General treatment of τ semileptonic decays by polarized-partial-width measurements

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(Received 29 December 1995)

The most general Lorentz-invariant spin-correlation functions for $\tau^-\rightarrow\rho^-\nu$, $a_1^-\nu$, $K^{*-}\nu$, $\pi^-\nu$, $K^-\nu$ are expressed in terms of eight semileptonic parameters. The parameters are physically defined in terms of τ-decay partial-width intensities for polarized final states. The parameters are also expressed in terms of a ''$(V-A)$ + additional chiral coupling'' structure in the $J_{\text{lepton}}^{\text{charged}}$ current, so as to bound effective-mass scales Λ for ''new physics'' such as arising from lepton compositeness, leptonic CP violation, leptonic T violation, τ weak magnetism, weak electricity, and/or second-class currents. The two tests for leptonic CP violation in $\tau\rightarrow\rho\nu$ decay are generalized to $\tau\rightarrow a_1\nu$ decay and to two additional tests if there are ν_R and $\overline{\nu_L}$ couplings. For $10^7(\tau^-,\tau^+)$ pairs at 10 GeV, from the $\{\rho^-,\rho^+\}$ mode and using the four-variable distribution I_4, the ideal statistical percentage errors are, for ξ, 0.6%, for ζ, 0.7%, for σ, 1.3%, and for ω, 0.6%. CP tests are typically $\sqrt{2}$ worse. Parameters sensitive to leptonic T violation are ω, and the following from the $\{a_1^-,a_1^+\}$ mode: using I_5^- the errors are, for η, 0.6%; using I_7, for η', 0.013; and using I_7^-, for ω', 0.002. In the future, by stage-two spin-correlation techniques, polarized-partial-width measurements should be useful in studying top quark, $W^\pm$, Z^0, and Higgs boson decays. [S0556-2821(96)05509-9]

PACS number(s): 13.35.Dx, 11.30.Er, 14.60.Fg

I. INTRODUCTION

The principal purpose of this paper is to provide a general treatment of two-body τ decays [1] which only assumes Lorentz invariance and exploits the treelike structure of the dominant contributions to the $\tau^-\tau^+$ production-decay sequence. In particular, CP invariance and a mixed $(V\mp A)$ structure of the τ charged current is not assumed. For instance, we introduce eight parameters to describe the most general spin-correlation function for the decay sequence $Z^0,\gamma^*\rightarrow\tau^-\tau^+\rightarrow(\rho^-\nu)(\rho^+\overline{\nu})$ followed by $\rho^{\text{ch}}\rightarrow\pi^{\text{ch}}\pi^0$ including both ν_L,ν_R helicities and both $\overline{\nu_R},\overline{\nu_L}$ helicities. Thus, by including the ρ polarimetry information that is available from the $\rho^{\text{ch}}\rightarrow\pi^{\text{ch}}\pi^0$ decay distribution, the polarized partial widths for $\tau^-\rightarrow\rho^-\nu$ are directly measurable. Depending principally on the absence of other interfering decay modes, direct measurements of polarized partial widths and of the associated ''longitude-transverse'' interference intensities should also be possible in top quark decays, in $W^\pm$ and Z^0 boson decays, and in Higgs meson decays.

The eight τ semileptonic decay parameters for $\tau^-\rightarrow\rho^-\nu,\ldots,$ are defined for the four polarized $\rho_{L,T}\nu_{L,R}$ final states: The first parameter is simply $\Gamma\equiv\Gamma_L^++\Gamma_T^+$, i.e., the full partial width for $\tau^-\rightarrow\rho^-\nu$. The second is the chirality parameter $\xi\equiv(\Gamma_L^-+\Gamma_T^-)/\Gamma$. Equivalently, $\xi\equiv$(Prob ν_τ is ν_L)$-$(Prob ν_τ is ν_R), or

$$\xi\equiv|\langle\nu_L|\nu_\tau\rangle|^2-|\langle\nu_R|\nu_\tau\rangle|^2. \tag{1}$$

So a value $\xi=1$ means the coupled ν_τ is pure ν_L. ν_L (ν_R) means the emitted neutrino has left- (L-)handed (R-handed) polarization. For the special case of a mixture of only V and A couplings and $m_{\nu_\tau}=0$, $\xi\rightarrow(|g_L|^2-|g_R|^2)/(|g_L|^2+|g_R|^2)$ and the ''stage-one spin correlation'' parameter $\zeta\rightarrow\xi$; see below.

The subscripts on the Γ's denote the polarization of the final ρ^-, either ''L=longitudinal'' or ''T=transverse;'' superscripts denote ''$\pm$ for sum/difference of the ν_L versus ν_R contributions.'' Such final-state-polarized partial widths are physical observables and, indeed, the equivalent semileptonic parameters $\xi,\zeta,\ldots$ can be measured by various spin-correlation techniques.

The remaining partial-width parameters are defined by

$$\zeta\equiv(\Gamma_L^--\Gamma_T^-)/(\mathscr{S}_\rho\Gamma),\quad \sigma\equiv(\Gamma_L^+-\Gamma_T^+)/(\mathscr{S}_\rho\Gamma). \tag{2}$$

To describe the interference between the ρ_L and ρ_R amplitudes, we define

$$\omega\equiv I_{\mathscr{R}}^-/(\mathscr{R}_\rho\Gamma),\quad \eta\equiv I_{\mathscr{R}}^+/(\mathscr{R}_\rho\Gamma),$$
$$\omega'\equiv I_{\mathscr{I}}^-/(\mathscr{R}_\rho\Gamma),\quad \eta'\equiv I_{\mathscr{I}}^+/(\mathscr{R}_\rho\Gamma), \tag{3}$$

where the measurable LT-interference intensities are

$$I_{\mathscr{R}}^\pm=|A(0,-\tfrac{1}{2})||A(-1,-\tfrac{1}{2})|\cos\beta_\alpha \pm|A(0,\tfrac{1}{2})||A(1,\tfrac{1}{2})|\cos\beta_a^R,$$
$$I_{\mathscr{I}}^\pm=|A(0,-\tfrac{1}{2})||A(-1,-\tfrac{1}{2})|\sin\beta_\alpha \pm|A(0,\tfrac{1}{2})||A(1,\tfrac{1}{2})|\sin\beta_a^R. \tag{4}$$

Here $\beta_a\equiv\phi_{-1}^a-\phi_0^a$, and $\beta_a^R\equiv\phi_{-1}^a-\phi_0^{aR}$ are the measurable phase differences of the associated helicity amplitudes $A(\lambda_\rho,\lambda_\nu)=|A|\exp(\iota\phi)$.

The definition for σ in Eq. (2) implies that $\tilde{\sigma}\equiv$(Prob ρ is ρ_L)$-$(Prob ρ is ρ_T), where

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TABLE I. Comparison of parameters' values for unique Lorentz couplings: entries are for ρ^- (a_1^-, if value differs). Numerical values are to one digit. The first three parameters (ξ,ζ,σ), plus the full partial width $\Gamma(\tau^-\to\rho^-\nu)$, give the polarized-final-state partial widths, Γ, for the four $\rho_{L,T}\nu_{L,R}$ final-state combinations. The latter four parameters ($\omega,\eta,\omega',\eta'$) give the complete $\rho_L-\rho_T$ interference intensities, Γ_{LT}.

	$V\mp A$	$S\pm P$	f_M+f_E	f_M-f_E
Γ's				
ξ	±1	±1	1	-1
$\mathscr{S}_\rho\zeta$	$\pm0.5(0)$	±1	$-0.8(-0.6)$	0.3
$\mathscr{S}_\rho\sigma$	0.5(0)	1	$-0.8(-0.6)$	-0.3
Γ_{LT}'s				
$\mathscr{R}_\rho\omega$	$\pm0.4(\pm0.5)$	0	0.3(0.5)	$-0.2(-0.7)$
$\mathscr{R}_\rho\eta$	0.4(0.5)	0	0.3(0.5)	0.2(0.7)
ω'	0	0	0	0
η'	0	0	0	0

$$\tilde{\sigma}=\mathscr{S}_\rho\sigma,$$

is the analogue of the neutrino's chirality parameter in Eq. (1). Thus the parameter σ, or $\tilde{\sigma}$, measures the degree of polarization of the emitted ρ. The parameter $\tilde{\zeta}\equiv\mathscr{S}_\rho\zeta$ characterizes the remaining odd-odd mixture of the ν and ρ spin polarizations. The full partial-width Γ characterizes the even-even mixture. Notice that we introduce ''tilde'' accents to denote the relative-partial-width-intensity parameters which occur when the hadronic factors $\mathscr{S}_\rho$, or $\mathscr{R}_\rho$, are factored out. Similarly, we define $\tilde{\omega}\equiv\mathscr{R}_\rho\omega$, $\tilde{\omega}'\equiv\mathscr{R}_\rho\omega'$, $\tilde{\eta}\equiv\mathscr{R}_\rho\eta$, $\tilde{\eta}'\equiv\mathscr{R}_\rho\eta'$.

In Sec. II, there is further discussion of these polarized partial widths in the helicity formalism (in the Jacob-Wick phase convention).

Important remarks

(1) The numerical values of ''$\xi,\zeta,\sigma,\ldots$'' are very distinct for different unique Lorentz couplings; see Tables I and II.

(2) Primed parameters $\omega'\neq0$ and/or $\eta'\neq0\Rightarrow\tilde{T}_{\rm FS}$ is violated (see Sec. II below).

(3) Barred parameters $\bar{\xi},\bar{\zeta},\ldots$ have the analogous definitions, see Sec. II, for the $\overline{CP}$ conjugate modes, $\tau^+\to\rho^+\bar{\nu},\ldots$. Therefore, any $\xi\neq\bar{\xi},\zeta\neq\bar{\zeta},\ldots\Rightarrow CP$ is violated. That is, ''slashed parameters'' $\not{\xi}\equiv\xi-\bar{\xi},\ldots$, could be introduced to characterize and quantify the degree of CP violation.

(4) These same parameters appear in the general angular distributions for the polarized $\tau^-\to\rho^-\nu\to(\pi^-\pi^0)\nu$ decay chain,

$$\frac{dN}{d(\cos\theta_1^\tau)d(\cos\tilde{\theta}_a)d\tilde{\phi}_a}=\mathbf{n}_a[1\pm\mathbf{f}_a\cos\theta_1^\tau]$$
$$\mp(1/\sqrt{2})\sin\theta_1^\tau\sin2\tilde{\theta}_a\mathscr{R}_\rho$$
$$\times[\omega\,\cos\tilde{\phi}_a+\eta'\sin\tilde{\phi}_a],\qquad(5)$$

with upper (lower) signs for a L-handed (R-handed) τ^-, where

TABLE II. Analytic form of the semileptonic parameters for unique Lorentz couplings: In this and following tables, the mass ratios are denoted by $\rho/\tau\equiv m_\rho/m_\tau$, etc.; for the other exclusive τ decay modes, such as $\tau\to a_1\nu$, simply replace ρ by a_1, $\mathscr{S}_\rho$ by $\mathscr{S}_{a_1}$, etc. We do not tabulate ω' and η' because $\omega'=\eta'=0$ if either (i) there is a unique Lorentz coupling, (ii) there is no leptonic T violation, and/or (iii) there is a ''V and $A,m_\nu=0$'' masking mechanism; see remark (5) in Sec. I.

	$V\mp A$	$S\pm P$	f_M+f_E	f_M-f_E
Γ's				
ξ	±1	±1	1	-1
$\mathscr{S}_\rho\zeta$	$\pm\mathscr{S}_\rho$	±1	$\frac{-2+(\rho^2/\tau^2)}{2+(\rho^2/\tau^2)}$	$+\frac{1}{3}$
$\mathscr{S}_\rho\sigma$	$\mathscr{S}_\rho$	1	$\frac{-2+(\rho^2/\tau^2)}{2+(\rho^2/\tau^2)}$	$-\frac{1}{3}$
Γ_{LT}'s				
$\mathscr{R}_\rho\omega$	$\pm\mathscr{R}_\rho$	0	$\frac{\sqrt{2}(\rho/\tau)}{2+(\rho^2/\tau^2)}$	$-\frac{\sqrt{2}\rho}{3\tau}$
$\mathscr{R}_\rho\eta$	$\mathscr{R}_\rho$	0	$\frac{\sqrt{2}(\rho/\tau)}{2+(\rho^2/\tau^2)}$	$\frac{\sqrt{2}\rho}{3\tau}$

$$\mathbf{n}_a=\tfrac{1}{8}(3+\cos2\tilde{\theta}_a+\sigma\mathscr{S}_\rho[1+3\cos2\tilde{\theta}_a]),$$
$$\mathbf{n}_a\mathbf{f}_a=\tfrac{1}{8}(\xi[1+3\cos2\tilde{\theta}_a]+\zeta\mathscr{S}_\rho[3+\cos2\tilde{\theta}_a]).\qquad(6)$$

Such formulas for the associated ''stage-two spin-correlation'' (S2SC) functions in terms of these eight semileptonic parameters are discussed below.

(5) The hadronic factors $\mathscr{S}_\rho$ and $\mathscr{R}_\rho$ have been explicitly inserted into the definitions of the semi-leptonic decay parameters, so that qualities such as $q_\rho^2=m_\rho^2$ can be smeared over in application due to the finite ρ width. For the ρ mode they are given by ($m\equiv m_\tau$)

$$\mathscr{S}_\rho=\frac{1-2(m_\rho^2/m^2)}{1+2(m_\rho^2/m^2)},\quad\mathscr{R}_\rho=\frac{\sqrt{2}(m_\rho/m)}{1+2(m_\rho^2/m^2)}.\qquad(7)$$

We have introduced the important factors $\mathscr{S}_\rho$ and $\mathscr{R}_\rho$ because, guided by experiment, we are analyzing versus a reference $J^{\rm charged}_{\rm lepton}$ theory consisting of ''a mixture of only V and A couplings with $m_\nu=0$.'' For such a theory these hadronic factors have a simple physical interpretation: for $\tau^-\to\rho^-_{L,T}\nu$ the factor $\mathscr{S}_\rho=(\text{Prob }\rho_L)-(\text{Prob }\rho_T)$, and the factor $\mathscr{R}_\rho=$the ''geometric mean of these probabilities''$=\sqrt{(\text{Prob }\rho_L)(\text{Prob }\rho_T)}$. These factors are not independent since $\mathscr{S}_\rho^2+4\mathscr{R}_\rho^2=1$.

If experiments had suggested instead a different dominant Lorentz structure than $V-A$, say ''f_M+f_E,'' then per Table II we would have replaced $\mathscr{S}_\rho$ everywhere by $(-2+\rho^2/\tau^2)/(2+\rho^2/\tau^2)$, etc.

It is reasonable at present to perform a general analysis versus a ''reference theory'' consisting of ''a mixture of only V and A couplings with $m_\nu=0$'': From experiments [1] by the ALEPH, ARGUS, CLEO II, and OPAL Collaborations, the leading contribution in the τ's $J^{\rm charged}_{\rm lepton}$ current is consistent with $(V-A)$ to better than the 5% level. For the nominal 10^7 event rates, we find that the S2SC function I_4 is insensitive, see Table I in Ref. [2], to $m_\nu \lesssim 23.1$ GeV, the present ALEPH bound.

In such a reference theory, each of the eight semileptonic parameters has a simple probabilistic significance for they are each directly proportional to Γ, ξ, $\mathcal{S}_\rho$, or $\mathcal{R}_\rho$: $\tilde{\sigma} \equiv \mathcal{S}_\rho \sigma \to \mathcal{S}_\rho$, $\tilde{\eta} \equiv \mathcal{R}_\rho \eta \to \mathcal{R}_\rho$, $\zeta \to \xi$, $\omega \to \xi$. Note in this reference theory, $\xi = (|g_L|^2 - |g_R|^2)/(|g_L|^2 + |g_R|^2)$ and $\Gamma = 2 m_\nu q_\rho (|g_L|^2 + |g_R|^2)(2 + m_\tau^2/m_\rho^2)$ in units of the Appendix.

Note in this reference theory, any leptonic T violation is ''masked'' since $\omega' = \eta' = 0$ (i.e., $\beta^a = \beta^b = 0$) automatically. This ''V and A, $m_\nu = 0$'' masking mechanism could be at least partially the cause for why leptonic T violation has not been manifest in previous experiments even if it were not suppressed in the fundamental Lagrangian.

(6) The ''additional structure'' due to additional Lorentz couplings in $J^{\rm charged}_{\rm lepton}$ can show up experimentally because of its interference with the $(V-A)$ part which, we assume, arises as predicted by the standard lepton model. Inclusion of the ρ polarimetry information that is available from the $\rho^{\rm ch} \to \pi^{\rm ch}\pi^0$ decay distribution, generalizes the ''stage-one spin-correlation'' (S1SC) function [3,4] $I(E_\rho, E_{\bar{B}})$. Since this adds on spin-correlation information from the next stage of decays in the decay sequence, we call such an energy-angular distribution a ''stage-two spin-correlation'' (S2SC) function[5].

The simplest useful S2SC is $I_4 = I(E_\rho, E_{\bar{\rho}}, \tilde{\theta}_1, \tilde{\theta}_2)$. The kinematic variables in I_4 are the usual ''spherical'' ones which naturally appear in the helicity formalism in describing such a decay sequence. The first stage of the decay sequence $\tau^-, \tau^+ \to (\rho^- \nu_\tau)(\rho^+ \bar{\nu}_\tau)$ is described by the three variables θ_1^τ, θ_2^τ, $\cos\phi$, where ϕ is the opening $\angle$ between the two decay planes. These are equivalent to the Z^0, or γ^* center-of-mass variables, E_ρ, $E_{\bar{\rho}}$, $\cos\psi$. Here ψ= ''opening $\angle$ between the ρ^- and ρ^+ momenta in the Z/γ^* c.m.''. When the Lorentz ''boost'' to one of the ρ rest frames is directly from the Z/γ^* c.m. frame, the second stage of the decay sequence is described by the usual two spherical angles for the $\pi^{\rm ch}$ momentum direction in that ρ rest frame: $\tilde{\theta}_1$, $\tilde{\phi}_1$ for $\rho_1^- \to \pi_1^- \pi_1^0$, and $\tilde{\theta}_2, \tilde{\phi}_2$ for $\rho_2^+ \to \pi_2^+ \pi_2^0$. (See figures in Ref. [5].) Similarly, a_1 polarimetry information can be included from the $\tau^- \to a_1^- \nu \to (\pi^-\pi^-\pi^+)\nu, (\pi^0\pi^0\pi^-)\nu$ decay modes.

(7) In addition to model independence, a major open issue is whether or not there is an additional chiral coupling in the τ's charged current. A chiral classification of additional structure is a natural phenomenological extension of the symmetries of the standard $SU(2)_L \times U(1)$ electroweak lepton model. The requirement of $\overline{u(p_\nu)} \to \overline{u(p_\nu)}\frac{1}{2}(1+\gamma_5)$ and/or $u(k_\tau) \to \frac{1}{2}(1-\gamma_5)u(k_\tau)$ invariance of the vector and axial current matrix elements $\langle \nu | v^\mu(0) | \tau \rangle$ and $\langle \nu | a^\mu(0) | \tau \rangle$, allows only g_L, g_{S+P}, $g_{S^-+P^-}, g_+ = f_M + f_E$, and $\tilde{g}_+ = T^+ + T_5^+$ couplings. From this $SU(2)_L$ perspective, the relevant experimental question is what are the best current limits on such additional couplings? Similarly, $\overline{u(p_\nu)} \to \overline{u(p_\nu)}$ $\frac{1}{2}(1-\gamma_5)$ and/or $u(k_\tau) \to \frac{1}{2}(1+\gamma_5)$ $u(k_\tau)$ invariance selects the complimentary set of g_R, g_{S-P}, $g_{S^- - P^-}, g_- = f_M - f_E$, and $\tilde{g}_- = T^+ - T_5^+$ couplings. The absence of $SU(2)_R$ couplings is simply built into the standard model; it is not predicted by it. So, what are the best current limits on such $SU(2)_R$ couplings in τ physics?

(8) In a separate paper[6], it has been reported that Lorentz-structure effective-mass scales of $\Lambda_i \approx$ few 100 GeV for real coupling constants g_i $(i = V+A, S \pm P, f_M \pm f_E, \ldots)$ can be probed using I_4 at M_Z center-of-mass energy in unpolarized $e^- e^+$ collisions. Lorentz-structure scales of 1–2 TeV can be probed using I_4 at 10 or 4 GeV. Such scales would arise because of a fundamental additional chiral coupling or be induced as a consequence of τ lepton compositeness.

For pure imaginary couplings, the statistical error limits obtained in the present paper for the leptonic T violation parameters $\omega, \eta, \omega', \eta'$ show that there will be significant improvement by use of I_5, I_5^-, and/or I_7^- versus the Ref. [6] results which used I_4 and gave limits of $(\Lambda_i)^2 \cong (30\ \mathrm{GeV})^2$ for pure imaginary couplings.

In Sec. III, as a step towards a precision answer to the question of additional Lorentz structures, the semileptonic parameters are expressed in terms of a ''$(V-A)$ + additional chiral coupling'' structure in the $J^{\rm charged}_{\rm lepton}$ current[6]. Two tables display the resulting values of the parameters when the various additional chiral couplings $(g_i/2\Lambda_i)$ are small relative to the standard $V-A$ coupling (g_L).

Section IV gives the most general Lorentz-invariant spin-correlation functions for $e^- e^+ \to \tau^- \tau^+$ followed by $\tau \to \rho\nu$, $a_1 \nu$, $K^* \nu$ including both $\nu_{L,R}$ helicities and both $\bar{\nu}_{R,L}$ helicities. Since these same parameters appear in Eq. (5), they could someday be measured by means of longitudinally polarized beams at a τ-charm factory or at a B factory with longitudinally polarized beams. At the end of Sec. IV several independent tests for leptonic T violation are proposed.

In Sec. V, the two tests for leptonic CP violation in $\tau \to \rho\nu$ decay [5] are generalized to $\tau \to a_1 \nu$ decay and to two additional tests if there are ν_R and $\bar{\nu}_L$ couplings[7].

Section VI treats $\tau^- \to \pi^- \nu$, $K^- \nu$ decay. These modes [6] each generally provide less information since here only two of the semileptonic parameters can be measured. But from the π mode there is good separation (>127 GeV from CLEO II data) of $V-A$ from a $T^+ + T_5^+$ coupling, whereas these couplings cannot be separated in the ρ and a_1 modes. There is also direct measurement of the chirality parameter ξ_π, i.e., of the probability that the emitted ν_τ is L handed. Unfortunately, the fundamental S^- and P^- couplings which do not contribute to $\tau \to \rho\nu, a_1\nu, K^*\nu$ are found to be suppressed in $\tau^- \to \pi^- \nu, K^- \nu$ decay.

Section VII contains several tables giving the associated ideal statistical errors for measurement of these semileptonic parameters based on S2SC functions at 10 GeV, 4 GeV, and at M_Z.

In conclusion Section VIII contains some additional remarks.

In the Appendix we list the $A(\lambda_\rho, \lambda_\nu)$ helicity amplitudes for $\tau^- \to \rho^- \nu$ for the most general $J^{\rm charged}_{\rm lepton}$ current.

II. PARAMETRIZATION OF τ SEMILEPTONIC DECAY MODES

The reader should be aware that it is not necessary to use the helicity formalism [8] because the parameters are fundamentally defined in terms of τ decay partial width intensities for polarized final states. However, the helicity formalism does provide a lucid, neat, and flexible framework for connecting Lorentz-invariant couplings at the Lagrangian level with Lorentz-invariant spin-correlation functions. In practice, the helicity formalism also frequently provides insights and checks on the resulting formulas and their symmetries. We present the discussion for the $\rho\nu$ channel, but the same formulas hold for the $a_1\nu$ and $K^*\nu$ channels. See Sec. VI for the $\pi\nu$ and $K\nu$ channels.

In the τ^- rest frame, the matrix element for $\tau^-\to\rho^-\nu$ is

$$\langle \theta_1^\tau,\phi_1^\tau,\lambda_\rho,\lambda_\nu|\tfrac{1}{2},\lambda_1\rangle = D^{(1/2)*}_{\lambda_1,\mu}(\phi_1^\tau,\theta_1^\tau,0)A(\lambda_\rho,\lambda_\nu), \quad (8)$$

where $\mu=\lambda_\rho-\lambda_\nu$ and λ_1 is the τ^- helicity. For the *CP*-conjugate process, $\tau^+\to\rho^+\bar{\nu}\to(\pi^+\pi^0)\bar{\nu}$, in the τ^+ rest frame,

$$\langle \theta_2^\tau,\phi_2^\tau,\lambda_{\bar{\rho}},\lambda_{\bar{\nu}}|\tfrac{1}{2},\lambda_2\rangle = D^{(1/2)*}_{\lambda_2,\bar{\mu}}(\phi_2^\tau,\theta_2^\tau,0)B(\lambda_{\bar{\rho}},\lambda_{\bar{\nu}}), \quad (9)$$

with $\bar{\mu}=\lambda_{\bar{\rho}}-\lambda_{\bar{\nu}}$.

These formulas only assume Lorentz invariance and do not assume any discrete symmetry properties. Therefore, it is easy to use this framework for testing for the consequences of such additional symmetries. In particular, for $\tau^-\to\rho^-\nu$ and $\tau^+\to\rho^+\bar{\nu}$ a specific discrete symmetry implies a specific relation among the associated helicity amplitudes:

$$\begin{array}{ll}
P & A(-\lambda_\rho,-\lambda_\nu)=A(\lambda_\rho,\lambda_\nu),\\
 & B(-\lambda_{\bar{\rho}},-\lambda_{\bar{\nu}})=B(\lambda_{\bar{\rho}},\lambda_{\bar{\nu}}),\\
C & B(\lambda_{\bar{\rho}},\lambda_{\bar{\nu}})=A(\lambda_{\bar{\rho}},\lambda_{\bar{\nu}}),\\
CP & B(\lambda_{\bar{\rho}},\lambda_{\bar{\nu}})=A(-\lambda_{\bar{\rho}},-\lambda_{\bar{\nu}}),\\
\tilde{T}_{\mathrm{FS}} & A^*(\lambda_\rho,\lambda_\nu)=A(\lambda_\rho,\lambda_\nu),\\
 & B^*(\lambda_{\bar{\rho}},\lambda_{\bar{\nu}})=B(\lambda_{\bar{\rho}},\lambda_{\bar{\nu}}),\\
CP\tilde{T}_{\mathrm{FS}} & B^*(\lambda_{\bar{\rho}},\lambda_{\bar{\nu}})=A(-\lambda_{\bar{\rho}},-\lambda_{\bar{\nu}}).
\end{array}$$

Measurement of a nonreal helicity amplitude implies a violation of $\tilde{T}_{\mathrm{FS}}$ invariance when a first-order perturbation in an ''effective'' Hermitian Hamiltonian is reliable. So $\tilde{T}_{\mathrm{FS}}$ invariance is expected to be violated when there are significant final-state interactions. $\tilde{T}_{\mathrm{FS}}$ invariance is to be distinguished from canonical T invariance which requires interchanging ''final'' and ''initial'' states. Actual time-reversed reactions are required for a direct test of T invariance.

A. Remarks on definitions by partial-width intensities for polarized final states

The τ semileptonic decay parameters for $\tau^-\to\rho^-\nu$, and likewise for $\tau^-\to a_1^-\nu$ and $\tau^-\to K^*\nu$, are defined above in the Introduction. The helicity formalism is useful so as to be clear about the terminology and sign conventions[8]. For $\tau^-\to\rho^-\nu$ decay, in terms of the helicity amplitudes $A(\lambda_\rho,\lambda_\nu)$ the final-state-polarized partial widths are

$$\Gamma_L^\pm=|A(0,-\tfrac{1}{2})|^2\pm|A(0,\tfrac{1}{2})|^2,$$

$$\Gamma_T^\pm=|A(-1,-\tfrac{1}{2})|^2\pm|A(1,\tfrac{1}{2})|^2. \quad (10)$$

Recall [5] that by rotational invariance the $A(1,-\tfrac{1}{2})=A(-1,\tfrac{1}{2})=0$; similarly for the ρ^+ mode in τ^+ decay, $B(1,-\tfrac{1}{2})=B(-1,\tfrac{1}{2})=0$.

To describe the contributions from the interference between the longitudinal (L) and transverse (T) vector-meson amplitudes in the decay process, we introduce the four additional parameters $(\omega,\eta,\omega',\eta')$. These depend on the measurable *LT*-interference intensities:

$$\begin{aligned}
I_{\mathscr{R}}^\pm &= \mathrm{Re}\{A(0,-\tfrac{1}{2})^*A(-1,-\tfrac{1}{2})\pm A(0,\tfrac{1}{2})^*A(1,\tfrac{1}{2})\}\\
&=|A(0,-\tfrac{1}{2})||A(-1,-\tfrac{1}{2})|\cos\beta_\alpha\\
&\quad\pm|A(0,\tfrac{1}{2})||A(1,\tfrac{1}{2})|\cos\beta_a^R,
\end{aligned} \quad (11)$$

$$\begin{aligned}
I_{\mathscr{T}}^\pm &= \mathrm{Im}\{A(0,-\tfrac{1}{2})^*A(-1,-\tfrac{1}{2})\pm A(0,\tfrac{1}{2})^*A(1,\tfrac{1}{2})\}\\
&=|A(0,-\tfrac{1}{2})||A(-1,-\tfrac{1}{2})|\sin\beta_\alpha\\
&\quad\pm|A(0,\tfrac{1}{2})||A(1,\tfrac{1}{2})|\sin\beta_a^R,
\end{aligned} \quad (12)$$

where $\beta_a\equiv\phi_{-1}^a-\phi_0^a$, $\beta_a^R\equiv\phi_1^a-\phi_0^{aR}$ are the measurable phase differences of the associated helicity amplitudes $A=|A|\exp(\iota\phi)$.

For the *CP* conjugate modes, $\tau^+\to\rho^+\bar{\nu}$ and $\tau^+\to a_1^+\bar{\nu}$, the definitions for their semileptonic decay parameters are the same except that all quantities are ''barred,'' and there is the substitution of helicity amplitudes $A(x,y)\to B(-x,-y)$. For instance, $\bar{\xi}=(\mathrm{Prob}\ \bar{\nu}_\tau\ \mathrm{is}\ \bar{\nu}_R)-(\mathrm{Prob}\ \bar{\nu}_\tau\ \mathrm{is}\ \bar{\nu}_L)=(\Gamma_L^-+\Gamma_T^-)/\Gamma$, and

$$\begin{aligned}
\bar{\omega}=&\{|B(0,\tfrac{1}{2})||B(1,\tfrac{1}{2})|\cos\beta_b\\
&-|B(0,-\tfrac{1}{2})||B(-1,-\tfrac{1}{2})|\cos\beta_b^L\}/(\mathscr{R}_\rho\bar{\Gamma}),
\end{aligned}$$

where $\bar{\Gamma}_L^\pm=|B(0,\tfrac{1}{2})|^2\pm|B(0,-\tfrac{1}{2})|^2$, $\bar{\Gamma}_T^\pm=|B(1,\tfrac{1}{2})|^2\pm|B(-1,-\tfrac{1}{2})|^2$.

Depending on the experimental situation, and/or the new physics under investigation it may sometimes be advantageous to rewrite the spin-correlation function(s) of interest directly in terms of the above final-state-polarized partial widths and *LT*-interference intensities, instead of using the above τ semileptonic decay parameters. Likewise, in applications to top quark, $W^\pm$, Z^0, Higgs boson, etc., decays the polarized partial widths themselves may be the most useful and fundamental quantities.

Note that the trigonometric structure of Eqs. (11) and (12) implies the two constraints

$$(\tilde{\eta}\pm\tilde{\omega})^2+(\tilde{\eta}'\pm\tilde{\omega}')^2=\frac{1}{4}[(1\pm\xi)^2-(\tilde{\sigma}\pm\tilde{\zeta})^2] \quad (13)$$

or

$$2|\eta'\pm\omega'|=\sqrt{(1\pm\xi)^2-(\tilde{\sigma}\pm\tilde{\zeta})^2-4(\tilde{\eta}\pm\tilde{\omega})^2}$$

among the $\eta,\eta',\omega,\omega'$ parameters which test for leptonic T violation. Consistency, i.e., unitarity, requires the argument of the square root must be non-negative. Equivalently, there are the two right-triangle relations

$$(I_{\mathscr{R}}^{\nu_L})^2+(I_{\mathscr{T}}^{\nu_L})^2=\Gamma_L^{\nu_L}\Gamma_T^{\nu_L},$$

$$(I_{\mathscr{R}}^{\nu_R})^2+(I_{\mathscr{T}}^{\nu_R})^2=\Gamma_L^{\nu_R}\Gamma_T^{\nu_R}$$

in terms of the algebraically convenient

$$I_{\mathscr{R}}^{\nu_L,\nu_R} \equiv \frac{1}{2}(I_{\mathscr{R}}^{+} \pm I_{\mathscr{R}}^{-}) = \left|A\left(0,\mp\frac{1}{2}\right)\right|\left|A\left(\mp 1,\mp\frac{1}{2}\right)\right|\cos\beta_a^{L,R} = \frac{\Gamma}{2}(\tilde{\eta} \pm \tilde{\omega}),$$

$$I_{\mathscr{T}}^{\nu_L,\nu_R} \equiv \frac{1}{2}(I_{\mathscr{T}}^{+} \pm I_{\mathscr{T}}^{-}) = \left|A\left(0,\mp\frac{1}{2}\right)\right|\left|A\left(\mp 1,\mp\frac{1}{2}\right)\right|\sin\beta_a^{L,R} = \frac{\Gamma}{2}(\tilde{\eta}' \pm \tilde{\omega}'),$$

$$\Gamma_L^{\nu_L,\nu_R} \equiv \frac{1}{2}(I_L^{+} \pm I_L^{-}) = \left|A\left(0,\mp\frac{1}{2}\right)\right|^2 = \frac{\Gamma}{4}(1 + \tilde{\sigma} \pm \xi \pm \tilde{\zeta}),$$

$$\Gamma_T^{\nu_L,\nu_R} \equiv \frac{1}{2}(I_T^{+} \pm I_T^{-}) = \left|A\left(\mp 1,\mp\frac{1}{2}\right)\right|^2 = \frac{\Gamma}{4}(1 - \tilde{\sigma} \pm \xi \mp \tilde{\zeta}),$$

with $\beta_a^L = \beta_a$ (we normally suppress such L superscripts). If there are only ν_L couplings, $I_{\mathscr{R}}^{\nu_R} = I_{\mathscr{T}}^{\nu_R} = \Gamma_L^{\nu_R} = \Gamma_T^{\nu_R} = 0$; equivalently $\eta = \omega$, $\eta' = \omega'$, $\xi = 1$, $\zeta = \sigma$.

Since all partial widths must be positive, there implicitly are obvious inequalities among these semileptonic parameters which could be used empirically for analysis of systematic effects and in making cuts on the $\tau^- \to \rho^- \nu \to (\pi^- \pi^0)\nu$ data set. For example, there can be contamination from $a_1 \to \pi^- 2\pi^0$ where one π^0 is missed, from particle misidentification, or from interference between the π^0's from ρ^- and ρ^+ decays which has not been included in these S2SC functions.

The hadronic factors $\mathscr{S}_\rho$ and $\mathscr{R}_\rho$ do depend on the particular τ semileptonic decay channel. For the a_1, K^* modes, replace respectively $m_\rho \to m_{a_1}, m_{K^*}$. The treatment in this paper assumes that the momentum dependence (i.e., the dependence on q_ρ^2, etc.) of the form factors g_L and g_i is negligible. Depending on the application and on the desired experimental test, more sophisticated treatments of the q_ρ^2 etc. dependence could be used such as ones which incorporate results from recent QCD calculations for τ decays [9] and ones which include possible contributions from additional resonances such as the ρ'. Because of the smearing and the continually improving understanding of QCD methods in τ physics, we do not expect this to be a fundamental difficulty in practice, but rather a technical matter that requires sufficient care.

These factors numerically are $(\mathscr{S},\mathscr{R})_{\rho,a_1,K^*} = 0.454$, 0.445; -0.015, 0.500; 0.330, 0.472. Because of the finite a_1 width, $\Gamma_{a_1} \sim 400$ MeV, the $\mathscr{S}_{a_1}$ factor vanishes in the interval $(m_{a_1} \pm \Gamma/2)$ at the point $q_{a_1}^2 = m_\tau^2/2 = (1.257\ \text{GeV})^2$. So for a_1, in applying spin correlation distributions, tilde functions $\tilde{\xi}(q^2) = \mathscr{S}_{a_1}\xi$ and $\tilde{\sigma}(q^2) = \mathscr{S}_{a_1}\sigma$ should be constructed (e.g., using Table II), convoluted with the Breit-Wigner resonance(s), and then fit to determine if more than a $V-A$ coupling is present. For a_1, ξ and σ should not be treated in the same manner as the other semileptonic parameters.

Recall [10] that $\mathscr{S}_{\pi,K} = 1$ for $J=0$, so $\mathscr{S}_{\rho,a_1,K^*}$ suppresses the S1SC signatures when $J \neq 0$. On the other hand, $\mathscr{R}_{\rho,a_1,K^*}$ does not appear for $J=0$ channels since their sequential decay chains end with the first stage.

III. SIGNIFICANCE OF SEMILEPTONIC PARAMETERS VERSUS "CHIRAL COUPLINGS"

The most general Lorentz coupling for $\tau^- \to \rho^- \nu_{L,R}$ is

$$\rho_\mu^* \overline{u_{\nu_\tau}}(p) \Gamma^\mu u_\tau(k), \tag{14}$$

where $k_\tau = q_\rho + p_\nu$. It is convenient to treat the vector and axial vector matrix elements separately. In Eq. (14),

$$\Gamma_V^\mu = g_V \gamma^\mu + \frac{f_M}{2\Lambda} \iota\sigma^{\mu\nu}(k-p)_\nu + \frac{g_{S^-}}{2\Lambda}(k-p)^\mu + \frac{g_S}{2\Lambda}(k+p)^\mu + \frac{g_{T^+}}{2\Lambda}\iota\sigma^{\mu\nu}(k+p)_\nu,$$

$$\Gamma_A^\mu = g_A \gamma^\mu \gamma_5 + \frac{f_E}{2\Lambda}\iota\sigma^{\mu\nu}(k-p)_\nu\gamma_5 + \frac{g_{P^-}}{2\Lambda}(k-p)^\mu\gamma_5 + \frac{g_P}{2\Lambda}(k+p)^\mu\gamma_5 + \frac{g_{T_5^+}}{2\Lambda}\iota\sigma^{\mu\nu}(k+p)_\nu\gamma_5. \tag{15}$$

The parameter Λ = "the effective-mass scale of new physics." In effective field theory this is the scale at which new particle thresholds are expected to occur or where the theory becomes nonperturbatively strongly interacting so as to overcome perturbative inconsistencies. It can also be interpreted as a measure of a new leptonic compositeness scale. In old-fashioned renormalization theory Λ is the scale at which the calculational methods and/or the principles of "renormalization" breakdown; see for example [11]. While some terms of the above form do occur as higher-order perturbative corrections in the standard model, such standard model (SM) contributions are "small" versus the sensitivities of present tests in τ physics in the analogous cases of the τ's neutral-current and electromagnetic-current couplings; cf. [12]. For charged-current couplings, the situation should be the same.

Without additional theoretical, cf. [6], or experimental inputs, it is not possible to select what is the "best" minimal set of couplings for analyzing the structure of the τ's charged current. For instance, by Lorentz invariance, there are the equivalence theorems that for the vector current

$$S \approx V + f_M, \quad T^+ \approx -V + S^- \tag{16}$$

and for the axial-vector current

$$P \approx -A + f_E, \quad T_5^+ \approx A + P^-. \tag{17}$$

On the other hand, dynamical considerations such as lepton compositeness would suggest searching for an additional tensorial $g_+ = f_M + f_E$ coupling which would preserve $\xi = 1$ but otherwise give non-$(V-A)$ values to the semileptonic parameters. For instance, $\sigma = \zeta \neq 1$ and $\eta = \omega \neq 1$.

The matrix elements of the divergences of these charged currents are

TABLE III. Analytic forms and numerical values of the partial width intensities for polarized final states for unique Lorentz couplings. Numerical entries are to one digit and are for ρ^- (a_1, if value differs). For $V\mp A$, the entry before the semicolon is for $V-A$, after for $V+A$.

	$V\mp A$	$S\pm P$	f_M+f_E	f_M-f_E
	Analytic form			
Γ_L^-/Γ	$\pm\frac{1}{2}(1+\mathscr{S}_\rho)$	± 1	$\frac{\rho^2}{2\tau^2+\rho^2}$	$+\frac{1}{3}$
Γ_T^-/Γ	$\pm\frac{1}{2}(1-\mathscr{S}_\rho)$	0	$\frac{2\tau^2}{2\tau^2+\rho^2}$	$-\frac{2}{3}$
Γ_L^+/Γ	$\pm\frac{1}{2}(1\pm\mathscr{S}_\rho)$	1	$\frac{\rho^2}{2\tau^2+\rho^2}$	$-\frac{2}{3}$
Γ_T^+/Γ	$\pm\frac{1}{2}(1\mp\mathscr{S}_\rho)$	0	$\frac{2\tau^2}{2\tau^2+\rho^2}$	$-\frac{1}{3}$
	Numerical value			
Γ_L^-/Γ	$\pm 0.7(\pm 0.5)$	± 1	0.0(0.2)	+0.3
Γ_T^-/Γ	$\pm 0.3(\pm 0.5)$	0	1.0(0.8)	−0.7
Γ_L^+/Γ	0.7(0.5);−0.3(−0.5)	1	0.0(0.8)	−0.7
Γ_T^+/Γ	0.3(0.5);−0.7(−0.5)	0	1.0(0.8)	−0.3

$$(k-p)_\mu V^\mu=\left[g_V(m_\tau-m_\nu)+\frac{g_{S^-}}{2\Lambda}q^2+\frac{g_S}{2\Lambda}(m_\tau^2-m_\nu^2)+\frac{g_{T^+}}{2\Lambda}(q^2-[m_\tau-m_\nu]^2)\right]\overline{u_\nu}u_\tau, \tag{18}$$

$$(k-p)_\mu A^\mu=\left[-g_A(m_\nu+m_\tau)+\frac{g_{P^-}}{2\Lambda}q^2+\frac{g_P}{2\Lambda}(m_\tau^2-m_\nu^2)+\frac{g_{T_5^+}}{2\Lambda}(q^2-[m_\tau+m_\nu]^2)\right]\overline{u_\nu}\gamma_5 u_\tau. \tag{19}$$

Both the weak magnetism $f_M/2\Lambda$ and the weak electricity $f_E/2\Lambda$ terms are divergenceless. On the other hand, since $q^2=m_\rho^2$, even when $m_\nu=m_\tau$ there are nonvanishing terms due to the couplings S^-,T^+,A,P^-,T_5^+.

Table II gives the analytic form of the semileptonic parameters for unique Lorentz couplings. Table III gives the analytic forms and numerical values of the partial-width intensities for polarized final states for unique Lorentz couplings.

A. Semileptonic parameters' form in terms of g_L plus an ''additional chiral coupling'' ($m_\nu=0$)

We first display the expected forms for the above semileptonic parameters for the $\tau\to\rho\nu, a_1\nu, K^*\nu$ decay modes for the case of a pure $V-A$ chiral coupling as in the standard lepton model. We assume that the mass of the τ neutrino and antineutrino are negligible. Next we will give the form for the case of a single chiral coupling ($g_i/2\Lambda_i$) in addition to the standard $V-A$ coupling. In this case, we first list the formula for an arbitrarily large additional contribution.

In Tables IV and V we list the formulas assuming that the additional contribution is small versus the $V-A$ coupling. Throughout this paper, we usually suppress the entry in the ''i'' subscript on the new-physics coupling scale ''Λ_i'' when it is obvious from the context of interest.

In the case of ''multiadditional'' chiral contributions, the general formulas for $A(\lambda_\rho,\lambda_\nu)$ which are listed in the Appendix can be substituted into the above definitions so as to derive the expression(s) for the ''multiadditional'' chiral

TABLE IV. Semileptonic decay parameters for $\tau^-\to\rho^-\nu$, etc. in the case of a single additional chiral coupling (g_i) which is small relative to the standard $V-A$ coupling (g_L). This table is for the $V+A$ and for the $S\pm P$ couplings. The next table is for additional tensorial couplings. In this paper Re (Im) denote respectively the real (imaginary) parts of the quantity inside the parentheses. Expressions for ''$a,\ldots, f$'' are Eqs. (38).

	$V\pm A$		Additional $S\pm P$	
	Pure g_L	Plus g_R	Plus g_{S+P}	Plus g_{S-P}
Γ's				
ξ	1	$\frac{\lvert g_L\rvert^2-\lvert g_R\rvert^2}{\lvert g_L\rvert^2+\lvert g_R\rvert^2}$	1	1
ζ	1	ξ	$1+a\frac{\mathrm{Re}(g_L^* g_{S+P})}{\lvert g_L\rvert^2}$	$1-b\left\lvert\frac{g_{S-P}}{g_L}\right\rvert^2$
σ	1	1	ζ	$1+c\left\lvert\frac{g_{S-P}}{g_L}\right\rvert^2$
Γ_{LT}'s				
ω	1	ξ	$1-d\frac{\mathrm{Re}(g_L^* g_{S+P})}{\lvert g_L\rvert^2}$	$1-e\left\lvert\frac{g_{S-P}}{g_L}\right\rvert^2$
η	1	1	ω	ω
ω'	0	0	$-f\frac{\mathrm{Im}(g_L^* g_{S+P})}{\lvert g_L\rvert^2}$	0
η'	0	0	ω'	0

contributions. Frequently we will suppress the subscript on m_τ.

Pure V−A coupling:

$$\zeta=\sigma=\omega=\eta=\xi=1,$$
$$\omega'=\eta'=0. \tag{20}$$

V+A also present:

$$\zeta=\xi,\qquad \omega=\xi,$$
$$\sigma=1,\qquad \eta=1,$$
$$\xi=(|g_L|^2-|g_R|^2)/(|g_L|^2+|g_R|^2)\qquad \omega'=\eta'=0. \tag{21}$$

S+P also present:

$$\zeta=\sigma=\left(\begin{array}{c}\left(1-2\frac{m_\rho^2}{m^2}\right)|g_L|^2+\frac{m}{\Lambda}\left[1-\frac{m_\rho^2}{m^2}\right]\mathrm{Re}(g_L^*g_{S+P})\\ +\left\{\frac{m}{2\Lambda}\left[1-\frac{m_\rho^2}{m^2}\right]\right\}^2|g_{S+P}|^2\end{array}\right)\Bigg/(\mathscr{S}_\rho\mathscr{D}^+), \tag{22}$$

$$\xi=1, \tag{23}$$

$$\omega=\eta=\sqrt{2}\frac{m_\rho}{m}\left(|g_L|^2+\frac{m}{2\Lambda}\left[1-\frac{m_\rho^2}{m^2}\right]\mathrm{Re}(g_L^*g_{S+P})\right)\Bigg/(\mathscr{R}_\rho\mathscr{D}^+),$$
$$\omega'=\eta'=-\sqrt{2}\frac{m_\rho}{2\Lambda}\left[1-\frac{m_\rho^2}{m^2}\right]\mathrm{Im}(g_L^*g_{S+P})/(\mathscr{R}_\rho\mathscr{D}^+), \tag{24}$$

where

$$\mathscr{D}^+=\left(1+2\frac{m_\rho^2}{m^2}\right)|g_L|^2+\frac{m}{\Lambda}\left[1-\frac{m_\rho^2}{m^2}\right]\mathrm{Re}(g_L^*g_{S+P})+\left\{\frac{m}{2\Lambda}\left[1-\frac{m_\rho^2}{m^2}\right]\right\}^2|g_{S+P}|^2.$$

S−P also present:

$$\zeta,\sigma=\left(\left(1-2\frac{m_\rho^2}{m^2}\right)|g_L|^2\mp\left\{\frac{m}{2\Lambda}\left[1-\left(\frac{m_\rho^2}{m^2}\right)\right]\right\}^2|g_{S-P}|^2\right)\Bigg/(\mathscr{S}_\rho\mathscr{D}^-), \tag{25}$$

where the upper (lower) sign on the right-hand side (RHS) goes with the first (second) entry on the left-hand side (LHS):

$$\xi=1, \tag{26}$$

$$\omega=\eta=\sqrt{2}\frac{m_\rho}{m}|g_L|^2/(\mathscr{R}_\rho\mathscr{D}^-),\qquad \omega'=\eta'=0, \tag{27}$$

where

$$\mathscr{D}^-=\left(1+2\frac{m_\rho^2}{m^2}\right)|g_L|^2+\left\{\frac{m}{2\Lambda}\left[1-\frac{m_\rho^2}{m^2}\right]\right\}^2|g_{S-P}|^2.$$

f_M+f_E also present: for this case we write the coupling constant of the sum of the weak magnetism and the weak electricity couplings as

$$g_+=f_M+f_E.$$

In this notation,

$$\zeta=\sigma=\left(\begin{array}{c}\left(1-2\frac{m_\rho^2}{m^2}\right)|g_L|^2+\frac{m_\rho^2}{m\Lambda}\mathrm{Re}(g_L^*g_+)\\ +\frac{m_\rho^2}{4\Lambda^2}\left[-2+\frac{m_\rho^2}{m^2}\right]|g_+|^2\end{array}\right)\Bigg/(\mathscr{S}_\rho\mathscr{D}_{\mathscr{T}}^+), \tag{28}$$

$$\xi=1,$$

$$\omega=\eta=\sqrt{2}\frac{m_\rho}{m}\left(|g_L|^2-\frac{m}{2\Lambda}\left[1+\frac{m_\rho^2}{m^2}\right]\mathrm{Re}(g_L^*g_+)+\frac{m_\rho^2}{4\Lambda^2}|g_+|^2\right)\Bigg/(\mathscr{R}_\rho\mathscr{D}_{\mathscr{T}}^+),$$
$$\omega'=\eta'=-\frac{m_\rho}{\sqrt{2}\Lambda}\left[1-\frac{m_\rho^2}{m^2}\right]\mathrm{Im}(g_L^*g_+)/(\mathscr{R}_\rho\mathscr{D}_{\mathscr{T}}^+), \tag{29}$$

where

$$\mathscr{D}_T^+=\left(1+2\frac{m_\rho^2}{m^2}\right)|g_L|^2-3\frac{m_\rho^2}{m\Lambda}\mathrm{Re}(g_L^*g_+)+\frac{m_\rho^2}{4\Lambda^2}\left[2+\frac{m_\rho^2}{m^2}\right]|g_+|^2.$$

TABLE V. Same as Table IV except this table is for additional tensorial couplings. Here $g_\pm = f_M \pm f_E$ involves $k_\tau - p_\nu$, whereas $\tilde{g}_\pm = g^+_{T^+ \pm T_5^+}$ involves $k_\tau + p_\nu$; see Eqs. (14) and (15). Expressions for ''$f,\ldots,o$'' are Eqs. (39).

	Additional $f_M \pm f_E$		Additional $T^+ \pm T_5^+$	
	Plus g_+	Plus g_-	Plus $\tilde{g}_+$	Plus $\tilde{g}_-$
Γ's				
ξ	1	$1-k\left\|\frac{g_-}{g_L}\right\|^2$	1	$\frac{\|g_L\|^2-\|m\tilde{g}_-/2\Lambda\|^2}{\|g_L\|^2+\|m\tilde{g}_-/2\Lambda\|^2}$
ζ	$1+g\frac{\mathrm{Re}(g_L^* g_+)}{\|g_L\|^2}$	$1-h\left\|\frac{g_-}{g_L}\right\|^2$	1	ξ
σ	ζ	$1-j\left\|\frac{g_-}{g_L}\right\|^2$	1	1
Γ_{LT}'s				
ω	$1-l\frac{\mathrm{Re}(g_L^* g_+)}{\|g_L\|^2}$	$1-n\left\|\frac{g_-}{g_L}\right\|^2$	1	ξ
η	ω	$1-o\left\|\frac{g_-}{g_L}\right\|^2$	1	1
ω'	$-f\frac{\mathrm{Im}(g_L^* g_+)}{\|g_L\|^2}$	0	0	0
η'	ω'	0	0	0

$f_M - f_E$ also present: similarly, we write the coupling constant of the difference of the weak magnetism and the weak electricity couplings as

$$g_- = f_M - f_E$$

and so

$$\zeta,\sigma = \left(\left(1-2\frac{m_\rho^2}{m^2}\right)\left|g_L\right|^2 \pm \frac{m_\rho^2}{4\Lambda^2}\left|g_-\right|^2\right) \Big/ (\mathscr{S}_\rho \mathscr{D}_T^-), \tag{30}$$

where the upper(lower) sign on the RHS goes with the first-(second) entry on the LHS. Also,

$$\xi = \left(\left(1+2\frac{m_\rho^2}{m^2}\right)\left|g_L\right|^2 - 3\frac{m_\rho^2}{4\Lambda^2}\left|g_-\right|^2\right) \Big/ \mathscr{D}_T^-, \tag{31}$$

$$\omega,\eta = \sqrt{2}\frac{m_\rho}{m}\left(|g_L|^2 \mp \frac{m_\rho^2}{4\Lambda^2}|g_-|^2\right)/(\mathscr{R}_\rho \mathscr{D}_T^-), \quad \omega' = \eta' = 0. \tag{32}$$

Here

$$\mathscr{D}_T^- = \left(1+2\frac{m_\rho^2}{m^2}\right)|g_L|^2 + 3\frac{m_\rho^2}{4\Lambda^2}|g_-|^2.$$

$T^+ + T_5^+$ also present: we let

$$\tilde{g}_+ = g^+_{T^+ + T_5^+}.$$

In this notation,

$$\zeta = \sigma = \xi = 1. \tag{33}$$

Also

$$\omega = \eta = 1; \quad \omega' = \eta' = 0. \tag{34}$$

A single additional $\tilde{g}_+ = g^+_{T^+ + T_5^+}$ coupling does not change the values from that of the pure $V-A$ coupling.

$T^+ - T_5^+$ also present: we let

$$\tilde{g}_- = g^+_{T^+ - T_5^+}$$

and so

$$\zeta = \xi, \quad \sigma = 1, \tag{35}$$

$$\xi = \frac{|g_L|^2 - |m\tilde{g}_-/2\Lambda|^2}{|g_L|^2 + |m\tilde{g}_-/2\Lambda|^2}, \tag{36}$$

$$\omega = \xi, \quad \eta = 1, \quad \omega' = \eta' = 0. \tag{37}$$

A single additional $\tilde{g}_- = g^+_{T^+ - T_5^+}$ coupling is equivalent to a single additional $V+A$ coupling, except for the interpretation of their respective chirality parameters.

B. Semileptonic parameters when ''additional chiral coupling'' is small

In Table IV for the $V+A$ and for the $S \mp P$ couplings, we list the ''expanded forms'' of the above expressions for the case in which there is a single additional chiral coupling ($g_i/2\Lambda_i$) which is small relative to the standard $V-A$ coupling (g_L). Similarly, in Table V is listed the formulas for the additional tensorial couplings. The tensorial couplings include the sum and difference of the weak magnetism and electricity couplings, $g_{\pm}=f_M \pm f_E$, which involve the momentum difference $q_\rho = k_\tau - p_\nu$. The alternative tensorial couplings $\tilde{g}_{\pm} = g^{+}_{T^+ \pm T_5^+}$ instead involve $k_\tau + p_\nu$.

Notice that, except for the following coefficients, the formulas tabulated in these two tables are short and simple. As above we usually suppress the entry in the ''i'' subscript on ''Λ_i''. For Table IV these coefficients are

$$a=\frac{4m_\rho^2}{m\Lambda}\frac{(1-m_\rho^2/m^2)}{(1-4m_\rho^4/m^4)},$$

$$d=\frac{m}{4\Lambda}\left(1-\frac{m_\rho^2}{m^2}\right)\frac{(1-2m_\rho^2/m^2)}{(1+2m_\rho^2/m^2)},$$

$$b=\frac{m^2}{2\Lambda^2}\frac{(1-m_\rho^2/m^2)^2}{(1-4m_\rho^4/m^4)},$$

$$e=\frac{m^2}{4\Lambda^2}\frac{(1-m_\rho^2/m^2)^2}{(1+2m_\rho^2)},$$

$$c=\frac{m_\rho^2}{\Lambda^2}\frac{(1-m_\rho^2/m^2)^2}{(1-4m_\rho^4/m^4)},\quad f=\frac{m}{2\Lambda}\left(1-\frac{m_\rho^2}{m^2}\right). \tag{38}$$

The additional coefficients for Table V are

$$g=\frac{2m_\rho^2}{m\Lambda}\frac{(1-4m_\rho^2/m^2)}{(1-4m_\rho^4/m^4)},$$

$$l=\frac{m(1+9m_\rho^2/m^2+2m_\rho^4/m^4)}{2\Lambda(1+2m_\rho^2/m^2)},$$

$$h=\frac{m_\rho^2}{2\Lambda^2}\frac{(1-4m_\rho^2/m^2)}{(1-4m_\rho^4/m^4)},\quad n=\frac{m_\rho^2(2+m_\rho^2/m^2)}{2\Lambda^2(1+2m_\rho^2/m^2)},$$

$$j=\frac{m_\rho^2}{\Lambda^2}\frac{(1-m_\rho^2/m^2)}{(1-4m_\rho^4/m^4)},\quad o=\frac{m_\rho^2(1-m_\rho^2/m^2)}{2\Lambda^2(1+2m_\rho^2/m^2)},$$

$$k=\frac{3m_\rho^2}{2\Lambda^2(1+2m^2/m_\rho^2)}. \tag{39}$$

Notice that $\mathcal{O}(1/\Lambda)$ coefficients occur in the case of an interference with the g_L coupling, and that otherwise $\mathcal{O}(1/\Lambda^2)$ coefficients occur. Should experimental measurements indicate other than a pure g_L value of a semileptonic parameter, a smearing and more sophisticated treated of these coefficients may be warranted. For a_1, as discussed at the end of Sec. II, tilde functions $\tilde{\xi}(q^2)$ and $\tilde{\sigma}(q^2)$ should be fit. For K^*, the $(1-4m_\rho^2/m^2)$ factor in the numerators of g and h almost vanishes. For ρ, these ''a to o'' coefficients are 0.1–1.8 except for $h\sim0.08$; there is at most a factor of 0.6 change over $(m_\rho \pm \Gamma/2)$.

In comparing the entries in these two tables, notice that (i) a single additional $\tilde{g}_+ = g^{+}_{T^+ + T_5^+}$ coupling does not change the values from that of the pure $V-A$ coupling, and that (ii) a single additional $\tilde{g}_- = g^{+}_{T^+ - T_5^+}$ is equivalent to a single additional $V+A$ coupling, except for the interpretation of their respective chirality parameters. This follows as a consequence of Eqs. (16) and (17) and the absence of contributions from the S^- and P^- couplings to the ρ, a_1, and K^* modes.

We have displayed this equivalence in Table V to emphasize that while the commonly assumed total absence of $\tilde{g}_\pm$ couplings in τ lepton decays is supported by the generally weaker tests of the experimental/theoretical normalization of the decay rates (such as by universality tests in lepton physics), $V-A(V+A)$ empirical values of the semileptonic decay parameters discussed in the tables for the ρ, a_1, K^* modes will not imply the absence of $\tilde{g}_+(\tilde{g}_-)$ couplings.

IV. SPIN-CORRELATION FUNCTIONS IN TERMS OF THE SEMILEPTONIC PARAMETERS

A. The full seven-variable S2SC function

For the production decay sequence $e^-e^+\to Z^0, \gamma^*\to\tau^-\tau^+\to(\rho^-\nu)(\rho^+\bar{\nu})$ followed by $\rho^{ch}\to\pi^{ch}\pi^0$ the full S2SC function including both ν_L,ν_R helicities and both $\bar{\nu}_R$, $\bar{\nu}_L$ helicities is given by

$$\begin{aligned}\mathbf{I}_7 &= \mathbf{I}(E_1,E_2,\phi;\tilde{\theta}_a,\tilde{\phi}_a;\tilde{\theta}_b,\tilde{\phi}_b)\\ &=\sum |T(h_1,h_2)|^2\mathbf{R}_{h_1,h_1}\overline{\mathbf{R}}_{h_2,h_2}\\ &\quad + e^{\iota\phi}T(++)T^*(--)\mathbf{r}_{+-}\overline{\mathbf{r}}_{+-}\\ &\quad + e^{-\iota\phi}T(--)T^*(++)\mathbf{r}_{-+}\overline{\mathbf{r}}_{-+},\end{aligned} \tag{40}$$

where $h_1=2\lambda_1$ with λ_1 the τ^- helicity, etc. The amplitudes $T(h_1,h_2)$ are the production helicity amplitudes given in Ref. [13] which describe $Z^0,\gamma^*\to\tau^-\tau^+$. This formula also holds if either, or both, $\tau^{\pm}\to a_1^{\pm}\nu$ followed by $a_1^{\pm}\to(3\pi)^{\pm}$. The specific $\tau^{\mp}$ decay channel determines which ''composite decay density matrix'' $\mathbf{R}_{h_1,h_1}$, or $\overline{\mathbf{R}}_{h_2,h_2}$, is to be inserted. The θ_a, ϕ_a angles describe the π^- momentum direction in the ρ^- rest frame for $\rho^-\to\pi^-\pi^0$ when the Lorentz boost is from the τ^- rest frame, etc. See Figs. 3 and 4 and the discussion in Ref. [5].

The literature on polarimetry methods and spin-correlation functions in τ physics includes Refs. [3–5,7,14]. The S2SC functions given in the present paper were derived in the same manner as those in Ref. [5].

Formulas for $\tau\to\rho\nu$

Including both ν_L and ν_R helicities and using a ''compact boldface formalism'' to denote this inclusion of both ν helicities, we find [7] the composite decay density matrix for $\tau^-\to\rho^-\nu\to(\pi^-\pi^0)\nu$ is

$$\mathbf{R}=\begin{pmatrix}\mathbf{R}_{++} & e^{\iota\phi_1^\tau}\mathbf{r}_{+-}\\ e^{-\iota\phi_1^\tau}\mathbf{r}_{-+} & \mathbf{R}_{--}\end{pmatrix}. \tag{41}$$

In terms of the semileptonic parameters, the diagonal elements are

$$\mathbf{R}_{\pm\pm}=\mathbf{n}_a[1\pm\mathbf{f}_a\cos\theta_1^\tau]\mp(1/\sqrt{2})\sin\,\theta_1^\tau\,\sin 2\tilde{\theta}_a\mathcal{R}_\rho[\omega\,\cos\tilde{\phi}_a + \eta'\sin\,\tilde{\phi}_a]. \tag{42}$$

These give the angular distributions $dN/d(\cos\theta_1^\tau)d(\cos\tilde{\theta}_a)d\tilde{\phi}_a$ for the polarized τ^- decay chain; see Eq. (5) above. The off-diagonal elements depend on

$$\mathbf{r}_{+-}=(\mathbf{r}_{-+})^*=\mathbf{n}_a\mathbf{f}_a\sin\theta_1^\tau + (1/\sqrt{2})\sin 2\tilde{\theta}_a\mathscr{R}_\rho\{\cos\theta_1^\tau[\omega\cos\tilde{\phi}_a+\eta'\sin\tilde{\phi}_a] + \iota[\omega\sin\tilde{\phi}_a-\eta'\cos\tilde{\phi}_a]\}. \quad (43)$$

In Eqs. (40) and (41),

$$\begin{pmatrix}\mathbf{n}_a\\ \mathbf{n}_a\mathbf{f}_a\end{pmatrix}=\cos^2\tilde{\theta}_a\frac{\Gamma_L^\pm}{\Gamma}\pm\frac{1}{2}\sin^2\tilde{\theta}_a\frac{\Gamma_T^\pm}{\Gamma} \quad (44)$$

or equivalently

$$\mathbf{n}_a=\tfrac{1}{8}(3+\cos 2\tilde{\theta}_a+\sigma\mathscr{S}_\rho[1+3\cos 2\tilde{\theta}_a]),$$

$$\mathbf{n}_a\mathbf{f}_a=\tfrac{1}{8}(\xi[1+3\cos 2\tilde{\theta}_a]+\zeta\mathscr{S}_\rho[3+\cos 2\tilde{\theta}_a]). \quad (45)$$

Similarly, for the conjugate process $\tau^+\to\rho^+\overline{\nu}\to(\pi^+\pi^0)\overline{\nu}$ including both $\overline{\nu_R}$ and $\overline{\nu_L}$ helicities,

$$\overline{\mathbf{R}}=\begin{pmatrix}\overline{\mathbf{R}}_{++} & e^{\iota\phi_2^\tau}\overline{\mathbf{r}}_{+-}\\ e^{-\iota\phi_2^\tau}\overline{\mathbf{r}}_{-+} & \overline{\mathbf{R}}_{--}\end{pmatrix}. \quad (46)$$

In terms of the semileptonic parameters, the diagonal elements are

$$\overline{\mathbf{R}}_{\pm\pm}=\mathbf{n}_b[1\mp\mathbf{f}_b\cos\theta_2^\tau] \pm(1/\sqrt{2})\sin\theta_2^\tau\sin 2\tilde{\theta}_b\mathscr{R}_\rho[\overline{\omega}\cos\tilde{\phi}_b-\overline{\eta'}\sin\tilde{\phi}_b] \quad (47)$$

and

$$\overline{\mathbf{r}}_{+-}=(\overline{\mathbf{r}}_{-+})^*=-\mathbf{n}_b\mathbf{f}_b\sin\theta_2^\tau-(1/\sqrt{2})\sin 2\tilde{\theta}_b\mathscr{R}_\rho\{\cos\theta_2^\tau \times[\overline{\omega}\cos\tilde{\phi}_b-\overline{\eta'}\sin\tilde{\phi}_b] + \iota[\overline{\omega}\sin\tilde{\phi}_b+\overline{\eta'}\cos\tilde{\phi}_b]\}. \quad (48)$$

In Eqs. (47) and (48),

$$\begin{pmatrix}\mathbf{n}_b\\ \mathbf{n}_b\mathbf{f}_b\end{pmatrix}=\cos^2\tilde{\theta}_b\frac{\overline{\Gamma}_L^\pm}{\overline{\Gamma}}\pm\frac{1}{2}\sin^2\tilde{\theta}_b\frac{\overline{\Gamma}_T^\pm}{\overline{\Gamma}} \quad (49)$$

or equivalently

$$\mathbf{n}_b=\tfrac{1}{8}(3+\cos 2\tilde{\theta}_b+\overline{\sigma}\mathscr{S}_\rho[1+3\cos 2\tilde{\theta}_b]),$$

$$\mathbf{n}_b\mathbf{f}_b=\tfrac{1}{8}(\overline{\xi}[1+3\cos 2\tilde{\theta}_b]+\overline{\zeta}\mathscr{S}_\rho[3+\cos 2\tilde{\theta}_b]). \quad (50)$$

Formulas for $\tau\to a_1\nu$

For the kinematic description of $\tau^-\to a_1^-\nu\to(\pi_1^-\pi_2^-\pi_3^+)\nu$, the normal to the $(\pi_1^-\pi_2^-\pi_3^+)$ decay triangle is used in place of the π^- momentum direction of the $\tau^-\to\rho^-\nu\to(\pi^-\pi^0)\nu$ sequential decay [15].

Including both ν_L and ν_R helicities, we find the composite decay density matrix for $\tau^-\to a_1^-\nu\to(\pi_1^-\pi_2^-\pi_3^+)\nu$ is

$$\mathbf{R}^\nu=S_1^+\mathbf{R}^+ + S_1^-\mathbf{R}^-, \quad (51)$$

where $\mathbf{R}^\pm$ have the same the same form as the earlier matrix, Eq. (41), except the elements now also have ''±'' superscripts; see below. $S_1^\pm$ depend on the strong-interaction form factors used to describe the decay $a_1^-\to\pi_1^-\pi_2^-\pi_3^+$. However, when the three-body Dalitz plot is integrated over, only the S_1^+ term remains, so it can be absorbed into the overall normalization factor which removes any arbitrary form-factor dependence. In Eq. (49), the $\mathbf{R}^+$ composite decay matrix elements are

$$\mathbf{R}^+_{\pm\pm}=\{\text{Eq. (42) with } (1/\sqrt{2})\to(-1/\sqrt{2})\},$$

$$\mathbf{r}^+_{+-}=(\mathbf{r}^+_{-+})^*=\{\text{Eq. (43) with } (1/\sqrt{2})\to(-1/\sqrt{2})\}, \quad (52)$$

with

$$\begin{pmatrix}\mathbf{n}_a\\ \mathbf{n}_a\mathbf{f}_a\end{pmatrix}=\sin^2\tilde{\theta}_a\frac{\Gamma_L^\pm}{\Gamma}\pm\left(1-\frac{1}{2}\sin^2\tilde{\theta}_a\right)\frac{\Gamma_T^\pm}{\Gamma} \quad (53)$$

or equivalently

$$\mathbf{n}_a=\tfrac{1}{16}(10-2\cos 2\tilde{\theta}_a-\sigma\mathscr{S}_\rho[5+3\cos 2\tilde{\theta}_a]),$$

$$\mathbf{n}_a\mathbf{f}_a=\tfrac{1}{16}(-\xi[5+3\cos 2\tilde{\theta}_a]+\zeta\mathscr{S}_\rho[10-2\cos 2\tilde{\theta}_a]). \quad (54)$$

Similarly, the $\mathbf{R}^-$ composite decay matrix elements are

$$\mathbf{R}^-_{\pm\pm}=-\mathbf{n}_a^-[1\mp\mathbf{f}_a^-\cos\theta_1^\tau]\mp(\sqrt{2})\sin\theta_1^\tau\sin\tilde{\theta}_a\mathscr{R}_{a_1}[\eta\cos\tilde{\phi}_a + \omega'\sin\tilde{\phi}_a], \quad (55)$$

with

$$\begin{pmatrix}\mathbf{n}_a^-\\ \mathbf{n}_a^-\mathbf{f}_a^-\end{pmatrix}=\cos\tilde{\theta}_a\frac{\Gamma_T^\mp}{\Gamma} \quad (56)$$

or

$$\mathbf{n}_a^-=\tfrac{1}{2}\cos\tilde{\theta}_a[\xi-\zeta\mathscr{S}_\rho],$$

$$\mathbf{n}_a^-\mathbf{f}_a^-=\tfrac{1}{2}\cos\tilde{\theta}_a[1-\sigma\mathscr{S}_\rho]. \quad (57)$$

Also

$$\mathbf{r}^-_{+-}=(\mathbf{r}^-_{-+})^*=\sin\theta_1^\tau\mathbf{n}_a^-\mathbf{f}_a^-+\sqrt{2}\sin\tilde{\theta}_a\mathscr{R}_{a_1}\{\cos\theta_1^\tau[\eta\cos\tilde{\phi}_a + \omega'\sin\tilde{\phi}_a]+\iota[\eta\sin\tilde{\phi}_a-\omega'\cos\tilde{\phi}_a]\}. \quad (58)$$

For the conjugate process $\tau^+\to a_1^+\overline{\nu}\to(\pi_1^+\pi_2^+\pi_3^0)\overline{\nu}$,

$$\overline{\mathbf{R}^\nu}=\overline{S_1^+}\overline{\mathbf{R}^+}+\overline{S_1^-}\overline{\mathbf{R}^-}. \quad (59)$$

The $\overline{\mathbf{R}}^+$ matrix elements are

$$\overline{\mathbf{R}}^+_{\pm\pm}=\{\text{Eq. (47) with } (1/\sqrt{2})\to(-1/\sqrt{2})\},$$

$$\overline{\mathbf{r}}^+_{+-}=(\overline{\mathbf{r}}^+_{-+})^*=\{\text{Eq. (48) with } (1/\sqrt{2})\to(-1/\sqrt{2})\}, \quad (60)$$

with

$$\begin{pmatrix} \mathbf{n}_b \\ \mathbf{n}_b \mathbf{f}_b \end{pmatrix} = \sin^2\tilde{\theta}_b \frac{\overline{\Gamma_L^{\pm}}}{\overline{\Gamma}} \pm \left(1 - \frac{1}{2}\sin^2\tilde{\theta}_b\right) \frac{\overline{\Gamma_T^{\pm}}}{\overline{\Gamma}} \quad (61)$$

or

$$\mathbf{n}_b = \tfrac{1}{16}(10 - 2\cos 2\tilde{\theta}_b - \overline{\sigma \mathcal{S}_\rho}[5 + 3\cos 2\tilde{\theta}_b]),$$

$$\mathbf{n}_b \mathbf{f}_b = \tfrac{1}{16}(-\overline{\xi}[5 + 3\cos 2\tilde{\theta}_b] + \overline{\zeta \mathcal{S}_\rho}[10 - 2\cos 2\tilde{\theta}_b]). \quad (62)$$

The $\overline{\mathbf{R}^-}$ matrix elements are

$$\overline{\mathbf{R}^-_{\pm\pm}} = -\mathbf{n}_b^-[1 \pm \mathbf{f}_b^- \cos\theta_2^\tau] \mp \sqrt{2}\sin\theta_2^\tau \sin\tilde{\theta}_b \mathcal{R}_{a_1}[\overline{\eta}\cos\tilde{\phi}_b - \overline{\omega^\tau}\sin\tilde{\phi}_b], \quad (63)$$

and

$$\overline{\mathbf{r}^-_{+-}} = (\overline{\mathbf{r}^-_{-+}})^* = \sin\theta_2^\tau \mathbf{n}_b^- \mathbf{f}_b^- + \sqrt{2}\sin\tilde{\theta}_b \mathcal{R}_{a_1} \times \{\cos\theta_2^\tau[\overline{\eta}\cos\tilde{\phi}_b - \overline{\omega^\tau}\sin\tilde{\phi}_b] + \iota[\overline{\eta}\sin\tilde{\phi}_b + \overline{\omega^\tau}\cos\tilde{\phi}_b]\}, \quad (64)$$

with

$$\begin{pmatrix} \mathbf{n}_b^- \\ \mathbf{n}_b^- \mathbf{f}_b^- \end{pmatrix} = \cos\tilde{\theta}_b \frac{\overline{\Gamma_T^{\mp}}}{\overline{\Gamma}} \quad (65)$$

or

$$\mathbf{n}_b^- = \tfrac{1}{2}\cos\tilde{\theta}_b[\overline{\xi} - \overline{\zeta \mathcal{S}_\rho}],$$

$$\mathbf{n}_b^- \mathbf{f}_b^- = \tfrac{1}{2}\cos\tilde{\theta}_b[1 - \overline{\sigma \mathcal{S}_\rho}]. \quad (66)$$

B. The simpler four-variable S2SC function

The simpler four-variable S2SC function including both ν and both $\overline{\nu}$ helicities is

$$\mathbf{I}_4 = \mathbf{I}(E_1, E_2, \tilde{\theta}_1, \tilde{\theta}_2) = |T(+,-)|^2 \rho_{++}\overline{\rho_{--}} + |T(-,+)|^2 \rho_{--}\overline{\rho_{++}} + |T(+,+)|^2 \rho_{++}\overline{\rho_{++}} + |T(-,-)|^2 \rho_{--}\overline{\rho_{--}}. \quad (67)$$

This formula is in terms of the *integrated* composite decay density matrices for the $\tau^\pm \to \rho^\pm \nu$ and/or for the $\tau^\pm \to a_1^\pm \nu$ decay chains with $\rho^\pm \to (2\pi)^\pm$ and $a_1^\pm \to (3\pi)^\pm$. Note that as for the **R**'s in the preceding section, in Eq. (67) the ρ's include both neutrino helicities. Here for convenience, unlike in Sec. IV A, we suppress a ''boldface font'' for the ρ's.

Formulas for $\tau \to \rho\nu$

For $\tau^- \to \rho^- \nu \to (\pi^- \pi^0)\nu$, with τ^- helicity $\lambda_1 = h/2$

$$\rho_{hh} \equiv \frac{1}{\Gamma} \frac{dN}{d(\cos\theta_1^\tau) d(\cos\tilde{\theta}_1)} = \frac{1}{8}(3 + \cos 2\tilde{\theta}_1) S + \frac{1}{32}(1 + 3\cos 2\tilde{\theta}_1) D, \quad (68)$$

where

$$S = 1 + h \zeta \mathcal{S}_\rho \cos\theta_1^\tau, \quad (69)$$

$$D = -S(1 - \cos 2\omega_1) + (\sigma \mathcal{S}_\rho + h\xi \cos\theta_1^\tau)(1 + 3\cos 2\omega_1) + h\omega \mathcal{R}_\rho 4\sqrt{2}\sin 2\omega_1 \sin\theta_1^\tau. \quad (70)$$

It is very important to note that the S contribution only appears in stage-one spin-correlation functions. This is the reason for the breakup in to S and D contributions in this section. Formulas for the Wigner rotation angles $\omega_{1,2}$ which respectively are solely functions of the $\rho^\mp$ energies $E_{1,2}$ are given in[5]. If $\tilde{\theta}_1$ is integrated out, i.e., if the polarimetry information from the $\rho^- \to (2\pi)^-$ stage is not included, then D does not contribute. In this manner, ζ is measurable. Then inclusion of the $\tilde{\theta}_1$ dependence gives D and also enables separation of ξ and ω because of their differing dependence on $\tilde{\theta}_1$.

For the CP conjugate process $\tau^+ \to \rho^+ \overline{\nu} \to (\pi^+ \pi^0)\overline{\nu}$, with τ^+ helicity $\lambda_1 = h/2$,

$$\overline{\rho_{hh}} \equiv \frac{1}{\overline{\Gamma}} \frac{dN}{d(\cos\theta_2^\tau) d(\cos\tilde{\theta}_2)} = \frac{1}{8}(3 + \cos 2\tilde{\theta}_2)\overline{S} + \frac{1}{32}(1 + 3\cos 2\tilde{\theta}_2)\overline{D}, \quad (71)$$

where

$$\overline{S} = 1 - h\overline{\zeta \mathcal{S}_\rho}\cos\theta_2^\tau, \quad (72)$$

$$\overline{D} = -\overline{S}(1 - \cos 2\omega_2) + (\overline{\sigma \mathcal{S}_\rho} - h\overline{\xi}\cos\theta_2^\tau)(1 + 3\cos 2\omega_2) - h\overline{\omega \mathcal{R}_\rho} 4\sqrt{2}\sin 2\omega_2 \sin\theta_2^\tau. \quad (73)$$

Formulas for $\tau \to a_1 \nu$

For $\tau^- \to a_1^- \nu \to (3\pi)^- \nu$, with τ^- helicity $\lambda_1 = h/2$ where

$$\rho_{hh} \equiv \frac{1}{\Gamma} \frac{dN}{d(\cos\theta_1^\tau) d(\cos\tilde{\theta}_1)} = \frac{1}{4}(3 + \cos 2\tilde{\theta}_1) S_{a_1} - \frac{1}{32}(1 + 3\cos 2\tilde{\theta}_1) D_{a_1}, \quad (74)$$

$$S_{a_1} = 1 + h\zeta \mathcal{S}_{a_1}\cos\theta_1^\tau, \quad (75)$$

$$D_{a_1}=S_{a_1}(3+\cos 2\omega_1)+(\sigma\mathscr{S}_{a_1}+h\xi\cos\theta_1^\tau)\times(1+3\cos 2\omega_1)+h\omega\mathscr{R}_{a_1}4\sqrt{2}\sin 2\omega_1\sin\theta_1^\tau. \quad (76)$$

The remarks above, following the analogous formulas in the ρ case, also apply here.

For the CP conjugate process $\tau^+\to a_1^+\overline{\nu}\to(3\pi)^+\overline{\nu}$, with τ^+ helicity $\lambda_2=h/2$,

$$\overline{\rho_{hh}}\equiv\frac{1}{\Gamma}\frac{dN}{d(\cos\theta_2^\tau)d(\cos\tilde{\theta}_2)}=\frac{1}{4}(3+\cos 2\tilde{\theta}_2)\overline{S_{a_1}}-\frac{1}{32}(1+3\cos 2\tilde{\theta}_2)\overline{D_{a_1}}, \quad (77)$$

where

$$\overline{S_{a_1}}=1-h\overline{\zeta}\mathscr{S}_{a_1}\cos\theta_2^\tau, \quad (78)$$

$$\overline{D_{a_1}}=\overline{S_{a_1}}(3+\cos 2\omega_2)+(\overline{\sigma}\mathscr{S}_\rho-h\overline{\xi}\cos\theta_2^\tau)(1+3\cos 2\omega_2)-h\overline{\omega}\mathscr{R}_{a_1}4\sqrt{2}\sin 2\omega_2\sin\theta_2^\tau. \quad (79)$$

C. The five-variable S2SC functions

In order to measure some of the parameters which test for leptonic T violation, we use the ϕ dependence of the spin correlation plus the variables in I_4. Recall that ϕ is the opening angle between the two τ decay planes. For the ideal statistical errors considered later in this paper, we assume that the τ lepton direction is known from a silicon vertex detector and so both $\cos(\phi)$ and $\sin(\phi)$ are known. The five-variable S2SC function is listed in Eq. (4.15) in Ref. [5], so here we only list the additional composite density matrix elements:

For the symmetric ''plus'' Dalitz distribution, for the ρ^- mode there is

$$\rho_{+-}=\frac{1}{32}\xi\sin\theta_1^\tau(1+3\cos 2\omega_1)(1+3\cos 2\tilde{\theta}_1)+\frac{1}{32}\zeta\mathscr{S}_\rho\sin\theta_1^\tau[11+\cos 2\tilde{\theta}_1+\cos 2\omega_1(1+3\cos 2\tilde{\theta}_1)]-\frac{1}{4\sqrt{2}}\mathscr{R}_\rho[\omega\cos\theta_1^\tau-\iota\eta']\times\sin 2\omega_1(1+3\cos 2\tilde{\theta}_1) \quad (80)$$

and for the CP conjugate mode there is $\overline{\rho_{+-}}=-[\rho_{+-}]^*$ with the usual subscript changes $1\to 2$, $a\to b$ as in Ref. [5].

Similarly, for the a_1^- mode there is

$$\rho_{+-}=-\frac{1}{32}\xi\sin\theta_1^\tau(1+3\cos 2\omega_1)(1+3\cos 2\tilde{\theta}_1)+\frac{1}{32}\zeta\mathscr{S}_{a_1}\sin\theta_1^\tau[21-\cos 2\tilde{\theta}_1-\cos 2\omega_1(1+3\cos 2\tilde{\theta}_1)]+\frac{1}{4\sqrt{2}}\mathscr{R}_{a_1}[\omega\cos\theta_1^\tau-\iota\eta']\sin 2\omega_1(1+3\cos 2\tilde{\theta}_1) \quad (81)$$

and again for the CP conjugate a_1^+ mode, $\overline{\rho_{+-}}=-[\rho_{+-}]^*$ with the same subscript substitutions.

For the ''minus'' Dalitz distribution for the a_1^- mode, there are, *with the same normalization as for the ''plus'' expressions given above*,

$$\rho_{hh}^-=\cos\tilde{\theta}_1\{\tfrac{1}{2}\cos\omega_1[-\xi+\zeta\mathscr{S}_{a_1}+h(1-\sigma\mathscr{S}_{a_1})\cos\tilde{\theta}_1]+h\sqrt{2}\sin\omega_1\sin\theta_1^\tau\eta\mathscr{R}_{a_1}\} \quad (82)$$

and for the CP conjugate a_1^+ there is $\overline{\rho_{hh}^-}=-[\rho^-_{-h,-h}]$. For the five-variable distribution for the ''minus'' Dalitz distribution, there is

$$\rho_{+-}^-=\tfrac{1}{2}(1-\sigma\mathscr{S}_{a_1})\sin\theta_1^\tau\cos\omega_1\cos\tilde{\theta}_1-\sqrt{2}\mathscr{R}_{a_1}[\eta\cos\theta_1^\tau-\iota\omega]\sin\omega_1\cos\tilde{\theta}_1 \quad (83)$$

and $\overline{\rho_{+-}^-}=[\rho_{+-}^-]^*$.

D. Tests for leptonic T violation

By unitarity and the assumption that only the minimal helicity amplitudes are needed, one can easily derive other expressions for measuring the phase differences between the helicity amplitudes.

In the case of only ν_L neutrinos, it follows that

$$\cos\beta_a=\frac{I_{\mathscr{R}}}{\sqrt{\Gamma_L\Gamma_T}}=\frac{2\omega\mathscr{R}_\rho}{\sqrt{1-(\zeta\mathscr{S}_\rho)^2}} \quad (84)$$

and that

$$\sin\beta_a=\frac{I_{\mathscr{T}}}{\sqrt{\Gamma_L\Gamma_T}}=\frac{2\omega'\mathscr{R}_\rho}{\sqrt{1-(\zeta\mathscr{S}_\rho)^2}}. \quad (85)$$

In the case of both ν_L and ν_R couplings, there are instead

$$\cos\beta_a=\frac{I_{\mathscr{R}}^{\nu_L}}{\sqrt{\Gamma_L^{\nu_L}\Gamma_T^{\nu_L}}}=\frac{2\mathscr{R}_\rho(\omega+\eta)}{\sqrt{(1+\xi)^2-(\mathscr{S}_\rho[\sigma+\zeta])^2}}. \quad (86)$$

Though physically transparent from their indices, the polarized partial widths and intensities in these expressions, $\Gamma_{R,I}^{\nu_L},\ldots$ and $I_{R,I}^{\nu_L},\ldots$ are explicitly defined in Sec. II. For the ν_R phase difference

$$\cos\beta_a^R=\frac{I_{\mathscr{R}}^{\nu_R}}{\sqrt{\Gamma_L^{\nu_R}\Gamma_T^{\nu_R}}}=\frac{2\mathscr{R}_\rho(\eta-\omega)}{\sqrt{(1-\xi)^2-(\mathscr{S}_\rho[\sigma-\zeta])^2}}. \quad (87)$$

Also

$$\sin\beta_a = \frac{I_{\mathscr{T}}^{\nu_L}}{\sqrt{\Gamma_L^{\nu_L}\Gamma_T^{\nu_L}}} = \frac{2\mathscr{R}_\rho(\omega' + \eta')}{\sqrt{(1+\xi)^2 - (\mathscr{S}_\rho[\sigma+\zeta])^2}}, \quad (88)$$

with

$$\sin\beta_a^R = \frac{I_{\mathscr{T}}^{\nu_R}}{\sqrt{\Gamma_L^{\nu_R}\Gamma_T^{\nu_R}}} = \frac{2\mathscr{R}_\rho(\eta' - \omega')}{\sqrt{(1-\xi)^2 - (\mathscr{S}_\rho[\sigma-\zeta])^2}}. \quad (89)$$

The two constraint equations, Eq. (13), in Sec. III A immediately follow from these expressions.

Measurement of $\beta_a \neq 0$ ($\beta_b \neq 0$) implies a violation of T invariance in $\tau^- \to \rho^- \nu (\tau^+ \to \rho^+ \bar{\nu})$ or the presence of an unexpected final-state interaction between the ν and ρ^-. Because of the further assumption of no unusual final state interactions, see Sec. II, one is actually testing for $\tilde{T}_{FS}$ invariance. Canonical T invariance relates $\tau^- \to \rho^- \nu$ and the actual time-reversed process $\rho^- \nu \to \tau^-$ which is not directly accessible by present experiments.

V. TESTS FOR NON-CKM-TYPE LEPTONIC *CP* VIOLATION

By *CP* invariance each of the barred semileptonic parameters should equal, within experimental errors, its unbarred associate. However, as was shown in Ref. [5], if only ν_L and $\overline{\nu_R}$ exist, there are two simple tests for ''non-Cabibbo-Kobayashi-Maskawa(CKM-)type'' leptonic *CP* violation in $\tau \to \rho \nu$ decay. Normally a CKM leptonic phase will contribute equally at tree level to both the τ^- decay amplitudes (for exceptions, see footnotes 14 and 15 in Ref. [5]). These two tests follow because by *CP* invariance $B(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}}) = \gamma_{CP} A(-\lambda_{\bar{\rho}}, -\lambda_{\bar{\nu}})$. So the two tests for leptonic *CP* violation are

$$\beta_a = \beta_b \quad \text{first test}, \quad (90)$$

where $\beta_a = \phi_{-1}^a - \phi_0^a$, $\beta_b = \phi_1^b - \phi_0^b$, and

$$r_a = r_b \quad \text{second test}, \quad (91)$$

where

$$r_a = \frac{|A(-1,-\frac{1}{2})|}{|A(0,-\frac{1}{2})|}, \quad r_b = \frac{|B(1,\frac{1}{2})|}{|B(0,\frac{1}{2})|}. \quad (92)$$

For sensitivity levels for $\tau \to \rho \nu$ decay, see Ref. [6].

This analysis can be easily generalized [7] to the $\tau \to a_1 \nu$ decay mode in which the a_1 has the opposite *CP* quantum number to that of the ρ: For the $\tau^- \to a_1^- \nu \to (\pi^- \pi^- \pi^+)\nu$, $(\pi^0 \pi^0 \pi^-)\nu$ modes, the composite-decay-density matrix is given by

$$\rho_{hh} = (1 + h\cos\theta_1^\tau)\left[\sin^2\omega_1 \cos^2\tilde{\theta}_1 + \left(1 - \frac{1}{2}\sin^2\omega_1\right)\sin^2\tilde{\theta}_1\right] + \frac{r_a^2}{2}(1 - h\cos\theta_1^\tau)\left[(1+\cos^2\omega_1)\cos^2\tilde{\theta}_1 + \left(1 + \frac{1}{2}\sin^2\omega_1\right)\sin^2\tilde{\theta}_1\right] - h\frac{r_a}{\sqrt{2}}\cos\beta_a \sin\theta_1^\tau \sin 2\omega_1\left[\cos^2\tilde{\theta}_1 - \frac{1}{2}\sin^2\tilde{\theta}_1\right]. \quad (93)$$

Table VI shows that the sensitivity of the a_1 mode, versus that of the ρ mode, is about two times better for the r_a measurement and is about five times worse for the β measurements. The simpler I_4 function was used for the error $\delta(r_a)$ and the full I_7 was used for the other δ's. The *CP* and $CP\tilde{T}_{\rm FS}$ predictions for the phase relation between β_a and β_b are opposite, see Table III in [2], so this provides a method for distinguishing between a new physics effect due to an unusual *CP*-violating final state interaction ($\beta_a = -\beta_b$) and one with a different mechanism of *CP* violation ($|\beta_a| \neq |\beta_b|$).

It is also easy to generalize these simple tests so as to include ν_R and $\overline{\nu_L}$ couplings. The necessary four-variable S2SC is given by

$$I(E_\rho, E_{\bar{\rho}}, \tilde{\theta}_1, \tilde{\theta}_2)\Big|_{\nu_R, \bar{\nu}_L} = I_4 + (\lambda_R)^2 I_4(\rho \to \rho^R) + (\overline{\lambda_L})^2 I_4(\overline{\rho} \to \overline{\rho^L}) + (\lambda_R \overline{\lambda_L})^2 I_4(\rho \to \rho^R, \overline{\rho} \to \overline{\rho^L}), \quad (94)$$

where $\lambda_R \equiv |A(0,1/2)|/|A(0,-1/2)|$, $\overline{\lambda_L} \equiv |B(0,-1/2)|/|B(0,1/2)|$ give the moduli's of the ν_R and $\overline{\nu_L}$ amplitudes versus the standard amplitudes. The corresponding composite density matrices for $\tau \to \rho \nu$ with ν_R and $\overline{\nu_L}$ final state particles are given by the substitution rules:

$$\rho_{hh}^R = \rho_{-h,-h}(r_a \to r_a^R, \beta_a \to \beta_a^R), \quad (95)$$

TABLE VI. The ideal statistical errors for the two tests for ''non-CKM-type'' leptonic *CP* violation in $\tau^- \to a_1^- \nu_L$ decay, including both $a_1^- \to (2\pi^- \pi^+)$ and $(2\pi^0 \pi^-)$ modes. The $CP\tilde{T}_{FS}$, CP, $\tilde{T}_{\rm FS}$ labels denote the symmetries which would respectively be violated if $r_a \neq r_b$, $\tilde{\beta} \neq 0$, etc. Note that $\tilde{\beta} \equiv \beta_a - \beta_b$ and $\beta' \equiv \beta_a + \beta_b$. At M_Z we assume 10^7 Z bosons and assume 10^7 $\tau^- \tau^+$ pairs at each of the other center-of-mass energies. To compare with the analogous values for $\tau^- \to \rho^- \nu$; see the tables in Ref. [5].

$E_{\rm c.m.}$	$\sigma(r_a)$ $CP\tilde{T}_{\rm FS}$, CP	$\sigma(\tilde{\beta}) \simeq \sigma(\beta_a)$ CP $\tilde{T}_{\rm FS}$	$\sigma(\beta')$ $CP\tilde{T}_{\rm FS}$, CP
M_Z	0.3%	~10°	~15°
10 GeV	0.05%	~3°	~3°
4 GeV	0.05%	~4°	~5°

$$\overline{\rho_{hh}^{L}}=\overline{\rho_{-h,-h}}(r_b\to r_b^L,\beta_b\to\beta_b^L), \tag{96}$$

where the ν_R and $\overline{\nu_L}$ moduli ratios and phase differences are defined by $r_a^R\equiv|A(1,1/2)|/|A(0,1/2)|$, $r_b^L\equiv|B(-1,-1/2)|/|B(0,-1/2)|$, $\beta_a^R\equiv\phi_1^a-\phi_0^{aR}$, $\beta_b^L\equiv\phi_{-1}^b-\phi_0^{bL}$. The two additional tests for ''non-CKM-type'' leptonic CP violation if R-handed ν and L-handed $\overline{\nu}$ exist are

$$\beta_a^R=\beta_b^L \quad \text{first } \nu_R/\overline{\nu_L} \text{ test}, \tag{97}$$

$$r_a^R=r_b^L \quad \text{second } \nu_R/\overline{\nu_L} \text{ test}. \tag{98}$$

VI. DESCRIPTION OF $\tau^-\to\pi^-\nu$, $K^-\nu$

The only observables for each of the $\tau^-\to\pi^-\nu$, $K^-\nu$ modes which can be measured by spin correlations are the chirality parameter

$$\xi_{\pi,K}=[|A(-1/2)|^2-|A(1/2)|^2]/[|A(-1/2)|^2+|A(1/2)|^2]$$

and the $\Gamma(\tau^-\to\pi^-\nu)$, or $\Gamma(\tau^-\to K^-\nu)$, partial width. The relative phase of the $A(\lambda_\nu)=A(\mp\frac{1}{2})$ amplitudes cannot be measured unless, e.g., the ν_L and ν_R have a common final decay channel. For $\tau^-\to\pi^-\nu$, or $K^-\nu$, the $(k_\tau+p_\nu)$ effective couplings $(k_\tau+p_\nu)_\alpha V_{\nu\tau}^\alpha$ and $(k_\tau+p_\nu)_\alpha A_{\nu\tau}^\alpha$ are equivalent to the standard $q_{\pi,\alpha}V_{\nu\tau}^\alpha$ and $q_{\pi,\alpha}A_{\nu\tau}^\alpha$ couplings. Here $V_{\nu\tau}^\alpha$ and $A_{\nu\tau}^\alpha$ are as in Eqs. (14) and (15). The f_M and f_E couplings do not contribute. The S^- and P^- couplings can contribute to the π^- and K^- channels, whereas they do not for the ρ,a_1,K^* modes. However, since $q\cdot V\sim(m_\pi^2/2\Lambda)g_{S^-}$ and $q\cdot A\sim(m_\pi^2/2\Lambda)g_{P^-}$ their contribution is strongly suppressed for $\Lambda>(\sim 1$ GeV) scales.

By Lorentz invariance, there are the equivalence theorems that $S^-\approx S\approx T^+\approx V$ and $P^-\approx P\approx T_5^+\approx A$. The general helicity amplitudes for $\tau^-\to\pi^-\nu$, or $K^-\nu$, for the above $q\cdot V$ and $q\cdot A$ couplings are

$$\begin{aligned}
A\left(\mp\frac{1}{2}\right)=&\,g_L(E_\rho\pm q_\pi)\sqrt{m_\tau(E_\nu\pm q_\pi)}+g_R(E_\rho\mp q_\pi)\sqrt{m_\tau(E_\nu\mp q_\pi)}+\left(\frac{m_\tau}{2\Lambda_i}\right)\Bigg[g_{S+P}+g_{S-P}+(g_{S^-+P^-}+g_{S^--P^-})\\
&\times\left(\frac{m_\pi^2}{m_\tau^2-m_\nu^2}\right)\Bigg]\{(E_\rho\pm q_\pi)\sqrt{m_\tau(E_\nu\pm q_\pi)}+(E_\rho\mp q_\pi)\sqrt{m_\tau(E_\nu\mp q_\pi)}\}\\
&+\tilde{g}_+\left(\frac{m_\tau}{2\Lambda}\right)\left\{\left(-1+\frac{m_\pi^2}{m_\tau^2-m_\nu^2}\right)(E_\rho\pm q_\pi)\sqrt{m_\tau(E_\nu\pm q_\pi)}+\left(1+\frac{m_\pi^2}{m_\tau^2-m_\nu^2}\right)(E_\rho\mp q_\pi)\sqrt{m_\tau(E_\nu\mp q_\pi)}\right\}\\
&+\tilde{g}_-\left(\frac{m_\tau}{2\Lambda}\right)\left\{\left(1+\frac{m_\pi^2}{m_\tau^2-m_\nu^2}\right)(E_\rho\pm q_\pi)\sqrt{m_\tau(E_\nu\pm q_\pi)}+\left(-1+\frac{m_\pi^2}{m_\tau^2-m_\nu^2}\right)(E_\rho\mp q_\pi)\sqrt{m_\tau(E_\nu\mp q_\pi)}\right\}.
\end{aligned} \tag{99}$$

The ξ_π parameter can be measured by the stage-one energy-correlation function $I(E_1^\pi,E_2^\pi)$ where $\rho_{\pm\pm}=1\pm\xi_\pi\cos\theta_1^\tau$, $\overline{\rho}_{\pm\pm}=1\mp\overline{\xi_\pi}\cos\theta_2^\tau$.

The associated ideal statistical errors for ξ_π are given in Table VII in Ref. [6]. These errors for ξ_π are about three times worse than those from I_4 for ξ_ρ at each of the three $E_{\text{c.m.}}$. The three variable $I(E_1,E_2,\phi)$ is identical in structure to Eq. (4.15) in Ref. [5], including the $\sin\phi$ term. Because $\rho_{+-}=\rho_{-+}=-\xi_\pi\sin\theta_1^\tau$, I_3 does not depend on additional semileptonic parameters beyond Γ_π and ξ_π.

From Eq. (99) the effective $\lambda=|g_{\text{eff}}/g_L|^2$ value follows for

$$\frac{\Gamma(\tau\to\pi\nu_\tau)}{\Gamma(\pi\to\mu\nu_\mu)}=\frac{\lambda}{2}\,\frac{m_\tau^3}{m_\mu^2 m_\pi}\left(\frac{1-m_\pi^2/m_\tau^2}{1-m_\mu^2/m_\pi^2}\right)^2. \tag{100}$$

For example,

$$\lambda_{S+P}=\left|1+\frac{m_\tau}{2\Lambda}\,\frac{g_{S+P}}{g_L}\right|^2,\quad \lambda_{\tilde{g}_+}=\left|1-\frac{m_\tau}{2\Lambda}\,\frac{\tilde{g}_+}{g_L}\left(1-\frac{m_\pi^2}{m_\tau^2}\right)\right|^2.$$

VII. ASSOCIATED IDEAL STATISTICAL ERRORS

For the 10^7 (τ^-,τ^+)'s at 10 GeV, and a like number at 4 GeV, we determine the ideal statistical errors in the same manner as in our earlier papers; see Ref. [4]. The results are tabulated in the following tables. We concentrate on the S2SC distributions with the fewest variables.

See Table VII for the errors for $(\xi,\zeta,\sigma,\omega)$ based on I_4. In general, the values for the ρ^- mode are slightly less than 1%. The CP tests for these semileptonic parameters are about $\sqrt{2}$ worse. Typically the a_1 values are about three times worse than the ρ values. The a_1 errors are generally smaller when obtained from the $\{\rho,a_1\}$ modes than from $\{a_1^-,a_1^+\}$. For comparison, use of the full seven-variable I_7 would only give a factor of about 2 improvement; see Tables VIII and IX. Notice that the statistical errors for ξ using the simpler $I(E_1,E_2)$ distributions in Ref. [4] cannot be directly compared with those listed here because Ref. [4] assumes a mixture of only V and A couplings in $J_{\text{lepton}}^{\text{charged}}$. Except for I_4, in this section in using S2SC functions to determine ideal statistical errors, we assume that the τ^- direction is known from a silicon vertex detector. Otherwise a Wigner rotation [5] must be included in using I_7 and the $\sin(\phi)$ correlation is not available in I_5 and I_5^-. For completeness, Tables X and XI give the analogous ideal statistical errors at M_Z for a 10^7 sample of Z^0 events.

To test for leptonic T violation, besides the ω parameter which can be measured from I_4 in both the ρ and a_1 modes, there is the η' parameter which can be obtained from I_5 in both the ρ and a_1 modes. Also there are the η and ω' pa-

TABLE VII. At 10 GeV and at 4 GeV: ideal statistical errors for measurements of the fundamental parameters ξ, ζ, σ, and ω by the stage-two spin-correlation function I_4 for the sequential decay of an off-mass-shell photon $\gamma^* \to \tau^- \tau^+$ with $\tau^- \to \rho^- \nu$ and $\tau^+ \to \rho^+ \bar{\nu}$, etc. For each parameter, the first row assumes CP invariance, for instance $\xi = \bar{\xi}$; then the following row contains the corresponding statistical errors for measurement of the same parameter not assuming CP invariance. The column headed by ''$\{\rho, a_1\}$ modes'' is for the $\{\rho^-, a_1^+\}$ and the $\{a_1^-, \rho^+\}$ sequential decay modes, etc.

	ρ^- values		a_1^- values	
	$\{\rho^-,\rho^+\}$	$\{\rho,a_1\}$ modes	$\{a_1^-,a_1^+\}$	$\{a_1,\rho\}$ modes
No. of events	605 160	867 925	324 000	885 600
At 10 GeV:				
ξ	0.0060	0.021	0.046	0.020
CP for ξ	0.0100	0.030	0.081	0.028
ζ	0.0070	0.022		
CP for ζ	0.011	0.031		
σ	0.013	0.033		
CP for σ	0.016	0.047		
ω	0.0057	0.020	0.037	0.017
CP for ω	0.010	0.028	0.069	0.024
At 4 GeV:				
ξ	0.013	0.033	0.080	0.044
CP for ξ	0.020	0.047	0.14	0.062
ζ	0.016	0.039		
CP for ζ	0.024	0.056		
σ	0.028	0.046		
CP for σ	0.028	0.064		
ω	0.015	0.041	0.059	0.034
CP for ω	0.025	0.058	0.11	0.049

rameters which only appear in S2SC distributions for the a_1 modes. See Tables XII–XIV. To normalize the ''minus'' Dalitz distributions I_5^- and I_7^-, we have assumed that the form factor dependence in Eq. (51) leads to an extra factor of $\frac{1}{5}$ in the overall ''minus'' distribution normalization versus

TABLE VIII. At 10 GeV: for measurements based on the seven-variable S2SC function I_7, ideal statistical errors for ξ, ζ, σ, ω, and η'. In this and following 10 GeV tables, the number of events for each sequential decay mode is the same as in Table VII.

	ρ^- values		a_1^- values	
	$\{\rho^-,\rho^+\}$	$\{\rho,a_1\}$ modes	$\{a_1^-,a_1^+\}$	$\{a_1,\rho\}$ modes
At 10 GeV:				
ξ	0.0032	0.0078	0.013	0.0076
CP for ξ	0.0052	0.011	0.022	0.011
ζ	0.0055	0.012		
CP for ζ	0.0080	0.017		
σ	0.0026	0.0032		
CP for σ	0.0037	0.0045		
ω	0.0031	0.0081	0.014	0.0086
CP for ω	0.0054	0.011	0.024	0.012
η'	0.0031	0.0085	0.013	0.0088
CP for η'	0.0060	0.012	0.025	0.012

TABLE IX. At 4 GeV: for measurements based on the seven-variable S2SC function I_7, ideal statistical errors for ξ, ζ, σ, ω, and η'. In this and following 4 GeV tables, the number of events for each sequential decay mode is the same as in Table VII.

	ρ^- values		a_1^- values	
	$\{\rho^-,\rho^+\}$	$\{\rho,a_1\}$ modes	$\{a_1^-,a_1^+\}$	$\{a_1,\rho\}$ modes
At 4 GeV:				
ξ	0.0055	0.013	0.020	0.012
CP for ξ	0.0089	0.018	0.035	0.017
ζ	0.0091	0.019		
CP for ζ	0.013	0.027		
σ	0.0027	0.0033		
CP for σ	0.0039	0.0046		
ω	0.0053	0.013	0.022	0.013
CP for ω	0.0092	0.018	0.038	0.019
η'	0.0047	0.013	0.019	0.014
CP for η'	0.0094	0.019	0.038	0.019

the corresponding ''plus'' distribution's normalization. For instance, to normalize I_5^-, the I_5 plus distribution's relative normalization was needed; so the I_5 ideal statistical errors were investigated but the errors were generally not significantly better than those for I_4 so we have not listed them. All the ''plus'' distributions are normalized by the available number of events. Only by actual experimental analyses can it be shown whether the ''minus'' distributions will have enough sensitivity for interesting tests of this type. See Ref. [16] for detailed treatments of the hadronic form factors. For determination of the I_5^- and I_7^- statistical errors, we use the minus distribution only for one side, e.g., the τ^- in γ^*, $Z^0 \to \tau^- \tau^+$ and use the plus distribution for the other side, e.g., the τ^+.

Table XV shows that the ω' parameter can be much better measured by the full I_7^- distribution. This suggests that for measurement of ω' the best few-parameter distributions are not I_4 or either the I_5's. From Table VIII, one similarly concludes that there should be a better observable for η' at 10 GeV.

TABLE X. At M_Z: the ideal statistical errors for ξ, ζ, σ, and ω for measurements based on the stage-two spin-correlation function I_4 for the sequential decay $Z^0 \to \tau^- \tau^+$ with $\tau^- \to \rho^- \nu$ and $\tau^+ \to \rho^+ \bar{\nu}$, etc. The entries can be compared with those in Table VII for 10 and 4 GeV center of mass energies.

	ρ^- values		a_1^- values	
	$\{\rho^-,\rho^+\}$	$\{\rho,a_1\}$ modes	$\{a_1^-,a_1^+\}$	$\{a_1,\rho\}$ modes
No. of events	20 303	29 119	10 870	29 119
At M_Z:				
ξ	0.027	0.081	0.20	0.094
CP for ξ	0.045	0.11	0.32	0.13
ζ	0.032	0.084		
CP for ζ	0.048	0.12		
σ	0.059	0.13		
CP for σ	0.073	0.18		
ω	0.026	0.076	0.16	0.082
CP for ω	0.045	0.11	0.27	0.12

TABLE XI. At M_Z: for measurements based on the seven-variable S2SC function I_7, ideal statistical errors for ξ, ζ, σ, ω, and η'. In this and following M_Z tables, the number of events for each sequential decay mode is the same as in Table X.

	ρ^- values $\{\rho^-,\rho^+\}$	$\{\rho,a_1\}$ modes	a_1^- values $\{a_1^-,a_1^+\}$	$\{a_1,\rho\}$ modes
At M_Z:				
ξ	0.013	0.033	0.056	0.034
CP for ξ	0.022	0.047	0.097	0.048
ζ	0.023	0.051		
CP for ζ	0.034	0.072		
σ	0.014	0.017		
CP for σ	0.019	0.025		
ω	0.013	0.034	0.065	0.040
CP for ω	0.022	0.048	0.11	0.056
η'	0.014	0.037	0.059	0.041
CP for η'	0.027	0.053	0.11	0.058

Typically the errors are a factor of 2 worse at 4 GeV than at 10 GeV. However, there is an important exception: the error for the η' parameter for $\tau^-\rightarrow\rho^-\nu$ using I_5 is about five times better at 4 GeV than at 10 GeV.

Notice that the S2SC functions do not enable a measurement of any relative phase between the ν_L and ν_R helicity amplitudes, so the helicity amplitudes can only ''almost'' be completely determined from knowledge of the eight semileptonic parameters [2].

VIII. OTHER CONCLUSIONS

The major conclusions are given in the abstract, Introduction, and in the ideal statistical errors given in the preceding section, so here we will only make a few additional remarks.

In the context of modern Monte Carlo simulations such as KORALB and TAUOLA, it should be simple and straightforward to build in the amplitudes for production of *L*-polarized and *T*-polarized ρ's or a_1's from distinct Lorentz-structure sources. Thereby, the results in Tables IV and V in this paper can be used for many systematic checks. For example, they could be used to experimentally test the *CP* and *T* invariance ''purity'' of detector components and of the data analysis by distinguishing which coefficients are or are not equal between various experimental data sets analyzed separately for the $\tau^{\mp}$ modes [17].

TABLE XII. Ideal statistical errors for measurements of the η' parameter by the $\sin(\phi)$ term in the S2SC function I_5 for the sequential decay of an off-mass-shell photon $\gamma^*\rightarrow\tau^-\tau^+$ with $\tau^-\rightarrow\rho^-\nu$ and $\tau^+\rightarrow\rho^+\nu$, etc. At M_Z, the corresponding errors are several orders of magnitude larger than unity.

	ρ^- values $\{\rho^-,\rho^+\}$ mode	a_1^- values $\{a_1^-,a_1^+\}$	$\{a_1,\rho\}$ modes
At 10 GeV:			
η'	0.11	0.61	0.35
CP for η'	0.23	2.1	0.49
At 4 GeV:			
η'	0.026	0.13	0.056
CP for η'	0.040	0.60	0.079

TABLE XIII. For measurements of $\tau^-\rightarrow a_1^-\nu$ from the sequential decay mode $\{a_1^-,a_1^+\}$ by the five-variable S2SC function I_5^-. Ideal statistical errors for η and ω'.

$\{a_1^-,a_1^+\}$	At 10 GeV	At 4 GeV	At M_Z
η	0.0062	0.017	0.044
CP for η	0.0087	0.025	0.062
ω'	0.25	0.23	
CP for ω'	0.35	0.32	

Assuming only ν_L couplings, a simple way for one to use a Monte Carlo simulation to test [18] for possible *CP* violation is to add an $S+P$ coupling (to the standard $V-A$ coupling) in the ρ decay mode such that the $S+P$ contribution has an overall complex coupling factor ''c'' in the τ^- mode and a complex factor ''d'' in the τ^+ mode. By Table VIII of Ref. [2], c.f. Eqs. (A1) and (A5) below, this will generate a difference in moduli and phases between the $\tau^{\mp}$ modes. Then the two tests for *CP* violation [5] are whether $|c|=|d|$ and $\arg(c)=\arg(d)$ experimentally.

Second, to be model independent and of greater use to theorists, experimental analyses should not assume a mixture of only *V* and *A* current couplings in τ decays. For the ρ, a_1, and K^* modes, by consideration of polarized partial widths there are several fundamental quantities besides the chirality parameter and the total partial width which can be directly measured. For example, for the a_1 mode there are three logically independent tests for only ν_L couplings: $\xi=1$, $\zeta=\sigma$, and $\omega=\eta$; if *T* violation occurred then the nonzero parameters $\omega'=\eta'$ if there are only ν_L couplings. For the ρ mode there are also these tests except that only ω and η' can be directly measured by S2SC functions; both η and ω' must be determined indirectly by the two constraint equations of Sec. II A.

It would be particularly interesting, as well as straightforward, to search for evidence for lepton compositeness in the most massive lepton, the τ, where naively such structure might be expected to be most easily observed. In analogy with the Pauli anomalous magnetic moment, such structure could show up as an additional tensorial $g_+=f_M+f_E$ coupling which would preserve $\xi=1$ (only ν_L couplings) but give non$(V-A)$ values to the other semileptonic parameters. From Table V, or Eqs. (28) and (29), there is the prediction that $\sigma=\zeta\neq 1$ and $\eta=\omega\neq 1$ with the constraint for Λ large that

TABLE XIV. For measurements of $\tau^-\rightarrow a_1^-\nu$ from the sequential decay modes $\{a_1^-,\rho^+\}+\{\rho^-,a_1^+\}$ by the five-variable S2SC function I_5^-. Ideal statistical errors for η and ω'.

$\{a_1,\rho\}$	At 10 GeV	At 4 GeV	At M_Z
η	0.0021	0.0036	0.014
CP for η	0.0030	0.0051	0.020
ω'	0.13	0.062	
CP for ω'	0.19	0.088	

TABLE XV. For measurements of $\tau^- \rightarrow a_1^- \nu$ from the sequential decay mode $\{a_1^-, a_1^+\}$ by the seven-variable S2SC function I_7^-. Ideal statistical errors for η and ω'.

$\{a_1^-, a_1^+\}$	At 10 GeV	At 4 GeV	At M_Z
η	0.0026	0.0050	0.0094
CP for η	0.0036	0.0071	0.013
ω'	0.0021	0.0045	0.011
CP for ω'	0.0030	0.0064	0.016

$$(\zeta-1)=(1-\omega)\frac{g}{l}. \tag{101}$$

By Eqs. (39) the ratio ''g/l'' is a known function of m_ρ and m_τ. Numerically$(g/l)_\rho$=0.079 [20]. At 10 or 4 GeV, by the ρ^- (a_1^-) modes, compositeness in the τ lepton could be respectively probed [5] to 1.2 TeV (1.5 TeV).

Third , in a completely general analysis, the values of the semileptonic parameters should not be carelessly combined from different modes because that could inadvertently mask an additional Lorentz contribution. For instance, measurement of ξ_π from the π mode only strongly constrains the $V+A$ chiral coupling; see Table VII of [6]; its measurement in $\tau \rightarrow \pi\nu$ *does not* significantly constrain the presence of scalar or tensorial couplings.

Lastly, in contrast to the purely leptonic modes [19], the τ semileptonic modes are qualitatively distinct [21] since they enable a second-stage spin correlation. This important difference is a tool that can be used in many important reactions of contemporary interest. It can and must be exploited in searching for new physics.

ACKNOWLEDGMENTS

For helpful discussions, we thank experimentalists and theorists at Cornell, DESY, Valencia, Montreux, KEK, and Beijing. We also thank the organizers and participants for a useful and intellectually stimulating WHEPP4 in Calcutta, where the manuscript was finished. For assistance, particularly with the checking, we thank Minseob Kim, Daryl P. Nazareth, and Hui-Chun (Hannah) Yang. This work was partially supported by U.S. Dept. of Energy Contract No. DE-FG 02-96ER40291.

APPENDIX: THE HELICITY AMPLITUDES IN TERMS OF THE CHIRAL COUPLINGS

In Sec. II, the simple symmetry relations among the amplitudes are possible because of the Jacob-Wick phase conventions that were built into the helicity formalism [8]. In combining these amplitudes with results from calculations of similar amplitudes by diagrammatic methods, care must be exercised to insure that the same phase conventions are being used (c.f. appendix in first paper in [13]).

The helicity amplitudes for $\tau^- \rightarrow \rho^- \nu_{L,R}$ for both $(V \mp A)$ couplings and m_ν arbitrary are for ν_L so $\lambda_\nu=-\frac{1}{2}$,

$$A\left(0,-\frac{1}{2}\right)=g_L\frac{E_\rho+q_\rho}{m_\rho}\sqrt{m_\tau(E_\nu+q_\rho)} - g_R\frac{E_\rho-q_\rho}{m_\rho}\sqrt{m_\tau(E_\nu-q_\rho)}, \tag{A1}$$

$$A\left(-1,-\frac{1}{2}\right)=g_L\sqrt{2m_\tau(E_\nu+q_\rho)}-g_R\sqrt{2m_\tau(E_\nu-q_\rho)} \tag{A2}$$

and for ν_R so $\lambda_\nu=\frac{1}{2}$,

$$A\left(0,\frac{1}{2}\right)=-g_L\frac{E_\rho-q_\rho}{m_\rho}\sqrt{m_\tau(E_\nu-q_\rho)} + g_R\frac{E_\rho+q_\rho}{m_\rho}\sqrt{m_\tau(E_\nu+q_\rho)}, \tag{A3}$$

$$A\left(1,\frac{1}{2}\right)=-g_L\sqrt{2m_\tau(E_\nu-q_\rho)}+g_R\sqrt{2m_\tau(E_\nu+q_\rho)}. \tag{A4}$$

Note that g_L, g_R denote the ''chirality'' of the coupling and $\lambda_\nu=\mp\frac{1}{2}$ denote the handedness of $\nu_{L,R}$. For $(S\pm P)$ couplings, the additional contributions are

$$A\left(0,-\frac{1}{2}\right)=g_{S+P}\left(\frac{m_\tau}{2\Lambda}\right)\frac{2q_\rho}{m_\rho}\sqrt{m_\tau(E_\nu+q_\rho)} + g_{S-P}\left(\frac{m_\tau}{2\Lambda}\right)\frac{2q_\rho}{m_\rho}\sqrt{m_\tau(E_\nu-q_\rho)},$$

$$A\left(-1,-\frac{1}{2}\right)=0, \tag{A5}$$

$$A\left(0,\frac{1}{2}\right)=g_{S+P}\left(\frac{m_\tau}{2\Lambda}\right)\frac{2q_\rho}{m_\rho}\sqrt{m_\tau(E_\nu-q_\rho)} + g_{S-P}\left(\frac{m_\tau}{2\Lambda}\right)\frac{2q_\rho}{m_\rho}\sqrt{m_\tau(E_\nu+q_\rho)}, \quad A\left(1,\frac{1}{2}\right)=0. \tag{A6}$$

The two types of tensorial couplings, $g_\pm=f_M\pm f_E$ and $\tilde{g}_\pm = g^{+}_{T^+\pm T_5^+}$, give the additional contributions

$$A\left(0,\mp\frac{1}{2}\right)=\mp g_{+}\left(\frac{m_{\tau}}{2\Lambda}\right)\left[\frac{E_{\rho}\mp q_{\rho}}{m_{\rho}}\sqrt{m_{\tau}(E_{\nu}\pm q_{\rho})}-\frac{m_{\nu}}{m_{\tau}}\frac{E_{\rho}\mp q_{\rho}}{m_{\rho}}\sqrt{m_{\tau}(E_{\nu}\mp q_{\rho})}\right]\pm g_{-}\left(\frac{m_{\tau}}{2\Lambda}\right)\left[-\frac{m_{\nu}}{m_{\tau}}\frac{E_{\rho}\pm q_{\rho}}{m_{\rho}}\sqrt{m_{\tau}(E_{\nu}\pm q_{\rho})}\right.$$

$$\left.+\frac{E_{\rho}\pm q_{\rho}}{m_{\rho}}\sqrt{m_{\tau}(E_{\nu}\mp q_{\rho})}\right]\mp\tilde{g}_{+}\left(\frac{m_{\tau}}{2\Lambda}\right)\left[\frac{E_{\rho}\pm q_{\rho}}{m_{\rho}}\sqrt{m_{\tau}(E_{\nu}\pm q_{\rho})}+\frac{m_{\nu}}{m_{\tau}}\frac{E_{\rho}\mp q_{\rho}}{m_{\rho}}\sqrt{m_{\tau}(E_{\nu}\mp q_{\rho})}\right]$$

$$\pm\tilde{g}_{-}\left(\frac{m_{\tau}}{2\Lambda}\right)\left[\frac{m_{\nu}}{m_{\tau}}\frac{E_{\rho}\pm q_{\rho}}{m_{\rho}}\sqrt{m_{\tau}(E_{\nu}\pm q_{\rho})}+\frac{E_{\rho}\mp q_{\rho}}{m_{\rho}}\sqrt{m_{\tau}(E_{\nu}\mp q_{\rho})}\right],$$

$$A\left(\mp 1,\mp\frac{1}{2}\right)=\mp\sqrt{2}g_{+}\left(\frac{m_{\tau}}{2\Lambda}\right)\left[\sqrt{m_{\tau}(E_{\nu}\pm q_{\rho})}-\frac{m_{\nu}}{m_{\tau}}\sqrt{m_{\tau}(E_{\nu}\mp q_{\rho})}\right]\pm\sqrt{2}g_{-}\left(\frac{m_{\tau}}{2\Lambda}\right)\left[-\frac{m_{\nu}}{m_{\tau}}\sqrt{m_{\tau}(E_{\nu}\pm q_{\rho})}+\sqrt{m_{\tau}(E_{\nu}\mp q_{\rho})}\right]$$

$$\mp\sqrt{2}\tilde{g}_{+}\left(\frac{m_{\tau}}{2\Lambda}\right)\left[\sqrt{m_{\tau}(E_{\nu}\pm q_{\rho})}+\frac{m_{\nu}}{m_{\tau}}\sqrt{m_{\tau}(E_{\nu}\mp q_{\rho})}\right]\pm\sqrt{2}\tilde{g}_{-}\left(\frac{m_{\tau}}{2\Lambda}\right)\left[\frac{m_{\nu}}{m_{\tau}}\sqrt{m_{\tau}(E_{\nu}\pm q_{\rho})}+\sqrt{m_{\tau}(E_{\nu}\mp q_{\rho})}\right].$$

[1] For recent experimental measurements of τ couplings see ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **337**, 383 (1994); ALEPH Collaboration, D. Buskulic *et al.*, *ibid.* **321**, 168 (1994); M. Schmidtler, in *Tau 94*, Proceedings of the Third Workshop on Tau Lepton Physics, Montreux, Switzerland, 1994, edited by L. Rolandi [Nucl. Phys. B (Proc. Suppl.) **40**, 265 (1995)]; J. Raab, *ibid.*, p. 413; M. Davier, *ibid.*, p. 395; R. Stroynowski, *ibid.*, p. 569; P. B. Renton, presented at 1995 Lepton-Photon Symposium, Beijing, China (unpublished). For hadronic aspects of τ decay modes, see and references therein, J. H. Kuhn, E. Mirkes, and M. Finkemeier, Report No. hep-ph/9511268 (unpublished). For a review of experiment and theory for the purely leptonic modes, see M. Perl, in *Tau 94*, this reference, p. 5; A. Pich and J. P. Silva, Phys. Rev. D **52**, 4006 (1995), and see A. Pich, Report No. hep-ph/9509253 (unpublished). See also, M. W. Grunewald, Humboldt University (Berlin) Report No. HUB-IEP-95/13 (unpublished), W. Lohmann and J. Raab, Report No. DESY 95-188 (unpublished), S. Gentile and M. Pohl, Report No. CERN-PPE/95-147 (unpublished). In the present paper, as previously, we assume a 10^7 $(\tau^-\tau^+)$ pair data sample at 10 GeV and separately at 4 GeV, and at M_Z, 10^7 Z^0's with $\mathscr{B}(Z^0\to\tau^-\tau^+)=0.03355$. For determination of ideal statistical errors we use the branching ratios of 24.6% for the ρ mode, and 18% for the sum of the neutral and charged a_1 modes, with an a_1 mass of 1.275 GeV.

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[6] C. A. Nelson, Phys. Lett. B **355**, 561 (1995); C. A. Nelson, M. Kim, D. P. Nazareth, and H.-C. Yang, Report No. hep-ph/9508367, SUNY BING 7/1/95, (unpublished). We list the ideal statistical error for the presence of an additional $V+A$ coupling as an error $\delta(\xi_A)$ on the chirality parameter ξ_A for $\tau^-\to A^-\nu$. Equivalently, if one ignores possible different L and R leptonic CKM factors, the effective lower bound on an additional $W_R^{\pm}$ boson (which couples only to right-handed currents) is

$$M_R=\{\delta(\xi_A)/2\}^{-1/4}M_L\,.$$

For the $\{\rho^-,\rho^+\}$ ($\{a_1^-,a_1^+\}$) mode the first cited paper for 10^7 $(\tau^-\tau^+)$ pairs gives $\delta(\xi_\rho)=0.0012(0.0018)$ for an additional $V+A$ coupling at 10 or 4 GeV, or equivalently $M_R>514$ GeV (464 GeV). Probably, $10^8(\tau^-\tau^+)$ pairs will be accumulated by a τ-charm factory [1] at 4 GeV, so all the potential 4 GeV bounds in the present paper and in our earlier analyzes, including Refs. [5,6], will be improved by a factor of 3.2.

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[18] Historically, for a complete Lorentz-invariant characterization of the charged current, there have been two popular choices for the minimal sets of couplings: Express the vector matrix element $\langle \nu | v^{\mu}(0) | \tau \rangle$ either in terms of V, f_M, and S^- (recall S^- does not contribute to the ρ, a_1, K^* modes) or of V, S, and S^-. Correspondingly, express the axial $\langle \nu | a^{\mu}(0) | \tau \rangle$ either in terms of A, f_E, and P^- (P^- does not contribute to the ρ, a_1, K^* modes) or of A, P, and P^-. So if there are only ν_L couplings, then the chiral combinations of $V-A$ and $S+P$ can contribute significantly to the ρ, a_1, K^* modes since $m_{\nu_\tau} \cong 0$. To include ν_R couplings, one would add $V+A$ and $S-P$ via an additional "$g_R \gamma^{\mu}(1+\gamma_5) + e(k+p)^{\mu}(1-\gamma_5)$" for the τ^- mode and "$\overline{g_R} \gamma^{\mu}(1+\gamma_5) + f(k+p)^{\mu}(1-\gamma_5)$" for the τ^+ mode where "e" and "f" are different complex coupling factors. In each case this gives the expected number of independent variables. The relative phase of g_L and g_R ($\overline{g_L}$ and $\overline{g_R}$) is not measurable by S2SC's per the last sentence in Sec. VII.

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[20] For the ρ, the ratio (g/l) varies from 0.108 to 0.034 across $(m_{\rho} \pm \Gamma/2)$. Equation (101) also occurs for an additional $S+P$ coupling except with the ratio (a/l) which varies from 5.07 to 12.1 across $(m_{\rho} \pm \Gamma/2)$.

[21] From existing results, a quantitative comparison with the ideal sensitivity in the purely leptonic case is possible if we assume an arbitrary mixture of V and A couplings with $m_{\nu}=0$. Then the semileptonic chirality parameter ξ_{ρ} and the chiral polarization parameter ξ_{lepton} can be compared since then they both equal $(|g_L|^2 - |g_R|^2)/(|g_L|^2 + |g_R|^2)$. By using I_4 to obtain ξ_{ρ} from $\{\rho^- \rho^+\}$, the statistical error [6] is $\delta(\xi_{\rho}) = 0.006$ at M_Z. This is a factor of 8 better than the pure leptonic mode's $\delta(\xi_{\text{lepton}}) = 0.05$ error [4] from averaging over the μ and e modes and using $I_3(E_1, E_2, \cos\psi_{12})$ where ψ_{12} is the opening angle between the two final charged leptons in the c.m. frame. A complete determination of the purely leptonic parameters for $\tau^- \to \mu^- \bar{\nu}_{\mu} \nu_{\tau}$ will require a difficult measurement of the μ polarization; see Fetscher [4].