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Partial $U(1)_A$ restoration and η enhancement in high-energy heavy-ion collisions

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We calculate the thermally averaged rates for η-π conversion and η scattering using the Di Vecchia–Veneziano and 't Hooft models, which incorporate explicitly the $U(1)_A$ anomaly. Assuming an exponential suppression of the $U(1)_A$ anomaly, we also take into account the partial restoration of $U(1)_A$ symmetry at high temperatures. We find that the chemical equilibrium between η and π breaks up considerably earlier than the thermal equilibrium. Two distinct scenarios for the η freeze-out are discussed and the corresponding chemical potentials are calculated. We predict an enhancement of the thermal η production as a possible signal of the partial $U(1)_A$ restoration in central high-energy heavy-ion collisions.

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I. INTRODUCTION

At the Lagrangian level, QCD has, in addition to $SU(N_f)\times SU(N_f)$ chiral symmetry, an approximate $U(1)_A$ symmetry, under which all left-handed quark fields are rotated by a common phase while the right-handed quark fields are rotated by an opposite phase. It is well known that the $U(1)_A$ symmetry is violated by the axial anomaly present at the quantum level and thus cannot give rise to the Goldstone boson which would occur when $U(N_f)\times U(N_f)$ chiral symmetry is spontaneously broken. The $U(1)_A$ particle, known as $\eta'(958)$ in the $N_f=3$ case, acquires an additional mass through the quantum tunneling effects mediated by instantons [1], breaking up the mass degeneracy with pions, kaons, and η in the chiral limit when all quarks (u, d, and s) are massless. The $\eta(547)$ particle also acquires an additional mass through the mixing with η'. It is believed that at high temperatures the instanton effects are suppressed due to the Debye-type screening [2]. Then one expects a practical restoration of $U(1)_A$ at high temperatures. If the restoration occurs at a temperature lower than the chiral phase transition temperature T_χ, there may be some interesting phenomenological implications in high-energy heavy-ion collisions, as suggested first by Pisarski and Wilczek [3] and more recently by Shuryak [4]. One of the consequences of $U(1)_A$ restoration is the enhancement of η particle production at small and intermediate transverse momenta due to the softening of its mass at high temperatures. However, the final yield of the η particles and their p_t distributions both depend crucially on the chemical and thermal equilibrating processes involving the η.

In this paper, we shall examine the rates of various processes relevant for the thermal η particle production, in particular, whether or not the η can decouple early enough from the thermal system expected to be produced in relativistic heavy-ion collisions. We shall present a theoretical calculation of the thermal cross sections for the processes $\eta\eta\leftrightarrow\eta\eta$, $\pi\eta\leftrightarrow\pi\eta$, and $\eta\eta\leftrightarrow\pi\pi$, essential to the thermal and chemical equilibration. Our calculations are based on models which explicitly incorporate the $U(1)_A$ anomaly. We also assume an exponential suppression of the $U(1)_A$ anomaly due to the Debye-type screening of the instanton effect [2], which leads to the temperature dependence of the η and η' masses. Our results suggest that the chemical equilibrium breaks up for η particles long before the thermal freeze-out. We suggest a modest enhancement of thermal η production as a signal for the relic of $U(1)_A$ restoration.

This paper is organized as follows: In Sec. II we compute the mass spectrum of η and η' using the Di Vecchia–Veneziano model, which incorporates the $U(1)_A$ anomaly and the η-η' mixing effect. We obtain the low-energy theorems for various scattering amplitudes. In Sec. III we incorporate the σ and the δ resonances using the 't Hooft model and reevaluate the η scattering cross sections. In Sec. IV we study the thermal averaged cross sections responsible for maintaining thermal and chemical equilibria, and suggest that the chemical equilibrium between η and π breaks up considerably earlier than the η thermal equilibrium. We discuss two scenarios for the η freeze-out and their corresponding signals for the η production. We briefly comment on the roles of η' and the QCD sphalerons in Secs. V and VI, respectively.

II. NONLINEAR σ MODEL: LOW-ENERGY THEOREMS

Up to now, there has been no direct experimental measurement of the η scattering cross sections (or the scattering lengths). One has to rely on theoretical models to calculate the interaction rates which are complicated by many uncertainties. Nevertheless, the scattering amplitudes at low energy can be more or less precisely predicted if the meson masses are soft, thanks to the soft-meson theorems which are based on the symmetry of the interactions and depend very little on the detailed dynamics. The current algebra predictions of these scattering amplitudes have been made very

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early by Osborn [5] based on SU(3)×SU(3), where the anomalous $\mathrm{U}(1)_A$ and the η and η' mixing are not included. In the light of softening of η and η' masses at high temperatures, we argue that the symmetry can be extended to U(3)×U(3). We shall rederive the low-energy amplitudes incorporating the anomalous $\mathrm{U}(1)_A$ using the nonlinear σ model that at the lowest order should give us the low-energy theorems. The standard Di Vecchia–Veneziano model [6,7], which incorporates the explicit $\mathrm{U}(1)_A$ anomaly, reads, after integrating out the gluon field,

$$\mathscr{L}_{\rm eff}=\frac{f_\pi^2}{4}\mathrm{Tr}(\partial^\mu U\partial_\mu U^\dagger)+\frac{f_\pi^2}{4}\mathrm{Tr}(MU+MU^\dagger)+\frac{f_\pi^2}{4}\frac{a}{4N_c}(\ln\det U-\ln\det U^\dagger)^2, \qquad (1)$$

where $U=\exp(i\Phi/f_\pi)$, $f_\pi=93$ MeV, $M=\mathrm{diag}(m_\pi^2,m_\pi^2,2m_K^2-m_\pi^2)$, and

$$\Phi=\begin{pmatrix} \pi^0+\eta_8/\sqrt{3}+\sqrt{2}\,\eta_1/\sqrt{3} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0+\eta_8/\sqrt{3}+\sqrt{2}\,\eta_1/\sqrt{3} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\overline{K^0} & -2\,\eta_8/\sqrt{3}+\sqrt{2}\,\eta_1/\sqrt{3} \end{pmatrix}. \qquad (2)$$

The last term in Eq. (1) is the anomaly term which breaks $\mathrm{U}(1)_A$ explicitly. It is easy to check that Eq. (1) satisfies the anomalous Ward identity which is crucial for determining the form of $\mathrm{U}(1)_A$ breaking [8]. In Eq. (1), a is related to the topological charge correlation function in pure Yang-Mills theory:

$$a=-i\frac{6}{f_\pi^2}\int d^4x\langle T[F_{\mu\nu}\widetilde{F}^{\mu\nu}(x)F_{\mu\nu}\widetilde{F}^{\mu\nu}(0)]\rangle_{\rm YM}, \qquad (3)$$

where $\widetilde{F}^{\mu\nu}$ is the dual gluon field strength tensor and angular brackets stand for the vacuum expectation value at zero temperature or the thermal average at finite temperature. The integral a is identically zero in perturbation theory; it only receives nonperturbative contributions arising from the topologically nontrivial instanton configurations. The calculation of a at both zero and finite temperatures has been done by Gross, Pisarski, and Yaffe [2] using a dilute gas approximation, and by Dyakonov and Petrov and by Shuryak [9] using an instanton liquid model. For our purpose, the phenomenological value of a at $T=0$ can be fixed by the meson mass spectroscopy, while $a(T\neq 0)$ will be modeled by assuming an exponential suppression shown by Pisarski and Yaffe [10] at high T.

The quadratic terms for the octet η_8 and the singlet η_1 from the Lagrangian reads

$$\mathscr{L}_{\rm mass}=-\frac{1}{2}\left[\left(-\frac{m_\pi^2}{3}+\frac{4m_K^2}{3}\right)\eta_8^2+\left(\frac{2m_K^2}{3}+\frac{m_\pi^2}{3}+a\right)\eta_1^2+\frac{2\sqrt{2}}{3}(2m_\pi^2-2m_K^2)\,\eta_8\eta_1\right]. \qquad (4)$$

Clearly, there is a mixing between the octet η_8 and the singlet η_1. The physical $\eta(547)$ and $\eta'(958)$ are defined by

$$\eta=\eta_8\cos\theta+\eta_1\sin\theta, \quad \eta'=-\eta_8\sin\theta+\eta_1\cos\theta \qquad (5)$$

to diagonalize the quadratic terms with the mixing angle

$$\tan\theta=\frac{4m_K^2-m_\pi^2-3m_\eta^2}{2\sqrt{2}(m_K^2-m_\pi^2)}, \qquad (6)$$

and the physical masses are

$$m_\eta^2=(m_K^2+a/2)-\frac{1}{2}\sqrt{(2m_K^2-2m_\pi^2-a/3)^2+8a^2/9}, \qquad (7)$$

$$m_{\eta'}^2=(m_K^2+a/2)+\frac{1}{2}\sqrt{(2m_K^2-2m_\pi^2-a/3)^2+8a^2/9}. \qquad (8)$$

The mixing angle θ, as well as m_η^2 and $m_{\eta'}^2$, depend on the instanton-induced quantity a which is a function of temperature.

The precise form of $a(T)$ at a temperature lower than the chiral phase transition temperature (T_χ) is not known. It has been shown by Shuryak and Velkovsky [11] that at a very low $T<f_\pi$, the instanton density shows a rather weak dependence on T. It is also argued by Pisarski and Wilczek [3] and by Shuryak [4] that at T_χ the instanton effect should be suppressed at least by an order of magnitude if the instanton is responsible for the spontaneous chiral symmetry breaking. This indicates a rather strong temperature dependence of the instanton effect as T approaches T_χ from below. To model such a dependence, we use a phenomenological parametrization in an exponential form [10,12,13]

$$a(T)=a(0)e^{-(T/T_0)^2}, \qquad (9)$$

where $T_0\simeq 100-200$ MeV, while keeping the masses of the pion and kaon approximately temperature independent, since they change very slowly with the temperature. It is known that mixing angle θ, m_η^2, and $m_{\eta'}^2$ at $T=0$ cannot be simultaneously fit to their experimental values by a single parameter $a(0)$. The best fit is to use the measured value of $m_\eta^2+m_{\eta'}^2$ as an input to determine $a(0)=(m_\eta^2+m_{\eta'}^2)-2m_K^2$ and use this $a(0)$ to predict θ, m_η^2, and $m_{\eta'}^2$ using Eqs. (6) and (7). At $T=0$, the predicted values are $\theta=18.3°$, $m_\eta=500$ MeV, and $m_{\eta'}=984$ MeV, compared to

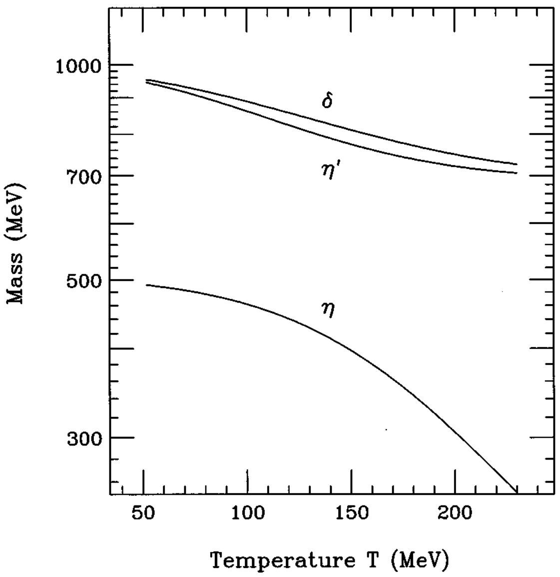

FIG. 1. The temperature dependence of m_η, $m_{\eta'}$, and m_δ. The parameter in the exponential suppression of the instanton effect is taken to be $T_0 = 150$ MeV.

the measured values $\theta^{\rm expt} \simeq 20°$ from $\eta, \eta' \to \gamma\gamma$, $m_\eta^{\rm expt} = 547$ MeV, and $m_{\eta'}^{\rm expt} = 958$ MeV. The temperature dependence of m_η and $m_{\eta'}$ is completely determined by a temperature-dependent $a(T)$ given in Eq. (9). Throughout this paper, we take $T_0 = 150$ MeV in Eq. (9). It should be emphasized that in relativistic heavy-ion collisions, the thermal system freezes out at about $T_{\rm th} = 130-150$ MeV, when the collision time scale exceeds the size of the system mainly determined by the nuclear radius $R = 4-8$ fm for central S+S or Pb+Pb collisions. Below the freeze-out temperature $T_{\rm th}$, the finite-temperature calculation of $a(T)$ does not make sense and the behavior of a is determined by the nonequilibrium dynamics. Figure 1 schematically plots such a temperature dependence. Clearly, the η becomes soft at high T and eventually is degenerate with the pions. The mass of η' also decreases. However, it does not become degenerate with the pion because of the large strange-quark mass, as is seen from Eq. (8). From Fig. 1 we see that the η' mass at high temperatures is still higher than the η mass at zero temperature. In Fig. 1 we also plot the temperature dependence of the δ-resonance mass which we will discuss in the following section. At temperatures higher than T_χ, the masses of these excitation modes will all rise again.

The interaction terms are obtained by expanding U in Eq. (1). In contrast with the pion field, there are no derivative couplings involving η and η'. We shall ignore the interactions of η and η' with kaons since they are heavy compared with pions. To the lowest order, the quartic terms involving π, η, and η' are

$$\begin{aligned}\mathscr{L}_{\rm int} = \frac{1}{2\times 4!\, f_\pi^2}[&2m_\pi^2(\eta\sin\chi + \eta'\cos\chi)^4 \\ &+ (4m_K^2 - 2m_\pi^2)(\eta'\sin\chi - \eta\cos\chi)^4 \\ &+ 12m_\pi^2 \boldsymbol{\pi}^2(\eta\sin\chi + \eta'\cos\chi)^2], \end{aligned} \tag{10}$$

where $\chi = \theta + \arctan 1/\sqrt{2}$. At very high T, as $\theta \to \arctan\sqrt{2}$ and $\chi \to \pi/2$, we can see that the η' decouples from interactions with π and η. The low-energy theorems on the two-body scattering amplitudes can be easily derived from Eq. (10):

$$\begin{aligned}
&\mathscr{A}(\eta\eta \leftrightarrow \eta\eta) = \frac{1}{f_\pi^2}[m_\pi^2 \sin^4\chi + (4m_K^2 - 2m_\pi^2)\cos^4\chi], \\
&\mathscr{A}(\eta\eta \leftrightarrow \pi^a\pi^a) = \mathscr{A}(\pi^a\eta \leftrightarrow \pi^a\eta) = \frac{1}{f_\pi^2} m_\pi^2 \sin^2\chi, \\
&\mathscr{A}(\eta'\eta' \leftrightarrow \pi^a\pi^a) = \mathscr{A}(\pi^a\eta' \leftrightarrow \pi^a\eta') = \frac{1}{f_\pi^2} m_\pi^2 \cos^2\chi, \\
&\mathscr{A}(\eta\eta' \leftrightarrow \pi^a\pi^a) = \mathscr{A}(\pi^a\eta' \leftrightarrow \pi^a\eta) = \frac{1}{f_\pi^2} m_\pi^2 \sin\chi\cos\chi, \\
&\mathscr{A}(\eta'\eta' \leftrightarrow \eta'\eta') = \frac{1}{f_\pi^2}[m_\pi^2 \cos^4\chi + (4m_K^2 - 2m_\pi^2)\sin^4\chi], \\
&\mathscr{A}(\eta'\eta' \leftrightarrow \eta\eta) = \mathscr{A}\eta\eta' \leftrightarrow \eta\eta' \\
&\qquad = \frac{1}{f_\pi^2}(4m_K^2 - m_\pi^2)\sin^2\chi\cos^2\chi, \\
&\mathscr{A}(\eta\eta \leftrightarrow \eta\eta') = \frac{1}{f_\pi^2}[m_\pi^2 \sin^3\chi\cos\chi \\
&\qquad - (4m_K^2 - 2m_\pi^2)\sin\chi\cos^3\chi], \\
&\mathscr{A}(\eta\eta' \leftrightarrow \eta'\eta') = \frac{1}{f_\pi^2}[m_\pi^2 \sin\chi\cos^3\chi \\
&\qquad - (4m_K^2 - 2m_\pi^2)\sin^3\chi\cos\chi].
\end{aligned} \tag{11}$$

The results calculated by Osborn [5] based on the current algebra can be recovered by taking $\theta = 0$ and using the Gell-Mann–Okubo relation $4m_K^2 = 3m_\eta^2 + m_\pi^2$. These low-energy theorems must be satisfied by any dynamical model, because they are solely based on the symmetry properties of the theory.

III. LINEAR σ MODEL: INCLUSION OF RESONANCES

The amplitudes listed in Eq. (11) grossly underestimate the strength of scatterings at higher energies, especially in the resonance regions. However, the inclusion of resonances introduces many uncertainties, such as which resonances should be included and what are the couplings of these resonances to the mesons. In addition, there is no guarantee that a naive lowest-order calculation will preserve the unitarity because of the strong interactions. Fortunately, the low-energy theorems provide us some guidelines as to how the amplitudes should approach their low-energy limits. The linear σ model based on the chiral symmetry is known to satisfy the low-energy theorems, and at the same time to be able to incorporate the resonances. To further reduce the input parameters, we consider the σ and $\delta(980)$ [now called $a_0(980)$] resonances, which, together with π and the $\eta_{\rm ns}$ to be defined below, form a complete representation of U(2)×U(2). We shall concentrate on the η particle, since there is no dramatic change of the η' mass with temperature, as shown in Fig. 1. We study the most relevant processes for the η production: $\eta\eta\leftrightarrow\pi\pi$, $\pi\eta\leftrightarrow\pi\eta$, and $\eta\eta\leftrightarrow\eta\eta$. In this case, U(3)×U(3) reduces to U(2)×U(2) except for the mixing effects which we have already calculated.

Let us introduce the nonstrange mode $\eta_{\text{ns}}=(u\overline{u}+d\overline{d})/\sqrt{2}$ and take m_s to be heavy. Then η_{ns} is approximately a mass eigenstate, $\eta_{\text{ns}}=\eta\sin\chi+\eta'\cos\chi$, whose mass is determined from Eq. (4) to be $m^2_{\text{ns}}\simeq 2a/3+m^2_\pi$. At zero temperature, $m_{\text{ns}}\simeq 709$ MeV. We then define the (2,2) representation multiplet of U(2)×U(2) as

$$\Phi=\frac{1}{2}(\sigma+i\,\eta_{\text{ns}})+\frac{1}{2}(\boldsymbol{\delta}+i\,\boldsymbol{\pi})\cdot\boldsymbol{\tau}. \tag{12}$$

The most general U(2)×U(2)-invariant potential is

$$V_0=-\mu^2\text{Tr}(\Phi^\dagger\Phi)+\frac{1}{2}(\lambda_1-\lambda_2)(\text{Tr}\Phi^\dagger\Phi)^2+\lambda_2\text{Tr}(\Phi^\dagger\Phi)^2 \tag{13}$$

and the mass term is

$$V_m=\frac{m^2_\pi f_\pi}{4}\text{Tr}(\Phi^\dagger+\Phi), \tag{14}$$

where λ_1, λ_2 are dimensionless constants. The U(1)$_A$-breaking term, consistent with the Ward identity, is introduced by 't Hooft [14] as

$$V_a=\frac{a}{3}(\det\Phi^\dagger+\det\Phi), \tag{15}$$

and the coefficient in V_a is chosen such that it gives the correct mass for η_{ns}. The mass spectrum can be derived from Eqs. (13), (14), and (15) by making a shift $\sigma\rightarrow f_\pi+\sigma$:

$$m^2_\sigma=\lambda_1 f^2_\pi+m^2_\pi,\quad m^2_\delta=\lambda_2 f^2_\pi+m^2_{\text{ns}}. \tag{16}$$

The decay widths are

$$\Gamma_\sigma=\frac{3}{32\pi}(m^2_\sigma-4m^2_\pi)^{1/2}\frac{(m^2_\sigma-m^2_\pi)^2}{f^2_\pi m^2_\sigma}, \tag{17}$$

$$\Gamma_\delta=\{[m^2_\delta-(m_{\text{ns}}+m_\pi)^2][m^2_\delta-(m_{\text{ns}}-m_\pi)^2]\}^{1/2}\times\frac{(m^2_\delta-m^2_{\text{ns}})^2}{16\pi f^2_\pi m^3_\delta}. \tag{18}$$

At zero temperature, $\Gamma_\sigma\sim 1$ GeV (if $m_\sigma\sim 700$ MeV) and $\Gamma_\delta\sim 200$ MeV. In principle, we should also take into account the temperature dependence of f_π and m_σ below T_χ. Here, we assume that the chiral phase transition is very rapid after which f_π and m_σ have very slow temperature dependences. Furthermore, due to the large width of the σ, the slow temperature dependence of m_σ will not change our results significantly. Under such an assumption, the linear σ model predicts also some softening of the δ resonance as T increases, because δ is the chiral partner of π and acquires some mass from the U(1)$_A$ anomaly. The temperature dependence of m_δ is plotted in Fig. 1.

The interaction terms are

$$\mathscr{L}_{\text{int}}=\frac{\lambda_1 f_\pi}{2}(\sigma^2+\eta^2_{\text{ns}}+\boldsymbol{\delta}^2+\boldsymbol{\pi}^2)\sigma+\frac{\lambda_1}{8}(\sigma^2+\eta^2_{\text{ns}}+\boldsymbol{\delta}^2+\boldsymbol{\pi}^2)^2+\lambda_2 f_\pi(\sigma\boldsymbol{\delta}+\eta_{\text{ns}}\boldsymbol{\pi})\cdot\boldsymbol{\delta}+\frac{\lambda_2}{2}(\sigma\boldsymbol{\delta}+\eta_{\text{ns}}\boldsymbol{\pi})^2+\frac{\lambda_2}{2}(\boldsymbol{\delta}\times\boldsymbol{\pi})^2. \tag{19}$$

The coupling constants λ_1 and λ_2 can be obtained from the mass relations of Eq. (16). It is worth pointing out that the above model should not be used to estimate the pion-pion scattering amplitude, because it does not include the important vector resonances such as ρ and A_1. However, since $\eta\eta$ and $\pi\eta$ scatterings cannot go through $J=1$ channel, they do not directly affect the interaction rates for η. Similarly, we have also neglected the η-ρ interaction.

To calculate the scattering amplitudes at the lowest order, we have to remove a pole singularity encountered when a resonance appears in the s channel. A naive introduction of Breit-Wigner resonance width will spoil the delicate cancellation between the contact interaction and the pole exchange at low energy, leading to the violation of the low-energy theorems. We adopt a minimal prescription to save the low-energy limit developed by Chanowitz and Gaillard [15], making the replacement

$$\lambda_1+\frac{\lambda^2_1 f^2_\pi}{s-m^2_\sigma+im_\sigma\Gamma_\sigma}\rightarrow\lambda_1(1-i\Gamma_\sigma/m_\sigma)\frac{s-m^2_\pi}{s-m^2_\sigma+im_\sigma\Gamma_\sigma}. \tag{20}$$

The scattering amplitudes are calculated as

$$\begin{aligned}\mathscr{A}(\eta\eta\leftrightarrow\eta\eta)&=\sin^4\chi\mathscr{A}(\eta_{\text{ns}}\eta_{\text{ns}}\leftrightarrow\eta_{\text{ns}}\eta_{\text{ns}})\\&=\sin^4\chi\lambda_1(1-i\Gamma_\sigma/m_\sigma)\left[\frac{s-m^2_\pi}{s-m^2_\sigma+im_\sigma\Gamma_\sigma}+\frac{t-m^2_\pi}{t-m^2_\sigma+im_\sigma\Gamma_\sigma}+\frac{u-m^2_\pi}{u-m^2_\sigma+im_\sigma\Gamma_\sigma}\right],\end{aligned}$$

$$\begin{aligned}\mathscr{A}(\eta\eta\leftrightarrow\pi^a\pi^a)&=\sin^2\chi\mathscr{A}(\eta_{\text{ns}}\eta_{\text{ns}}\leftrightarrow\pi^a\pi^a)\\&=\sin^2\chi\lambda_1(1-i\Gamma_\sigma/m_\sigma)\,\frac{s-m^2_\pi}{s-m^2_\sigma+im_\sigma\Gamma_\sigma}\\&\quad+\sin^2\chi\lambda_2(1-i\Gamma_\delta/m_\delta)\\&\quad\times\left[\frac{t-m^2_{\text{ns}}}{t-m^2_\delta+im_\delta\Gamma_\delta}+\frac{u-m^2_{\text{ns}}}{u-m^2_\delta+im_\delta\Gamma_\delta}\right],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(\eta\pi^a\leftrightarrow\eta\pi^a)&=\sin^2\chi\mathcal{A}(\eta_{\rm ns}\pi^a\leftrightarrow\eta_{\rm ns}\pi^a)\\&=\sin^2\chi\lambda_1(1-i\Gamma_\sigma/m_\sigma)\,\frac{t-m_\pi^2}{t-m_\sigma^2+im_\sigma\Gamma_\sigma}\\&\quad+\sin^2\chi\lambda_2(1-i\Gamma_\delta/m_\delta)\left[\frac{s-m_{\rm ns}^2}{s-m_\delta^2+im_\delta\Gamma_\delta}\right.\\&\quad\left.+\frac{u-m_{\rm ns}^2}{u-m_\delta^2+im_\delta\Gamma_\delta}\right].\end{aligned}\tag{21}$$

The cross sections for these processes are readily calculated by integrating out the scattering angle in u and t, most conveniently in the c.m. frame:

$$\sigma=\frac{f}{32\pi s}\frac{|\mathbf{p}_{3\,\rm c.m.}|}{|\mathbf{p}_{1\,\rm c.m.}|}\int_{-1}^{1}|\mathcal{A}|^2 d\cos\theta,\tag{22}$$

where $f=(1)1/2$ for (non)identical particles in the final state. We should mention that while η scattering rate at moderate temperatures might be underestimated in the nonlinear σ model, the linear σ model could also overestimate the rate. The real value might lie somewhere in between.

IV. THERMAL PRODUCTION OF THE η PARTICLE

We are interested in the production of η from a thermal source. To learn about the thermal history of the η, one needs to calculate the thermal averaged cross sections for various reaction channels. Since we are only concerned with the qualitative picture, we assume throughout the rest of this paper Boltzmann distribution functions for thermalized π's and η's and ignore the quantum Bose-Einstein enhancement. The thermal averaged cross section for $i+j\to k+l$ is

$$\langle v_{ij}\sigma_{ij}(T)\rangle=\frac{1}{8T}\frac{\int_{\sqrt{s_0}}^{\infty}d\sqrt{s}\,\sigma_{ij}(\sqrt{s})\lambda(s,m_i,m_j)K_1(\sqrt{s}/T)}{m_i^2m_j^2K_2(m_i/T)K_2(m_j/T)},\tag{23}$$

where $\lambda(s,m_i,m_j)=[s-(m_i+m_j)^2][s-(m_i-m_j)^2]$ and $\sqrt{s_0}$ is the reaction threshold. The reactions $\eta\eta\to\eta\eta$ and $\pi\eta\to\pi\eta$ determine the collision time scale responsible for maintaining the thermal equilibrium while $\eta\eta\to\pi\pi$ is responsible for the chemical equilibrium between π's and η's. We define the time scales $\tau_{\rm ther}$ and $\tau_{\rm chem}$ as

$$\begin{aligned}\tau_{\rm ther}^{-1}&=\langle v\sigma(\eta\eta\to\eta\eta)\rangle n_\eta+\langle v\sigma(\eta\eta\to\pi\pi)\rangle n_\eta\\&\quad+\langle v\sigma(\pi\eta\to\pi\eta)\rangle n_\pi,\\\tau_{\rm chem}^{-1}&=\langle v\sigma(\eta\eta\to\pi\pi)\rangle n_\eta,\end{aligned}\tag{24}$$

respectively, where n_π and n_η are the number densities for π and η, and the summation over different pion states is understood. We have performed a numerical integration in Eq. (23) and plotted $\tau_{\rm ther}$ and $\tau_{\rm chem}$ as functions of the temperature in Fig. 2. In the calculation, we have explicitly taken into account the temperature dependence of $m_\eta(T)$, $m_\delta(T)$, $m_{\rm ns}(T)$, and $\Gamma_\delta(T)$ as calculated in Secs. II and III. We take a typical value $R=6$ fm for the transverse freeze-out radius of the system. We define the thermal and chemical freeze-out temperatures $T_{\rm th}$ and $T_{\rm ch}$, respectively, as $\tau_{\rm ther}(T_{\rm th})=R$ and $\tau_{\rm chem}(T_{\rm ch})=R$. One finds, from Fig. 2,

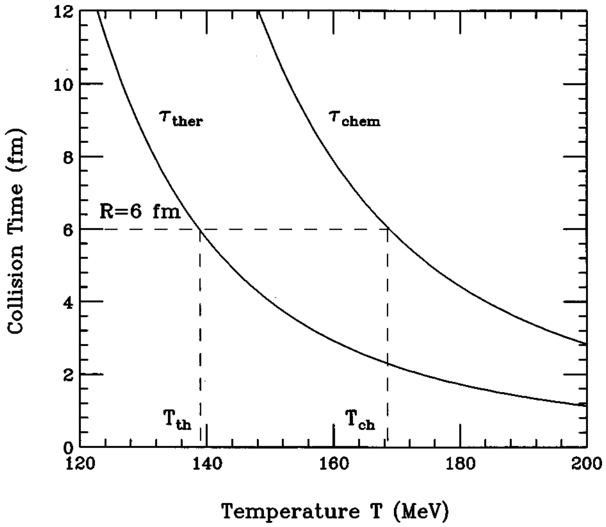

FIG. 2. The characteristic time scales of the thermal and chemical equilibrations for the η particle.

$$T_{\rm th}\simeq 139\ \text{MeV and } T_{\rm ch}\simeq 168\ \text{MeV},\tag{25}$$

which are the temperatures at which the thermal and chemical equilibria start to break up, respectively. It is worth noting that $T_{\rm th}$ is comparable to the decoupling temperature of the thermal pions.

The result that $T_{\rm ch}$ is considerably higher than $T_{\rm th}$ offers an interesting possibility to detect the suppression of the $\mathrm{U}(1)_A$ anomaly effect at high temperatures caused by the Debye-type screening. At sufficiently high temperatures $T>T_{\rm ch}$, the η rescattering and the π-η conversion are frequent so that the system possesses both thermal and chemical equilibria. As the system expands and the temperature falls into the range $T_{\rm th}<T<T_{\rm ch}$, the π-η conversion process becomes slow and is effectively turned off; the system can no longer maintain the chemical equilibrium. There is an approximate conservation of the total number of η's since neither $\eta\eta\to\eta\eta$ nor $\pi\eta\to\pi\eta$ can change the total η number. The number density of η at the chemical break-up temperature $T=T_{\rm ch}$ is determined by the mass of η at such a temperature $m_\eta(T_{\rm ch})$:

$$n_\eta[m_\eta(T_{\rm ch}),T_{\rm ch}]=\frac{1}{2\pi^2}m_\eta(T_{\rm ch})^2T_{\rm ch}K_2\left[\frac{m_\eta(T_{\rm ch})}{T_{\rm ch}}\right],\tag{26}$$

and the momentum distribution is just the Boltzmann distribution with zero chemical potential. However, this is not the final particle distribution, because the thermal collisions can still alter the momentum distribution. Nevertheless, the total number N_η given by

$$N_\eta=\pi R^2\tau_c n_\eta[m_\eta(T_{\rm ch}),T_{\rm ch}]\tag{27}$$

is conserved at any time $\tau<\tau_c$ since $\eta\eta\leftrightarrow\pi\pi$ is turned off. Here $m_\eta(T_{\rm ch})\simeq 360$ MeV and τ_c is the proper time when the temperature of the system reaches $T_{\rm ch}$.

As the system cools down to $T_{\rm th}$, the mass of η should tend to $m_\eta(T_{\rm th})\simeq 413$ MeV, according to Fig. 1. If the rate for increasing $m_\eta(T)$ is comparable to the thermal collision rate, the η particle adiabatically relaxes to $m_\eta(T_{\rm th})$. In this case, which we shall call Scenario A, one expects a standard thermal distribution for η at the freeze-out temperature $T_{\rm th}$ with a mass $m_\eta(T_{\rm th})$. The total number conservation requires η to develop a chemical potential $\mu>0$ such that (neglecting the transverse expansion)

$$\tau_d e^{\mu/T_{\rm th}} n_\eta[m_\eta(T_{\rm th}),T_{\rm th}]=\tau_c n_\eta[m_\eta(T_{\rm ch}),T_{\rm ch}], \quad (28)$$

where τ_d is the freeze-out time when $T=T_{\rm th}$. The momentum distribution function in the local comoving frame is

$$f(\mathbf{p})=e^{\mu/T_{\rm th}}e^{-\frac{\sqrt{m_\eta^2(T_{\rm th})+\mathbf{p}^2}}{T_{\rm th}}}. \quad (29)$$

The chemical potential μ is a function of temperature, whose value at freeze-out can be determined from Eq. (28), once τ_d/τ_c is known. We assume that pions dominate the energy-momentum tensor (in fact, we explicitly checked the contribution from η and found it negligible) so that τ_d/τ_c can be estimated by solving the ideal (1+1)-dimensional hydrodynamic equation

$$\frac{d\epsilon}{d\tau}+\frac{\epsilon+P}{\tau}=0, \quad (30)$$

where ϵ is the energy density and P is the pressure, for massive pions. We find that $\tau_d/\tau_c\simeq 1.53$, given $T_{\rm ch}/T_{\rm th}=1.21$. Substituting the ratio back in Eq. (28), one finds $\lambda_\eta=e^{\mu/T_{\rm th}}=1.58$. We thus predict that if there is a partial U(1)$_A$ restoration at high temperatures, the thermal η production given by Eq. (29) will be enhanced in this scenario due to both the finite chemical potential $\lambda_\eta\simeq 1.58$ and a smaller η mass $m_\eta(T_{\rm th})\simeq 413$ MeV at the thermal freeze-out temperature $T_{\rm th}$. To quantify such an enhancement, we use Eq. (29) to calculate the p_t distribution of η particle, employing the fireball model and taking into account the transverse flow effects as described in Ref. [16]:

$$\frac{dN_\eta}{p_t dp_t}\propto\lambda_\eta\int_0^R r dr\sqrt{m_\eta^2+p_t^2}\,I_0(p_t\sinh\rho/T_{\rm th})$$
$$\times K_1(\sqrt{m_\eta^2+p_t^2}\cosh\rho/T_{\rm th}), \quad (31)$$

where $\rho=\tanh^{-1}(\beta_t)$ and $\beta_t=\beta_s(r/R)^\alpha$ (with $\beta_s=0.5$, $\alpha=2$) is the transverse flow velocity profile [16]. To reduce the possible normalization ambiguity, we also calculate the p_t distribution for pions at the same freeze-out temperature $T_{\rm th}$, taking into account only the dominant resonance decays, $\rho\rightarrow 2\pi$, and plot the ratio

$$\frac{\eta}{\pi^0}\equiv\frac{dN_\eta/p_t dp_t}{dN_\pi/p_t dp_t} \quad (32)$$

as a function of p_t in Fig. 3 as the dot-dashed line. It should be noted that the thermal ratio is only relevant when p_t is small. At very large p_t, hard processes become important and the fireball model is no longer applicable. For comparison, we also plot the same ratio for a normal case in which the η particles freeze out at the same temperature $T_{\rm th}$ but with the zero-temperature mass $m_\eta=540$ MeV.

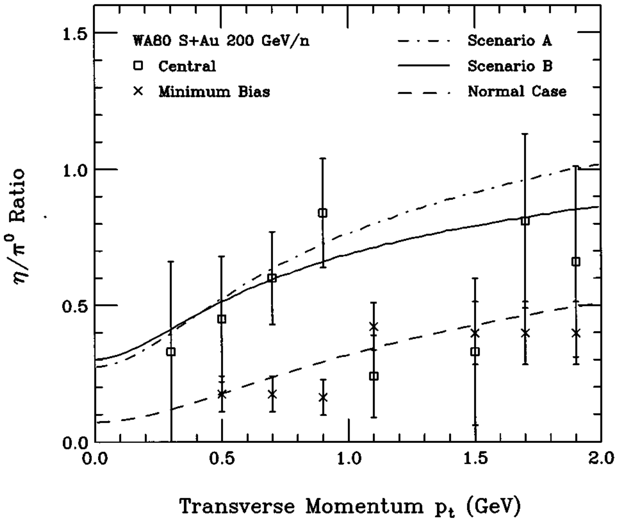

FIG. 3. The predicted ratio η/π^0 as a function of the transverse momentum p_t in three scenarios as discussed in the text: Scenarios A and B include the temperature dependence of η mass due to a partial U(1)$_A$ restoration. The zero-temperature η mass is used in the normal case. The experimental data for both central and minimus-biased events from WA80 [17,18] are also indicated.

Another situation, which we shall call Scenario B, is that where the rate for increasing $m_\eta(T)$ when $T_{\rm th}<T<T_{\rm ch}$ is considerably smaller than the thermal collision rate. In this case, things get more complicated because the screening process is out of equilibrium. The η number conservation still holds, but the momentum distribution is quite different from that in Scenario A. Roughly, one may imagine that even though the temperature drops to $T_{\rm th}$ after the chemical breakup, m_η will still have the value $m_\eta(T_{\rm ch})$, in close analogy to a "quenching" situation. The number density at the thermal freeze-out temperature is then $n_\eta[m_\eta(T_{\rm ch}),T_{\rm th}]$, and the chemical potential is determined by

$$\tau_d e^{\mu/T_{\rm th}} n_\eta[m_\eta(T_{\rm ch}),T_{\rm th}]=\tau_c n_\eta[m_\eta(T_{\rm ch}),T_{\rm ch}], \quad (33)$$

yielding $\lambda_\eta=e^{\mu/T_{\rm th}}\simeq 1.24$. The momentum distribution function is

$$f(\mathbf{p})=e^{\mu/T_{\rm th}}e^{-\frac{\sqrt{m_\eta^2(T_{\rm ch})+\mathbf{p}^2}}{T_{\rm th}}}, \quad (34)$$

which predicts larger η enhancement at low p_t than that at high p_t. We also plot the ratio η/π^0 based on this scenario in Fig. 3 as the solid line.

What happens after the thermal freeze-out? It is clear that there must exist some mechanism for the η to relax from the "temporary" entity whose mass is either $m_\eta(T_{\rm th})$ or $m_\eta(T_{\rm ch})$ to its true identity at zero temperature with $m_\eta=540$ MeV. A possible picture might be that the η particles still feel a negative potential in the fireball. The height of the potential barrier is determined by the mass difference $\Delta m=m_\eta-m_\eta(T_{\rm ch})$. The η particles with p_t smaller than Δm will be trapped in the potential well until the rarefaction wave reaches the center of the interaction volume. Such a picture has been suggested by Shuryak [4] and is similar to the mechanism of cold kaon production [20]. At this stage, we do not attempt to address this nonequilibrium issue, but just to remark that our calculation here may have underestimated the enhancement effect at small $p_t \lesssim \Delta m \sim 100-200$ MeV.

As a comparison, we also plot the thermal η/π ratio and its p_T dependence assuming the zero-temperature η mass at the thermal freeze-out as the dashed line in Fig. 3. The η/π ratio and its p_T dependence in this normal case is compatible with the data of minimus-biased $S+Au$ events from WA80 [17] experiments where peripheral collisions dominate and we believe there is no partial $\mathrm{U}(1)_A$ restoration. Comparing to this normal case, both Scenarios A and B predict an enhancement of the thermal η production due to a partial $\mathrm{U}(1)_A$ symmetry restoration. Although preliminary data from WA80 [18] on η/π^0 ratio in central $S+Au$ collisions at the CERN Super Proton Synchroron (SPS) energy, as indicated in Fig. 3, have shown such a trend of enhancement over the minimus-biased events, one certainly needs better statistics of the central collisions in order to make a definite conclusion. A related matter is the enhanced dilepton pair production via η Dalitz decay $\eta \rightarrow \ell^+ \ell^- \gamma$. If the η production is enhanced about three times, as we have predicted, the observed dilepton enhancement with the invariant mass below 500 MeV at the CERN SPS [19] may be partially accounted for.

V. ROLE OF THE η'

There should be also some enhancement of the ratio η'/π^0, since the mass of η' also decreases as the temperature increases. Moreover, since the couplings of η' to η and π in our model become small and eventually goes to zero when $\mathrm{U}(1)_A$ is completely restored, η' might decouple from the system earlier than η. The decay $\eta' \rightarrow \pi\pi\eta$ can also enhance the η production. However, in our model, we postulate that the $\mathrm{U}(1)_A$ restoration occurs at a temperature below the chiral phase transition temperature. Therefore, the kaon mass m_K is large and the η' does not become very soft. At $T=T_{\mathrm{ch}}$, the η' mass $m_{\eta'}$ is about 750 MeV. Even without the effect of chiral symmetry breaking, the large strange-quark mass can give rise to a large mixing between η and η' according to Eq. (4). This mixing gives η' a mass $m_{\eta'}^2 = 2m_K^2 - m_\pi^2$ even if $\mathrm{U}(1)_A$ is completely restored. This mass is significantly larger than the η mass at any temperature. Therefore, in the context of our model, the η' effects are only moderate. The ratio η'/π^0 should never exceed that of η/π^0.

VI. ROLE OF QCD SPHALERONS

So far we have confined ourselves to the possible suppression of the instanton effects at finite temperature that causes the softening of the masses arising from the topological charge transitions. At very high temperatures, it is known that such transitions can occur without going through the instanton configurations. In fact, they are dominated by sphaleronlike transitions whose electroweak counterparts have been extensively studied in the literature [21]. It is pointed out by McLerran, Mottola, and Shaposhnikov [22] that the rate of a QCD sphaleron transition should be estimated in analogy to the electroweak theory for temperatures above the symmetry restoration, which may not be quite suppressed. In the range of temperatures discussed in this paper, the rate of the QCD sphaleron transition may be unimportant. A rough estimate by Giudice and Shaposhnikov [23] is

$$\Gamma_{\mathrm{sph}}^{\mathrm{QCD}} = \frac{8}{3}(\alpha_s/\alpha_W)^4 \Gamma_{\mathrm{sph}}^{\mathrm{EW}} = \frac{8\kappa}{3}(\alpha_s T)^4, \tag{35}$$

where κ is the strength of the transition. The characteristic time scale of the sphaleron transition is

$$\tau_{\mathrm{sph}} = (192\kappa\alpha_s^4 T)^{-1} \sim \frac{50}{\kappa T}. \tag{36}$$

There is some evidence for κ to be $O(1)$ from lattice calculations [23]. Unless κ is really big, greater than 10, the sphalerons should be decoupled from the system in the hadronic phase, where the instanton effect is most dominant.

Note added. After completing this work we learned of a recent paper by Kapusta, Kharzeev, and McLerran on the effect of $\mathrm{U}(1)_A$ symmetry restoration on η' particle production [24].

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