

QCD equalities for baryon current matrix elements

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An examination of the symmetries manifest in the QCD path integral for current matrix elements reveals various equalities among the quark sector contributions. QCD equalities among octet baryon magnetic moments lead to a determination of the disconnected sea-quark contribution to nucleon magnetic moments, which is the most reliable determination in the literature. Matching QCD equalities to recent calculations of decuplet baryon magnetic moments in chiral perturbation theory (χ PT) reveals an equivalence between χ PT to $O(p^2)$ and the simple quark model with an explicit disconnected sea-quark contribution. New insights into SU(3)-flavor symmetry breaking, sea contributions, and constituent quark composition are obtained. The strangeness contribution to nucleon magnetic moments is found to be large at $G_M^S(0) = -0.75 \pm 0.30 \mu_N$. The QCD equalities must be followed by any model which hopes to capture the essence of nonperturbative QCD. Not all models are in accord with these symmetries.

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I. INTRODUCTION

The Euclidean path integral formulation of quantum field theory is the origin of fundamental approaches to the study of quantum chromodynamics (QCD) in the nonperturbative regime. While the lattice regularization of the path integral is the only known approach to *ab initio* determinations of hadron properties, the actual implementation of the numerical simulations is met with formidable technical difficulties. To proceed one must work with unphysically heavy current quarks and most simulations still involve the quenched approximation. Understanding the physics associated with these approximations is an industry in itself [1–5].

The QCD sum rule approach to nonperturbative QCD may also be formulated in Euclidean space [6,7]. However, the Borel-improved spectral sum rules are more widely known and provide better suppression of excited state contaminations. Here, one encounters difficulties in determining the vacuum expectation values of the operators of the operator product expansion (OPE). Additional uncertainties surround the viability of the continuum model, utilized to remove excited state contaminations from the OPE. An in-depth examination of these issues may be found in Ref. [8].

In contrast, this investigation focuses on fundamental QCD equalities based directly on the symmetries of the QCD path integral. In the following, it will be clearly stated which properties are fundamental symmetries of QCD and which properties depend on the actual dynamics of QCD. As such, the following equalities must be met by any model which hopes to capture the essence of nonperturbative QCD.

The only approximation made in the following discussion of QCD equalities is the equivalence of the u and d current quark masses. Hence, we will focus on observables which are not dominated by isospin violation at the QCD level. Since the QCD scale parameter $\Lambda_{\text{QCD}} \gg m_d - m_u$, this approximation is generally accepted to be excellent, and is shared by many approaches probing hadron structure.

The outline of this paper is as follows. Section II reviews the path integral formalism and the operators used to probe current matrix elements of low-lying baryons. Correlation

functions are calculated to reveal the QCD equalities. Section III discusses the QCD equalities for octet baryons and demonstrates their utility by calculating the disconnected sea-quark loop contributions to magnetic moments. Section IV addresses the QCD equalities for decuplet baryons and their relationship to predictions from chiral perturbation theory (χ PT). Here, quantitative relationships between SU(3) breaking in the valence and sea sectors are obtained. Sea contributions are evaluated and the predictions for baryon magnetic moments from lattice QCD are updated. The strangeness contribution to nucleon magnetic moments is established and compared with other approaches in Sec. V. A comprehensive breakdown of the quark sector contributions to baryon magnetic moments which may be useful in the development of more sophisticated models is provided here. Finally, Sec. VI reviews the highlights of QCD equalities.

II. FORMALISM

A. Path integral

The determination of hadron properties in field theoretic approaches are centered around the QCD vacuum expectation values of appropriately chosen operators O_i . In the Euclidean path integral formulation, these vacuum expectation values are given by

$$\begin{aligned} \langle 0 | O_1(A_\mu, \psi, \bar{\psi}) O_2(A_\mu, \psi, \bar{\psi}) \cdots | 0 \rangle \\ = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S_G(A_\mu) - \bar{\psi} M(A_\mu) \psi} \\ \times [\bar{\psi} \bar{\psi} \cdots \Gamma(A_\mu) \psi \psi \cdots], \end{aligned} \quad (2.1)$$

with

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S_G(A_\mu) - \bar{\psi} M(A_\mu) \psi}. \quad (2.2)$$

Here, $S_G(A_\mu)$ is the gauge action, and $M(A_\mu) = (\gamma_\mu D_\mu + m)$ where D_μ is the covariant derivative.

A sum over quark flavors with appropriate masses is implicit. The explicit dependence of the various terms on the gauge field A_μ has been illustrated. $\Gamma(A_\mu)$ simply signifies the underlying matrix multiplications in Dirac space and the operator dependence on A_μ . The fermion fields are described by a Grassmann algebra. The functional integration may be done analytically to give the well-known form

$$\begin{aligned} & \langle 0 | O_1(A_\mu, \psi, \bar{\psi}) O_2(A_\mu, \psi, \bar{\psi}) \cdots | 0 \rangle \\ &= \frac{1}{Z} \int \mathcal{D}A_\mu e^{-S_G(A_\mu)} \det M(A_\mu) \\ & \quad \times [\Gamma(A_\mu) M^{-1}(A_\mu) M^{-1}(A_\mu) \cdots], \end{aligned} \quad (2.3)$$

where each pair of ψ and $\bar{\psi}$ in the square brackets of (2.1) produce $M^{-1}(A_\mu)$, the nonperturbative quark propagator hereafter referred to as S . The origin of the QCD equalities lies in the detailed structure of the term in brackets in (2.3). The remaining factors,

$$\mathcal{D}A_\mu e^{-S_G(A_\mu)} \det M(A_\mu), \quad (2.4)$$

are common to all the individual terms in the brackets and more generally are common to any hadron considered in the approach. Hence all the physics differentiating one hadron from the next is contained in the bracketed term of (2.3), which we shall now turn our attention to in detail.

B. Operators

Current matrix elements of hadrons are extracted from the consideration of a time-ordered product of three operators. Generally, an operator exciting the hadron of interest from the QCD vacuum is followed by the current of interest, which in turn is followed by an operator annihilating the hadron back to the QCD vacuum. The operator can take the form

$$\begin{aligned} O_{\sigma\tau}^\mu(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma) &= \Gamma^{\beta\alpha} T \left[\int d\mathbf{x}_2 e^{-i\mathbf{p}' \cdot \mathbf{x}_2} \chi_\sigma^\alpha(x_2) \right. \\ & \quad \left. \times \int d\mathbf{x}_1 e^{+i(\mathbf{p}' - \mathbf{p}) \cdot \mathbf{x}_1} j^\mu(x_1) \bar{\chi}_\tau^\beta(0) \right]. \end{aligned} \quad (2.5)$$

Here, χ is a hadron interpolating field usually composed of quark (and possibly gluon) field operators and Dirac γ matrices designed to isolate the quantum numbers of the hadron under consideration. The subscripts σ and τ are optional Lorentz indices which are required for the excitation of spin-3/2 baryons or vector or axial-vector mesons. The Lorentz index μ is also provided for vector or axial-vector probes of the hadron structure. Of course, scalar, pseudoscalar, or tensor probes may also be considered. The Dirac matrix Γ provides for the projection of various Dirac γ structures, and α and β are Dirac indices. The vectors \mathbf{p} and \mathbf{p}' provide for the design of any momentum transfer. Euclidean time evolution in t_1 and t_2 provides a mechanism for the isolation of ground state properties. To reveal the QCD equalities, it is sufficient to work in coordinate space with the operator

$$O_{\sigma\tau}^\mu(x_2, x_1) = T[\chi_\sigma^\alpha(x_2) j^\mu(x_1) \bar{\chi}_\tau^\beta(0)]. \quad (2.6)$$

C. Interpolating fields

A gauge-invariant operator having maximal overlap with the ground-state hadron is local and of minimal dimension. These criteria alone are sufficient to uniquely define the Δ -baryon interpolating field. For Δ^{++} the form is

$$\chi_\mu^{\Delta^{++}}(x) = \epsilon^{abc} [u^{Ta}(x) C \gamma_\mu u^b(x)] u^c(x). \quad (2.7)$$

Here u denotes the up-quark field, and the indices a, b , and c are color indices. The antisymmetric tensor ϵ^{abc} places the three quarks in a color singlet state. C is the charge conjugation operator, and T denotes transpose. The interpolators coupling to other charge states of Δ may be obtained from Eq. (2.7) by using the isospin-three-component lowering operator. In particular,

$$\begin{aligned} \chi_\mu^{\Delta^+}(x) &= \frac{1}{\sqrt{3}} \epsilon^{abc} \{ 2[u^{Ta}(x) C \gamma_\mu d^b(x)] u^c(x) \\ & \quad + [u^{Ta}(x) C \gamma_\mu u^b(x)] d^c(x) \}. \end{aligned} \quad (2.8)$$

Other decuplet baryon interpolating fields are obtained with the appropriate substitutions of $u(x), d(x) \rightarrow u(x), d(x)$ or $s(x)$.

The uniqueness of the Δ^{++} interpolator is easily demonstrated [9] by considering the general local current with $I=3/2$ and $J=3/2$. Consider

$$\chi_\mu^{\Delta^{++}} = \epsilon^{abc} (u^{Ta} C \Gamma_\epsilon u^b) \gamma_\mu \Gamma^{\epsilon'} u^c, \quad (2.9)$$

where Γ_ϵ may take on any of the sixteen Dirac γ matrices. By transposing the scalar quantity in parentheses one finds

$$\epsilon^{abc} u^{Ta} C u^b = \epsilon^{abc} u^{Ta} C \gamma_5 u^b = \epsilon^{abc} u^{Ta} C \gamma_\mu \gamma_5 u^b = 0. \quad (2.10)$$

An application of the Fierz transformations reveals

$$\begin{aligned} \epsilon^{abc} (u^{Ta} C \sigma_{\rho\lambda} u^b) \gamma_\mu \sigma^{\rho\lambda} u^c &= -\frac{1}{2} \epsilon^{abc} (u^{Ta} C \sigma_{\rho\lambda} u^b) \gamma_\mu \sigma^{\rho\lambda} u^c \\ &= 0. \end{aligned} \quad (2.11)$$

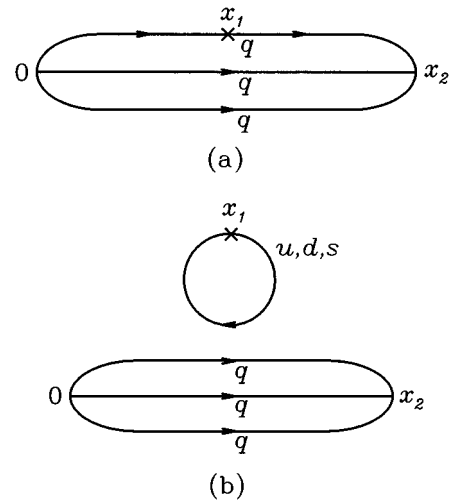


FIG. 1. Diagrams illustrating the two topologically different insertions of the current indicated by \times . These skeleton diagrams for the connected (a) and disconnected (b) current insertions may be dressed by an arbitrary number of gluons.

Another application of the Fierz transformations provides the relation

$$\epsilon^{abc}(u^{Ta}C\gamma_\lambda u^b)\sigma^{\lambda\mu}u^c = i\epsilon^{abc}(u^{Ta}C\gamma_\mu u^b)u^c, \quad (2.12)$$

equating the only nonzero variants of (2.9).

It is well established that there are two local interpolating fields of minimal dimension for the nucleon. In lattice calculations, the commonly used interpolating field for the proton has the form

$$\chi_1^p(x) = \epsilon^{abc}[u^{Ta}(x)C\gamma_5 d^b(x)]u^c(x). \quad (2.13)$$

In the QCD sum rule approach, it is common to find linear combinations of this interpolating field and

$$\chi_2^p(x) = \epsilon^{abc}[u^{Ta}(x)Cd^b(x)]\gamma_5 u^c(x), \quad (2.14)$$

which vanishes in the nonrelativistic limit. Interpolating fields for the other members of the baryon octet containing a doubly represented quark flavor may be obtained from (2.13) and (2.14) with the appropriate substitutions of quark field operators.

D. Correlation functions

To begin, consider the two-point function for Δ^+ . Using (2.7) and contracting out pairs of quark field operators in accord with (2.3) one has

$$\begin{aligned} T[\chi_\mu^{\Delta^+}(x)\bar{\chi}_\nu^{\Delta^+}(0)] &= \frac{1}{3}\epsilon^{abc}\epsilon^{a'b'c'}[4S_u^{aa'}\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_d^{cc'} + 4S_u^{aa'}\gamma_\nu C S_d^{Tbb'}C\gamma_\mu S_u^{cc'} + 4S_d^{aa'}\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_u^{cc'} \\ &\quad + 2S_u^{aa'}\text{tr}(\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_d^{cc'}) + 2S_u^{aa'}\text{tr}(\gamma_\nu C S_d^{Tbb'}C\gamma_\mu S_u^{cc'}) + 2S_d^{aa'}\text{tr}(\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_u^{cc'})], \end{aligned} \quad (2.15)$$

where the quark propagator $S_u^{aa'} = T[u^a(x), \bar{u}^{a'}(0)]$ and similarly for other quark flavors. SU(3)-flavor symmetry is obviously displayed in this equation.

In determining the three-point function, one encounters two topologically different ways of performing the current insertion. Figure 1 displays skeleton diagrams for these two insertions. These diagrams may be dressed with an arbitrary number of gluons. Diagram (a) illustrates the connected insertion of the current to one of the valence quarks¹ of the baryon. It is here that the Pauli-blocking in the sea contributions are taken into account. Diagram (b) accounts for the alternative time ordering where the current first produces a disconnected $q\bar{q}$ pair which in turn interacts with the valence quarks of the baryon via gluons.

The number of terms in the three-point function is four times that in (2.15). The correlation function relevant to the Δ^+ current matrix element is

$$\begin{aligned} T[\chi_\mu^{\Delta^+}(x_2)j^\mu(x_1)\bar{\chi}_\nu^{\Delta^+}(0)] &= \frac{1}{3}\epsilon^{abc}\epsilon^{a'b'c'}[4\hat{S}_u^{aa'}\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_d^{cc'} + 4\hat{S}_u^{aa'}\gamma_\nu C S_d^{Tbb'}C\gamma_\mu S_u^{cc'} + 4\hat{S}_d^{aa'}\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_u^{cc'} \\ &\quad + 4S_u^{aa'}\gamma_\nu C \hat{S}_u^{Tbb'}C\gamma_\mu S_d^{cc'} + 4S_d^{aa'}\gamma_\nu C \hat{S}_u^{Tbb'}C\gamma_\mu S_u^{cc'} + 4S_u^{aa'}\gamma_\nu C \hat{S}_d^{Tbb'}C\gamma_\mu S_u^{cc'} \\ &\quad + 4S_u^{aa'}\gamma_\nu C S_d^{Tbb'}C\gamma_\mu \hat{S}_u^{cc'} + 4S_d^{aa'}\gamma_\nu C S_u^{Tbb'}C\gamma_\mu \hat{S}_u^{cc'} + 4S_u^{aa'}\gamma_\nu C S_u^{Tbb'}C\gamma_\mu \hat{S}_d^{cc'} \\ &\quad + 2\hat{S}_u^{aa'}\text{tr}(\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_d^{cc'}) + 2\hat{S}_u^{aa'}\text{tr}(\gamma_\nu C S_d^{Tbb'}C\gamma_\mu S_u^{cc'}) + 2\hat{S}_d^{aa'}\text{tr}(\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_u^{cc'}) \\ &\quad + 2S_u^{aa'}\text{tr}(\gamma_\nu C \hat{S}_u^{Tbb'}C\gamma_\mu S_d^{cc'}) + 2S_d^{aa'}\text{tr}(\gamma_\nu C \hat{S}_u^{Tbb'}C\gamma_\mu S_u^{cc'}) + 2S_u^{aa'}\text{tr}(\gamma_\nu C \hat{S}_d^{Tbb'}C\gamma_\mu S_u^{cc'}) \\ &\quad + 2S_u^{aa'}\text{tr}(\gamma_\nu C S_d^{Tbb'}C\gamma_\mu \hat{S}_u^{cc'}) + 2S_d^{aa'}\text{tr}(\gamma_\nu C S_u^{Tbb'}C\gamma_\mu \hat{S}_u^{cc'}) + 2S_u^{aa'}\text{tr}(\gamma_\nu C S_u^{Tbb'}C\gamma_\mu \hat{S}_d^{cc'})] \\ &\quad + \sum_{q=u,d,s} e_q \sum_i \text{tr}(S_q^{ii}(x_1, x_1)\gamma_\mu) \frac{1}{3}\epsilon^{abc}\epsilon^{a'b'c'}[4S_u^{aa'}\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_d^{cc'} \\ &\quad + 4S_u^{aa'}\gamma_\nu C S_d^{Tbb'}C\gamma_\mu S_u^{cc'} + 4S_d^{aa'}\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_u^{cc'} + 2S_u^{aa'}\text{tr}(\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_d^{cc'}) \\ &\quad + 2S_u^{aa'}\text{tr}(\gamma_\nu C S_d^{Tbb'}C\gamma_\mu S_u^{cc'}) + 2S_d^{aa'}\text{tr}(\gamma_\nu C S_u^{Tbb'}C\gamma_\mu S_u^{cc'})], \end{aligned} \quad (2.16)$$

where

$$\hat{S}_q^{aa'}(x_2, x_1, 0) = e_q \sum_i S_q^{ai}(x_2, x_1)\gamma_\mu S_q^{ia'}(x_1, 0) \quad (2.17)$$

¹It should be noted that the term ‘‘valence’’ used here differs with that commonly used surrounding discussions of deep-inelastic structure functions. Here ‘‘valence’’ simply describes the quark whose quark flow line runs continuously from $0 \rightarrow x_2$. These lines can flow backwards as well as forwards in time and therefore have a sea contribution associated with them [1].

denotes the connected insertion of the probing current to a quark of charge e_q . Here we have explicitly selected the electromagnetic current. However, the following discussion may be generalized to any quark-field-based current operator bilinear in the quark fields.

The latter term of (2.16) accounts for the loop contribution depicted in Fig. 1(b). The sum over the quarks running around the loop has been restricted to the flavors relevant to the ground-state baryon octet and decuplet. In the SU(3)-flavor limit the sum vanishes for the electromagnetic current. However, the heavier strange quark mass allows for a non-trivial result. Because of the technical difficulties of numerically estimating M^{-1} for the squared lattice volume of diagonal spatial indices at $q^2 \neq 0$, these contributions have been omitted from previous lattice calculations of electromagnetic structure. For other observables such as the scalar density or forward matrix elements of the axial vector current relevant to the spin of the baryon, the ‘‘charges’’ running around the

loop do not sum to zero. In this case the second term of (2.16) can be just as significant as the connected term [10,11].

An examination of (2.16) reveals complete symmetry among the quark flavors in the correlation function. For example, wherever a d quark appears in the correlator, a u quark also appears in the same position in another term. An interesting consequence of this is that the connected insertion of the electromagnetic current for Δ^0 vanishes. All electromagnetic properties of the Δ^0 have their origin strictly in the disconnected loop contribution. Physically, what this means is that the valence wave function for each of the quarks in the Δ resonances are identical.

To further illustrate this symmetry, we turn to another system in which this symmetry is broken, namely the nucleon. The correlation function relevant to proton matrix elements obtained from (2.13) is

$$\begin{aligned}
T[\chi_1^p(x_2)j^\mu(x_1)\bar{\chi}_1^p(0)] &= \epsilon^{abc}\epsilon^{a'b'c'}[\hat{S}_u^{aa'}\text{tr}(S_u^{bb'}\gamma_5CS_d^{Tcc'}C^{-1}\gamma_5) + \hat{S}_u^{aa'}\gamma_5CS_d^{Tbb'}C^{-1}\gamma_5S_u^{cc'} + S_u^{aa'}\text{tr}(\hat{S}_u^{bb'}\gamma_5CS_d^{Tcc'}C^{-1}\gamma_5) \\
&+ S_u^{aa'}\gamma_5CS_d^{Tbb'}C^{-1}\gamma_5S_u^{cc'} + S_u^{aa'}\text{tr}(S_u^{bb'}\gamma_5\hat{S}_d^{Tcc'}C^{-1}\gamma_5) + S_u^{aa'}\gamma_5CS_d^{Tbb'}C^{-1}\gamma_5S_u^{cc'}] \\
&+ \sum_{q=u,d,s}e_q\sum_i\text{tr}(S_q^{ii}(x_1,x_1)\gamma_\mu)\epsilon^{abc}\epsilon^{a'b'c'}[S_u^{aa'}\text{tr}(S_u^{bb'}\gamma_5CS_d^{Tcc'}C^{-1}\gamma_5) \\
&+ S_u^{aa'}\gamma_5CS_d^{Tbb'}C^{-1}\gamma_5S_u^{cc'}]. \tag{2.18}
\end{aligned}$$

Here we see very different roles played by u and d quarks in the correlation function. The neutron correlation function is obtained with the exchange of $u \leftrightarrow d$ in (2.18). The absence of equivalence for u and d contributions allows the connected quark sector to give rise to a nontrivial neutron charge radius, a large neutron magnetic moment, or a violation of the Gottfried sum rule. Perhaps it is also worth noting that the relative contributions of u and d quarks to nucleon moments depends on the dynamics of QCD. Numerical simulations indicate that the relative contributions of the u and d valence quarks to nucleon magnetic moments are very different from the SU(6)-spin-flavor symmetric wave functions of the simple quark model [12].

Another interesting point to emphasize is that there is no simple relationship between the properties of a particular

quark flavor bound in different baryons. For example, the correlator for Σ^+ is given by (2.18) with $d \rightarrow s$. Hence, a u -quark propagator in Σ^+ is multiplied by an s -quark propagator, whereas in the proton the u -quark propagators are multiplied by a d -quark propagator. The different mass of the neighboring quark gives rise to an environment sensitivity in the u -quark contributions to observables [12–18]. This point sharply contrasts the concept of an intrinsic quark property that is independent of the quark’s environment. This concept of an intrinsic quark property is a fundamental foundation of many constituent-based quark models and is not in accord with QCD.

The symmetry of the correlation function obtained from χ_2^p of (2.14) is identical to that of (2.18). The relevant correlation function is

$$\begin{aligned}
T[\chi_2^p(x_2)j^\mu(x_1)\bar{\chi}_2^p(0)] &= \epsilon^{abc}\epsilon^{a'b'c'}[\gamma_5\hat{S}_u^{aa'}\gamma_5\text{tr}(S_u^{bb'}CS_d^{Tcc'}C^{-1}) + \gamma_5\hat{S}_u^{aa'}CS_d^{Tbb'}C^{-1}S_u^{cc'}\gamma_5 + \gamma_5S_u^{aa'}\gamma_5\text{tr}(\hat{S}_u^{bb'}CS_d^{Tcc'}C^{-1}) \\
&+ \gamma_5S_u^{aa'}CS_d^{Tbb'}C^{-1}\hat{S}_u^{cc'}\gamma_5 + \gamma_5S_u^{aa'}\gamma_5\text{tr}(S_u^{bb'}CS_d^{Tcc'}C^{-1}) + \gamma_5S_u^{aa'}CS_d^{Tbb'}C^{-1}S_u^{cc'}\gamma_5] \\
&+ \sum_{q=u,d,s}e_q\sum_i\text{tr}(S_q^{ii}(x_1,x_1)\gamma_\mu)\epsilon^{abc}\epsilon^{a'b'c'}[\gamma_5S_u^{aa'}\gamma_5\text{tr}(S_u^{bb'}CS_d^{Tcc'}C^{-1}) \\
&+ \gamma_5S_u^{aa'}CS_d^{Tbb'}C^{-1}S_u^{cc'}\gamma_5]. \tag{2.19}
\end{aligned}$$

The interference of χ_1^p and χ_2^p provide a similar symmetry.

As a final point, it is important to note that the symmetry of $u \leftrightarrow d$ for describing the current matrix elements of the neutron in terms of the proton is always satisfied when the disconnected loop contribution is separated from the valence sector as in (2.18) or (2.19). Many models fail to include the loop contribution explicitly, but rather break isospin symmetry between the u and d quarks by absorbing the loop contribution into the definition of the constituent quark. This leads to the common misinterpretation that isospin symmetry breaking in octet baryons is large. When the loop contribution is absorbed into the valence quark contribution, an exchange of u and d contributions does not necessarily correctly describe the neutron in terms of the proton, or vice-versa.

It is important to estimate the size of such disconnected loop contributions in the nucleon. As we shall see, an estimate of the strangeness contribution to nucleon matrix elements can be obtained from the following QCD equalities.

III. OCTET BARYON SYMMETRIES AND LOOP CONTRIBUTIONS

Equations (2.18) and (2.19) provide the following equalities for current matrix elements of octet baryons:

$$p = e_u D_N + e_d S_N + O_N, \quad (3.1a)$$

$$n = e_d D_N + e_u S_N + O_N, \quad (3.1b)$$

$$\Sigma^+ = e_u D_\Sigma + e_s S_\Sigma + O_\Sigma, \quad (3.1c)$$

$$\Sigma^- = e_d D_\Sigma + e_s S_\Sigma + O_\Sigma, \quad (3.1d)$$

$$\Xi^0 = e_s D_\Xi + e_u S_\Xi + O_\Xi, \quad (3.1e)$$

$$\Xi^- = e_s D_\Xi + e_d S_\Xi + O_\Xi. \quad (3.1f)$$

Here D , S , and O represent contributions from the doubly represented valence quark flavor, the singly represented valence flavor, and the disconnected loop sector, respectively. Subscripts allow for environment sensitivity, e indicates the quark flavor ‘‘charge’’ and is not restricted to electric charge, and the baryon label represents the observable corresponding to the charge, momentum transfer, and Lorentz component(s) of the probing current. These are QCD equalities and must be reproduced by any model which hopes to reflect the properties of QCD when the light-current quark masses are isospin symmetric.

There are two important features here which are often neglected in model formulations. Isospin symmetric models based on the valence sector omit the disconnected sea-quark loop contribution, O . Often, no provision is made for the environment sensitivity of the valence sector contributions.

On the surface, it appears we have described six quantities in terms of nine parameters. However, the QCD equalities are more generally applicable. They are not restricted to the bulk baryon properties, but also provide information on the individual quark sector contributions, to be measured at the Continuous Electron Beam Accelerator Facility (CEBAF).

The equalities define a pattern for quark sector contributions which may be useful in the development of more sophisticated models.

Focusing now on electromagnetic properties, the disconnected loop contributions may be isolated in the favorable form

$$O_N = \frac{1}{3} \left[2p + n - \frac{D_N}{D_\Sigma} (\Sigma^+ - \Sigma^-) \right], \quad (3.2a)$$

$$O_N = \frac{1}{3} \left[p + 2n - \frac{S_N}{S_\Xi} (\Xi^0 - \Xi^-) \right], \quad (3.2b)$$

$$O_\Sigma = \frac{1}{3} \left[\Sigma^+ + 2\Sigma^- + \frac{S_\Sigma}{S_\Xi} (\Xi^0 - \Xi^-) \right], \quad (3.2c)$$

$$O_N = \frac{1}{3} \left[\Xi^0 + 2\Xi^- + \frac{D_\Xi}{D_\Sigma} (\Sigma^+ - \Sigma^-) \right]. \quad (3.2d)$$

The ratios of doubly or singly represented quark contributions appearing in (3.2) account for the environment sensitivity of the quark sector contribution. In terms of quark flavors, the ratios appearing in (3.2a) and (3.2b) for the nucleon are

$$\frac{D_N}{D_\Sigma} = \frac{u_p}{u_\Sigma} = \frac{d_n}{d_\Sigma}, \quad \text{and} \quad \frac{S_N}{S_\Xi} = \frac{d_p}{d_\Xi} = \frac{u_n}{u_\Xi}. \quad (3.3)$$

In many quark models, these ratios are simply taken to be one. Hence, by identifying the loop contributions in which the leading terms of (3.2) dominate the total contribution, a determination of the disconnected sea contribution may be obtained with minimal model dependence.

For magnetic moments, $O \equiv \mu_l$, and Eq. (3.2) takes the form

$$\mu_l^N = \frac{1}{3} \left[3.673 - \frac{\mu_u^p}{\mu_u} (3.618) \right], \quad (3.4a)$$

$$\mu_l^N = \frac{1}{3} \left[-1.033 - \frac{\mu_u^n}{\mu_u} (-0.599) \right], \quad (3.4b)$$

$$\mu_l^\Sigma = \frac{1}{3} \left[0.138 + \frac{\mu_s^\Sigma}{\mu_d} (-0.599) \right], \quad (3.4c)$$

$$\mu_l^\Xi = \frac{1}{3} \left[-2.551 + \frac{\mu_s^\Xi}{\mu_d} (3.618) \right], \quad (3.4d)$$

where the moments are in units of nuclear magnetons (μ_N). Equation (3.4b) provides a favorable case for a determination of the disconnected sea contribution to the nucleon’s magnetic moment with minimal model dependence. Taking the simple quark model ratio of $\mu_u^n / \mu_u = 1$ provides $\mu_l^N = -0.14\mu_N$.

To improve on this estimate, we turn to the lattice QCD calculations of environment sensitivity for these moments [12,14]. Because of the nature of the ratios involved, the systematic uncertainties in the lattice QCD calculations are expected to be small relative to the statistical uncertainties. Statistical uncertainties for the relevant ratios are estimated

via a third-order single-elimination jackknife [12,14]. The following ratios of magnetic moments are found:

$$\begin{aligned} \frac{D_N}{D_\Sigma} &= 1.136 \pm 0.078, & \frac{S_N}{S_\Xi} &= 0.721 \pm 0.457, \\ \frac{S_\Sigma}{S_\Xi} &= 0.390 \pm 0.244, & \frac{D_\Xi}{D_\Sigma} &= 0.585 \pm 0.039. \end{aligned} \quad (3.5)$$

The latter two ratios are to be compared with 0.65 in the simple quark model [19]. A combination of these uncertainties with the experimental uncertainties in quadrature, provides the following predictions, in units of μ_N , corresponding to (3.4a) through (3.4d):

$$\begin{aligned} \mu_l^N &= -0.15 \pm 0.09, & \mu_l^N &= -0.20 \pm 0.09, \\ \mu_l^\Sigma &= -0.03 \pm 0.05, & \mu_l^\Xi &= -0.14 \pm 0.05. \end{aligned} \quad (3.6)$$

Despite very different uncertainties in the quark sector ratios, both (3.4a) and (3.4b) yield similar estimates for the disconnected sea contribution to the nucleon's magnetic moment. A weighted average provides

$$\mu_l^N = -0.17 \pm 0.07 \mu_N. \quad (3.7)$$

In view of the minimal model dependence associated with (3.4b) this estimate is the most reliable estimate for the disconnected loop contribution to the nucleon's magnetic moment to date. These results for the nucleon agree with another estimate obtained through an examination of experimental violations of the Sachs sum rule for magnetic moments [13]. There $\mu_l^N = -0.19 \pm 0.09 \mu_N$. The disconnected sea contribution to the nucleon moment is of the order of 10%. In light of the precision data, such a contribution is relatively significant.

The results for μ_l^Σ and μ_l^Ξ suggest contrasting views for the environment sensitivity of the disconnected loop contributions. However, the results are similar at the one standard deviation level. The relation of (3.4c) for μ_l^Σ is particularly unfavorable and the result depends more sensitively on the ratio of quark flavor contributions.

IV. DECUPLET BARYON SYMMETRIES AND LOOP CONTRIBUTIONS

A. QCD equalities and constituent quark composition

The maximal symmetry of the decuplet baryons, and the four charge states of the Δ provide a much richer environment for the foundation of useful QCD equalities. The symmetries of (2.16) provide the following relationships among current matrix elements of decuplet baryons:

$$\Delta^{++} = (3 e_u + 0 e_d) L_\Delta + O_\Delta, \quad (4.1a)$$

$$\Delta^+ = (2 e_u + 1 e_d) L_\Delta + O_\Delta, \quad (4.1b)$$

$$\Delta^0 = (1 e_u + 2 e_d) L_\Delta + O_\Delta, \quad (4.1c)$$

$$\Delta^- = (0 e_u + 3 e_d) L_\Delta + O_\Delta, \quad (4.1d)$$

$$\Sigma^{*+} = 2 e_u L_{\Sigma^*} + e_s H_{\Sigma^*} + O_{\Sigma^*}, \quad (4.1e)$$

$$\Sigma^{*0} = (e_u + e_d) L_{\Sigma^*} + e_s H_{\Sigma^*} + O_{\Sigma^*}, \quad (4.1f)$$

$$\Sigma^{*-} = 2 e_d L_{\Sigma^*} + e_s H_{\Sigma^*} + O_{\Sigma^*}, \quad (4.1g)$$

$$\Xi^{*0} = 2 e_s H_{\Xi^*} + e_u L_{\Xi^*} + O_{\Xi^*}, \quad (4.1h)$$

$$\Xi^{*-} = 2 e_s H_{\Xi^*} + e_d L_{\Xi^*} + O_{\Xi^*}, \quad (4.1i)$$

$$\Omega^- = 3 e_s H_\Omega + O_\Omega. \quad (4.1j)$$

Here L , H , and O denote light, heavy (strange), and disconnected sea-quark loops, respectively. The subscript allows for environment sensitivity. As before, the charge factors are not necessarily restricted to electromagnetic charge.

Specializing to electromagnetic properties, the symmetries are

$$\Delta^{++} = +2 L_\Delta + O_\Delta, \quad (4.2a)$$

$$\Delta^+ = +1 L_\Delta + O_\Delta, \quad (4.2b)$$

$$\Delta^0 = +0 L_\Delta + O_\Delta, \quad (4.2c)$$

$$\Delta^- = -1 L_\Delta + O_\Delta, \quad (4.2d)$$

$$\Sigma^{*+} = +\frac{4}{3} L_{\Sigma^*} - \frac{1}{3} H_{\Sigma^*} + O_{\Sigma^*}, \quad (4.2e)$$

$$\Sigma^{*0} = +\frac{1}{3} L_{\Sigma^*} - \frac{1}{3} H_{\Sigma^*} + O_{\Sigma^*}, \quad (4.2f)$$

$$\Sigma^{*-} = -\frac{2}{3} L_{\Sigma^*} - \frac{1}{3} H_{\Sigma^*} + O_{\Sigma^*}, \quad (4.2g)$$

$$\Xi^{*0} = -\frac{2}{3} H_{\Xi^*} + \frac{2}{3} L_{\Xi^*} + O_{\Xi^*}, \quad (4.2h)$$

$$\Xi^{*-} = -\frac{2}{3} H_{\Xi^*} - \frac{1}{3} L_{\Xi^*} + O_{\Xi^*}, \quad (4.2i)$$

$$\Omega^- = -H_\Omega + O_\Omega. \quad (4.2j)$$

The Δ^0 properties are a direct reflection of SU(3)-flavor symmetry breaking in the disconnected sea. The central point here is that the disconnected sea-quark loop contribution cannot be absorbed into the connected contribution. This observation provides some rather interesting insight into the composition of a constituent quark.

In the simple constituent quark model, decuplet baryon magnetic moments are obtained by summing the intrinsic moments of the constituent quarks. In this case, L and H of (4.2) are assigned global values independent of the baryon, and the disconnected loop contribution, O , vanishes. Thus the constituent quark is void of any disconnected sea-quark loop physics. It is composed of a current quark dressed with nonperturbative glue, and as such, has a sea-quark component associated with the Z graphs of the current quark. This discussion carries over to octet baryons provided the constituent quark properties are isospin symmetric, as in $\mu_u = -2 \mu_d$, etc.

In the simplest constituent quark model based on SU(6) symmetry, the proton moment is given by

$$\mu_p = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d + \mu_l, \quad (4.3)$$

with $\mu_u = -2 \mu_d$ and the disconnected loop contribution, $\mu_l = 0$. However, since the factors $4/3$ and $-1/3$ sum to one,

it is possible to absorb a finite loop contribution into the property of the constituent quark

$$\mu_p = \frac{4}{3}(\mu_u + \mu_l) - \frac{1}{3}(\mu_d + \mu_l), \quad (4.4)$$

and still correctly describe the neutron by an exchange of u and d quarks as

$$\mu_n = \frac{4}{3}(\mu_d + \mu_l) - \frac{1}{3}(\mu_u + \mu_l). \quad (4.5)$$

Of course, the absorption of μ_l into the constituent quark property breaks isospin symmetry among the quark moments. Since most constituent quark models of octet baryons do break this symmetry, the physics of the $\bar{q}q$ sea is included in the constituent quark. Thus, the simple quark model always did predict some strangeness in the nucleon. As we will see, the amount may be determined through a measure of isospin symmetry violation and ratios of light to strange constituent quark masses. Since constituent quarks in octet baryons already have the physics of sea-quarks included intrinsically, the concept of a Fock-space expansion of $qqq(\bar{q}q)^n$ constituent quarks seems redundant.

The connected insertions of the current denoted by L or H in (4.2) are symmetric within an isospin multiplet. Hence taking octet baryon properties with loop contributions absorbed into the effective degrees of freedom and applying them to decuplet baryon properties [20] violates the QCD equalities among connected insertions of the probing current.

B. Chiral perturbation theory

The effective Lagrangian approach of chiral perturbation theory (χ PT) describes baryons as an octet-meson cloud of approximate Goldstone bosons coupled to nonrelativistic baryons. Despite the phenomenological need for the introduction of an explicit Δ resonance [21], as well as the Roper multiplet [22], the approach is argued to capture the essence of nonperturbative QCD.

Recently, relationships among the decuplet baryon magnetic moments were established in a rigorous one-loop, $O(p^2)$, χ PT analysis [22] where careful regard was given to the renormalizability of the approach. The relationships among the magnetic moments are [22]

$$\Delta^{++} = -2 \Omega^- - 3 \Delta^0, \quad (4.6a)$$

$$\Delta^+ = -1 \Omega^- - 1 \Delta^0, \quad (4.6b)$$

$$\Delta^0 = +0 \Omega^- + 1 \Delta^0, \quad (4.6c)$$

$$\Delta^- = +1 \Omega^- + 3 \Delta^0, \quad (4.6d)$$

$$\Sigma^{*+} = -1 \Omega^- - 2 \Delta^0, \quad (4.6e)$$

$$\Sigma^{*0} = +0 \Omega^- + 0 \Delta^0, \quad (4.6f)$$

$$\Sigma^{*-} = +1 \Omega^- + 2 \Delta^0, \quad (4.6g)$$

$$\Xi^{*0} = +0 \Omega^- - 1 \Delta^0, \quad (4.6h)$$

$$\Xi^{*-} = +1 \Omega^- + 1 \Delta^0, \quad (4.6i)$$

$$\Omega^- = +1 \Omega^- + 0 \Delta^0. \quad (4.6j)$$

At first glance, it would appear that (4.6) fails to follow the QCD equalities. Moreover, it appears on the surface that even the simple physics of SU(3)-flavor symmetry breaking in the valence sector is absent in (4.6). However, demanding these relationships among decuplet baryon magnetic moments satisfy the QCD equalities of (4.2) establishes relationships between SU(3) breaking in the disconnected loop sector and SU(3) breaking in the valence sector. The following relationships are consistent with χ PT to one-loop order.

The χ PT results satisfy the QCD equalities by allowing the Δ^0 moment, reflecting the breaking of SU(3)-flavor in the disconnected sea contribution, to introduce SU(3)-flavor symmetry breaking into the valence sector. Equating the two representations provides the following particularly interesting relation:

$$L_\Delta = L_\Sigma = L_\Xi = -\Omega^- - 2 \Delta^0. \quad (4.7)$$

This relation indicates χ PT has failed to resolve any environment sensitivity in the light quark sector contributions to magnetic moments. This is truly disappointing, as this is where the interesting physics resides in decuplet baryon properties. The only consolation is that lattice QCD calculations [14] suggest that such an approximation is reasonable in decuplet baryons, whereas it is not for octet baryons. However, the symmetries of χ PT to this order are not the symmetries of QCD.

In the Σ and Ξ systems, there are two linearly independent equations and three unknown quark sector contributions. While we are fortunate to isolate the light valence sector, it is not possible to simultaneously ascertain the environment independence of both H and O contributions. In any event, the χ PT results are in accord with the relations

$$\Delta^{++} = 3 \mu_u + \mu_l, \quad (4.8a)$$

$$\Delta^+ = 2 \mu_u + \mu_d + \mu_l, \quad (4.8b)$$

$$\Delta^0 = \mu_u + 2 \mu_d + \mu_l, \quad (4.8c)$$

$$\Delta^- = 3 \mu_d + \mu_l, \quad (4.8d)$$

$$\Sigma^{*+} = 2 \mu_u + \mu_s + \mu_l, \quad (4.8e)$$

$$\Sigma^{*0} = \mu_u + \mu_d + \mu_s + \mu_l, \quad (4.8f)$$

$$\Sigma^{*-} = 2 \mu_d + \mu_s + \mu_l, \quad (4.8g)$$

$$\Xi^{*0} = 2 \mu_s + \mu_u + \mu_l, \quad (4.8h)$$

$$\Xi^{*-} = 2 \mu_s + \mu_d + \mu_l, \quad (4.8i)$$

$$\Omega^- = 3 \mu_s + \mu_l, \quad (4.8j)$$

where

$$\mu_u = \frac{2}{3} L = \frac{2}{3} (-\Omega^- - 2 \Delta^0), \quad (4.9a)$$

$$\mu_d = -\frac{1}{3} L = -\frac{1}{3} (-\Omega^- - 2 \Delta^0), \quad (4.9b)$$

$$\mu_s = -\frac{1}{3} H = -\frac{1}{3} (-\Omega^- + \Delta^0), \quad (4.9c)$$

$$\mu_l = \Delta^0. \quad (4.9d)$$

Of course this is simply the naive constituent quark model result with an explicit disconnected sea-quark contribution. Moreover, the qualitative agreement between the valence quark sector results of lattice QCD calculations and the simple quark model has already been established in [14].

The new information obtained from χ PT is the explicit relationship between SU(3) breaking in the disconnected loop and that in the valence sector. The vanishing Σ^{*0} moment in χ PT trivially provides the Δ^0 moment as a sum of constituent quark moments

$$\Delta^0 \simeq -(\mu_u + \mu_d + \mu_s), \quad (4.10a)$$

$$\simeq -\frac{1}{3}(L-H). \quad (4.10b)$$

The valence sectors of Σ^{*0} and Ξ^{*0} may be used to estimate the Δ^0 magnetic moment. The lattice QCD calculations of Ref. [14] indicate

$$\Delta^0 = -\Sigma_{\text{val}}^{*0} = -0.29 \pm 0.05 \mu_N, \quad (4.11a)$$

$$= -\frac{1}{2}\Xi_{\text{val}}^{*0} = -0.33 \pm 0.07 \mu_N, \quad (4.11b)$$

and the weighted average of these results is

$$\mu_l^\Delta = \Delta^0 = -0.30 \pm 0.04 \mu_N. \quad (4.12)$$

This result is approximately three times the result obtained for octet baryons under the assumption of no environment sensitivity for the disconnected sea-quark loop [13]. There it was found

$$\mu_l^N = -0.10 \pm 0.06 \mu_N. \quad (4.13)$$

Naively, one might expect such a factor of three as the spin of the Δ is three times the spin of the nucleon.

The disconnected sea-quark loop contribution of (4.12) applied to the Ω^- is precisely the contribution required to augment the earlier reported [14] valence sector contribution of $-1.73(22)\mu_N$ to $-2.03(22)\mu_N$ and restore agreement with the new precision measurement of the Ω^- magnetic moment [23] of $-2.024(56)\mu_N$.

The disconnected sea-quark loop contribution to decuplet baryon magnetic moments is not small. Given the recent flurry of activity surrounding decuplet baryon magnetic moments, a summary of the predictions for these moments obtained from lattice QCD [12,14] including the disconnected sea contributions is provided in Table I. The chief assumption in obtaining the loop contribution for decuplet baryons is the independence of loop contributions from environment effects, an assumption supported by the agreement of estimates for μ_l^Δ from Σ^{*0} and Ξ^{*0} .

The scale parameter accounting for lattice artifacts is selected to reproduce the quantity $\mu_{\Sigma^+} - \mu_{\Sigma^-}$ which is independent of disconnected sea-quark loops and provides minimal statistical uncertainties in the predictions. This scale parameter differs slightly from the parameter reproducing the proton moment used up to this point. The Δ^0 moment indicates the best estimate for $\mu_l^\Delta = -0.319 \pm 0.053$.

Octet baryon moments with the disconnected sea-quark loop contributions of (3.6) are also indicated in Table I to provide some reference on the reliability of the predictions.

TABLE I. Predictions for octet and decuplet baryon magnetic moments from lattice QCD are compared with experimental results [19,23] where available. A single scale parameter accounting for lattice artifacts has been introduced to reproduce the quantity $\mu_{\Sigma^+} - \mu_{\Sigma^-}$, which is independent of disconnected sea-quark loops. Uncertainties in the last digit(s) are indicated in parentheses.

Baryon	Lattice QCD (μ_N)	Experiment (μ_N)
Octet ^a		
p	2.72(26)	2.793
n	-1.82(34)	-1.913
Σ^+	2.47(9)	2.458(10)
Σ^0	0.66(8)	
Σ^-	-1.15(9)	-1.160(25)
Ξ^0	-1.27(14)	-1.250(14)
Ξ^-	-0.63(7)	-0.6507(25)
Decuplet		
Δ^{++}	5.98(93)	3.7→7.5
Δ^+	2.83(45)	
Δ^0	-0.319(53)	
Δ^-	-3.47(52)	
Σ^{*+}	2.95(33)	
Σ^{*0}	0.021(12)	
Σ^{*-}	-2.91(33)	
Ξ^{*0}	0.276(39)	
Ξ^{*-}	-2.47(24)	
Ω^-	-2.11(20)	-2.024(56)

^a Λ^0 is omitted as the disconnected sea-quark loop contribution cannot be estimated from the QCD equalities.

It is remarkable that a single scale parameter applied to the lattice QCD results is all that is required to reproduce the octet baryon magnetic moments within uncertainties. The essential ingredients to any model which hopes to reproduce these moments are an environment sensitivity of the quark sector contributions and a small but essential disconnected sea-quark contribution.

V. STRANGENESS CONTRIBUTIONS TO NUCLEON MOMENTS

The idea that the contributions of various flavors running around the disconnected sea loop might be estimated by considering ratios of constituent quark masses is not new [24]. However, the connection between quarks running around the disconnected loop and constituent quarks is obscured by the fact that constituent quarks are usually associated with the valence quark sector carrying the quantum numbers of the hadron. However, χ PT and the QCD equalities presented here have provided a quantitative link between the two, consistent to $O(p^2)$ and one-loop in χ PT.

With this new relationship, the strangeness contribution to nucleon moments may be isolated. Denoting μ_l^u as the u -quark contribution to the disconnected sea loop in the nucleon, etc.,

TABLE II. Various estimates for strange quark contributions to nucleon magnetic moments.

Approach	Reference	$G_M^s(0) = -3\mu_l^s$ (μ_N)
QCD equalities	This work	-0.75 ± 0.30
Lattice QCD	Leinweber [18,13]	-0.33
Poles	Hammer <i>et al.</i> [25]	-0.24 ± 0.03
SU(3) NJL	Kim <i>et al.</i> [26]	-0.45
Kaon loops	Musolf and Burkardt [27]	$-0.31 \rightarrow -0.40$
Kaon loops	Cohen <i>et al.</i> [28]	$-0.24 \rightarrow -0.32$
Vector dom.	Cohen <i>et al.</i> [28]	$-0.24 \rightarrow -0.32$
SU(3) Skyrme	Park <i>et al.</i> [29]	-0.13
Poles	Jaffe [30]	-0.31 ± 0.09
Simple quark model	This work and Ref. [19]	-0.20

$$\mu_l^N = \mu_l^u + \mu_l^d + \mu_l^s, \quad (5.1a)$$

$$= -\left(\frac{\mu_{\text{val}}^d}{\mu_{\text{val}}^s} - 1\right) \mu_l^s. \quad (5.1b)$$

Here, the equivalence of light to heavy magnetic moment ratios in the valence and loop sector has been assumed, in accord with the discussion surrounding (4.10). The doubly represented quark sector contributions have smaller statistical uncertainties and are more like constituent quarks than the singly represented quark sector [14]. Using the doubly represented quark sectors of Σ and Ξ discussed in Sec. III to estimate the ratio of quark moments,

$$\begin{aligned} \mu_l^s &= -\frac{D_{\Xi}/D_{\Sigma}}{(1-D_{\Xi}/D_{\Sigma})} \mu_l^N, \\ &= +0.25 \pm 0.10 \mu_N. \end{aligned} \quad (5.2)$$

TABLE III. Predictions for the quark sector contributions to baryon magnetic moments. Quark charges are included, such that the row sum reproduces the baryon moment. A single scale parameter accounting for lattice artifacts has been introduced to reproduce the quantity $\mu_{\Sigma^+} - \mu_{\Sigma^-}$. Uncertainties in the last digit(s) are indicated in parentheses. All moments are in units of μ_N .

Baryon	Valence sector			Disconnected loop sector		
	u	d	s	u	d	s
p	2.74(20)	0.15(11)	0	-0.84(24)	0.42(12)	0.25(10)
n	-0.30(21)	-1.37(10)	0	-0.84(24)	0.42(12)	0.25(10)
Σ^+	2.42(4)	0	0.08(5)	-0.15(20)	0.07(10)	0.04(7)
Σ^0	1.21(2)	-0.60(1)	0.08(5)	-0.15(20)	0.07(10)	0.04(7)
Σ^-	0	-1.21(2)	0.08(5)	-0.15(20)	0.07(10)	0.04(7)
Ξ^0	-0.43(9)	0	-0.71(5)	-0.67(18)	0.34(9)	0.20(7)
Ξ^-	0	0.22(5)	-0.71(5)	-0.67(18)	0.34(9)	0.20(7)
Δ^{++}	6.30(96)	0	0	-1.54(21)	0.77(10)	0.45(9)
Δ^+	4.20(64)	-1.05(16)	0	-1.54(21)	0.77(10)	0.45(9)
Δ^0	2.10(32)	-2.10(32)	0	-1.54(21)	0.77(10)	0.45(9)
Δ^-	0.	-3.15(48)	0	-1.54(21)	0.77(10)	0.45(9)
Σ^{*+}	3.90(44)	0	-0.64(7)	-1.54(21)	0.77(10)	0.45(9)
Σ^{*0}	1.95(22)	-0.98(11)	-0.64(7)	-1.54(21)	0.77(10)	0.45(9)
Σ^{*-}	0	-1.96(22)	-0.64(7)	-1.54(21)	0.77(10)	0.45(9)
Ξ^{*0}	1.83(18)	0	-1.24(5)	-1.54(21)	0.77(10)	0.45(9)
Ξ^{*-}	0	-0.91(9)	-1.24(5)	-1.54(21)	0.77(10)	0.45(9)
Ω^-	0	0	-1.79(16)	-1.54(21)	0.77(10)	0.45(9)

Table II compares this result with other estimates for the strangeness contribution to nucleon magnetic moments. This new result is large relative to other estimates.

The large value found here relative to that in Ref. [18] reflects the environment sensitivity of the disconnected sea-quark loop contribution and the ratio of strange to light quark contributions assumed in resolving the strangeness contribution. In Ref. [18] D_{Ξ}/D_{Σ} was crudely estimated to be 1/2. Using the value of (3.5) provides $G_M^s(0) = -0.42(25)\mu_N$ assuming environment independence of the disconnected sea-quark loop contributions. This correction is small relative to that obtained by allowing for environment effects in the extraction of loop contributions.

In Sec. IV we established that constituent quark models which break isospin symmetry between the u and d quark moments have the physics of the disconnected sea-quark loops included intrinsically. Taking the usual fit of μ_u^F , μ_d^F , and μ_s^F intrinsic quark moments to p , n , and Λ baryon moments [19], the disconnected sea-quark loop contribution may be isolated from

$$\begin{aligned} \mu_l^N &= \frac{1}{3}(2\mu_d^F + \mu_u^F), \\ &= -0.031\mu_N. \end{aligned} \quad (5.3)$$

This result is small enough that one might begin to worry about real isospin violation in the current quark masses. Accounting for a 5 MeV current quark mass difference, $m_d - m_u$, increases this result to $\sim -0.04\mu_N$. Application of (5.2) using $D_{\Xi}/D_{\Sigma} = \mu_s^F/\mu_d^F$, provides the simple quark model prediction of the strangeness contribution to nucleon magnetic moments indicated in Table II.

Table III provides a breakdown of the various quark sector contributions to baryon magnetic moments. It will be interesting to confront the predictions for the nucleon with the experimental determinations anticipated from CEBAF. The sector contributions $u:d:s$ of 1.90(30):0.57(16):0.25(10) for the proton are sufficiently different from the traditionally viewed simple quark model contributions of 2.47:0.32:0 to be interesting. Similarly, the neutron sector contributions are $-1.14(32):-0.95(16):0.25(10)$ to be compared with $-1.30:-0.62:0$ in the traditional simple quark model. It is interesting to note that when isospin symmetry breaking in the u - d moments is used to isolate the strangeness contribution in the simple quark model, the quark sector contributions of $u:d:s=2.30:0.42:0.07$ for the proton and $-1.13:-0.85:0.07$ for the neutron look more similar to the predictions from lattice QCD and QCD equalities.

VI. SUMMARY

Using valence sector information coupled with experiment where available, or chiral perturbation theory, we have seen how QCD equalities can be used to gain insight into sea-quark contributions, and in particular, strangeness contributions to nucleon properties. The QCD equalities were derived for a general quark current, bilinear in the quark fields.

Here, the focus in practice has been on the magnetic properties of baryons. In particular, the disconnected sea-quark loop contribution to the nucleon moment is determined to be $-0.17\pm 0.07\mu_N$, and the strange quark contributes $+0.25\pm 0.10\mu_N$ to the loop. For decuplet baryons, the disconnected sea-quark loop contribution is estimated to be

$-0.32\pm 0.05\mu_N$, of which $+0.45\pm 0.09\mu_N$ has its origin from the strange quark.

The QCD equalities have also resolved the equivalence of χ PT to $O(p^2)$ and the simple quark model with an explicit disconnected sea-quark contribution. New relationships between SU(3)-flavor symmetry breaking in valence and sea contributions were obtained. Moreover, an implicit strangeness contribution was identified in simple constituent quark models where isospin symmetry between the u and d -quark moments, $\mu_u = -2\mu_d$, is broken.

The techniques demonstrated here may be applied to other observables of quark current operators. For example, the strangeness radius of the nucleon might be extracted from a study of experimentally measured nucleon form factors and models of Ξ and Σ form factors. Favorable combinations could be found which minimize the model dependence.

Finally, it is hoped that these equalities will be useful in the development of more sophisticated models of QCD. As one considers the possible refinements of quark models, it is important to maintain the symmetries of QCD. Not all QCD-inspired models have succeeded in doing this.

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