 3 TeV [5]. This makes the 3-3-1 model an interesting possibility for physics beyond the standard model, in particular if future experiences confirm in more detail an $SU(2) \otimes U(1)$ model (for instance, if the $Z \to b\bar{b}$ decay and several Z-pole asymmetries confirm the value expected in the model) and no new quarks with charge 2/3 and −1/3 were found.

However, if in the future new quarks are observed having the same charge as the quarks already known or if the third generation turns to be in fact different from the other two generations (say, with different interactions), it will be necessary to consider modifications of the original 3-3-1 model. For instance, a model with five charge −1/3 and four charge 2/3 quarks has already been considered in Ref. [6]. There are, however, other representation contents: (A) four charge −1/3 and five charge 2/3 quarks or, (B) the third generation in a vector-like representation of the electroweak symmetry. We will give below two examples of such sort of models. One of them is an extension of one of the models proposed some years ago by Georgi and Pais [7].

Once we are convinced that theories based on the 3-3-1 gauge symmetry are interesting possibilities for physics at the TeV range, we must study how the basic ideas of this sorts of models can be generalized.

Hence, in this work we wish to generalize these sorts of models in several ways. First, by expanding the color degrees of freedom (n) and the electroweak sector (m), i.e., we will consider models based on the gauge symmetry:

$$SU(n)_C \otimes SU(m)_L \otimes U(1)_N. \tag{1.1}$$

In most of these extensions the anomaly cancellation occurs among all generations together, and not generation per generation. However, if a 3-3-1 model has the third generation not anomalous, also it will be so in its extension.

In (almost) all these models, which we recall that are indistinguishable from the standard model at low energies, in order to cancel anomalies the number of families (N_f) must be divisible by the number of color degrees of freedom (3). Hence the simplest alternative is $N_f = 3$. By denoting N_q and N_l the number of quark and lepton families, respectively, we will see that the relation $N_q = N_l = N_f = 3$ is a particular feature of 3-m-1 models. When $n \neq 3$, N_q and N_l are still related to each other but it is not necessary that $N_q = N_l$ in order to have anomaly cancellation.

We will use the criterion that the values for m in Eq. (1.1) are determined by the leptonic sector. It means that if each generation is treated separately, SU(4) is the highest symmetry group to be considered in the electroweak sector. Thus, there is no room for $SU(5)_L \times U(1)_N$ if we restrict ourselves to the case of leptons with charges $\pm1, 0$.

In the color sector, for simplicity, in addition to the usual case of $n = 3$, we will comment the cases $n = 4, 5$. These extensions have been considered in the context of the $SU(2)_L \otimes U(1)_Y$ model [8,9].

Next, models with left-right symmetry and/or with horizontal symmetry are also considered. We discuss too a SU(6) grand unified theory in which one of these models

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may be embedded.

We must stress that all extensions of 3-3-1 models have flavor changing neutral currents (FCNC). However, up to now in all models of this kind which have been considered in detail, it was always possible to have, in the sector of the model which coincides with the observed one, natural conservation of flavor in the neutral currents. Hence, FCNC effects are restricted to the exotic sectors of the models. The only exception is model B below since in this case there are right-handed currents coupled to the Z^0 which do not conserve flavor but they involve an arbitrary right-handed mixing matrix. Since all extensions of the 3-3-1 model that we will consider here have an $\mathrm{SU}(3)_L$ subgroup, we think that the suppression of the FCNC in the observed part of the particle spectrum is a general feature of this kind of model. This is far from being an obvious fact but it was shown in several 3-3-1 models and recently in the 3-4-1 case. For details see Refs. [6,10,11].

This work is organized as follows. In Sec. II we consider two new possibilities of 3-3-1 models. In Sec. III we consider models with $n = 3$, $m = 3,4$ (Sec. III A). We will also discuss the cases for $n = 4,5$ (Sec. III B). In Sec. IV we give general features of the extensions with left-right symmetry (Sec. IV A) and with horizontal symmetries (Sec. IV B). We also consider (Sec. IV C) possible embedding in SU(6). The last section is devoted to our conclusions.

II. TWO 3-3-1 MODELS

Here, we will treat two interesting possibilities of 3-3-1 models with the electric charge operator defined as $2Q = \lambda_3 + \lambda_8/\sqrt{3} + 2N$. Both models have the same gauge boson sector. They differ slightly in the scalar sector but they are quite different in the fermion sector. One of the models (model A) has five charge 2/3 quarks and four charge $-1/3$ ones; the other model (model B) has four charge 2/3 and five charge $-1/3$ quarks and the third generation in a vector-like representation of SU(3). The lepton sector is also different in both models. Model B is an extension with three quark generations of one of the models put forward in Ref. [7]. In model A anomalies cancel out only among all generations with each generation being anomalous. In model B only the third generation is not anomalous.

A. Model A

All leptons generations transform as triplets of SU(3):

$$\Psi_{aL} = \begin{pmatrix} \nu_a \\ l_a^- \\ E_a^- \end{pmatrix}_L \sim (\mathbf{3}, -2/3), \quad a = e, \mu, \tau; \tag{2.1}$$

while quarks transform as follows:

$$Q_{iL} = \begin{pmatrix} d_i \\ u_i \\ u_i' \end{pmatrix}_L \sim (\mathbf{3}^*, 1/3), \quad i = 1,2;$$
$$Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ d_4 \end{pmatrix}_L \sim (\mathbf{3}, 0), \tag{2.2}$$

and all charged right-handed fields in singlets. Neutrinos remain massless as long as no right-handed components are introduced. We have omitted the color index.

In the quark sector the phenomenological states in Eqs. (2.2) are linear combinations of the mass eigenstates (u,c,t,t',t'') and (d,s,b,b') for the charge 2/3 and charge $-1/3$ sectors, respectively. Three of the left-handed charge 2/3 ($-1/3$) quark fields are part of SU(2) doublets $u_{1,2,3}$ ($d_{1,2,3}$). The other fields $u'_{1,2}$ and d_4 are in singlets of SU(2).

Let us introduce the antitriplets of Higgs bosons:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta_1^+ \\ \eta_2^+ \end{pmatrix} \sim (\mathbf{3}^*, 2/3), \quad \sigma = \begin{pmatrix} \sigma^- \\ \sigma_1^0 \\ \sigma_2^0 \end{pmatrix} \sim (\mathbf{3}^*, -1/3), \tag{2.3}$$

and a third one σ' transforming like σ.

The most general quark Yukawa couplings are

$$\begin{aligned}-\mathcal{L}_{qY} = &\sum_{i\alpha} A_{i\alpha}\bar{Q}_{iL}\mathcal{D}_{\alpha R}\eta \\ &+ \sum_{i\beta}[B_{i\beta}\bar{Q}_{iL}\sigma + B'_{i\beta}\bar{Q}_{iL}\sigma']\mathcal{U}_{\beta R} \\ &+ \sum_{\alpha}[E_{3\alpha}\bar{Q}_{3L}\sigma^* + E'_{3\alpha}\bar{Q}_{3L}\sigma'^*]\mathcal{D}_{\alpha R} \\ &+ \sum_{\beta} F_{3\beta}\bar{Q}_{3L}\mathcal{U}_{\beta R}\eta^* + \text{H.c.}\,,\end{aligned} \tag{2.4}$$

where $i = 1,2$; $\alpha = 1,2,3,4$; $\beta = 1,2,3,4,5$ and we have chosen the basis $\mathcal{D}_{\alpha R} = d_{1,2,3,4R}$; $\mathcal{U}_{\beta R} = u_{1,2,3R}, u'_{1,2R}$; with η^*, σ^* the respective antitriplets and we have omitted SU(3) indices.

Let us assume the following vacuum expectation values (VEVs):

$$\langle\sigma_1^0\rangle \neq 0,\ \langle\sigma_2^0\rangle = 0, \quad \langle{\sigma'}_1^0\rangle = 0,\ \langle{\sigma'}_2^0\rangle \neq 0; \tag{2.5}$$

and we also assume that the mass scale characteristic of the SU(3) symmetry is rather high:

$$\langle{\sigma'}_2^0\rangle \gg \langle\sigma_1^0\rangle, \langle\eta^0\rangle. \tag{2.6}$$

Before going on, let us consider the neutral currents coupled to Z^0. We must determine which fields have the same couplings than in the SU(2)⊗U(1) effective theory i.e., when the condition in Eq. (2.6) is satisfied. The photon field is

$$A_\mu = s_W\left(W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8\right) + \frac{1}{\sqrt{3}}\left(3 - 4s_W^2\right)^{\frac{1}{2}} B_\mu, \tag{2.7a}$$

while the massive neutral bosons are

$$Z_\mu = c_W\, W^3_\mu - \frac{1}{\sqrt{3}} \tan\theta_W \left[s_W\, W^8_\mu + (3-4s^2_W)^{\frac{1}{2}} B_\mu \right], \tag{2.7b}$$

which correspond to the usual Z^0, and the heaviest one is given by

$$Z'_\mu = \frac{1}{\sqrt{3}c_W} \left[-(3-4s^2_W)^{\frac{1}{2}} W^8_\mu + s_W\, B_\mu \right], \tag{2.7c}$$

where $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, and θ_W is the usual weak mixing angle. From the electric charge definition we get

$$\left(\frac{g'}{g}\right)^2 = \frac{3s^2_W}{3-4s^2_W}, \tag{2.8}$$

hence, $\sin^2\theta_W < 3/4$ [6].

The neutral current interactions of fermions (ψ_i) can be written as usual

$$\begin{aligned}\mathcal{L}^{\rm NC} &= -\frac{g}{2\cos\theta_W} \left[\sum_i L_i \bar\psi_{iL}\gamma^\mu\psi_{iL} + R_i \bar\psi_{iR}\gamma^\mu\psi_{iR} \right] Z_\mu \\ &= -\frac{g}{2\cos\theta_W} \sum_i \bar\psi_i\gamma^\mu(g_{Vi} - \gamma^5 g_{Ai})\psi_i Z_\mu, \end{aligned} \tag{2.9}$$

where $g_{Vi} \equiv \frac{1}{2}(L_i + R_i)$ and $g_{Ai} \equiv \frac{1}{2}(L_i - R_i)$. For Z^0, we obtain for the charge $-1/3$ sector

$$L_{d_1} = L_{d_2} = L_{d_3} = -1 + \frac{2}{3}\, s^2_W, \quad L_{d_4} = \frac{2}{3}\, s^2_W, \tag{2.10a}$$

$$R_{d_1} = R_{d_2} = R_{d_3} = R_{d_4} = \frac{2}{3}\, s^2_W. \tag{2.10b}$$

Similarly, for the charge 2/3 sector

$$L_{u_1} = L_{u_2} = L_{u_3} = 1 - \frac{4}{3}\, s^2_W, \quad L_{u'_1} = L_{u'_2} = -\frac{4}{3}\, s^2_W, \tag{2.11a}$$

$$R_{u_1} = R_{u_2} = R_{u_3} = R_{u'_1} = R_{u'_2} = -\frac{4}{3}\, s^2_W. \tag{2.11b}$$

From Eqs. (2.10) we see that d_1, d_2, and d_3 have the same couplings to the Z^0 as the d, s, b quarks in the standard electroweak model, but d_4 has a pure vector coupling. In the charge 2/3 sector we observe from Eqs. (2.11) that u_1, u_2, and u_3 have the same couplings of the usual u, c, t quarks in the standard model, while u'_1, u'_2 have pure vector couplings to the Z^0. The mass eigenstates will be denoted by u, c, t, t', t'' and d, s, b, b'. Hence, if we avoid a general mixing in the mass matrix we will implement a GIM mechanism [12] in the model.

Finally, for leptons we have

$$L_{\nu_a} = 1, \quad L_{l_a} = -1 + 2\, s^2_W, \quad L_{E_a} = 2\, s^2_W, \tag{2.12a}$$

$$R_{\nu_a} = 0, \quad R_{l_a} = R_{E_a} = 2\, s^2_W. \tag{2.12b}$$

We see that neutrinos and e^-, μ^-, τ^- have the same couplings to the Z^0 than in the SU(2)⊗U(1) model. The heavy leptons E_a have vector-like couplings. Lepton couplings with the Z'^0 conserve flavor in each sector: ν_a, l^-_a, and E^-_a. This is not a surprise since lepton generations are treated democratically.

Knowing the neutral current couplings, given in Eqs. (2.10) and (2.11), in order to avoid a general mixing in the mass matrices, we will introduce the following discrete symmetries:

$$\begin{aligned} &d_{1,2,3R} \to d_{1,2,3R};\ d_{4R} \to -d_{4R}, \\ &u_{1,2,3R} \to u_{1,2,3R};\ u'_{1,2R}, \to -u'_{1,2R}, \end{aligned} \tag{2.13a}$$

$$Q_{1,2,3L} \to Q_{1,2,3L}, \quad \eta, \sigma \to \eta, \sigma, \quad \sigma' \to -\sigma', \tag{2.13b}$$

$$\Psi_{aL} \to \Psi_{aL}, \quad l^-_{aR} \to l^-_{aR}, \quad E^-_{aR} \to -E^-_{aR}. \tag{2.13c}$$

From Eq. (2.4) the quark mass matrices

$$\bar{\mathcal{D}}_{\alpha L} M^D_{\alpha\alpha'} \mathcal{D}_{\alpha' R}, \quad \bar{\mathcal{U}}_{\beta L} M^U_{\beta\beta'} \mathcal{U}_{\beta' R}, \tag{2.14}$$

with the discrete symmetries in Eq. (2.13) become

$$M^U = \begin{pmatrix} M^U_1 & 0 \\ 0 & M^U_2 \end{pmatrix}, \tag{2.15}$$

for the charge 2/3 sector, and

$$M^D = \begin{pmatrix} M^D_1 & 0 \\ 0 & m_{d_4} \end{pmatrix}, \quad m_{d_4} = E'_{34}\langle \sigma'^0_2 \rangle \tag{2.16}$$

for the charge $-1/3$ one. M^U_1 and M^D_1 are arbitrary 3×3 matrices in the basis u_1, u_2, u_3 and d_1, d_2, d_3, respectively; M^U_2 is an arbitrary 2×2 matrix in the basis u'_1, u'_2. We see that d_4 does not mix with the other quarks of charge $-1/3$. The mass matrices in Eqs. (2.14)–(2.16) can be diagonalized. In terms of the mass eigenstates they become

$$\bar D_L \hat M^D D_R, \quad \bar U_L \hat M^U U_R, \tag{2.17}$$

where $\hat M^U = \mathrm{diag}(m_u, m_c, m_t, m_{t'}, m_{t''})$, and $\hat M^D = \mathrm{diag}(m_d, m_s, m_b)$ and U and D denote (u, c, t, t', t'') and (d, s, b, b'), respectively.

The mass matrices in Eqs. (2.15) and (2.16) are diagonalized by performing the transformations

$$\begin{aligned} \mathcal{U}_L = V^U_L U_L, \quad \mathcal{U}_R = V^U_R U_R, \\ \mathcal{D}_L = V^D_L D_L, \quad \mathcal{D}_R = V^D_R D_R, \end{aligned} \tag{2.18}$$

with

$$V^U_L = \begin{pmatrix} V^U_{1L} & 0 \\ 0 & V^U_{2L} \end{pmatrix}, \quad V^U_{1R} = \begin{pmatrix} V^U_{1R} & 0 \\ 0 & V^U_{2R} \end{pmatrix}, \tag{2.19a}$$

$$V_L^D = \begin{pmatrix} V_{1L}^D & 0 \\ 0 & 1 \end{pmatrix}, \quad V_R^U = \begin{pmatrix} V_{1R}^D & 0 \\ 0 & 1 \end{pmatrix}. \qquad (2.19b)$$

$V_{1L,R}^D$, $V_{1L,R}^U$ are unitary 3×3 matrices and $V_{2L,R}^U$ 2×2 ones. By considering the interactions in the quark sector we will see which of the matrices in Eqs.(2.19) survive in the Lagrangian.

The vector boson-quark interactions are

$$\mathcal{L}_q = -\frac{g}{\sqrt{2}} \times \Big(-\bar{u}'_{iL}\gamma^\mu u_{iL} X_\mu^0 - \bar{u}'_{iL}\gamma^\mu d_{iL} V_\mu^+ - \bar{u}_{iL}\gamma^\mu d_{iL} W_\mu^+ + \bar{u}_{3L}\gamma^\mu d_{3L} W_\mu^+ + \bar{u}_{3L}\gamma^\mu d_{4L} V_\mu^+ + \bar{d}_{3L}\gamma^\mu d_{4L} X_\mu^0 \Big) + \text{H.c.} \qquad (2.20)$$

In particular, the interaction with the W^+ boson can be written as usual with the mixing matrix defined as $V_{KM} = V_L^{U\dagger} V_L^D$. On the other hand, we have the currents coupled to the V_μ^+:

$$\mathcal{L}_{qV}^{CC} = -\frac{g}{\sqrt{2}} (\bar{u}_1\, \bar{u}_2\, \bar{u}_3\, \bar{u}'_1\, \bar{u}'_2)_L \Delta \gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ 0 \end{pmatrix}_L V_\mu^+ + \text{H.c.}, \qquad (2.21)$$

where Δ is a 5×5 matrix with $\Delta_{41} = \Delta_{51} = -\Delta_{34} = 1$ and all other elements vanish. In terms of the mass eigenstates we can write Eq. (2.21) as

$$\mathcal{L}_{qV}^{CC} = -\frac{g}{\sqrt{2}} (\bar{u}\ \ \bar{c}\ \ \bar{t}\ \ \bar{t}'\ \ \bar{t}'')_L V_L^{U\dagger} \Delta V_L^D \gamma^\mu \begin{pmatrix} d \\ s \\ b \\ b' \\ 0 \end{pmatrix}_L V_\mu^+ + \text{H.c.}, \qquad (2.22)$$

V_L^U, V_L^D being the matrices in Eqs. (2.19a) and (2.19b). [Here V_L^D is a 5×5 extension, through zeros on the fifth line and row, of the matrix in Eq. (2.19b).] In these sorts of models it is not interesting to define V_L^U as being the unit matrix as it is usually done in the SU(2)⊗U(1) model. This is because this matrix appears also in the neutral currents with the extra Z'^0 present in the model [10]. So we do not assume that the charge 2/3 mass eigenstates appear unmixed. There are similar interactions with the X^0 boson but in this case there are mixture involving matrix elements of V_{1L}^U and V_{2L}^U in the charge 2/3 sector and V_{1L}^D in the charge $-1/3$ sector.

In the lepton sector, neutrinos and the usual charged leptons have the same couplings to the W^+ boson as in the SU(2)⊗U(1) model,

$$\mathcal{L}_l = -\frac{g}{2}\sum_a \Big[\bar{\nu}_{aL}\gamma^\mu l_{aL}^- W_\mu^+ + \bar{\nu}_{aL}\gamma^\mu E_{aL}^- V_\mu^+ + \bar{l}_{aL}\gamma^\mu E_{aL}^- X_\mu \Big] + \text{H.c.} \qquad (2.23)$$

The charged leptons get a mass via the interaction with the σ^* and σ'^* scalars. With discrete symmetries in Eq. (2.13c) the leptonic Yukawa interactions are

$$-\mathcal{L}_{lY} = \sum_{ab} \bar{\Psi}_{aL} \left[h_{ab} l_{bR} \sigma^* + h'_{ab} E_{bR} \sigma'^* \right] + \text{H.c.}, \quad (2.24)$$

h_{ab}, h'_{ab} are arbitrary 3×3 matrices and neutrinos remain massless if right-handed neutrinos are not introduced. We can define the neutrino fields in such a way that there is not mixing in the W^+ interactions but there are mixings in the V^+, X^0 ones. There is not mixing among l_a^- and E_a^- in the mass matrix since the discrete symmetries in (2.13c) forbid it. So there is not flavor violation in Higgs-boson couplings too.

B. Model B

As we said before, this is an extended version of the model proposed some years ago by Georgi and Pais [7]. Here we have to consider a third quark generation since today there is evidence of the existence of a t quark [1]. However, as we will see later, the phenomenology of this model is rather different from that of Georgi and Pais's model in the quark sector. Also, altough both models have the same lepton sector we will allow a general mixing in the charged lepton sector.

Let us first consider leptons. This is the same of Ref. [7] with four antitriplets $(\mathbf{3}^*, -1/3)$:

$$\Psi_{eL}: \begin{pmatrix} e^- \\ \nu_e \\ \nu_e^c \end{pmatrix}_L, \quad \Psi_{\mu L}: \begin{pmatrix} \mu^- \\ \nu_\mu \\ \nu_\mu^c \end{pmatrix}_L;$$

$$\Psi_{\tau L}: \begin{pmatrix} \tau^- \\ \nu_\tau \\ \nu_\tau^c \end{pmatrix}_L, \quad \Psi_{TL}: \begin{pmatrix} T^- \\ \nu_T \\ \nu_T^c \end{pmatrix}_L, \qquad (2.25a)$$

two other antitriplets with $(\mathbf{3}^*, 2/3)$

$$\Psi'_{eL}: \begin{pmatrix} N_1^0 \\ \tau^+ \\ e^+ \end{pmatrix}_L, \quad \Psi'_{\mu L}: \begin{pmatrix} N_2^0 \\ T^+ \\ \mu^+ \end{pmatrix}_L, \qquad (2.25b)$$

and the neutral singlets $(N_{1L}^0)^c, (N_{2L}^0)^c$.

The quark fields of the first two generations (suppressing the color indices) are in two left-handed triplets $(\mathbf{3}, 0)$

$$Q_{iL} = \begin{pmatrix} u_i \\ d_i \\ d'_i \end{pmatrix}_L, \quad i = 1, 2 \qquad (2.26)$$

and the right-handed components in singlets $u_{iR} \sim (\mathbf{1}, 2/3)$ and $d_{iR}, d'_{iR} \sim (\mathbf{1}, -1/3)$. Finally, the third quark generation transforms in a vector-like representation. We can choose $(\mathbf{3},0)_{L+R}$, and the model has three charge 2/3 and six charge $-1/3$ quarks. Or

$$Q_{3L}=\begin{pmatrix} d_3 \\ u_3 \\ u_4 \end{pmatrix}_L \sim (\mathbf{3}^*,1/3),\qquad Q_{3R}=\begin{pmatrix} d_3 \\ u_3 \\ u_4 \end{pmatrix}_R \sim (\mathbf{3}^*,1/3), \tag{2.27}$$

with four charge 2/3 and five charge $-1/3$ quarks. In this work we shall treat in detail only the last one. Besides the scalar fields in Eq. (2.3) we introduce σ'' which transforms like σ and σ' and has the VEV like the last one, finally a singlet neutral scalar ϕ with VEV $\langle\phi\rangle \neq 0$. It is straightforward to verify that σ' and σ'' (and obviously ϕ) do not contribute to the mass of the $W^\pm$ bosons so they are not constrained by this parameter. Thus, we can assume the following hierarchy:

$$\langle{\sigma''}^0_2\rangle \gg \langle{\sigma'}^0_2\rangle > 246\ \text{GeV} > \langle\eta^0\rangle, \langle\sigma^0_1\rangle,\quad \langle\eta^0\rangle^2+\langle\sigma^0_1\rangle^2 \approx (246\ \text{GeV})^2. \tag{2.28}$$

The scalar σ'', which breaks the SU(3) symmetry, will not couple to the fermions.

As we said before, the gauge bosons are the same of model A. In particular, the neutral ones are given in Eqs. (2.7). Thus, in this model we get the neutral current couplings defined in Eq. (2.9), for the case of Z^0

$$L_{d_1}=L_{d_2}=L_{d_3}=-1+\frac{2}{3}s^2_W,\quad L_{d'_1}=L_{d'_2}=\frac{2}{3}s^2_W, \tag{2.29a}$$

$$R_{d_1}=R_{d_2}=\frac{2}{3}\sin^2\theta_W,\quad R_{d_3}=-1+\frac{2}{3}s^2_W,\quad R_{d'_1}=R_{d'_2}=\frac{2}{3}s^2_W, \tag{2.29b}$$

for the charge $-1/3$ quarks. For the charge 2/3 sector we have

$$L_{u_1}=L_{u_2}=L_{u_3}=1-\frac{4}{3}s^2_W,\quad L_{u_4}=-\frac{4}{3}s^2_W, \tag{2.30a}$$

$$R_{u_1}=R_{u_2}=-\frac{4}{3}s^2_W,\quad R_{u_3}=1-\frac{4}{3}s^2_W,\quad R_{u_4}=-\frac{4}{3}s^2_W \tag{2.30b}$$

and also for leptons

$$L_{\nu_a}=1,\quad L_{l_a}=-1+2s^2_W,\quad L_{N_1}=L_{N_2}=-1, \tag{2.31a}$$

$$R_{\nu_a}=0,\ R_e=R_\mu=2s^2_W,\quad R_\tau=R_T=-1+2s^2_W,\ R_{N_1}=R_{N_2}=0, \tag{2.31b}$$

where $\nu_a=\nu_e,\nu_\mu,\nu_\tau,\nu_T$ and $l_a=e,\mu,\tau,T$. We see that neutrinos, electron, and muon have the same couplings than in the standard model, N_i have right-handed couplings, and the lepton τ and T have both only vector couplings. We will return to this point later on.

The neutral current coefficients in Eqs. (2.29), (2.30), and (2.31) suggest the following discrete symmetries:

$$\eta,\sigma\to\eta,\sigma;\quad \sigma',\phi\to-\sigma',-\phi; \tag{2.32a}$$

$$d_{1,2R}\to d_{1,2R},\quad d'_{1,2R}\to -d'_{1,2R},\quad u_{1,2R}\to u_{1,2R}; \tag{2.32b}$$

$$Q_{1,2,3L}\to Q_{1,2,3L},\quad Q_{3R}\to -Q_{3R}. \tag{2.32c}$$

For σ'' we assume $\sigma''\to i\sigma''$, which ensures that σ'' does not couple to fermions.

Hence, the most general, compatible with the symmetries (2.32), Yukawa couplings are

$$-\mathcal{L}_{qY}=\sum_{i,j}\bar{Q}_{iL}\left[A_{ij}u_{jR}\eta^*+B_{ij}\,d_{jR}\,\sigma^*\right]+\sum_{i,\alpha}\bar{Q}_{iL}\,B'_{i\alpha}\,d'_{\alpha R}\,\sigma'^*+\lambda\,\bar{Q}_{3L}\,Q_{3R}\,\phi + \sum_i\left[\epsilon\,\lambda_i\bar{Q}_{iL}\,Q_{3R}\,\sigma'+h_{3i}\,\bar{Q}_{3L}\,u_{iR}\,\sigma+k_{3i}\bar{Q}_{3L}\,d_{iR}\,\eta\right]+\text{H.c.}, \tag{2.33}$$

where $i,j,\alpha=1,2$; η^*,σ^* are the respective antitriplets; and ϵ is the completely antisymmetric SU(3) tensor.

From Eq. (2.33) we obtain, with the VEVs as in Eq. (2.5), the following mass matrices: one 4×4 matrix for the charge 2/3 sector and one 5×5 for the charge $-1/3$ sector. Explicitly, in the d_1,d_2,d_3,d'_1,d'_2 basis we get

$$M^D=\begin{pmatrix} B_{11}\langle\sigma_1\rangle & B_{12}\langle\sigma_1\rangle & -\lambda_1\langle\sigma'_2\rangle & 0 & 0\\ B_{21}\langle\sigma_1\rangle & B_{22}\langle\sigma_1\rangle & -\lambda_2\langle\sigma'_2\rangle & 0 & 0\\ k_{31}\langle\eta\rangle & k_{32}\langle\eta\rangle & \lambda\langle\phi\rangle & 0 & 0\\ 0 & 0 & 0 & B'_{11}\langle\sigma'_2\rangle & B'_{12}\langle\sigma'_2\rangle\\ 0 & 0 & 0 & B'_{21}\langle\sigma'_2\rangle & B'_{22}\langle\sigma'_2\rangle \end{pmatrix}. \tag{2.34}$$

On the other hand, in the charge 2/3 sector and in the u_1, u_2, u_3, u_4 basis we get

$$M^U = \begin{pmatrix} A_{11}\langle\eta\rangle & A_{12}\langle\eta\rangle & \lambda_1\langle\sigma_2'\rangle & 0 \\ A_{21}\langle\eta\rangle & A_{22}\langle\eta\rangle & \lambda_2\langle\sigma_2'\rangle & 0 \\ h_{31}\langle\sigma_1\rangle & h_{32}\langle\sigma_1\rangle & \lambda\langle\phi\rangle & 0 \\ 0 & 0 & 0 & \lambda\langle\phi\rangle \end{pmatrix}. \tag{2.35}$$

Since the VEV $\langle\phi\rangle$, does not contribute to the vector boson masses, it is not constrained but is not necessarily very large. We see that both mass matrices are of the direct sum form and there is no mixing of the d_1, d_2, d_3 fields with the d_1', d_2' ones; and u_4 also does not couple to u_1, u_2, u_3.

Hence, the diagonalization of the mass matrices is done using the unitary matrices of the direct sum form

$$V_L^D = \begin{pmatrix} V_{1L}^D & 0 \\ 0 & V_{2L}^D \end{pmatrix}, \quad V_R^D = \begin{pmatrix} V_{1R}^D & 0 \\ 0 & V_{2R}^D \end{pmatrix}, \tag{2.36}$$

$$V_L^U = \begin{pmatrix} V_{1L}^U & 0 \\ 0 & 1 \end{pmatrix}, \quad V_R^U = \begin{pmatrix} V_{1R}^D & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.37}$$

where $V_{1L,1R}^D, V_{1L,1R}^U$ are 3×3 and $V_{2L,2R}^D$ 2×2 unitary matrices. Another possibility in order to give a correct mass to the quarks is to introduce a $(\mathbf{6}^*, -1/3)$ Higgs multiplet.

From Eqs. (2.34) and (2.35) it is possible to obtain the right mass spectrum for quarks, including a heavy top quark. However, we recall that the value $m_t \simeq 174$ GeV is obtained assuming the standard model decay $t \to W^+ b$. In fact, without the SM assumption the lower limit on the top mass is 62 GeV [13]. In the present model there are several new induced top decays, say by neutral and charged scalars, and this implies that if the experimental result is interpreted in terms of this model, the top quark is not necessarily too heavy. In fact, it has been suggested recently that the mass region of interest can be [14]

$$M_Z/2 \lesssim m_t \lesssim M_W + m_b, \tag{2.38}$$

but even this region may be relaxed if there is an extra charge 2/3 quark like t'. In particular, models with decays such as $t \to ch^0$ and $t \to bH^+$ may allow m_t in the range of Eq. (2.38). Theoretical analysis [14] of this situation has been done in terms of two Higgs doublets non-SUSY extensions of the standard model. Experimental searches for $t \to H^+ b$ have been interpreted in several two-Higgs-doublet extensions [13].

Finally, let us consider the lepton-scalar couplings. Instead of assuming a global symmetry which prevents the transitions μ and T into e and τ, as in Ref. [7], we will allow a general mixing among all charged leptons. Hence, the Yukawa interactions are

$$\sum_{ab,x} \epsilon\,\Gamma^x_{ab} \overline{(\Psi_{aL})^c}\, \Psi'_{bL} \sigma^x + \sum_{bi} H_{bi} \overline{\Psi'_{bL}}\, N^0_{iR} \eta + \sum_{ai,x} h^x_{ai} \bar{\Psi}_{aL} N_{iR} \sigma^x + \text{H.c.}, \tag{2.39}$$

where $a = e, \tau, \mu, T$; $b = e, \mu$ and $i = 1, 2$; the superscript x denotes σ or σ' and their couplings h, Γ and h', Γ', respectively. Here Ψ^c is the charge conjugated field. From Eq. (2.39) we can verify that a general 4×4 mass matrix arise in the charged lepton sector. We will assume that

$$H_{bi}\langle\eta^0\rangle \gg \Gamma_{ab}\langle\sigma_1^0\rangle, \Gamma'_{ab}\langle\sigma'^0_2\rangle, \tag{2.40}$$

$$h_{ai}\langle\sigma_1^0\rangle, h'_{ai}\langle\sigma_2'^0\rangle,$$

which allows us to neglect the mixing of the ν fields with the massive $N_{1,2}$ ones. It also implies that $N_{1,2}$ are heavier than the charged leptons.

In this model charged and non-Hermitian neutral currents are as follows:

$$\mathcal{L}_q = -\frac{g}{\sqrt{2}} \Bigg[\sum_i \left(\bar{u}_{iL}\gamma^\mu d_{iL} W^+_\mu + \bar{u}_{iL}\gamma^\mu d'_{iL} V^+_\mu + \bar{d}_{iL}\gamma^\mu d'_{iL} X^0_\mu \right) - \bar{u}_{4L}\gamma^\mu u_{3L} X^0_\mu - \bar{u}_{4L}\gamma^\mu d_{3L} V^+_\mu$$
$$- \bar{u}_{3L}\gamma^\mu d_{3L} W^+_\mu - \bar{u}_{4R}\gamma^\mu u_{3R} X^0_\mu - \bar{u}_{4R}\gamma^\mu d_{3R} V^+_\mu - \bar{u}_{3R}\gamma^\mu d_{3R} W^+_\mu \Bigg] + \text{H.c.} \tag{2.41}$$

for quarks, and

$$\mathcal{L}_l = -\frac{g}{\sqrt{2}} \Bigg[\sum_a \left(\bar{\nu}^c_{aL}\gamma^\mu \nu_{aL} X^0_\mu + \bar{\nu}^c_{aL}\gamma^\mu l^-_{aL} V^+_\mu + \bar{\nu}_{aL}\gamma^\mu l^-_{aL} W^+_\mu \right) + \bar{e}^+_L\gamma^\mu \tau^+_L X^0_\mu + \bar{e}^+_L\gamma^\mu N^0_{1L} V^+_\mu$$
$$+ \bar{\tau}^+_L\gamma^\mu N^0_{1L} W^+_\mu + \bar{\mu}^+_L\gamma^\mu T^+_L X^0_\mu + \bar{\mu}^+_L\gamma^\mu N^0_{2L} V^+_\mu + \bar{T}^+_L\gamma^\mu N^0_{2L} W^+_\mu \Bigg] + \text{H.c.} \tag{2.42}$$

for leptons. As long as neutrinos remain massless there is no mixing in the charged currents coupled to W^+. However, there is mixing in the currents coupled to V^+ and X^0. Notice that the right-handed currents coupled to W^+ involve the charged leptons τ and T and the heavy neutral fermions $N_{1,2}$. After neutrinos getting mass through radiative corrections, mixing will appear in the interactions with W^+. Recall that the scalar η_2^+ is an SU(2) singlet and there are three SU(2) doublets in the scalar multiplets given in Eq. (2.3), hence the Zee mechanism for generating neutrino masses may be implemented in this model [15].

The charged currents coupled to the W^+ boson can be written in the quark sector as

$$\mathcal{L}^{CC}_{qW} = -\frac{g}{\sqrt{2}}(\bar{u}_1\,\bar{u}_2\,\bar{u}_3)_L\gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L W^+_\mu - \frac{g}{\sqrt{2}}\bar{u}_{3R}\gamma^\mu d_{3R}W^+_\mu + \text{H.c.} \tag{2.43}$$

The left-handed currents in Eq. (2.43) can be parametrized in terms of mass eigenstates and Kobayashi-Maskawa mixing matrix, $V_{\rm KM} = V^{U\dagger}_L V^D_L$. The right-handed current in Eq. (2.43) can be written in terms of the mass eigenstates:

$$\mathcal{L}^{CC}_{qWR} = -\frac{g}{\sqrt{2}}(\bar{u}\;\bar{c}\;\bar{t})_R\gamma^\mu V^{U\dagger}_{1R}\tilde{\Delta}\,V^D_{1R}\begin{pmatrix} d \\ s \\ b \end{pmatrix}_R W^+_\mu + \text{H.c.} \tag{2.44}$$

where $\tilde{\Delta} = \text{diag}(0,0,1)$. The other matrices V^U_L, V^D_L also survive in the interactions involving the bosons V^+ and X^0.

Notice that, besides the neutral currents coupled to the Z^0 given in Eq. (2.9), we have additional right-handed couplings $[\bar{u}_{3R}\gamma^\mu u_{3R} - \bar{d}_{3R}\gamma^\mu d_{3R}]Z^0_\mu$, or, written in terms of the mass eigenstates

$$\mathcal{L}''^{\rm NC} = \mathcal{L}^{\rm NC} + \mathcal{L}'^{\rm NC}, \tag{2.45}$$

with $\mathcal{L}^{\rm NC}$ being parametrized like in Eq. (2.9) and

$$\mathcal{L}'^{\rm NC}_U = -\frac{g}{2\cos\theta_W}(\bar{u}\;\bar{c}\;\bar{t})_R\gamma^\mu V^{U\,\dagger}_{1R}\tilde{\Delta}\,V^U_{1R}\begin{pmatrix} u \\ c \\ t \end{pmatrix}_R Z^0_\mu, \tag{2.46}$$

for the charge 2/3 sector,

$$\mathcal{L}'^{NC}_D = +\frac{g}{2\cos\theta_W}(\bar{d}\;\bar{s}\;\bar{b})_R\gamma^\mu V^{D\,\dagger}_{1R}\tilde{\Delta}\,V^D_{1R}\begin{pmatrix} d \\ s \\ b \end{pmatrix}_R Z^0_\mu, \tag{2.47}$$

for the charge $-1/3$ sector. There are similar currents coupled to the V^+, X^0, Z'^0 bosons.

Although Eqs. (2.46) and (2.47) are FCNC, all flavors have the same dependence on the weak mixing angle. This is not the case of the neutral currents coupled to Z'^0; here each flavor has a different dependence on that angle.

In the leptonic sector we have the GIM mechanism at the tree level in the left-handed neutral currents coupled to the Z^0 and Z'^0. However, there are FCNC in the neutral right-handed currents. Hence, in terms of the mass eigenstates e, μ, τ, T we have the following interactions:

$$\mathcal{L}^{\rm NC}_{lR} = +\frac{g}{2\cos\theta_W}(\bar{e}\;\bar{\mu}\;\bar{\tau}\;\bar{T})_R\gamma^\mu V^{l\dagger}_R Y^l_R V^l_R \begin{pmatrix} e \\ \mu \\ \tau \\ T \end{pmatrix}_R Z_\mu\,, \tag{2.48}$$

where V^l_R is the 4×4 right-handed mixing matrix in the charged lepton sector and we have introduced $Y^l_R = \text{diag}(2s^2_W, 2s^2_W, -1+2s^2_W, -1+2s^2_W)$. We see that after diagonalizing the mass matrix of the charged leptons, the right-handed mixing matrix survives in Eq. (2.48). The value of this new mixing parameters can be chosen in such a way that the model be consistent with the measurements of τ asymmetries at LEP. In fact, it has been measured as the ratio of vector to axial-vector neutral couplings and it has been found to be consistent with the hypothesis of $e-\tau$ universality [16,17]. Thus at the tree level we can impose

$$(V^{l\dagger}_R Y^l_R V^l_R)_{ee} \approx (V^{l\dagger}_R Y^l_R V^l_R)_{\mu\mu} \approx (V^{l\dagger}_R Y^l_R V^l_R)_{\tau\tau} \approx 2s^2_W, \tag{2.49}$$

with the other elements of the matrix $V^{l\dagger}_R Y^l_R V^l_R$ to be set up by experiments. Notice that the charged currents of the neutrinos ν_a with all charged leptons are purely left-handed as it can be seen in Eq. (2.42). There are measurements of the τ-neutrino helicity [18] and Michel parameters [19] which confirm a dominant V-A structure in the charged current for the τ and its neutrino.

Mixing among the charged leptons appears in the current coupled to X^0 and those coupled to V^+ induce transitions $l_a \leftrightarrow N_i$ which are sensible on the Cabibbo-like mixing in the charged leptons.

In this model there are additional neutral leptons $N^0_{1,2}$. We have seen that they are heavier than the charged leptons. In fact, they must be heavy enough not to contribute to the invisible Z^0 width [17].

Notice that the quark mass matrices in model B are different from those of the model of Ref. [7]. We can see the discrete symmetries in Eqs. (2.33) only as an indication of which ones are the dominant mixings. Eventually, we can allow them to be broken.

We can also build a model in which there are two quark generations transforming as $(\mathbf{3}^*, 1/3)_L$ and one as $(\mathbf{3}, 0)_{L+R}$. In this case there are two leptonic antitriplets $(\mathbf{3}, 1/3)$ and four ones transforming as $(\mathbf{3}, -2/3)$. In this case it is necessary, however, to include right-handed charged leptons in singlets.

III. MODELS WITH EXTENDED COLOR AND ELECTROWEAK SECTORS

A. Models with extended electroweak sector

First, let us consider $n = 3$ models. When $m = 2, 3$ we have the standard model and the 3-3-1 models, respectively. Next, there is a 3-4-1 model in which the electric charge operator is defined as

$$Q=\frac{1}{2}\left(\lambda_3-\frac{1}{\sqrt{3}}\lambda_8-\frac{2}{3}\sqrt{6}\lambda_{15}\right)+N, \tag{3.1}$$

where the λ matrices are [20]

$$\lambda_3=\mathrm{diag}(1,-1,0,0),\ \lambda_8=\left(\frac{1}{\sqrt{3}}\right)\mathrm{diag}(1,1,-2,0),$$

$$\lambda_{15}=\left(\frac{1}{\sqrt{6}}\right)\mathrm{diag}(1,1,1,-3).$$

Leptons transform as $(\mathbf{1},\mathbf{4},0)$, two of the three quark families, say Q_{iL}, $i=1,2$, transform as $(\mathbf{3},\mathbf{4}^*,-1/3)$, and one family, Q_{3L}, transforms as $(\mathbf{3},\mathbf{4},+2/3)$:

$$\psi_{aL}=\begin{pmatrix}\nu_a\\ l_a\\ \nu_a^c\\ l_a^c\end{pmatrix}_L,$$

$$Q_{iL}=\begin{pmatrix}d_i\\ u_i\\ d_i'\\ j_i\end{pmatrix}_L,\qquad Q_{3L}=\begin{pmatrix}u_3\\ d_3\\ u_3'\\ J\end{pmatrix}_L, \tag{3.2}$$

where u_3' and J are new quarks with charge $+2/3$ and $+5/3$, respectively; j_i and d_i', $i=1,2$ are new quarks with charge $-4/3$ and $-1/3$, respectively. We remind the reader that in Eq. (3.2) all fields are still symmetry eigenstates. Right-handed quarks transform as singlets under SU(4).

A model with $\mathrm{SU}(4)_L$ symmetry and leptons transforming as in Eq. (3.2) was proposed by Voloshin some years ago [21]. In this context it can be possible to understand the existence of neutrinos with large magnetic moment and small mass. However, in Ref. [21] the quark sector was not considered.

Quark masses are generated by introducing the following Higgs $\mathrm{SU}(4)_L\otimes\mathrm{U}(1)_N$ multiplets: $\chi\sim(\mathbf{4},-1),\rho\sim(\mathbf{4},+1)$, η and $\eta'\sim(\mathbf{4},0)$.

In order to obtain massive charged leptons it is necessary to introduce a $(\mathbf{10}^*,0)$ Higgs multiplet because the lepton mass term transforms as $\bar{\psi}_L^c\psi_L\sim(\mathbf{6}_A\oplus\mathbf{10}_S)$. The $\mathbf{6}_A$ will leave some leptons massless and some others mass degenerate. Therefore we will choose $H=\mathbf{10}_S$. Neutrinos remain massless at least at the tree level but the charged leptons gain mass. The corresponding VEVs are $\langle\eta\rangle=(v,0,0,0)$, $\langle\rho\rangle=(0,w,0,0)$, $\langle\eta'\rangle=(0,0,v',0)$, $\langle\chi\rangle=(0,0,0,u)$, and $\langle H\rangle_{42}=v''$ for the decuplet. In this way the symmetry breaking of the $\mathrm{SU}(4)_L\otimes\mathrm{U}(1)_N$ group down to $\mathrm{SU}(3)_L\otimes\mathrm{U}(1)_{N'}$ is induced by the χ Higgs. The $\mathrm{SU}(3)_L\otimes\mathrm{U}(1)_{N'}$ symmetry is broken down into $\mathrm{U}(1)_{em}$ by the ρ,η,η' and H Higgs bosons. As in the models of Sec. II, it is necessary to introduce some discrete symmetries which ensure that the Higgs fields give a quark mass matrix in the charge $-1/3$ and $2/3$ sectors of the direct sum form in order to avoid general mixing among quarks of the same charge.

In fact, we have the symmetry breaking pattern, including the SU(3) of color,

$$\begin{array}{c}\mathrm{SU}(3)_C\otimes\mathrm{SU}(4)_L\otimes\mathrm{U}(1)_N\\ \downarrow\langle\chi\rangle\\ \mathrm{SU}(3)_C\otimes\mathrm{SU}(3)_L\otimes\mathrm{U}(1)_{N'}\\ \downarrow\langle\eta'\rangle\\ \mathrm{SU}(3)_C\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_{N''}\\ \downarrow\langle x\rangle\\ \mathrm{SU}(3)_C\otimes\mathrm{U}(1)_{em}\end{array} \tag{3.3}$$

where $\langle x\rangle$ means $\langle\rho\rangle$, $\langle\eta\rangle$, $\langle H\rangle$ [11].

The electroweak gauge bosons of this theory consist of a $\mathbf{15}$ W_μ^i, $i=1,\ldots,15$ associated with $\mathrm{SU}(4)_L$ and a singlet B_μ associated with $\mathrm{U}(1)_N$. There are four neutral bosons: a massless γ and three massive ones: Z,Z',Z''. The lightest one, say the Z, corresponds to the Weinberg-Salam-Glashow neutral boson. Assuming the approximation $u\gg v'\gg v,v'',w$ the extra neutral bosons, say Z',Z'', have masses which depend mainly on u,v'.

Concerning the charged vector bosons, as in the model of Ref. [4] there are doubly charged vector bosons and doublets of SU(2) (X_μ^+,X^0) and $(\bar{X}_\mu^0,X_\mu^-)$, which produce interactions like $\bar{\nu}_{aL}^c\gamma^\mu l_{aL}X_\mu^+$ and $\bar{\nu}_{aL}^c\gamma^\mu\nu_{aL}X_\mu^0$, as in model I of Ref. [6]. We have also the $V_{1,2}^{\pm}$ vector bosons with interactions like $\bar{l}_{aL}^c\gamma^\mu\nu_{aL}V_{1\mu}^+$ and $\bar{l}_{aL}^c\gamma^\mu\nu_{aL}^cV_{2L}^+$. All charged currents, including ones coupled with quarks, are given in Ref. [11].

B. $n=4,5$ models

Let us consider $n=4,5$ models. Although the $\mathrm{SU}(3)_C$ gauge symmetry is the best candidate for the theory of the strong interactions, there is no fundamental reason why the colored gauge group must be $\mathrm{SU}(3)_C$. In fact, it is possible to consider other Lie groups. In general we have the possibilities $\mathrm{SU}(n)$, $n\geq 3$ [22].

In particular, models in which quarks transform under the fundamental representations of $\mathrm{SU}(4)_C$ and $\mathrm{SU}(5)_c$ were considered in Refs. [8] and [9], respectively, in the context of the SM. These models preserve the experimental consistency of the SM at low energies. For instance, in the $\mathrm{SU}(5)_C\otimes\mathrm{SU}(m)_L\otimes\mathrm{U}(1)_N$ model a Higgs field transforming as the $\mathbf{10}$ representation of $\mathrm{SU}(5)_C$ breaks the symmetry as follows [9]:

$$\begin{array}{c}\mathrm{SU}(5)_C\otimes\mathrm{SU}(m)_L\otimes\mathrm{U}(1)_N\\ \downarrow\langle\mathbf{10}\rangle\\ \mathrm{SU}(3)_C\otimes\mathrm{SU}(2)'\otimes\mathrm{SU}(m)_L\otimes\mathrm{U}(1)_N.\end{array} \tag{3.4}$$

Later the electroweak symmetry will be broken and the remaining symmetry will be $\mathrm{SU}(3)_C\otimes\mathrm{SU}(2)'\otimes\mathrm{U}(1)_{em}$ as in the models with $m=3,4$ considered above. Notice that, due to the relation between the color degrees of freedom and the number of families, it is necessary to introduce four and five leptonic families for $n=4$ and $n=5$, respectively, if we assume that the number of quark families is still three. In general we have $N_l=|n(n_1-n_2)|$, where n_1 and n_2 are the number of quark multiplets transforming as $\mathbf{m}$ and $\mathbf{m}^*$, respectively, and $N_q=n_1+n_2$. If $n_1>n_2$ leptons must transform as $\mathbf{m}^*$ and if $n_1<n_2$ leptons are assigned to $\mathbf{m}$. It is still

possible to have $N_q = N_l$. Assuming this condition (and $n_1 > n_2$) for the case of even n, i.e., $n = 2p$, $p \geq 2$ we have $n_1/n_2 = (2p+1)/(2p-1)$; and for odd n, i.e., $n = 2p+1$, $p \geq 1$ we get $n_1/n_2 = (p+1)/p$. For $n = 4$ the condition $N_q = N_l$ is satisfied if $n_1/n_2 = 5/3$. Analogously, for $n = 5$ we have $n_1/n_2 = 3/2$. It means that if we let the number of quark families to be equal to the number of the lepton families, the minimal number of families is 8 for $n = 4$ and five for $n = 5$.

On the other hand, if we maintain $N_q = 3$ it is necessary, as we said before, to introduce new lepton families. Let us denote these additional families by (N_i, E_i, E_i^c) with $i = 1$ for $n = 4$; or $i = 1, 2$ when $n = 5$. The new leptons must be heavy enough in order to keep consistency with phenomenology. Since the right-handed neutrinos, transforming as singlets under the gauge group, do not contribute to the anomaly, their number is not constrained by the requirement of obtaining an anomaly-free theory. Hence, we can introduce an arbitrary number of such fields. When these singlets are added, the Z^0 invisible width is always smaller than the prediction of the minimal SM. In fact it has been shown that in this case [23]

$$\Gamma(Z \to \text{neutrinos}) \leq N_l \Gamma^0, \tag{3.5}$$

where N_l is the number of left-handed lepton families and Γ^0 is the standard width for one massless neutrino. Hence, it will be always possible to choose the neutrinos's mixing angles and masses in such a way that the theoretical value in (3.5) be consistent with the experimental one [17,24].

IV. OTHER POSSIBLE EXTENSIONS

Other possibilities are models with left-right symmetry in the electroweak sector $\mathrm{SU}(n)_C \otimes \mathrm{SU}(m)_L \otimes \mathrm{SU}(m)_R \otimes \mathrm{U}(1)_N$ and also models with horizontal symmetries G_H, i.e., $\mathrm{SU}(n)_C \otimes \mathrm{SU}(m)_L \otimes \mathrm{U}(1)_N \otimes G_H$.

A. Left-right symmetries

In models with left-right symmetry the V-A structure of weak interactions is related to the mass difference between the left- and right-handed gauge bosons, $W_L^\pm$ and $W_R^\pm$, respectively, as a result of the spontaneous symmetry breaking [25].

These sorts of models are easily implemented in the 3-3-1 context by adding a new charged lepton E. For instance, in models with left-handed leptons transforming as $(\nu_a, l_a^-, E_a^+)_L^T$ the right-handed triplet is $(\nu_a, l_a^-, E_a^+)_R^T$. In the quark sector, the left-handed components are as in Ref. [4] and similarly the right-handed components, in such a way that anomalies cancel in each chiral sector. Explicitly, the charge operator is defined as

$$Q = I_{3L} + I_{3R} + \frac{Y}{2}\,, \tag{4.1}$$

where $I_{3L(3R)}$ and $Y/2$ are of the form $(1/2)\lambda_3$ and $-(\sqrt{3}/2)\lambda_8 + N\mathbf{1}$, respectively, for the model of Refs. [4]. The Higgs multiplet $(\mathbf{3}, \mathbf{3}^*, 0)$ and its conjugate give mass to all fermions but in order to complete the symmetry breaking it is necessary to add more Higgs multiplets.

B. Horizontal symmetries

Particle mixture occurs in the standard model among particles which are equivalent concerning their position in the gauge multiplets. It was noted some years ago that it is possible to determine the weak mixing angles in terms of the quark masses, provided we assume that all equivalent multiplets of the vertical gauge symmetry transform in the same way under horizontal symmetries. Therefore, the three families are put into a single representation of the horizontal group [26].

That is, in the context of the SM the gauge symmetry in the horizontal direction was considered as a transformation among the left-handed doublets and among right-handed singlets. At first sight, horizontal symmetries are less interesting in the context of 3-3-1 models since quark generations transform in a different way under $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$. Apparently, the only possibility is the horizontal $G_H = \mathrm{SU}(2)_H$ symmetry. In this case there are no additional conditions for canceling gauge anomalies since SU(2) is a safe group. For instance, with $n = 3$, $m = 3, 4$, the three quark generations transform, in both left- and right- handed sectors, in the following way: two of them as a doublet and the third one as a singlet under $\mathrm{SU}(2)_H$ [27,28]. The same is valid for leptons but in this case the three lepton triplets can transform as the adjoint representation as well.

The horizontal gauge bosons and the extra Higgs bosons have to be heavier than the W bosons or very weakly coupled to the usual fermions in order to suppress appropriately flavor changing neutral transitions in both quark and lepton sectors.

C. Embedding in SU(6)

There are also the grand unified extensions of all the possibilities we have treated above. The group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$ has rank 5 and it is a subgroup of SU(6). It has been shown in the last group that the anomalies $\mathcal{A}$ of **15** and **6*** are such that $\mathcal{A}(\mathbf{15}) = -2\mathcal{A}(\mathbf{6}^*)$ [29]. Then, pairs of **15** and **15***; **6*** and **6** and, finally one **15** and two **6*** are the smallest anomaly-free irreducible representations in SU(6) [30]. On the other hand, the representation **20** is safe.

Just as an example, let us consider the SU(6) symmetry, which is a possible unified theory for model B. Using the notation of Ref. [31], in the entry $(\mathbf{a}, \mathbf{b})_f(N)$, $\mathbf{a}$ is an irreducible representation of $\mathrm{SU}(3)_C$ and $\mathbf{b}$ is an irreducible representation of $\mathrm{SU}(3)_L$. The subindex f means, in an obvious notation, the respective fields of the model and the second parenthesis contains the value of the $U(1)_N$ generator when acting on the states in the $(\mathbf{a},\mathbf{b})$.

In model B there are 74 degrees of freedom. Left-handed leptons and four of the right-handed d-type quarks are in $\mathbf{6}^*$:

$$\mathbf{6^*}_j = (\mathbf{3}^*,\mathbf{1})_{d^c_{jL}}(+1/3) + (\mathbf{1},\mathbf{3}^*)_{\Psi_{jL}}(-1/3), \qquad (4.2)$$

where $d^c_{jL} = d^c_{1L}, d^c_{2L}, d'^c_{1L}, d'^c_{2L}$; $\Psi_{jL} = \Psi_{eL}, \Psi_{\mu L}, \Psi_{\tau L}, \Psi_{TL}$ [see Eq.(2.25a)]. Two quark generations transforming as $(\mathbf{3},\mathbf{3},0)$ and the right-handed u-type quarks are in two $\mathbf{15}$

$$\mathbf{15}_{Q_{iL}} = (\mathbf{3}^*,\mathbf{1})_{u^c_{iL}}(-2/3) + (\mathbf{1},\mathbf{3}^*)_{\Psi'_{iL}}(2/3) + \\ +(\mathbf{3},\mathbf{3})_{Q_{iL}}(0), \qquad (4.3)$$

where $u^c_{iL} = u^c_{1L}, u^c_{2L}$; $\Psi'_{iL} = \Psi'_{eL}, \Psi'_{\mu L}$ [see Eq. (2.25b)]. Finally, the left-handed and right-handed quarks of the third generation are in one $\mathbf{20}$

$$\mathbf{20}_{Q_{3L}} = (\mathbf{1},\mathbf{1})_X(1) + (\mathbf{1},\mathbf{1})_{X^c}(-1) \\ +(\mathbf{3},\mathbf{3}^*)_{Q_{3L}}(+1/3) + (\mathbf{3}^*,\mathbf{3})_{Q^c_{3L}}(-1/3), \qquad (4.4)$$

with X a new charged lepton and the neutral leptons N_{iL} are singlets of SU(6). Thus, we have an anomaly-free theory with the fields of the first two generations in $\mathbf{6}^*$ and $\mathbf{15}$. The third generation is not anomalous.

Let us consider the prediction of the weak mixing angle, $\sin\theta_W$. In SU(N) theories we have

$$\sin^2\theta_W = \frac{\sum_a (I_{3a})^2}{\sum_a (Q_a)^2}, \qquad (4.5)$$

where I_3 is the third component of the weak isospin, Q is the electric charge, and the sum extends over all fields in a given representation. Hence, in SU(6) we have the prediction that $\sin^2\theta_W = 3/8$. This is the same value of the SU(5) model [32]. It is easy to verify that all representations $\mathbf{6}^*$, $\mathbf{15}$, and $\mathbf{20}$ in Eqs. (4.2)–(4.4) give the same answer as it must be. On the other hand, we recall that in models A and B it holds that $\sin^2\theta_W < 3/4$ [6]. Thus, the theory has a Landau pole when $\sin^2\theta_W = 3/4$. The theory might be, before getting this pole, unified in an SU(6) model.

However, it is not a trivial issue to show that the unification in SU(6) may actually occur [33]. This is so, because in 3-3-1 the couplings α_c and α_{3L} has $\beta_c > \beta_{3L}$.

Since the model has new particles, we may have to consider mass threshold corrections for the β functions, since these new particles could have masses below the unification energy scale, or even, we may not assume the decoupling theorem. We recall that in the standard model with two Higgs doublets the decoupling theorem [34] must not be necessarily valid, since there are physical effects proportional to $m^2_{\rm Higgs}$ [35]. Hence, it could be interesting to study the way in which the masses of the extra Higgs and exotic quarks in the model become large, as it has been done in the standard model scenario for an extended Higgs sector [35] or for the mass difference between fermions of a multiplet [36]. It means that there is no "grand desert" if 3-3-1 models are realized in nature.

How can we study the embedding of the SM in 3-3-1? The last model has fields which do not exist in the minimal SM, but which are present in the same multiplet of 3-3-1 with the known quarks. For instance, the quarks J's have to be added to the SM transforming as $(\mathbf{3},\mathbf{1},Q_J)$ under the 3-2-1 factors. The scalar and vector boson sectors of the SM have also to be extended with new fields. Hence, we must add scalar fields transforming as (i) four singlets $(\mathbf{1},\mathbf{1},Y_S)$: one with $Y_S=0$, one with $Y_S=1$ and two with $Y_S=2$, (ii) four doublets $(\mathbf{1},\mathbf{2},Y_D)$: one with $Y_D=-3$ and three with $Y_D=1$; finally, (iii) one triplet $(\mathbf{1},\mathbf{3},-2)$. It is also necessary to add extra vector bosons (U^{++},V^+) which transform as $(\mathbf{1},\mathbf{2},3)$. For this reason we believe that 3-3-1 models are not just an embedding of the SM but an alternative to describe these same interactions and new ones.

V. CONCLUSIONS

The 3-3-1 symmetry is in fact an interesting extension of the standard model. It gives answers to some questions put forward by the later model and new physics could arise at not too high energies, say in the TeV range.

In the previous sections we have examined two 3-3-1 models, both of them with extra heavy quarks and leptons, and also some of their possible extensions. What we want to do now is to discuss briefly some possible phenomenological consequences concerning models A and B dicussed in Secs. II A and II B, respectively.

(i) In the Higgs sector, by using the gauge invariance it is not possible to choose all VEVs to be real. Hence, there is CP violation via scalar exchange. Since the quark mass matrices receive contributions from two VEVs, there are also FCNCs in the Higgs boson couplings but their effects could be suppressed by fine tuning among some parameters [6] or by heavy scalars. The possibility of spontaneous CP violations in the context of the 3-3-1 model of Ref. [4] has been study recently in Ref. [37].

(ii) In model A, the left-handed quark mixing matrices V^U_L and V^D_L, defined in Eqs. (2.18) or (2.19), survive in the Lagrangian. See for instance Eqs. (2.22). Mixings are also different in the interactions with X^0_μ and with V^-_μ, as can be seen from Eq. (2.20). This induces new sources of CP violation since there are phases in these interactions which cannot be absorbed. This also happens in model B. However, in this case, as have seen in Sec. II, even the right-handed quark mixing matrices, V^U_R and V^D_R, survive. We recall that in the standard model although the matrices $V^{U,D}_{L,R}$ are needed, after the diagonalization of the quark mass matrices the only place in the Lagrangian where these matrices appear is in charged currents coupled to the W^+ and only in the form $V^{U\dagger}_L V^D_L$. In this case, V^D_L is identified with the usual Cabibbo-Kobayashi-Maskawa mixing matrix by choosing $V^U_L = \mathbf{1}$. Since this matrix does not appear in other places of the Lagrangian, this choice is enough. This is not the case for all 3-3-1 models [10].

(iii) It is well known that almost all Z^0-pole observables are in agreement with the standard model predictions [17]. There are, however, two of these observables which do not seem to agree with the model's predictions:

(a) The first one concerns the heavy quark production ratios,

$$R_f = \frac{\Gamma(Z^0 \to f\bar{f})}{\Gamma(Z^0 \to \text{hadrons})} \equiv \frac{\Gamma_f}{\Gamma_h}, \quad \Gamma_h = \sum_q \Gamma_q, \quad (5.1)$$

which have been measured for c and b quarks. Considering R_c as the SM prediction ($R_c \approx 0.171$), we have $R_b = 0.2192 \pm 0.0018$ which is about 2σ discrepancy with the expected value $R_b = 0.2156 \pm 0.0006$ [2]. These ratios depend on the effective vector and axial-vector couplings $\bar{g}_{Vf}$ and $\bar{g}_{Af}$, which in the SM scenario include radiative corrections. In the context of model B, $\bar{g}_{Vf}$ and $\bar{g}_{Af}$ refer to the couplings defined in terms of the R coefficients, which incorporate the right-handed neutral currents in Eqs. (2.47) and (2.48) and radiative corrections too. Here we will write them only at the tree level:

$$\bar{g}_{Vd} = g^{SM}_{Vd} - \frac{1}{2}\left|(V^D_{1R})_{3d}\right|^2,$$
$$\bar{g}_{Ad} = g^{SM}_{Ad} + \frac{1}{2}\left|(V^D_{1R})_{3d}\right|^2, \quad (5.2a)$$

$$\bar{g}_{Vs} = g^{SM}_{Vd} - \frac{1}{2}\left|(V^D_{1R})_{3s}\right|^2,$$
$$\bar{g}_{As} = g^{SM}_{As} - \frac{1}{2}\left|(V^D_{1R})_{3s}\right|^2, \quad (5.2b)$$

$$\bar{g}_{Vb} = g^{SM}_{Vd} - \frac{1}{2}\left[1 - \left|(V^D_{1R})_{3d}\right|^2 - \left|(V^D_{1R})_{3s}\right|^2,\right], \quad (5.2c)$$

$$\bar{g}_{Ab} = g^{SM}_{Ad} + \frac{1}{2}\left[1 - \left|(V^D_{1R})_{3d}\right|^2 - \left|(V^D_{1R})_{3s}\right|^2\right], \quad (5.2d)$$

$$\bar{g}_{Vu} = g^{SM}_{Vu} + \frac{1}{2}\left|(V^U_{1R})_{3u}\right|^2,$$
$$\bar{g}_{Au} = g^{SM}_{Au} - \frac{1}{2}\left|(V^U_{1R})_{3u}\right|^2, \quad (5.2e)$$

$$\bar{g}_{Vc} = g^{SM}_{Vu} + \frac{1}{2}\left|(V^U_{1R})_{3c}\right|^2,$$
$$\bar{g}_{Ac} = g^{SM}_{Au} - \frac{1}{2}\left|(V^U_{1R})_{3c}\right|^2, \quad (5.2f)$$

where $g^{SM}_{Vd} = -\frac{1}{2} + \frac{2}{3}s^2_W$, $g^{SM}_{Ad} = -\frac{1}{2}$; $g^{SM}_{Vu} = \frac{1}{2} - \frac{4}{3}s^2_W$, $g^{SM}_{Au} = \frac{1}{2}$ and in Eq. (5.2c) and (5.2d) we have used the unitarity of the V^D_{1R} matrix. We see that the Z^0 does not couple universally to right-handed fermions [see also Eq. (2.48)]. So, in this model it is possible that the theoretical predictions for the Z^0 width into light quarks and leptons and into $b\bar{b}$ as well would be modified [38]. However, like in the standard model, eventually we have to consider radiative corrections. The model is complex enough to allow several contributions to these radiative corrections besides that of the t quark to the $Zb\bar{b}$ vertex and they must be studied in detail. Here, we only wish to call attention for the possibilities of the model with respect to the present measurements of R_b.

It is also possible to considere a general mixing in both charged sectors. In this case V^U_{1R} and V^D_{1R} in Eqs. (2.47) and (2.48) would be 4×4 and 5×5 unitary matrices, respectively. In order to mantain the right-handed couplings in Eq. (2.46) compatible with the numerical values of the SM at the tree level, we might choose

$$\left(V^{D*}_R \tilde{\Delta} V^D_R\right)_{dd} \approx \left(V^{D*}_R \tilde{\Delta} V^D_R\right)_{ss} \approx \left(V^{D*}_R \tilde{\Delta} V^D_R\right)_{bb} \approx 0, \quad (5.3)$$

where $\tilde{\Delta} = \text{diag}(0,0,1,0,0)$. The $K_L - K_S$ mass difference constrains only the matrix element $(V^{D*}_R)_{d3}(V^D_R)_{3s}$. In order to determine the other elements of the matrix V^D_R it is necessary to study in detail B decays. A similar situation occurs in the charge 2/3 sector.

(b) The second one is the value of the left-right asymmetry $A^{0e}_{LR} = A^0_e$ obtained by LEP measurements of the forward-backward asymmetry. It corresponds to $\sin^2\theta_W = 0.2321 \pm 0.0005$ [39] while the SLD left-right asymmetry measurement implies $\sin^2\theta_W = 0.2292 \pm 0.0010$ [40]. Hence, there is an experimental discrepancy between the LEP and SLD values for A^0_e at the 2.7σ level. If this is confirmed, it could indicate new physics coupled in a different way to the third generation. For instance: (1) extended gauge structures with extra neutral bosons, like the Z'; (2) extra fermions like t', b', or even heavy leptons as E^-; (3) nonstandard Higgs particles, and (4) new heavy particles loop effects like exotic leptons [41], quarks, or supersymmetric particles.

In particular the Z' contribution is an interesting possibility. In fact, it has been pointed out recently that the discrepancy between both measurements can be reconciled if a new neutral gauge boson Z' nearly degenerate with the Z^0 and with appropriate couplings to the quarks do exist [42]. This new neutral gauge boson may also be responsible for the observed value of R_b. In our models Z' is not necessarily nearly degenerate with Z. However, constraints coming from the neutral K mass difference would not imply necessarily a heavy Z' since we can obtain a consistency with the observed value of this mass difference by choosing appropriately some of the matrix elements of V^D_L. These matrix elements are different from the mixing angles appearing in the observables R_b and A_{FB}. The couplings defined in Eq. (2.9) for the case of the Z' are all flavor violating [6] and, as we have extra mixing matrices in these models, it is possible that a global analysis of all data will show compatibility among the low energy processes like $K_L - K_S$ mass difference and line shape variables like Γ_Z, $R \equiv \Gamma_h/\Gamma_l$, σ_h, and others.

Hence, it is possible that the discrepancies in (a) and (b) will be an indication of new physics which generates a vertex correction to the Z couplings, new box and vector boson vacuum polarization contributions, or that there is a new physics at the tree level, or both of them. Models A and B are rich enough to allow the implementation of all the effects above.

(iv) Some time ago it was pointed out that since the left-handiness of the b quark has not been tested experimentally this quark may decay through, in the extreme case, purely right-handed couplings to the c and u quarks [43,44]. The chirality of the b quark can be tested by studying the decay of polarized Λ_b baryons [43,44]. These ideas were worked out in the context of an $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model. In such a model the smallness of the b to c coupling is due not to the value of the corresponding mixing angle but to the small value of the right-handed Fermi constant G_{FR}, and the right-handed W_R boson must be light since [45]

$$\frac{G_{FR}}{G_{FL}} \simeq \frac{1}{\sqrt{2}} \left(g_R^2/M_{W_R}^2\right) / \left(g_L^2/M_{W_L}^2\right) \simeq V_{bc} \simeq 0.04. \quad (5.4)$$

In model B (Sec. II B) an intermediate situation is realized. In Eq. (2.44) the charged left-handed currents are the usual ones. However, there are also right-handed currents coupled to the W^+ boson with the same strength G_F but it depends on some of the right-handed couplings V_R^U and V_R^D appearing in Eq. (2.45). Hence, the constraint in Eq. (5.4) implies only that $(V_R^{U*})_{3c}(V_R^D)_{3b} < 0.04$.

Notice that the left-handed couplings of the b quark to the c and u quarks are the same of the standard model [See Eq. (2.44)]. However, there are contributions to the semileptonic b decays in which (a) a right-handed b-to-$c(u)$ current couples to a left-handed lepton current [see Eqs. (2.43) and (2.45)]; (b) a left-handed b-to-$c(u)$ current couples to a right-handed lepton current, and (c) both currents are right-handed. Cases (b) and (c) involve the heavy lepton sector: $\bar{\tau}_L^c \gamma^\mu N_{1L} = -\bar{N}_{1R}^c \gamma^\mu \tau_R$ or $\bar{T}_L^c \gamma^\mu N_{2L} = -\bar{N}_{2R}^c \gamma^\mu T_R$, and can be suppressed if N_i and T are heavy.

Analyses of the B_d^0-$\bar{B}_c^0$ and B_s^0-$\bar{B}_s^0$ mixings must be done in our context too. The dominant contributions in our model come from two-t-quark box diagrams as in the standard model. This involves other matrix elements of V_L^D. Hence, as we said before, in our models it would be necessary to make a global analysis involving Z-pole observables, CP violation, semileptonic B decays, and other processes in order to fit the several parameters appearing in it.

(v) These models predict new processes in which the initial states have the same electric charge as $ff \to W^- V^-$. These types of processes have only recently begun to be studied [46–48]. Also in some extensions of these models, with spontaneous and/or explicit breaking of $L+B$ symmetry, it is possible to have kaon decays with $|\Delta L| = 2$, like $K^+ \to \pi^- \mu^+ \mu^+, \pi^- \mu^+ e^+$, similarly in D and B mesons decays. Experimental data imply $B(K^+ \to \pi^- \mu^+ \mu^+) < 1.5 \times 10^{-4}$ [47]. The process $e^- e^- \to W^- W^-$, which also could occur in some extensions of the 3-3-1 models, has been recently investigated in other context [48].

In summary, none of these models is severely constrained at low energies. For instance, in the leptonic sector both of them are consistent with the existence of three light neutrinos [17]. Neutrinos will get mass through radiative corrections and some of their properties as the magnetic moments will be studied elsewhere.

Another interesting feature of these kinds of models is that they include some extensions of the Higgs sector which have been considered in the context of the $SU(2) \otimes U(1)$ theory: more doublets, single and doubly charged singlets, triplets, etc. That is, all these extensions of the SM are subset of the full Higgs sector of 3-m-1 models.

Finally, we would like to say that the supersymmetric version of the model of Ref. [4] has been considered in Ref. [49].

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