B_c meson lifetime

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We investigate the total inclusive decay rate of the (ground state) B_c meson within the framework of an operator product expansion in inverse powers of the heavy quark masses and subsequent matching onto nonrelativistic QCD. The expansion is organized as a series in the strong coupling and in powers of the heavy quark velocities in the B_c, reflecting the nonrelativistic nature of a heavy-heavy bound state. In this aspect the character of the expansion differs from the more familiar case of heavy-light mesons. The framework incorporates systematically corrections to the leading b- and c-quark decays due to binding effects, as well as contributions from weak annihilation and Pauli interference. Based on this approach we find for the B_c meson lifetime $\tau_{B_c} = (0.4$–$0.7)$ ps, the dominant mechanism being the decay of the charm constituent. [S0556-2821(96)06109-1]

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I. INTRODUCTION

The motivation for studying weak decays of heavy hadrons is essentially twofold. First, one aims at understanding basic properties of the weak interaction at a fundamental level, including the precise determination of Cabibbo-Kobayashi-Maskawa (CKM) parameters. Second, systems containing heavy quarks allow us to test our understanding of QCD in an interesting limiting case where, due to the large mass scale involved, certain aspects of the dynamics simplify. Both topics are of course intimately connected as the analysis of weak decays of heavy hadrons is always faced with the problem of disentangling the interplay of the strong and weak forces. In this respect the B_c meson, the lowest $\bar{b}c$ quark bound state, is a particularly interesting system to study. Unlike in the case of heavy-light mesons, such as, for example, B^+, B_d, or B_s, the bound state dynamics of the B_c can be systematically treated in a nonrelativistic expansion, which has proved very successful for the description of the $c\bar{c}$ and $b\bar{b}$ families. At the same time, and by contrast to the $c\bar{c}$ and $b\bar{b}$ ground states, B_c is stable against strong or electromagnetic decay due to its flavor content and disintegrates only via weak interactions. With these properties, B_c is in fact a unique example, since the top quark lifetime is so short that the analogous $t\bar{b}$ or $t\bar{c}$ mesons do not exist.

Several features of the $\bar{b}c$ system have already been the subject of investigations in the past. A comprehensive analysis of the $\bar{b}c$ spectroscopy and the strong and electromagnetic decays of the excited states has been given in [1,2]. Weak decay properties of the ground state B_c, semileptonic, and various exclusive modes have also been discussed [3–5]. The lifetime of B_c, τ_{B_c} is briefly considered in [4,5] where a rough estimate has been presented. An estimate of τ_{B_c} using a modified spectator model and information gained from the calculation of dominant exclusive modes is described in [6]. The wide range of lifetimes, $\tau_{B_c} = (0.4 - 1.2)$ ps, reported in

these papers, reflects the various model assumptions on the modification of the free quark decay rates due to bound state effects.

In the present article we discuss a systematic approach to computing the B_c lifetime, which is based on the optical theorem for the inclusive decay rate, an operator product expansion of the transition operator and a subsequent nonrelativistic expansion of the operator matrix elements in the B_c meson state. In the first step, the operator product expansion of the transition operator, we rely on the fact that if both quarks can be considered as heavy (and their mass difference is also large), the energy release in the weak decay of either quark is large compared to the characteristic scale for the bound state dynamics. One may then expand in the ratio of these scales. Technically, this step copies the procedure for inclusive decays of heavy-light mesons, reviewed in [7]. In the second step, the expansion of matrix elements, we utilize the fact that the bound state of two heavy quarks is nonrelativistic in first approximation. This approximation can be systematically improved by means of nonrelativistic QCD [8], which organizes the evaluation of matrix elements in full QCD in an expansion in p/m_b and p/m_c, where $p = m_b v_b = m_c v_c \approx 1$ GeV is the typical quark three-momentum in the B_c meson. The finite set of matrix elements in nonrelativistic QCD, which incorporates all nonperturbative effects, must be determined from lattice calculations or, less rigorously, from potential models. At this point, our treatment differs from an analogous one for heavy-light mesons, whose bound state dynamics is essentially different. In this case the scale relevant to the bound state is Λ_{QCD}, while in a heavy-heavy system an additional scale, $p > \Lambda_{\text{QCD}}$ (for the case at hand), is dynamically generated. Consequently, the importance of different operators is not ordered according to their dimension alone, but follows from the "velocity scaling rules" derived in [9], when adapted to the case of two heavy quarks of different masses.

The procedure outlined above, when combined with the inevitable emission of hard gluons in the decay, results in a

double expansion in α_s and $v_Q \equiv p/m_Q$, with both parameters small, if both quarks are sufficiently heavy. Although this feature is conceptually very attractive, it is not *a priori* clear whether in the realistic case the parameters will be small enough to guarantee a reasonable behavior of the expansion. First, in the $c \to su\bar{d}$ transition, the ratio of the typical quark momentum in the bound state and the energy release is not a very small number. Second, although the reduced mass for the $\bar{b}c$ system falls in between the reduced masses of $\bar{b}b$ and $\bar{c}c$, the c quark velocity in the B_c meson is larger than in the J/ψ, because the c quark has to balance the momentum of a heavier b quark. Thus, relativistic corrections are expected to exceed those in the $\bar{c}c$ system. While it might appear that the B_c meson is a rather marginal case for the operator product expansion, its convergence properties can only be properly assessed after an explicit numerical investigation. As we shall see, there are no obvious indications that the nonrelativistic expansion does not work.

We estimate a short B_c lifetime (in comparison to earlier estimates),

$$\tau_{B_c} = (0.4 - 0.7) \text{ ps}, \tag{1}$$

the main uncertainty arising from the poorly known charm quark mass. B_c mesons are expected to be produced at the CERN Large Hadron Collider (LHC), if not before, in numbers that are sufficient to test this prediction. Recently, the first B_c meson candidates have already been reported from the CERN e^+e^- collider (LEP) [10]. The search for B_c at the Fermilab Tevatron is summarized in [11].

II. THE OPERATOR PRODUCT EXPANSION

The optical theorem relates the total decay width of a particle to the imaginary part of its forward scattering amplitude. Applied to the B_c meson total width Γ_{B_c}, this relationship can be written as

$$\Gamma_{B_c} = \frac{1}{2M_{B_c}} \langle B_c | \mathcal{T} | B_c \rangle, \tag{2}$$

where the transition operator $\mathcal{T}$ is defined by

$$\mathcal{T} = \text{Im } i \int d^4x \, T \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0). \tag{3}$$

Here, $\mathcal{H}_{\text{eff}}$ is the usual effective Hamiltonian describing the low energy weak interactions of b and c quarks. The B_c state in (2) is to be taken in conventional (relativistic) continuum normalization, $\langle B_c | B_c \rangle = 2EV$.

In the case of heavy quark decay, where the energy release is large, one may perform an operator product expansion (OPE) of $\mathcal{T}$. In this way the expression in (3) is expanded in a series of local operators of increasing dimension, whose contributions to Γ are suppressed by increasing inverse powers of the heavy quark masses. This formalism has already been applied to calculate the total rates for charm and bottom hadrons containing one single heavy quark [7]. For earlier work using similar methods see also [12,13]. Here, we shall extend this approach to the treatment of B_c, where both

constituents can be considered as heavy. The operator product expansion leads to the generic result

$$\mathcal{T} = \sum_n C_n(\mu) \mathcal{O}_n(\mu). \tag{4}$$

We now describe the operators that contribute to corrections up to order v^4 to the free quark decay, where v is the quark velocity in the B_c meson. For simplicity of discussion, we will for a moment not distinguish the b- and c-quark velocities and masses (in the following Q can be either b or c). The velocity scaling rules themselves will be discussed in the subsequent section. The dominant contribution to the decay width is generated by the operators $\mathcal{O}_{3Q} = \bar{Q}Q$. To leading order in the velocity expansion of its matrix element, this operator reproduces the free quark decay width of the quark Q. Using the equation of motion for the heavy quark fields to eliminate redundant operators, no operator of dimension four remains. There is one operator of dimension five $\mathcal{O}_{GQ} = \bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q$, whose contribution is suppressed by v^4 relative to the leading contribution. The most important correction comes from the dimension six operators $\mathcal{O}_{4Q} = \bar{b}\Gamma c \bar{c}\Gamma' b$, which scale as v^3 (Γ, Γ' collectively denote the Dirac and color structure). Finally, the dimension six operators $\mathcal{O}_{61Q} = \bar{Q}\sigma_{\mu\nu}\gamma_\rho D^\mu G^{\nu\rho}Q$ and $\mathcal{O}_{62Q} = \bar{Q}D_\mu G^{\mu\nu}\Gamma_\nu Q$ contribute also at order v^4. In the following we do not compute the coefficient functions of the latter two operators, so that the expansion will be complete to order v^3. We include the operators $\mathcal{O}_{GQ}$ and take their contributions as indicative of the error due to neglect of other contributions of order v^4.

Performing the OPE results in the expression

$$\mathcal{T} = \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{6,\text{PI}} + \mathcal{T}_{6,\text{WA}}, \tag{5}$$

$$\mathcal{T}_{6,\text{WA}} = \mathcal{T}_{cs} + \mathcal{T}_{ud} + \sum_l \mathcal{T}_{vl}, \tag{6}$$

where the first two terms in (5) account for the operators $\mathcal{O}_{3Q}$ and $\mathcal{O}_{GQ}$ and the other two for the four-fermion operators $\mathcal{O}_{4Q}$. Explicitly,

$$\mathcal{T}_{35b} = \Gamma_{b,\text{spec}} \bar{b}b - \frac{\Gamma_{0b}}{m_b^2}[2P_{c1} + P_{c\tau 1} + K_{0b}(P_{c1} + P_{cc1})$$
$$+ K_{2b}(P_{c2} + P_{cc2})]\mathcal{O}_{Gb}, \tag{7}$$

$$\mathcal{T}_{35c} = \Gamma_{c,\text{spec}} \bar{c}c - \frac{\Gamma_{0c}}{m_c^2}[(2 + K_{0c})P_{s1} + K_{2c}P_{s2}]\mathcal{O}_{Gc}, \tag{8}$$

where

$$\Gamma_{0b} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2, \quad \Gamma_{0c} = \frac{G_F^2 m_c^5}{192\pi^3}, \tag{9}$$

and

$$K_{0Q} = c_-^2 + 2c_+^2, \quad K_{2Q} = 2(c_+^2 - c_-^2). \tag{10}$$

The phase space factors P_i are given as [7,14]

$$P_{c1} = (1-y)^4, \quad P_{c2} = (1-y)^3, \tag{11}$$

$$P_{c\tau1} = \sqrt{1-2(r+y)+(r-y)^2}[1-3(r+y)+3(r^2+y^2)-r^3$$
$$-y^3-4ry+7ry(r+y)]$$
$$+ 12r^2y^2\ln\frac{[1-r-y+\sqrt{1-2(r+y)+(r-y)^2}]^2}{4ry},$$

$$\tag{12}$$

$$P_{cc1} = \sqrt{1-4y}(1-6y+2y^2+12y^3)+24y^4\ln\frac{1+\sqrt{1-4y}}{1-\sqrt{1-4y}}, \tag{13}$$

$$P_{cc2} = \sqrt{1-4y}\left(1+\frac{y}{2}+3y^2\right)-3y(1-2y^2)\ln\frac{1+\sqrt{1-4y}}{1-\sqrt{1-4y}}, \tag{14}$$

where $y = m_c^2/m_b^2$ and $r = m_\tau^2/m_b^2$. P_{s1} (P_{s2}) is identical to P_{c1} (P_{c2}), except that in this case $y = m_s^2/m_c^2$. Note also that $P_{c\tau1} = P_{c1}$ for $r=0$ and $P_{c\tau1} = P_{cc1}$ for $r=y$. In decays of the b quark, we neglect terms of order m_s^2/m_b^2 and set $m_s = 0$. Furthermore,

$$\mathcal{T}_{6,\mathrm{PI}} = \frac{G_F^2}{4\pi}|V_{cb}|^2 p_-^2 (1-z_-)^2[(c_+^2+c_-^2)(\bar{b}_i b_i)_{V-A}(\bar{c}_j c_j)_{V-A}$$
$$+ (c_+^2-c_-^2)(\bar{b}_i b_j)_{V-A}(\bar{c}_j c_i)_{V-A}], \tag{15}$$

$$\mathcal{T}_{cs} = -\frac{G_F^2}{4\pi}|V_{cb}|^2 p_+^2 \left[\frac{(1-z_+)^3}{12}g_{\alpha\beta}+\left(\frac{(1-z_+)^2}{2}\right.\right.$$
$$\left.\left. -\frac{(1-z_+)^3}{3}\right)\frac{p_{+\alpha}p_{+\beta}}{p_+^2}\right][(c_+-c_-)^2$$
$$\times (\bar{b}_i b_i)_{V-A}^\alpha(\bar{c}_j c_j)_{V-A}^\beta + (5c_+^2+c_-^2+6c_+c_-)$$
$$\times (\bar{b}_i b_j)_{V-A}^\alpha(\bar{c}_j c_i)_{V-A}^\beta], \tag{16}$$

$$\mathcal{T}_{\nu\tau} = -\frac{G_F^2}{\pi}|V_{cb}|^2 p_+^2 \left[\frac{(1-z_\tau)^3}{12}g_{\alpha\beta}+\left(\frac{(1-z_\tau)^2}{2}\right.\right.$$
$$\left.\left. -\frac{(1-z_\tau)^3}{3}\right)\frac{p_{+\alpha}p_{+\beta}}{p_+^2}\right](\bar{b}_i b_j)_{V-A}^\alpha(\bar{c}_j c_i)_{V-A}^\beta, \tag{17}$$

$$\mathcal{T}_{ud} = \mathcal{T}_{cs}(z_+ \to 0), \quad \mathcal{T}_{\nu e} = \mathcal{T}_{\nu\mu} = \mathcal{T}_{\nu\tau}(z_\tau \to 0), \tag{18}$$

$$p_\pm = p_b \pm p_c, \quad z_\pm = \frac{m_c^2}{p_\pm^2}, \quad z_\tau = \frac{m_\tau^2}{p_+^2}. \tag{19}$$

In the above equations, the QCD correction factors have been written generically:

$$c_+ = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{6/(33-2f)}, \quad c_- = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{-12/(33-2f)}, \tag{20}$$

where f is the number of flavors. The scale μ is approximately m_b in (7) and m_c in (8), respectively. For (15) and
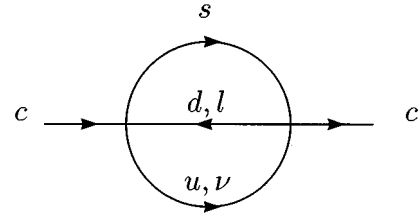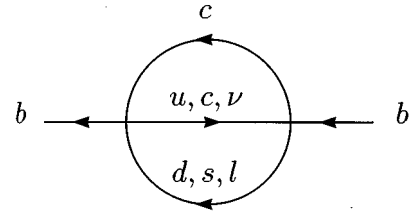

FIG. 1. Leading contributions in α_s to the spectator decays.

(16), one might anticipate a scale $\mu \approx 2m_b m_c/(m_b+m_c)$, with twice the reduced mass representing the characteristic scale of the $\bar{b}c$ bound state. Of course, the scales are not fixed precisely and one has to allow for a certain variation in μ, indicating an uncertainty that is due to neglected higher order QCD effects. A clarification of this issue requires the consideration of next-to-leading corrections.

The contributions of the leading operators $\bar{b}b$ and $\bar{c}c$ correspond to the imaginary part of the diagrams in Fig. 1, which are contained in expression (3). The coefficients of $\bar{b}b$ and $\bar{c}c$ in (7), (8) can be obtained in the usual way by matching the diagrams of Fig. 1, corresponding to the leading terms of the full expression (3), onto the operators $\bar{b}b$, $\bar{c}c$. These coefficients are equivalent to the free quark decay rate and are known in next-to-leading logarithmic approximation in QCD [15–19], including the charm quark mass effects to $O(\alpha_s)$ [19]. To include the next-to-leading log effects, the Wilson coefficients in the effective weak Hamiltonians are required at next-to-leading order and single gluon exchange corrections to the diagrams in Fig. 1 need to be considered. The complete next-to-leading order corrections are incorporated in the numerical analysis below and denoted by $\Gamma_{Q,\mathrm{spec}}$ in (7) and (8). Similarly, the contributions with $\mathcal{O}_{GQ}$ are obtained when an external gluon line is attached in all possible ways to the inner quark lines in Fig. 1. The corresponding coefficients are known in leading log approximation. Finally, the dimension six operators and their coefficients arise from all those contributions, where one of the internal charm lines is "cut" in the diagrams of Fig. 1. The resulting graphs are depicted in Fig. 2. These contributions are also known in the literature as "weak annihilation" [Fig. 2(a)] and "Pauli interference" [Fig. 2(b) — note the orientation of the fermion lines].

The expressions (7), (8) have been derived in [20] (see also [21]) and are also discussed in [7]. The coefficients for the dimension six operators given in (15)–(17) are new.
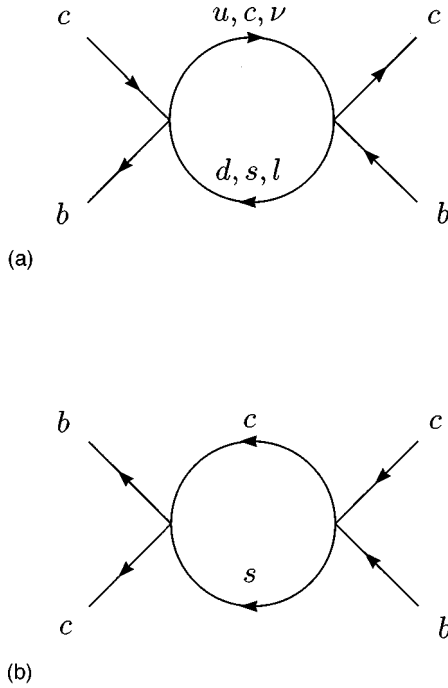
FIG. 2. ''Weak annihilation'' (a) and ''Pauli interference'' (b) contributions to the coefficient functions of four-fermion operators in the OPE.

They are valid to leading logarithmic accuracy in QCD and include charm quark and τ lepton mass effects.

In order to obtain the total decay width, the expansion of the transition operator (5) has to be taken between the B_c meson states. This step involves the calculation of matrix elements of the local operators listed above. In general, this is a difficult nonperturbative problem which in practice may introduce considerable uncertainties in the evaluation of the hadron lifetime. This is the case, for instance, for heavy-light b- or c-flavored mesons [7,22]. By contrast, an important advantage of the present system consisting of *two* heavy quarks is the applicability of a nonrelativistic treatment. Given the successes of this approach in describing $\bar{c}c$ and $\bar{b}b$ quarkonia, one can expect a rather reliable determination of the required B_c matrix elements using the nonrelativistic formalism, either within the framework of potential models or by employing lattice QCD. As we shall see, this treatment transforms the $1/m_Q$ expansion into an expansion in powers of the heavy quark velocities.

Before discussing the evaluation of matrix elements and the numerical results for the B_c meson lifetime, we address briefly the physical interpretation of the various terms in (7)–(15). First, let us consider the strictly asymptotic limit in which $m_Q/\Lambda_{QCD}\to\infty$. In this limit the $\bar{b}c$ system becomes extremely nonrelativistic, consisting of two weakly bound heavy quarks, slowly moving in a Coulomb-type potential. The total decay rate for that system is then simply given by the sum $\Gamma_{b,\mathrm{spec}}+\Gamma_{c,\mathrm{spec}}$ of the weak decay rates of the quasifreely and independently decaying heavy quarks. In the asymptotic limit we have $\langle B_c|\bar{Q}Q|B_c\rangle/(2M_{B_c})=1+O(v_Q^2)$, $Q=b,c$, and we indeed recover this simple picture within the formalism described above. Of course, the truly asymptotic case is unrealistic. However, the simple spectator decay

contribution is seen to emerge as the formally leading term in the operator product expansion. Bound state corrections are described in (5) by the operators $\mathscr{O}_{GQ}$ (and others of higher dimension which we omitted) and corrections to the asymptotic value of the matrix element of $\bar{Q}Q$. Since the contribution due to the chromomagnetic operator $\mathscr{O}_{GQ}$ can be related to the B_c-B_c^* mass splitting, it is actually of order v^4 in the nonrelativistic expansion and thus formally of subleading importance. The only correction of order v^2 arises from the kinetic energy of the quarks inside the bound state, which leads to a reduction of the matrix element of $\bar{Q}Q$ by a factor $1-v_Q^2/2$, obviously representing the effect of time dilatation.

The question of bound-state corrections to the decay rate of a system of two heavy fermions has also been studied, from a different point of view (using a Bethe-Salpeter formalism) in [23] (see also [24]). It has been shown in this work that the leading net correction to the rate due to bound-state effects is just from time dilatation, $-v^2/2$. Other conceivable contributions, such as phase space suppression and Coulomb enhancement, turn out to effectively cancel in the final answer. As we have seen, this picture is in accordance with the findings obtained here within the OPE approach, which incorporates this result in a rather simple and straightforward manner.

III. MATRIX ELEMENTS

Let us next complete and summarize the discussion of the required B_c matrix elements, part of which we have already encountered above. The operators described in the previous section are still expressed in terms of four-component fields Q. In a system containing a nonrelativistic quark, antiquarks cannot be produced, since this would require an energy larger than m_Q. It is then appropriate to ''integrate out'' the small components of the field and express the result in terms of a two-spinor ψ_Q. In this way, all contributions from scales larger than μ, where $m_Q>\mu>m_Qv_Q$, are made explicit and can be accounted for perturbatively. This procedure is standard from nonrelativistic approximations to QED and heavy quark effective theory (HQET) and leads to

$$\bar{Q}Q = \psi_Q^\dagger\psi_Q - \frac{1}{2m_Q^2}\psi_Q^\dagger(i\vec{D})^2\psi_Q + \frac{3}{8m_Q^4}\psi_Q^\dagger(i\vec{D})^4\psi_Q$$
$$- \frac{1}{2m_Q^2}\psi_Q^\dagger g\vec{\sigma}\cdot\vec{B}\psi_Q - \frac{1}{4m_Q^3}\psi_Q^\dagger(\vec{D}\cdot g\vec{E})\psi_Q + \cdots,$$

$$(21)$$

$$\bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q = -2\psi_Q^\dagger g\vec{\sigma}\cdot\vec{B}\psi_Q - \frac{1}{m_Q}\psi_Q^\dagger(\vec{D}\cdot g\vec{E})\psi_Q + \cdots,$$

$$(22)$$

where we have already omitted the term $\psi_Q^\dagger\vec{\sigma}\cdot(g\vec{E}\times\vec{D})\psi_Q$ (spin-orbit coupling), whose matrix element vanishes in the valence Fock state of a pseudoscalar meson. The two-spinor ψ_Q is here defined to have the same normalization as Q:

$$\int d^3x \; \psi_Q^\dagger \psi_Q = \int d^3x \; Q^\dagger Q. \qquad (23)$$

To the required order, ψ_Q is then related to the upper components ϕ of Q,

$$Q \equiv e^{-imt} \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \qquad (24)$$

through

$$\psi_Q = \left(1 + \frac{(i\vec{D})^2}{8m_Q^2} \right) \phi, \qquad (25)$$

which can be verified by using the equations of motion. Note also that the covariant derivative is understood to be in the adjoint representation when acting on the chromoelectric field:

$$(\vec{D} \cdot \vec{E}) = (\vec{\partial} T^a - gf^{abc} T^b \vec{A}^c) \vec{E}^a. \qquad (26)$$

Equations (21) and (22) are valid up to terms of $O(v^6)$. In the numerical analysis below we shall also neglect the small term of $O(\vec{p}^4/m_Q^4)$, which yields the $O(v^4)$ correction to time dilatation

$$1 - \frac{1}{2} \frac{\vec{p}^2}{m_Q^2} + \frac{3}{8} \frac{\vec{p}^4}{m_Q^4} + \cdots = 1 - \frac{1}{2} v_Q^2 - \frac{1}{8} v_Q^4 + \cdots = \sqrt{1 - v_Q^2}. \qquad (27)$$

Radiative corrections modify the coefficients of the chromomagnetic ($\vec{\sigma} \cdot \vec{B}$) and the "Darwin" ($\vec{D} \cdot \vec{E}$) term in (21). In the present context, these effects can be neglected consistently.

The matrix elements of the operators on the right-hand sides of (21) and (22) can be evaluated in the nonrelativistic effective theory [8], for example on the lattice, the obvious advantage being the possibility of using coarse lattices, since short-distance effects are already accounted for. To obtain a desired accuracy in v^2, the appropriate number of corrections to the leading order effective Lagrangian have to be retained. Preliminary studies of B_c mesons on the lattice have recently appeared [25], but are not yet competitive with phenomenological potential models.

To assess the importance of various contributions, we recall [9] that the quark field ψ_Q scales with the heavy quark three-velocity as $(m_Q v_Q)^{3/2}$, a spatial derivative as $m_Q v_Q$, the electric field gE as $m_Q^2 v_Q^3$, and the magnetic field gB as $m_Q^2 v_Q^4$ (in Coulomb gauge). The coupling g in a matrix element counts as $v_Q^{1/2}$. In the present case of a system with unequal quark masses, one has to keep in mind that additional factors from the mass ratio can enhance or suppress a given contribution. These scaling rules imply that the last term kept in (21) and (22) is of the same order as the chromomagnetic term $\vec{\sigma} \cdot \vec{B}$, in contrast with the analysis of heavy-light mesons where this term is suppressed by $\Lambda_{\mathrm{QCD}}/m_Q$ relative to the chromomagnetic interaction. The velocity counting relies on the inequality $p \sim 1$ GeV $> \Lambda_{\mathrm{QCD}}$, where p is the typical quark momentum in the bound state.

Let us now turn to the evaluation of matrix elements in potential models for the bound state. Denote by $T = m_c v_c^2/2 + m_b v_b^2/2$ the average total kinetic energy of the quarks in the bound state. Then, since $m_b v_b = m_c v_c$, we obtain[1]

$$\frac{\langle B_c | \psi_b^\dagger (i\vec{D})^2 \psi_b | B_c \rangle}{2M_{B_c} \cdot m_b^2} \simeq v_b^2 \simeq \frac{2m_c}{m_b(m_c + m_b)} T, \qquad (28)$$

$$\frac{\langle B_c | \psi_c^\dagger (i\vec{D})^2 \psi_c | B_c \rangle}{2M_{B_c} \cdot m_c^2} \simeq v_c^2 \simeq \frac{2m_b}{m_c(m_c + m_b)} T. \qquad (29)$$

The kinetic energy T is known rather precisely, within about 10%, $T = 0.37$ GeV [2]. It is also approximately independent of the reduced mass of the bound state (for the "logarithmic potential" this statement is exact). This value of T implies $v_c^2 \approx 0.38$ and $v_b^2 \approx 0.035$. A straightforward calculation yields the matrix elements

$$\frac{\langle B_c | \psi_c^\dagger g\vec{\sigma} \cdot \vec{B} \psi_c | B_c \rangle}{2M_{B_c}} = -\frac{4}{3} g^2 \frac{|\Psi(0)|^2}{m_b}, \qquad (30)$$

$$\frac{\langle B_c | \psi_c^\dagger (\vec{D} \cdot g\vec{E}) \psi_c | B_c \rangle}{2M_{B_c}} = \frac{4}{3} g^2 |\Psi(0)|^2, \qquad (31)$$

where $\Psi(0)$ denotes the wave function at the origin. The matrix elements for b-quark fields are obtained by interchanging m_b and m_c on the right-hand side. The second equation can also be obtained from using the equation of motion for the chromoelectric field. The matrix element of the resulting four-fermion operator factorizes to leading order in v^2 and can be evaluated in a straightforward way.

We note that in the potential model $\Psi(0)$, the decay constant f_{B_c} and the vector-pseudoscalar spin splitting are related by

$$f_{B_c}^2 = \frac{12|\Psi(0)|^2}{M_{B_c}}, \qquad M_{B_c^*} - M_{B_c} = \frac{8}{9} g^2 \frac{|\Psi(0)|^2}{m_b m_c}. \qquad (32)$$

For the numerical analysis to follow, we take $M_{B_c^*} - M_{B_c} = 73$ MeV from [1]. With the parameters of the Buchmüller-Tye potential, this spin splitting implies $f_{B_c} = 500$ MeV, which we use as our central value for the decay constant.

Combining these results, we estimate the matrix elements (21) and (22). Notice that the matrix element of $\bar{c} g \sigma_{\mu\nu} G^{\mu\nu} c$ is dominated by the divergence of the chromoelectric field rather than by the spin-spin interaction, because the latter is suppressed by m_c/m_b. It is interesting to detail the deviations of the matrix element of $\bar{Q}Q$ from unity for the c quark, where relativistic corrections are the largest. We find

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[1] To avoid additional notation, the matrix elements are written for b quarks rather than for antiquarks.

$$\frac{\langle B_c|\bar{c}c|B_c\rangle}{2M_{B_c}} = 1 - \frac{1}{2}v_c^2 + \frac{3}{4}\frac{M_{B_c^*} - M_{B_c}}{m_c}\left(1 - \frac{m_b}{2m_c}\right) + \cdots$$

$$\approx 1 - 0.190 + 0.037 - 0.061 + \cdots. \quad (33)$$

Despite the large c-quark velocity, this expansion appears quite well behaved. As anticipated, the largest reduction of the rate is due to time dilatation, representing the dominant bound-state effect. Furthermore, we have

$$\frac{\langle B_c|\bar{c}g\,\sigma_{\mu\nu}G^{\mu\nu}c|B_c\rangle}{2M_{B_c}} = 3m_c(M_{B_c^*} - M_{B_c})\left(1 - \frac{m_b}{2m_c}\right). \quad (34)$$

The matrix elements of $\bar{b}b$ and $\bar{b}g\,\sigma_{\mu\nu}G^{\mu\nu}b$ can be obtained by interchanging $m_b \leftrightarrow m_c$ in the above equations.

Finally, we need the matrix elements of the four-quark operators in (15)–(17). To estimate their values, we employ factorization. This method, which lacks a firm footing for heavy-light mesons, can in fact be justified in the present case of B_c. Deviations from factorization arise from higher Fock components of the B_c wave function and therefore are of higher order in the nonrelativistic expansion. The relevant matrix elements are then given by

$$\langle B_c|(\bar{b}_i b_j)_{V-A}^\alpha(\bar{c}_j c_i)_{V-A}^\beta|B_c\rangle = f_{B_c}^2\left(\frac{1}{2}q^2 g^{\alpha\beta} - q^\alpha q^\beta\right), \quad (35)$$

$$\langle B_c|(\bar{b}_i b_i)_{V-A}^\alpha(\bar{c}_j c_j)_{V-A}^\beta|B_c\rangle = \frac{f_{B_c}^2}{3}\left(\frac{1}{2}q^2 g^{\alpha\beta} - q^\alpha q^\beta\right), \quad (36)$$

where q is the B_c meson four-momentum, $q^2 = M_{B_c}^2 \approx (m_b + m_c)^2$, and f_{B_c} is the B_c meson decay constant (in the normalization in which $f_\pi = 131$ MeV). Deviations from factorization modify the decay rate only at order v^5 relative to the free quark decay. Using (35) and (36), the matrix elements of the dimension six contributions to the transition operator are found to be

$$\frac{\langle B_c|\mathscr{T}_{6,PI}|B_c\rangle}{2M_{B_c}} = \frac{G_F^2}{12\pi}|V_{cb}|^2 f_{B_c}^2 M_{B_c} p_-^2(1 - z_-)^2[2c_+^2 - c_-^2], \quad (37)$$

$$\frac{\langle B_c|\mathscr{T}_{cs}|B_c\rangle}{2M_{B_c}} = \frac{G_F^2}{24\pi}|V_{cb}|^2 f_{B_c}^2 M_{B_c} m_c^2(1 - z_+)^2[4c_+^2 + c_-^2 + 4c_+ c_-], \quad (38)$$

$$\frac{\langle B_c|\mathscr{T}_{\nu\tau}|B_c\rangle}{2M_{B_c}} = \frac{G_F^2}{8\pi}|V_{cb}|^2 f_{B_c}^2 M_{B_c} m_\tau^2(1 - z_\tau)^2. \quad (39)$$

The matrix elements of $\mathscr{T}_{ud}$, $\mathscr{T}_{\nu e}$, and $\mathscr{T}_{\nu\mu}$ are negligibly small as a consequence of helicity suppression.

In the following section, we evaluate the QCD correction factors $c_{+/-}$ in the above expressions at a scale $\mu = 2m_{red} \approx 2.3$ GeV, that is characteristic for a transition involving both bound-state quarks. If this scale were widely separated from m_b, one would switch to the effective theory

at a scale $\mu \approx m_b$, and scale the operators with their anomalous dimension to the lower scale. Since $2m_{red} \approx m_b/2$, we exercise our freedom to choose the matching scale within some variation of m_b and evaluate the QCD correction factors directly at $2m_{red}$.

IV. DISCUSSION

We are now ready to collect the various contributions presented above and to derive an estimate for the B_c meson lifetime. We first summarize the input parameters that we will use in our numerical analysis:

$$m_b = 5.0 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_s = 0.2 \text{ GeV},$$

$$|V_{cb}| = 0.04, \quad (40)$$

$$M_{B_c} = 6.26 \text{ GeV}, \quad M_{B_c^*} - M_{B_c} = 0.073 \text{ GeV},$$

$$T = 0.37 \text{ GeV}, \quad f_{B_c} = 0.5 \text{ GeV}. \quad (41)$$

The parameters M_{B_c}, $M_{B_c^*} - M_{B_c}$, and f_{B_c} are taken from [1] and T is taken from [2]. The numbers for m_b and m_c correspond to pole masses, the figures in (40) representing our central values. We comment on their choice below. The renormalization scales μ are chosen as follows: $\mu_1 = m_b$ in decays of the b constituent, $\mu_2 = m_c$ in decays of the c constituent, and $\mu_3 = 2m_{red}$ in modes involving both bound-state quarks.

As already mentioned, the pure spectator decay rates $\Gamma_{b,\text{spec}}$ and $\Gamma_{c,\text{spec}}$ are known to next-to-leading order in perturbative QCD [15–19]. The most complete calculation, including final state mass effects in the QCD corrections, is due to [19]. We use their results to evaluate the spectator quark decay contributions. Thereby, we take proper account of charm quark and tau lepton mass effects. The strange quark mass is kept for the charm decay modes, but neglected in b decays. The impact of electron and muon masses is likewise small and has been neglected throughout.

First, we would like to present an overview of the different B_c decay mechanisms and their relative importance as obtained within the nonrelativistic OPE-based framework we are advocating. To this end we fix all input parameters at their central values and calculate the partial, semi-inclusive B_c decay modes corresponding to the various underlying quark subprocesses. The results are collected in Table I. We observe a dominance of the charm decay modes over b-quark decay. Weak annihilation is sizable, yet still considerably smaller than b decay. Adding up all contributions yields a total width of $\Gamma_{B_c} = 1.914$ ps^{-1}, implying

$$\tau_{B_c} = 0.52 \text{ ps.} \quad (42)$$

Various branching fractions can also be inferred from Table I. For instance, the semileptonic branching ratio $B(B_c \to Xe\nu)$ is found to be about 12%.

These estimates involve considerable uncertainties and Table I is merely intended to convey the general trend and typical numbers for our favorite parameter set. The dominant uncertainty comes from the quark masses. To limit the related ambiguity in our phenomenological analysis, we em-

TABLE I. Contributions to the B_c meson decay rate in ps^{-1}. The partial $\bar{b}$- and c-quark decay rates are understood to include the nonperturbative corrections from dimension five operators as discussed in the text. Cabibbo-suppressed modes are implicitly taken into account with the corresponding allowed channels. The input parameters are specified at the beginning of Sec. IV. WA=weak annihilation, PI=Pauli interference.

Mode	Partial rate (ps^{-1})
$\bar{b} \to \bar{c}u\bar{d}$	0.310
$\bar{b} \to \bar{c}c\bar{s}$	0.137
$\bar{b} \to \bar{c}e\nu$	0.075
$\bar{b} \to \bar{c}\tau\nu$	0.018
$\Sigma\bar{b} \to \bar{c}$	0.615
$c \to su\bar{d}$	0.905
$c \to se\nu$	0.162
$\Sigma c \to s$	1.229
WA:$\bar{b}c \to c\bar{s}$	0.138
WA:$\bar{b}c \to \tau\nu$	0.056
PI	-0.124
Total	1.914

TABLE II. Parameter dependence of τ_{B_c} against (small) variations of the parameter set $\{X_c\}$ specified at the beginning of Sec. IV and repeated in the second column of the table. l_X is defined by $\delta\tau_{B_c}(X_c)/\tau_{B_c}(X_c) \equiv l_X(\delta X/X_c)$, where $\delta\tau_{B_c}(X_c) = \tau_{B_c}(X_c + \delta X) - \tau_{B_c}(X_c)$. Thus, for instance, the entry for $X = m_c$ tells us that increasing m_c from 1.5 GeV to 1.6 GeV decreases $\tau_{B_c}(X_c) = 0.52$ ps by 20.7%.

X	X_c	l_X
m_b	5.0 GeV	-1.96
m_c	1.5 GeV	-3.10
m_s	0.2 GeV	0.14
$|V_{cb}|$	0.04	-0.72
μ_1	5.0 GeV	0.02
μ_2	1.5 GeV	0.13
μ_3	2.3 GeV	-0.07
T	0.37 GeV	0.16
$M_{B_c^*} - M_{B_c}$	0.073 GeV	0.01
f_{B_c}	0.5 GeV	-0.07
M_{B_c}	6.26 GeV	-0.04

ploy the following strategy. We take the charm quark mass m_c as the basic input parameter, allowing it to vary within

$$1.4 \text{ GeV} \leq m_c \leq 1.6 \text{ GeV} \qquad (43)$$

which is in the ball park of the pole mass values available in the literature. For any given value of m_c, we then fix the value of m_b by the requirement that the measured B_d meson lifetime $\tau_{B_d} \approx 1.55$ ps is obtained. This is justified since the total B_d width is essentially determined by the b-quark decay contribution with very small pre-asymptotic bound-state corrections and the OPE formalism can be expected to be reliable in this case. For the b-quark spectator decay rate we again use the complete next-to-leading order expressions [19]. For completeness, we also incorporated the bound-state effects that are described in [7]. Imposing the τ_{B_d} constraint, yields a b-quark mass that effectively includes unknown higher order perturbative corrections in the decay of the bottom quark. It practically eliminates the m_b and V_{cb} dependence of the predicted value of τ_{B_c}. It turns out that a determination of m_b, for any given m_c, via the approach just described, is approximately equivalent to the relation

$$m_b = m_c + 3.5 \text{ GeV}, \qquad (44)$$

which is roughly consistent with the well-known formula relating m_b and m_c in heavy quark effective theory (HQET). Adopting this procedure and varying m_c between 1.4 and 1.6 GeV, results in the prediction

$$0.4 \text{ ps} \leq \tau_{B_c} \leq 0.7 \text{ ps}. \qquad (45)$$

Variations of our central value (42) due to variations of other input parameters can be inferred from Table II. This table

could also be used if, by the time τ_{B_c} is measured, a different parameter set appears preferred.

In addition to using the measured B_d lifetime, one could be tempted to use the D^0 lifetime to eliminate the strong dependence on m_c. Since a very large charm mass is required to reproduce the absolute D^0 lifetime and semileptonic width in the OPE approach, this would lead to the prediction of a very low B_c lifetime $\tau_{B_c} \approx 0.35$ ps. We refrain from this procedure, since the OPE applied to charmed mesons is less reliable than that for B_c and probably more qualitative than quantitative [22].

Let us conclude the discussion with the following remarks.

(a) As already discussed above, the expression for the B_c meson total width we have derived is consistent up to and including terms of $O(v^3)$ in the nonrelativistic expansion. Part of the $O(v^4)$ corrections, those due to the operators $\bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q$, have also been considered to obtain an estimate of the order of magnitude of these contributions, while the neglected contributions arise from the operators $\mathcal{O}_{61Q}$ and $\mathcal{O}_{62Q}$ listed earlier. The labor involved in calculating these remaining contributions does not appear justified in view of the uncertainties connected with the quark masses. To reinforce this point and to see the behavior of the expansion more explicitly, we write down the size of the various terms of a given order in v, using central parameter values. For the sum of the $c \to s$ decay modes, one obtains (1.526–0.289–0.008) ps^{-1}, writing separately the terms of order v^0, v^2, and v^4. For the total b-decay contribution, one finds (0.640–0.011–0.014) ps^{-1}. The sum of all v^3 effects amounts to typically 0.1 ps^{-1}. In the case of b decay, the corrections are particularly small, since they are additionally suppressed by inverse powers of the large b-quark mass. Generally speaking, the relevant velocity is the one of the lighter constituent, v_c, which determines the convergence properties of the nonrelativistic expansion. As we see, the series is rather well behaved. Note also that a further contribution of $O(v^4)$ comes from the expansion of the time dila-

tation factor. Numerically, $-v_c^4/8 \approx -2\%$. Although one has to be careful about drawing definitive conclusions, since the $O(v^4)$ contribution has not yet been calculated completely, these observations support the assumption that the total $O(v^4)$ term is rather small.

Let us re-emphasize that, technically, in the approach adopted here the velocity expansion distinguishes a heavy-heavy meson from a heavy-light meson, for which an expansion in Λ_{QCD}/m_Q is appropriate. This distinction was missed in the treatment of [26] (which also starts with an OPE), resulting in an incorrect evaluation of the matrix elements of $\bar{Q}g\sigma \cdot GQ$. As a consequence, a large correction was obtained from this formally sub-leading $[O(v^4)]$ contribution, which we found to be essentially negligible, even for charm. The leading correction terms to the spectator picture of $O(v^2)$ from the kinetic energy and of $O(v^3)$, from weak annihilation and Pauli interference, were not calculated explicitly in [26].

(b) The Pauli interference contribution exhibits a fairly substantial dependence on the renormalization scale. The number shown in Table I corresponds to $\mu=2m_{red} \approx 2.3$ GeV. Allowing a variation of μ from 1 to 5 GeV yields a range of values from -0.342 ps^{-1} to -0.036 ps^{-1} for this contribution. This large dependence is formally of $O(\alpha_s)$, which goes beyond the leading log approximation we are working in presently. It represents a theoretical uncertainty in this calculation. The pronounced sensitivity to the scale arises from sizable cancellations that occur between the Wilson coefficients in (37). Note that the Pauli interference contribution is *positive* in the limit $\alpha_s \to 0$ and changes sign because of the presence of important short-distance QCD effects. The situation could be improved by studying next-to-leading order QCD corrections to Pauli interference, including a proper matching of the operators to nonrelativistic QCD. By contrast, the scale dependence turns out to be very moderate in the case of weak annihilation of $\bar{b}c$ into $c\bar{s}$, only $\pm 10\%$ for the same range of μ as before.

(c) For the channels $\bar{b} \to \bar{c}c\bar{s}$ and $\bar{b} \to \bar{c}\tau\nu$, the weak annihilation term can be comparable to, or even exceed, the spectator contribution. This feature is related to the strong phase space suppression of the three-body decay mechanism, which is absent for weak annihilation. It would persist even in the limiting case where α_s is taken to be very small and the nonrelativistic bound-state description would be a perfect approximation. The charm mass could be so large that the spectator decay would be kinematically forbidden in which case weak annihilation would just be the leading contribution, but still reliably calculable in the nonrelativistic approach. For these reasons, the dominance of weak annihilation does not indicate a problem for the validity of the operator product expansion.

(d) An important conceptual point that needs to be mentioned is that the application of the OPE is based on the assumption of local quark hadron duality. By this we mean that the sum over all decay channels can be described in terms of partonic degrees of freedom. Although this assumption should be valid asymptotically as $m_Q/\Lambda_{QCD} \to \infty$, it is not *a priori* clear how well it is satisfied in practice. Little is known rigorously and quantitatively about the exact conditions to be met if duality is supposed to hold. Conceivably,

the issue of duality is related to the fact that the velocity expansion is at best asymptotic. This includes the possibility that it might not even be asymptotic at Minkowskian kinematics. Since, for B_c, the expansion is still convergent, we are concerned mainly with the second possibility, in which case the OPE is meaningless beyond a certain order, even if it appears to be convergent. Although duality has been questioned for the decay of D mesons [27], it seems, at least to the authors, that there is as yet no place in the charm and bottom hadron family where this possibility could be unambiguously separated from uncertainties due to unknown input parameters (quark masses, matrix elements) or convergence properties of the expansion. In this situation, one has to rely on a more pragmatic attitude towards the problem, assume that quark-hadron duality makes sense for a given case, explore the consequences and, if possible, check whether the emerging picture is at least consistent.

The critical point with regard to this issue for B_c is charm decay. Here, the energy release is not as comfortably large as it is in the case of bottom decay, and, in fact, slightly smaller than that in D meson decay. However, the reliability of a duality is channel dependent. Duality might still work reasonably well in the case of the charm transitions contributing to B_c decay, even if it were violated for D mesons. After all, as we have emphasized, the nature of the OPE is very different for B_c due to the nonrelativistic character of the heavy-heavy-type bound state.

A useful cross-check comes from comparing the inclusive decay width for a given channel with the sum of the corresponding exclusive decay modes. In the present case, $c \to su\bar{d}$, these are modes like $B_c \to B_s^{(*)}\pi$, $B_s^{(*)}\rho$, $B^{(*)}K^{(*)}, \ldots$. In [4,5], phenomenological models have been used to estimate the rates for twenty of these two-body decay modes. Adding their results, one finds a total of 0.49 ps^{-1} [Bauer-Stech-Wirbel (BSW) model] and 0.65 ps^{-1} [Isgur-Scora-Grinstein-Wise (ISGW) model] in [4] and 0.67 ps^{-1} in [5], to be compared with 0.91 ps^{-1} for the inclusive width from Table I. Viewing this comparison with due caution, regarding the model dependences and other uncertainties in the estimation of exclusive modes as well as the charm mass uncertainty for the inclusive prediction, it is reassuring that, first, the order of magnitude comes out to be consistent and, second, the sum of exclusive modes does not exceed the inclusive result. A similar observation holds for the semileptonic $c \to se\nu$ transitions.

Although the arguments we have given are of a somewhat heuristic nature, they all seem to indicate that the OPE approach makes sense for inclusive B_c decays. At the very least, the underlying assumptions do not seem to be obviously violated.

(e) In [4,6], the B_c lifetime has been estimated on the basis of a modified spectator model, where the phase space for the free quark decay is modified to account for the physically accessible kinematic region [4] or the b- and c-quark masses are reduced by the binding energy to incorporate bound-state effects [6]. Like the OPE formalism these approaches lead to a reduction of the free quark decay rates caused by binding, but the details of the resulting picture are markedly different. The modified spectator ansatz does not correctly approach the asymptotic limit of very heavy quark masses, where, as in the OPE-based calculation, the leading

bound-state corrections come only from time dilatation. Our bound-state corrections are numerically much smaller than the very large effects reported in [6], which invert the hierarchy between c and $\bar{b}$ decays, making the latter more prominent, and lead to a considerably longer lifetime of $\tau_{B_c} = 1.35 \pm 0.15$ ps [6]. It has been emphasized in [6] that the model calculation has to be regarded with caution, since the inclusive charm decay rate is found to be below the rate estimates of the corresponding exclusive modes. Because of this feature, the result for the lifetime quoted above has been corrected to $\tau_{B_c} = (1.1–1.2)$ ps [6]. This estimate is, however, still sizably larger than the one derived in the present analysis, where, as we have seen, the inclusive and the exclusive approach can be viewed as roughly consistent.

V. SUMMARY

In this article we have presented a detailed investigation of the B_c meson total width based on a systematic operator product expansion of the transition operator. In several important aspects this analysis goes beyond the estimates derived previously in the literature: First, we have emphasized the nonrelativistic nature of the heavy-heavy bound-state system B_c. As we have shown, this fact has crucial implications for the organization of the operator product expansion, which is different from the case of heavy-light mesons. In addition, the framework allows a systematic evaluation of the relevant matrix elements of local operators using the nonrelativistic bound-state wave functions. Matrix elements of four-quark operators, for instance, factorize exactly to leading order in the velocity expansion in contrast with the general situation.

Free-quark decay of $\bar{b}$ and c represents, both formally and numerically, the leading (in v) contribution to the B_c decay rate. For their evaluation the complete next-to-leading order expressions in renormalization-group-improved QCD perturbation theory have been employed. The dominant bound-state corrections come from time dilatation and are determined by the heavy quark velocities, which we have estimated from existing potential model calculations. A further new result is the complete $O(v^3)$ contributions to the decay rate, due to weak annihilation and Pauli interference, for which explicit expressions, valid to leading logarithmic accuracy in QCD, have been derived.

Numerically, our analysis yields $\tau_{B_c} = 0.4$–0.7 ps, where the measured B_d lifetime has been used to reduce the uncertainties due to the bottom quark mass and V_{cb}.

We have also briefly considered the reliability of the operator product expansion, whose applicability relies on the assumption of quark-hadron duality. Although this issue remains an important caveat to be kept in mind, we have found no indication for an obvious violation of its validity. On the contrary, the series itself is rather well behaved and model calculations of exclusive modes are consistent with the duality assumption. It will be interesting to compare the predictions for B_c decay based on local duality with future experimental results. These should help to assess to what extent the underlying theoretical assumptions are valid in practice. Clearly, experimental progress towards a measurement of the lifetime and other inclusive properties of B_c is eagerly awaited.

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