PQCD analysis of inclusive semileptonic decays of B mesons

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(Received 2 October 1995; revised manuscript received 13 December 1995)

We develop the perturbative QCD formalism for inclusive semileptonic B meson decays, which includes Sudakov suppression from the resummation of large radiative corrections near the high end of charged lepton energy. Transverse degrees of freedom of partons are introduced to facilitate the factorization of B meson decays. Ambiguities appearing in the quark-level analysis are then avoided. A universal distribution function, arising from the nonperturbative Fermi motion of the b quark, is constructed according to the heavy quark effective field theory based operator product expansion, through which the mean and the width of the distribution function are related to hadronic matrix elements of local operators. Charged lepton spectra of the $B\rightarrow X_u l\nu$ decay are presented. We find 50% suppression near the end point of the spectrum. The overall suppression on the total decay rate is 8% for the free quark model, and is less than 7% for the use of smooth distribution functions. With our predictions, it is then possible to extract the Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}|$ from experimental data. We also discuss possible implications of our analysis when confronted with the rather small observed semileptonic branching ratio in B meson decays. [S0556-2821(96)03809-X]

PACS number(s): 13.20.He, 12.38.Cy, 13.25.Hw

I. INTRODUCTION

The studies of semileptonic decays in heavy mesons within the framework of perturbative quantum chromodynamics (PQCD) dated back to the 1970s [1]. The mass M_Q of a heavy quark Q provides a rationale of this approach. The advantage of the PQCD formalism is that it provides a natural normalization of decay amplitudes. This point is of great importance to obtain a model-independent extraction of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, which are the key phenomenological parameters in understanding the symmetry-breaking physics of the standard model. We have shown that PQCD is applicable to the exclusive $B\rightarrow\pi l\nu$ and $B\rightarrow\rho l\nu$ decays [2], and an upper limit of $|V_{ub}|$ around 2.7×10^{-3} was extracted from experimental data [3] directly. Recently, there have been controversies concerning the inclusive semileptonic branching ratio of B meson decays [4,5]. Even new physics was proposed [4] to resolve the discrepancy between theoretical predictions and experimental outcomes. It has been argued that PQCD radiative corrections may play an important role in the semileptonic decays $B\rightarrow Xl\nu$ [5]. In view of this, it is essential to first sort out the correct PQCD contributions to these decays.

In the pioneering works of Chay *et al.* [6] and Shifman *et al.* [7] a systematic expansion of relevant hadronic matrix elements in semileptonic decays in the inverse power of M_Q was obtained by combining heavy quark effective field theory (HQEFT) and the method of operator product expansion (OPE). It was shown that the differential decay rate $d\Gamma/dq^2dE_l$ can be consistently calculated, when it is suitably averaged over the charged lepton energy E_l. Here $q^2=(p_l+p_\nu)^2$ is the lepton pair invariant mass with p_l and p_ν the charged lepton and neutrino momenta, respectively. At leading order, the expansion reproduces the naive parton model predictions [8] of free heavy-quark decays. Next-to-leading order corrections, starting at $O(\Lambda_{\rm QCD}^2/M_Q^2)$, are expected to be small in heavy meson decays. This approach has also been extended to the case of nonleptonic decays [9].

To extract V_{ub} from the $B\rightarrow X_u l\nu$ decay, one needs to measure the charged lepton spectrum near the high end of E_l, such that the huge $B\rightarrow X_c l\nu$ background stops contributing. The measurement must be performed within an accuracy of several hundred MeV, because the energy difference between the end points of the $b\rightarrow c$ and $b\rightarrow u$ transitions is only about 330 MeV. Unfortunately, OPE, the theoretical tool, breaks down in this region. Since the expansion parameter $1/M_Q$ should be replaced by $1/(M_Q-q\cdot v)$ when q is not small at the end point, OPE is not reliable. The heavy-quark velocity v is defined by $P_Q=M_Qv$, P_Q being the heavy-quark momentum. To circumvent the difficulty, Neubert [10] and Bigi *et al.* [11] have performed a resummation of OPE, that results in a model-independent ''shape function'' in the description of the charged lepton spectrum. This universal shape function can be determined in principle by an infinite tower of nonperturbative matrix elements expanded in the increasing power of $1/M_Q$, and has been employed in the study of the inclusive rare decay $B\rightarrow X_s\gamma$ [12,13]. One can therefore measure this shape function, say, in the $B\rightarrow X_s\gamma$ decay, and then apply it to the $B\rightarrow X_u l\nu$ decay to obtain a model-independent prediction of V_{ub}. However, under the present experimental situation, one has to make an ansatz for this shape function according to some QCD constraints. Data of the decay $B\rightarrow X_s\gamma$ will definitely remove this ambiguity in the choice of shape functions in the future.

On the parton model side, Bareiss and Paschos [14] argued that the phase space for the decays is dominated by distances near the light cone, and one can use the distribution function of the Q quark in the heavy meson H_Q in the infinite momentum frame to fill up the gap between the simple heavy quark kinematics and the heavy meson kinematics. This distribution function can be obtained by measuring the Q-quark fragmentation function in the heavy meson production from e^+e^- annihilation. To remove the singularities near the end point of the charged lepton spectrum, which are due to soft gluon bremsstrahlung, these authors simply resummed soft gluon corrections in the naive leading (double) logarithmic approximation and obtained a Sudakov suppression factor. To fill up the kinematic gap, Altarelli and Petrarca [15] regarded the light quark inside H_Q as a quasifree particle but with a Gaussian spectrum of Fermi momentum p:

$$f(p)=\frac{4p^2}{\sqrt{\pi}p_f^3}\exp\left(-\frac{p^2}{p_f^2}\right), \qquad (1)$$

where p_f is a free parameter that can be fixed by heavy-quark symmetry. To smooth out the end-point singularities, these authors also resummed leading soft gluon contributions into a Sudakov form factor.

Korchemsky and Sterman [16] gave the first PQCD treatment of the decays $B\to X_s\gamma$ and $B\to X_u l\nu$, in which higher-order corrections were factorized into a soft distribution function, a jet function, and a hard-scattering amplitude according to the kinematic regions of loop momenta. The equivalence between the shape function [10,11,13] and the distribution function was pointed out. Soft gluon corrections, which correspond to the nonperturbative origin of the distribution function, were resummed systematically up to next-to-leading logarithms using the Wilson-loop formalism. However, their analysis is appropriate only in the end-point region, and the effects of resummation were not estimated.

In all the above approaches, the factorization was formulated at the quark-level kinematics, and the missing states inside the kinematic window between M_Q and M_H, M_H being the H_Q meson mass, were populated by introducing extra heavy-quark Fermi motion, which arises from the recoil of the light partons in H_Q. Our approach is formulated at the meson-level kinematics directly, in which the heavy quark Q, carrying a fraction of the heavy meson momentum, has an invariant mass close to M_Q. Our formalism therefore removes the ambiguity in the definition of the heavy quark mass M_Q, and the kinematic gap is filled up naturally.

In this paper we shall derive the PQCD factorization formula for the semileptonic decay $B\to X_u l\nu$ in a rigorous way, which is suitable for the entire range of the spectrum. The factorization procedures demand the inclusion of the transverse degrees of freedom of partons. Hence, we perform the resummation of large perturbative corrections in the transversal configuration space using the technique developed in [2], which is also accurate up to next-to-leading logarithms. It can be shown that our resummation result coincides with that in [16] at the end point. The transverse momenta carried by the b quark inside the B meson, whose distribution is governed by the Sudakov form factor from the resummation, play an important role here. They diminish the on-shell probability of the outgoing u quark at the end point, and thus suppress the singularities.

We define the kinematics of the inclusive semileptonic decays of heavy mesons in Sec. II, and derive the factorization formula for the charged lepton spectrum, which incorporates the transverse degrees of freedom of partons. The formula is expressed as the convolution of a hard-scattering amplitude with a jet and a universal soft function. In Sec. III, we resum the large logarithms in these convolution factors by solving a set of evolution equations. The initial condition of the soft function is identified as the distribution function, which is equivalent to the shape function mentioned above. In Sec. IV, we construct a distribution function according to the HQEFT based OPE, and relate the mean and width of the distribution function to the hadronic matrix elements of the kinematic operator. Hence, both perturbative higher-order corrections and nonperturbative corrections from Fermi motion are included in our formalism. We present numerical results in Sec. V, and show that the Sudakov form factor from the resummation and the distribution function indeed render the end-point spectrum smoother as stressed in [12]. Section VI is the conclusion.

II. FACTORIZATION THEOREMS

We consider the semileptonic inclusive decays of a B meson:

$$B(P_B)\to l(p_l)+\overline{\nu(p_\nu)}+\text{hadrons}. \qquad (2)$$

The three independent kinematic variables are chosen as E_l, q^2, and q_0 in our discussion. E_l and $q=p_l+p_\nu$ have been defined in the Introduction, and q_0 is the energy of the lepton pair. With these variables, the triple differential decay rate is written as

$$\frac{d^3\Gamma}{dE_l dq^2 dq_0}=\frac{1}{256\pi^4 M_B}|\mathcal{M}(E_l,q^2,q_0)|^2, \qquad (3)$$

where M_B is the B meson mass, and the weak matrix element is given by

$$\mathcal{M}=V_{ub}\frac{G_F}{\sqrt{2}}\overline{l\Gamma}_\mu\nu_l\sum_X\langle X|j^\mu|B\rangle. \qquad (4)$$

In Eq. (4) V_{ub} is the corresponding CKM matrix element, and $j_\mu=\overline{u\Gamma}_\mu b$ is the electroweak current with $\Gamma_\mu=\gamma_\mu(1-\gamma_5)$.

We work in the rest frame of the B meson, and choose the following light-cone components for relevant momenta:

$$P_B=(P_B^+,P_B^-,\mathbf{0}_\perp),\quad p_l=(p_l^+,0,\mathbf{0}_\perp),\quad p_\nu=(p_\nu^+,p_\nu^-,\mathbf{p}_{\nu\perp}), \qquad (5)$$

with $P_B^+=P_B^-=M_B/\sqrt{2}$ and $p_\nu^2=0$. The independent variables are identified as p_l^+, p_ν^-, and p_ν^+, and their relations to E_l, q^2 and q_0 are $E_l=p_l^+/\sqrt{2}$, $q^2=2p_l^+p_\nu^-$, and $q_0=(p_l^++p_\nu^++p_\nu^-)/\sqrt{2}$, respectively. We define $P_b=P_B-p$ as the b quark momentum, which satisfies $P_b^2\approx M_b^2$, M_b being the b quark mass. p is the kicks from the light components inside the B meson, which has a large plus component p^+ and small transverse components $\mathbf{p}_\perp$. The purpose of introducing the transverse degrees of freedom

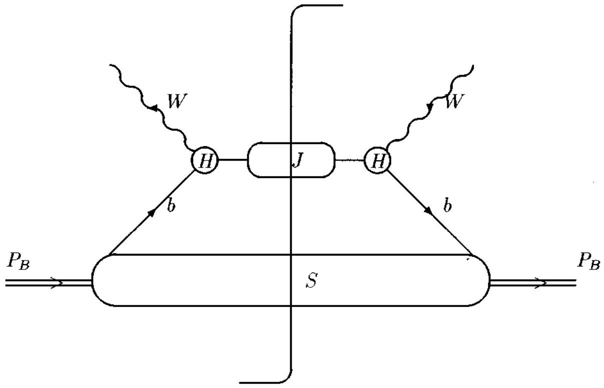

FIG. 1. Factorization of inclusive semileptonic decays of the B meson into a soft (S), a jet (J), and a hard (H) subprocess.

will become clear later. The b quark decays into a u quark with momentum $P_u = P_B - p - q$. We have distinguished the B meson momentum P_B from the b quark momentum P_b here.

It is more convenient to employ the scaling variables

$$x = \frac{2E_l}{M_B}, \quad y = \frac{q^2}{M_B^2}, \quad y_0 = \frac{2q_0}{M_B}, \tag{6}$$

instead of the dimensionful ones E_l, q^2, and q_0. Note that the scaling variables are defined in terms of the B meson mass M_B, since we formulate the factorization according to the B meson kinematics. This differs from the conventional treatment in the literature [16], where the scaling variables were defined in terms of the b quark mass. For massless leptons, it is easy to show, using the momentum configurations defined in Eq. (5), that the phase space is given by

$$0 \leq x \leq 1, \quad 0 \leq y \leq x, \quad \frac{y}{x} + x \leq y_0 \leq y + 1. \tag{7}$$

In the end-point region with $x \to 1$ ($p_l^+ \to M_B/\sqrt{2}$) and $y \to 0$ ($p_\nu^- \to 0$), we have $y_0 \to 1$ ($p_\nu^+ \to 0$) and $p \to 0$. The u quark then has a large minus component $P_u^- = (1 - y/x) M_B/\sqrt{2}$ but a very small plus component $P_u^+ = (1 - y_0 - y/x) M_B/\sqrt{2}$, and thus a very small invariant $P_u^2 = M_B^2(1 - y_0 + y)$, which forms an on-shell jet subprocess. The u quark travels a long distance of $O(1/\Lambda_{\rm QCD})$ before hadronized. Besides, the B meson is dominated by soft dynamics, which is the origin of the soft function stated in the Introduction. The remaining dominant subprocess is the hard one, which contains the weak decay vertices. Therefore, the important contributions are factorized into the soft (S), jet (J), and hard (H) subprocesses as shown in Fig. 1.

The factorization formula for the inclusive semileptonic decay $B \to X_u l \nu$ is written as

$$\frac{1}{\Gamma_l^{(0)}} \frac{d^3\Gamma}{dx\, dy\, dy_0} = M_B^2 \int_{z_{\rm min}}^{z_{\rm max}} dz \int d^2\mathbf{p}_\perp \times S(z, \mathbf{p}_\perp, \mu) J(z, P_u^-, \mathbf{p}_\perp, \mu) \times H(z, P_u^-, \mathbf{p}_\perp, \mu), \tag{8}$$

with the momentum fraction z defined by $z = P_b^+/P_B^+ = 1 - p^+/P_B^+$ and $\Gamma_l^{(0)} = (G_F^2/16\pi^3)|V_{ub}|^2 M_B^5$ [16]. μ in Eq. (8) is the renormalization and factorization scale. The triple differential decay rate is, of course, μ independent. Note that in the region $y \to x \sim 1$ the outgoing u quark becomes soft and Eq. (8) fails. We shall show that contributions from this dangerous region are suppressed by phase space. The upper limit of z takes the value $z_{\rm max} = 1$ in our analysis. If performing the factorization according to the b quark kinematics, one must assume $z_{\rm max} = M_B/M_b$, which is greater than 1, in order to fill up the kinematic window. It has been explained [16] that $z_{\rm max} > 1$ is not allowed in perturbation theory, and is thus of nonperturbative origin. From the kinematic constraints in Eq. (7) and the on-shell condition of the u-quark jet, the lower limit of z should be $z_{\rm min} = x$, instead of $z_{\rm min} = 0$.

The tree-level expressions for the convolution factors J and H are given by

$$J^{(0)} = \delta(P_u^2) = \delta\left(M_B^2 \left[1 - y_0 + y - (1 - z)\left(1 - \frac{y}{x}\right) - \frac{2\mathbf{p}_\perp \cdot \mathbf{p}_{\nu\perp}}{M_B^2} - \frac{\mathbf{p}_\perp^2}{M_B^2} \right] \right),$$
$$H^{(0)} \propto (P_b \cdot p_\nu)(p_l \cdot P_u) = [(P_B - p) \cdot p_\nu](p_l \cdot P_u) \propto (x - y)\left(y_0 - x - (1 - z)\frac{y}{x} + \frac{2\mathbf{p}_\perp \cdot \mathbf{p}_{\nu\perp}}{M_B^2} \right). \tag{9}$$

Equation (8) can be regarded as an expression at the intermediate stage in the derivation of conventional factorization theorems. If the $\mathbf{p}_\perp$ dependence in J and H is negligible, the variable $\mathbf{p}_\perp$ in S can be integrated over, and Eq. (8) reduces to the conventional factorization formula. However, it is obvious from Eq. (9) that at least the $\mathbf{p}_\perp$ dependence in J is not negligible, especially in the end-point region. This is the reason we introduce the transverse degrees of freedom into our analysis.

Suppose we consider higher-order corrections to Eq. (8) from a gluon crossing the final-state cut, and route the loop momentum l through, say, the jet subprocess. Without losing generality, we approximate $J(p^+ + l^+, P_u^- + l^-, \mathbf{p}_\perp + l_\perp) \approx J(p^+, P_u^-, \mathbf{p}_\perp + l_\perp)$ according to the kinimatic relations $l^+ < p^+$, $l^- < P_u^-$, and $l_\perp \approx \mathbf{p}_\perp$. Hence, the loop integral cannot be performed unless the dependence of J on transverse momentum is known. This difficulty can be removed by Fourier transform:

$$J(p^+, P_u^-, \mathbf{p}_\perp + l_\perp) = \int \frac{d^2\mathbf{b}}{(2\pi)^2} \tilde{J}(p^+, P_u^-, \mathbf{b}) e^{i(\mathbf{p}_\perp + l_\perp)\cdot \mathbf{b}}, \tag{10}$$

where the impact parameter b (Fourier conjugate variable of $p_\perp$) measures the transverse distance traveled by the jet. Using Eq. (10), the $l_\perp$ dependence is decoupled from the jet function, and the factor $e^{il_\perp \cdot \mathbf{b}}$ is absorbed into the loop integral, which can then be performed. Therefore, an extra factor $e^{il_\perp \cdot \mathbf{b}}$ is associated with each gluon crossing the final-state cut in our formalism.

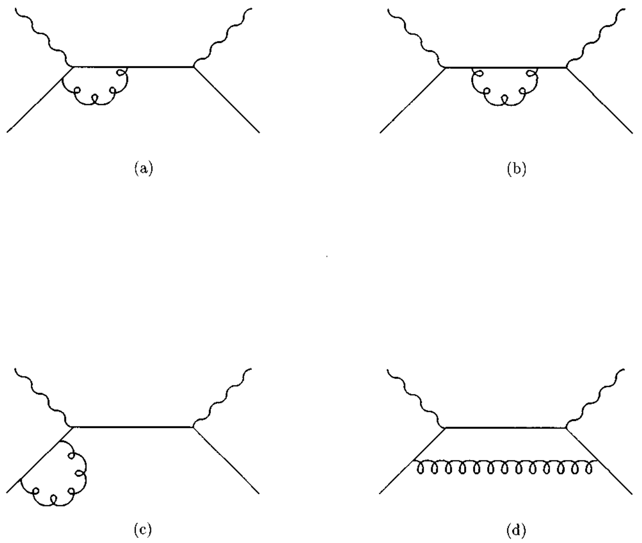

FIG. 2. Lowest-order radiative corrections to the inclusive B meson decays.

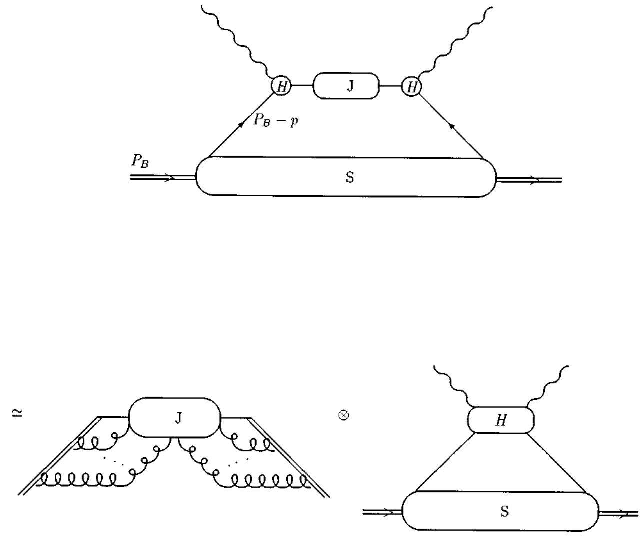

FIG. 3. Factorization of the jet subprocess.

To further simplify the factorization formula, we neglect those terms involving $\mathbf{p}_{\nu\perp}$ in J and H. This is a good approximation for $x \to 0$ and $x \to 1$, since for $x \to 0$ contributions from transverse momenta are not important, and for $x \to 1$ the magnitude $p_{\nu\perp} = M_B\sqrt{(y_0 - x - y/x)y/x}$ vanishes. Equation (8) then becomes

$$\frac{1}{\Gamma_l^{(0)}}\frac{d^3\Gamma}{dx\; dy\; dy_0} = M_B^2\int_x^1 dz\int\frac{d^2\mathbf{b}}{(2\pi)^2}\times\tilde{S}(z,\mathbf{b},\mu)\tilde{J}(z,P_u^-,\mathbf{b},\mu)H(z,P_u^-,\mu). \tag{11}$$

III. RESUMMING THE JET, SOFT, AND HARD SUBPROCESSES

It can be shown that the dominant subprocesses J, S, and H contain large logarithms form radiative corrections. In particular, J gives rise to double (leading) logarithms in the end-point region. These large corrections spoil the perturbation theory and must be organized. In what follows we shall demonstrate in detail how to resume these large corrections up to next-to-leading logarithms. The first step in resummation is to map out the leading regions of radiative corrections. We work in axial gauge $n \cdot A = 0$, where n is a spacelike vector. The $O(\alpha_s)$ diagrams that contain large double logarithms in axial gauge at the end point are Figs. 2(a) and 2(b). The self-energy diagram, Fig. 2(c), and the diagram with a soft gluon connecting the two heavy-quark lines, as shown in Fig. 2(d), give only single soft logarithms.

In the collinear region with the loop momentum l parallel to P_u and in the soft region with $l \to 0$ we can eikonalize the heavy b-quark line. Then Figs. 2(a) and 2(b) are factorized out of the cross section, and they are the diagrams that J absorbs, as shown in Fig. 3. With the eikonal approximation, the b-quark propagator is expressed as $1/v \cdot l$ to the order $1/M_B$ with $v = (1,1,\mathbf{0}_\perp)$. Hence, the factorization in Eq. (11) is in fact valid to $O(1/M_B)$. The physics involved in this approximation is that a soft gluon or a gluon moving parallel to P_u cannot explore the details of the b quark, and its dynamics can be factorized. This is consistent with the HQEFT, where the b quark is treated as a classical relativistic particle carrying color source. Since the large mass M_B does not appear in the eikonal propagator, the only large scale in J is P_u^-.

The remaining diagrams, Figs. 2(c) and 2(d), that give single soft logarithms are grouped into S. It is then obvious that S depends only on the properties of the bound state $|B\rangle$, but not on the particular short-distance subprocess. Therefore, S is a universal function describing the distribution of the b quark inside the B meson. In fact, in the $1/M_b$ limit one can identify S as the model-independent ''shape function'' or ''primodial function'' obtained from the resummation in [10,13,11], respectively.

The basic idea of the resummation technique is as follows. If the double logarithms are organized into an exponential form, $\tilde{J} \sim \exp[-\ln P_u^- \ln(\ln P_u^-/\ln b)]$, one can simplify the analysis by studying the derivative of $\tilde{J}$, $d\tilde{J}/d \ln P_u^- = C\tilde{J}$ [2]. Since the coefficient function C contains only single logarithms, it can be treated by renormalization-group (RG) methods. In this way, one reduces the complicated double-logarithm problem to a single-logarithm problem.

$\tilde{J}$ is scale invariant in the gauge vector n as shown by the gluon propagator in axial gauge, $-iN^{\mu\nu}(l)/(l^2 + i\epsilon)$, with

$$N^{\mu\nu}(l) = g^{\mu\nu} - \frac{n^\mu l^\nu + l^\mu n^\nu}{n \cdot l} + n^2\frac{l^\mu l^\nu}{(n \cdot l)^2}. \tag{12}$$

Therefore, $\tilde{J}$ must depend on P_u^- through the ratio $(P_u \cdot n)^2/n^2$. It is then easy to show that the differential operator $d/d \ln P_u^-$ can be replaced by d/dn using a chain rule:

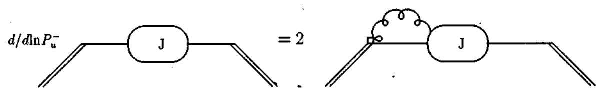

FIG. 4. Graphic representation of Eq. (16).

$$\frac{d\tilde{J}}{d\ln P_u^-} = -\frac{n^2}{v'\cdot n} v'^{\alpha} \frac{d}{dn^{\alpha}} \tilde{J} \tag{13}$$

with the vector $v' = (0,1,\mathbf{0}_\perp)$. This simplifies the task tremendously, because the momentum P_u flows through both quark and gluon lines, but n appears only in gluon propagators.

Applying d/dn to the gluon propagator, we obtain

$$\frac{d}{dn_\alpha} N^{\mu\nu} = -\frac{1}{l\cdot n}(N^{\mu\alpha}l^\nu + N^{\nu\alpha}l^\mu). \tag{14}$$

The momentum l that appears at both ends of the differentiated gluon line is contracted with the vertex, where the gluon attaches the u quark or the eikonal lines. Next we add up all diagrams with different differentiated gluon propagators, and apply the Ward identity. Finally, we arrive at a differential equation for $\tilde{J}$ as shown in Fig. 4, where the square vertex represents

$$gT^a \frac{n^2}{(v'\cdot n)(l\cdot n)} v'^{\alpha}, \tag{15}$$

with T^a the color matrix. The factor 2 counts the two external quark lines of $\tilde{J}$. An important feature of the square vertex is that the gluon momentum l does not give rise to collinear divergences because of the nonvanishing n^2. The leading regions of l are then soft and ultraviolet, in which the subdiagram containing the square vertex can be factorized as shown in Fig. 5 at $O(\alpha_s)$. Hence, the differentiation really turns the double-logarithm problem into a single-logarithm problem as stated before.

To separate the soft and ultraviolet scales in Fig. 5, we introduce a function $\mathcal{K}$ to organize the soft logarithms from the four diagrams, Figs. 5(a)–5(d), and a function $\mathcal{G}$ for the

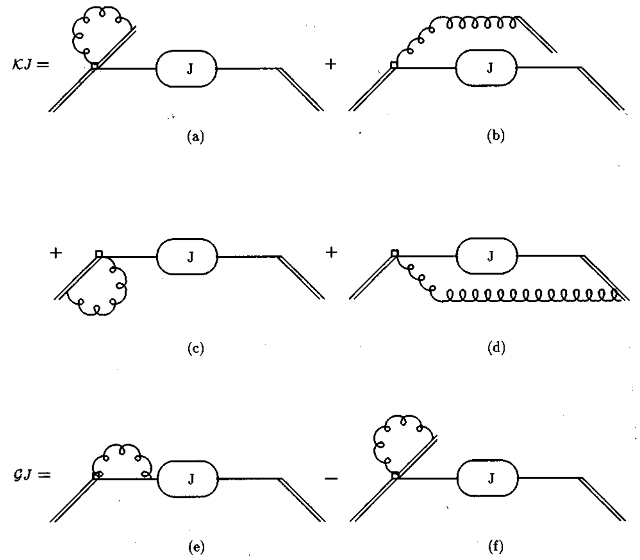

FIG. 5. Lowest-order diagrams for the functions $\mathcal{K}$ and $\mathcal{G}$.

ultraviolet divergences from Figs. 5(e) and 5(f). Double counting is avoided by the subtraction in $\mathcal{G}$. Generalizing $\mathcal{K}$ and $\mathcal{G}$ to all orders, we derive the differential equation for $\tilde{J}$,

$$\frac{d}{d\ln P_u^-}\tilde{J}(P_u^-,b,\mu) = 2\{\mathcal{K}[b\mu,\alpha_s(\mu)] + \mathcal{G}[P_u^-/\mu,\alpha_s(\mu)]\}\tilde{J}(P_u^-,b,\mu). \tag{16}$$

The scale b in $\mathcal{K}$ arises from Fig. 5(b), which contains an extra factor $e^{il_\perp\cdot\mathbf{b}}$ as explained in Sec. II.

At the one-loop level, the renormalized function $\mathcal{K}$ calculated in dimensional regularization for the gauge vector $n \propto (1,-1,\mathbf{0}_\perp)$ is given by

$$\mathcal{K} = \text{Fig. 5(a)} + \text{Fig. 5(b)} + \text{Fig. 5(c)} + \text{Fig. 5(d)} - \delta\mathcal{K}, \tag{17}$$

with

$$\text{Fig. 5(a)} + \text{Fig. 5(b)} = -g^2 C_F \mu^\epsilon \int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \left\{ \pi\delta(l^2)e^{il_\perp\cdot\mathbf{b}} - \frac{i}{l^2+i\epsilon} \right\} \frac{n^2 v'_\alpha v'_\beta}{(v'\cdot n)(l\cdot n)(l\cdot v')} N^{\alpha\beta}(l)$$
$$= -\frac{\alpha_s}{2\pi} C_F \left\{ \frac{2}{\epsilon} + \ln \pi\mu^2 b^2 e^{\gamma} \right\}, \tag{18}$$

$$\text{Fig. 5(c)} + \text{Fig. 5(d)} = -g^2 C_F \mu^\epsilon \int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \left\{ \pi\delta(l^2)e^{il_\perp\cdot\mathbf{b}} - \frac{i}{l^2+i\epsilon} \right\} \frac{n^2 v'_\alpha v_\beta}{(v'\cdot n)(l\cdot n)(l\cdot v)} N^{\alpha\beta}(l) = 0. \tag{19}$$

$\delta\mathcal{K}$ is the scheme-dependent counterterm which will be specified later. $C_F=4/3$ is the color factor, and γ is the Euler constant. Similarly, $\mathcal{G}$ is given by

$$\mathcal{G}=\text{Fig. 5(e)}+\text{Fig. 5(f)}-\delta\mathcal{G}, \tag{20}$$

with

$$\begin{aligned}\text{Fig. 5(e)}+\text{Fig. 5(f)}=&-g^2C_F\mu^\epsilon\int\frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}}\frac{\not P_u-\not l}{(P_u-l)^2+i\epsilon}\\&\times\left\{\gamma_\alpha+\frac{\not P_u-\not l}{l\cdot v'}v'_\alpha\right\}\\&\times\frac{n^2v'_\beta}{(v'\cdot n)(l\cdot n)}\frac{N^{\alpha\beta}(l)}{l^2+i\epsilon}\\=&-\frac{\alpha_s}{2\pi}C_F\left\{-\frac{2}{\epsilon}+\ln\frac{(P_u^-)^2\nu e^{\gamma-1}}{\pi\mu^2}\right\},\end{aligned} \tag{21}$$

and $\nu=(n\cdot v')^2/|n^2|$ is the gauge factor.

In the modified minimal subtraction ($\overline{\text{MS}}$) scheme with

$$\delta\mathcal{K}=-\delta\mathcal{G}=-\frac{\alpha_s}{2\pi}C_F\left(\frac{2}{\epsilon}+\ln 4\pi e^{-\gamma}\right), \tag{22}$$

the one-loop $\mathcal{K}$ and $\mathcal{G}$ are given by

$$\begin{aligned}\mathcal{K}&=-\frac{\alpha_s}{2\pi}C_F\ln\left(\frac{b^2\mu^2e^{2\gamma}}{4}\right),\\ \mathcal{G}&=-\frac{\alpha_s}{2\pi}C_F\ln\left(\frac{4(P_u^-)^2\nu}{\mu^2e}\right).\end{aligned} \tag{23}$$

We have separated the two scales in $\tilde{J}$, b, and P_u^-, into the functions $\mathcal{K}$ and $\mathcal{G}$, respectively, such that RG methods are applicable to the summation of the corresponding single logarithms. Although $\mathcal{K}$ and $\mathcal{G}$ possess individual ultraviolet pole, their sum is finite, and thus a RG invariant quantity. The RG equations for the renormalized $\mathcal{K}$ and $\mathcal{G}$ are

$$\begin{aligned}\mu\frac{d}{d\mu}\mathcal{K}(b\mu,\alpha_s(\mu))&=-\gamma_{\mathcal{K}}(\alpha_s(\mu))\\&=-\mu\frac{d}{d\mu}\mathcal{G}(P_u^-/\mu,\alpha_s(\mu))\end{aligned} \tag{24}$$

with

$$\gamma_{\mathcal{K}}=\mu\frac{d}{d\mu}\delta\mathcal{K}=-\mu\frac{d}{d\mu}\delta\mathcal{G}, \tag{25}$$

the anomalous dimension of $\mathcal{K}$. To two loops, $\gamma_{\mathcal{K}}$ is given by [17]

$$\gamma_{\mathcal{K}}=\frac{\alpha_s}{\pi}C_F+\left(\frac{\alpha_s}{\pi}\right)^2C_F\left[C_A\left(\frac{67}{36}-\frac{\pi^2}{12}\right)-\frac{5}{18}n_f\right], \tag{26}$$

where $n_f=4$ is the number of quark flavors and $C_A=3$ is the color factor.

Equation (24) has the solution

$$\begin{aligned}&\mathcal{K}(b\mu,\alpha_s(\mu))+\mathcal{G}(P_u^-/\mu,\alpha_s(\mu))\\&=\mathcal{K}(1,\alpha_s(P_u^-))+\mathcal{G}(1,\alpha_s(P_u^-))-\int_{1/b}^{P_u^-}\frac{d\bar{\mu}}{\bar{\mu}}A(\alpha_s(\bar{\mu}))\end{aligned} \tag{27}$$

with the anomalous dimension

$$A(\alpha_s)=\gamma_{\mathcal{K}}(\alpha_s)+\beta(g)\frac{\partial}{\partial g}\mathcal{K}(1,\alpha_s). \tag{28}$$

Substituting Eq. (27) into (16), we obtain the evolution of $\tilde{J}$ in P_u^- and b:

$$\tilde{J}(P_u^-,b,\mu)=\exp[-2s(P_u^-,b)]\tilde{J}(b,\mu). \tag{29}$$

The RG invariant Sudakov exponent is given by [18]

$$\begin{aligned}s(P_u^-,b)=&\frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right)+\frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right)-\frac{A^{(1)}}{2\beta_1}(\hat{q}-\hat{b})\\&-\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{b})+1}{\hat{b}}-\frac{\ln(2\hat{q})+1}{\hat{q}}\right]\\&-\left[\frac{A^{(2)}}{4\beta_1^2}-\frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right)\\&+\frac{A^{(1)}\beta_2}{8\beta_1^3}[\ln^2(2\hat{q})-\ln^2(2\hat{b})],\end{aligned} \tag{30}$$

with the variables

$$\hat{q}\equiv\ln\left(\frac{P_u^-}{\Lambda}\right),\quad \hat{b}\equiv\ln\left(\frac{1}{b\Lambda}\right). \tag{31}$$

The QCD scale $\Lambda\equiv\Lambda_{\text{QCD}}$ will be set to 0.2 GeV in the numerical study in Sec. V. The coefficients β_i and $A^{(i)}$ are

$$\begin{aligned}\beta_1&=\frac{33-2n_f}{12},\\ \beta_2&=\frac{153-19n_f}{24},\\ A^{(1)}&=\frac{4}{3},\\ A^{(2)}&=\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{27}n_f+\frac{8}{3}\beta_1\ln\left(\frac{e^\gamma}{2}\right).\end{aligned} \tag{32}$$

Having summed up the double logarithms, we concentrate on the single logarithms in $\tilde{S}$, H and the initial condition $\tilde{J}(b,\mu)$. Since both the differential decay rate and the Sudakov exponent $s(P_u^-,b)$ are RG invariant, we have the RG equations

$$\mathscr{D}\tilde{J}(b,\mu)=-2\gamma_q\tilde{J}(b,\mu),$$
$$\mathscr{D}\tilde{S}(b,\mu)=-\gamma_S\tilde{S}(b,\mu), \tag{33}$$
$$\mathscr{D}H(P_u^-,\mu)=(2\gamma_q+\gamma_S)H(P_u^-,\mu),$$

with

$$\mathscr{D}=\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}. \tag{34}$$

$\gamma_q=-\alpha_s/\pi$ is the quark anomalous dimension in axial gauge, and $\gamma_S=-(\alpha_s/\pi)C_F$ is the anomalous dimension of $\tilde{S}$. The function $\tilde{S}$ in fact contains soft single logarithms only from Fig. 2(d), because the contribution from Fig. 2(c) vanishes for the gauge vector $n\propto(1,-1,\mathbf{0}_\perp)$ under eikonal approximation. Hence, γ_S is derived from the evaluation of Fig. 2(d) straightforwardly.

Integrating Eq. (33), we obtain the evolution of all the convolution factors:

$$\tilde{J}(z,P_u^-,b,\mu)=\exp\left[-2s(P_u^-,b)-2\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right]\tilde{J}(z,b,1/b),$$
$$\tilde{S}(z,b,\mu)=\exp\left[-\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_S(\alpha_s(\bar{\mu}))\right]f(z,b,1/b), \tag{35}$$
$$H(z,P_u^-,\mu)=\exp\left[-\int_{\mu}^{P_u^-}\frac{d\bar{\mu}}{\bar{\mu}}[2\gamma_q(\alpha_s(\bar{\mu}))+\gamma_S(\alpha_s(\bar{\mu}))]\right]H(z,P_u^-,P_u^-).$$

We shall neglect the intrinsic b dependence of the distribution function f below. If Sudakov suppression in the large-b region is strong, we may drop the evolution of f and $\tilde{J}$ in b, which is proportional to $\alpha_s(1/b)$. Hence, we assume $f(z,b,1/b)=f(z)$, $\tilde{J}(z,b,1/b)=\tilde{J}^{(0)}(z,b)$, the Fourier transform of the tree-level expression in Eq. (9), and $H(z,P_u^-,P_u^-)=H^{(0)}(z,P_u^-)$.

Substituting Eq. (35) into (11), we derive the factorization formula for the inclusive semileptonic B meson decay:

$$\frac{1}{\Gamma_l^{(0)}}\frac{d^3\Gamma}{dx\,dy\,dy_0}=M_B^2\int_x^1 dz\int_0^\infty\frac{bdb}{2\pi}f(z)\tilde{J}^{(0)}(z,b)\times H^{(0)}(z,P_u^-)\exp[-S(P_u^-,b)]. \tag{36}$$

The complete Sudakov exponent S is given by

$$S(P_u^-,b)=2s(P_u^-,b)-\frac{5}{3\beta_1}\ln\frac{\hat{P}_u^-}{\hat{b}}, \tag{37}$$

with $\hat{P}_u^-=\ln(P_u^-/\Lambda)$, which combines all the exponents in Eq. (35) and includes both leading and next-to-leading logarithms. It is straightforward to observe from Eq. (37) that the Sudakov form factor e^{-S} falls off quickly at large $b\sim1/\Lambda$, where $\alpha_s(1/b)>1$ and perturbation theory fails. Hence, the Sudakov form factor guarantees that main contributions to the factorization formula come from the small b, or short-distance, region, and the perturbative treatment is indeed self-consistent.

We stress that our formalism is applicable to the entire range of the spectrum, if the $\mathbf{p}_{\nu\perp}$ dependence in J and H was not neglected, not only to the end-point region as in [16]. We neglected the $\mathbf{p}_{\nu\perp}$ dependence for the sake of simplicity of calculation. Since the Sudakov form factor is defined in the region

$$\left(1-\frac{y}{x}\right)\frac{M_B}{\sqrt{2}}=P_u^->\frac{1}{b}>\Lambda, \tag{38}$$

there exist the suppression effects as $y<(1-\sqrt{2}\Lambda/M_B)x$. The phase space with suppression expands to the largest extent for $x\to1$ and vanishes at $x\to0$. Hence, lowest-order predictions receive maximal suppression at the end point, corresponding to the presence of large logarithms, but are modified only slightly at small x, implying weaker logarithmic corrections. This is consistent with our expectation for the Sudakov effects in the whole range $0<x<1$.

IV. CONSTRUCTING THE UNIVERSAL SOFT FUNCTION

Since all the double-logarithmic corrections have been absorbed into the jet subprocess, the soft function $\tilde{S}$ contains only soft single logarithms from Fig. 2(d) [Fig. 2(c) does not contribute because of the choice of the gauge vector $n\propto(1,-1,\mathbf{0}_\perp)$]. These single logarithms can be summed by solving the RG equation $\mathscr{D}\tilde{S}=-\gamma_S\tilde{S}$ as shown in Eq. (33). The solution has been given in Eq. (35):

$$\tilde{S}(z,b,\mu)=\exp\left[-\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_S(\alpha_s(\bar{\mu}))\right]f(z), \tag{39}$$

where the initial condition $f(z)$ for the evolution of $\tilde{S}$ must be determined phenomenologically. From the definition of $\tilde{S}$, it is obvious that f depends only on the properties of the bound state $|B\rangle$, but not on the particular short-distance subprocess. Therefore, f is a process-independent universal function describing the distribution of the b quark inside a B meson.

As mentioned in the previous section, in the $1/M_b$ limit one can identify f as the model-independent shape function or primodial function obtained from the resummation of OPE in [10,11,13]. It was observed by Korchemsky and Sterman [16] that one can deduce some nonperturbative information of f from the perturbative resummation of soft gluons. Theses authors argued that the infrared renormalons appearing during the resummation procedure produce power-correction ambiguities. Hence, there should exist corresponding nonperturbative power-correction ambiguities to render the physical f well defined. To leading order, with a ''minimal'' ansatz for integrating the first infrared renormalon singularity, Korchemsky and Sterman [16] derived

$$f_{\rm KS}(z)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(1-z)^2}{2\sigma^2}\right], \tag{40}$$

which describes a Gaussian distribution with width σ around $z=1$. Requiring that the minimal ansatz be consistent with HQEFT and OPE, one fixes the width to be

$$\sigma^2=\frac{\mu_\pi^2}{3M_b^2}, \tag{41}$$

where $\mu_\pi^2=0.54\pm0.12$ GeV^2 [19] is obtained from QCD sum-rule estimation.

Recall that $f_{\rm KS}$ was derived by integrating only the first infrared renormalon. This fact is reflected in the nonvanishing property of $f_{\rm KS}$ in the unphysical region $z>1$, and in its validity in the vicinity of $z=1$. However, comparing $f_{\rm KS}$ with the phenomenological distribution function [20],

$$f_P(z)=N_P\,\frac{z(1-z)^2}{[(1-z)^2+\epsilon_P z]^2}, \tag{42}$$

one finds that they do share the same leading $(1-z)^2$ behavior. f_P was obtained from the experiments of the B meson production in e^+e^- annihilation by applying a crossing to the light quark and then a time-reversal transformation [20]. The constant $N_P=0.133\,068$ is the normalization, and $\epsilon_P=0.006$ is the shape parameter. Inspired by this observation, we postulate the following two-parameter distribution function for the B meson:

$$f_B(z)=N\,\frac{z(1-z)^2}{[(z-a)^2+\epsilon z]^2}\,\theta(1-z), \tag{43}$$

which has the correct leading $(1-z)^2$ behavior near $z=1$. The purpose of introducing one more parameter a in Eq. (43) is to allow a consistency check on f_P in Eq. (42).

We now relate the parameters a and ϵ to the hadronic matrix elements of some local operators derived from HQEFT-based OPE, which are standard techniques [10,12,13]. The distribution function in axial gauge is defined by

$$f_B(z)=\frac{1}{2\sqrt{2}}\int\frac{dy^-}{2\pi}\,e^{i(1-z)P_B^+y^-}\langle B|\overline{b_v}(0)b_v(y^-)|B\rangle, \tag{44}$$

in which the large momentum of the rescaled heavy-quark field $b_v(x)$ has been projected out as usual in HQEFT by

$$b_v(x)=e^{iM_b v\cdot x}b(x). \tag{45}$$

Hence, $f_B(z)$ is independent of the b-quark mass as it is written in terms of a matrix element in Eq. (44). Note that the Dirac matrix structure has been properly factorized as shown in Eq. (44).

To connect with HQEFT, we write the b-quark momentum as $P_b=P_B-p=M_b v+(M_B-M_b)v-p$, and identify the residual momentum of the b quark as $k=\Lambda v-p$, $\Lambda=M_B-M_b$ being the effective mass of the light partons in the B meson. The probability to find a b quark with light-cone residual momentum k^+ inside the B meson, $f_r(k^+)$, has been defined in [13]. The moments of $f_r(k^+)$,

$$A_n=\int k^{+n}f_r(k^+)dk^+, \tag{46}$$

have been derived in HQEFT-based OPE as [13]

$$A_0=1,\quad A_1=0,\quad A_2=\frac{M_B^2}{3}\,K_b,\dots\,. \tag{47}$$

These moments are expressed in terms of hadronic matrix elements corresponding to the structure of the $1/M_b$ expansion, with

$$K_b\equiv-\frac{1}{2M_B}\left\langle B\left|\overline{b_v}\,\frac{(iD)^2}{2M_b^2}\,b_v\right|B\right\rangle. \tag{48}$$

The vanishing $O(\Lambda_{\rm QCD}/M_B)$ contribution to the first moment A_1 is consistent with the conclusion from the renormalon analysis that the first nontrivial power correction begins at $O(\Lambda_{\rm QCD}^2/M_B^2)$ and with our intuition for vanishing average residual momentum of the b quark inside the B meson in the heavy-quark limit.

The moments of $f_B(z)$ can be expressed as local hadronic matrix elements by performing an OPE of the bilocal operator $b_v(0)b_v(y^-)$ in Eq. (44) in the power of $1/M_b$. The relation between f_B and f_r is then given by [13]

$$f_B(z)dz=[f_r(k^+)+O(1/M_b)]dk^+, \tag{49}$$

which reflects the difference of order $\overline{\Lambda}$ between the B meson kinematics and the b-quark kinematics. Using Eq. (46) and the definition $z=P_b^+/P_B^+$ or $k^+=P_B^+(z-M_b/M_B)$, it is straightforward to derive the moments of f_B. They are

$$\int_0^1 f_B(z)dz=1, \tag{50}$$

$$\int_0^1 dz(1-z)f_B(z)=\overline{\Lambda}/M_B+O(\Lambda_{\rm QCD}^2/M_B^2), \tag{51}$$

$$\int_0^1 dz(1-z)^2 f_B(z)=\frac{\overline{\Lambda}^2}{M_B^2}+\frac{2}{3}\,K_b+O(\Lambda_{\rm QCD}^3/M_B^3). \tag{52}$$

The first formula gives the correct normalization of f_B, which corresponds to the total number of b quarks inside a B meson. The second formula is related to the effective mass of light quarks, Λ. The third formula gives the hadronic matrix element of the kinematic operator, K_b.

To have a better insight, we examine if the distribution function is consistent with our physical intuition for the behavior of the heavy b quark inside a B meson. We calculate the mean μ and the variance σ^2 of $f_B(z)$ from Eqs. (51) and (52), and derive

$$\mu=1-\frac{\overline{\Lambda}}{M_B}+O\left(\frac{\Lambda_{\rm QCD}^2}{M_B^2}\right), \tag{53}$$

$$\sigma^2=\frac{2K_b}{3}+O\left(\frac{\Lambda_{\rm QCD}^3}{M_B^3}\right). \tag{54}$$

Substituting the QCD sum rule [19] and B^*-B mass-splitting [21] results,

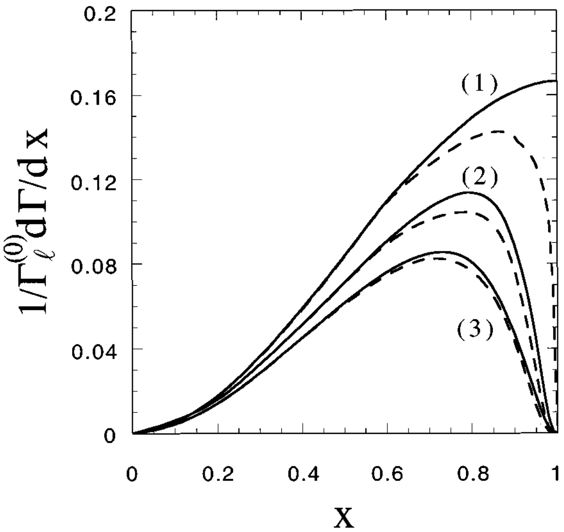

FIG. 6. Charged lepton spectra of the $B \to X_u l \nu$ decay for (1) $f(z)=\delta(1-z)$, (2) $f(z)=f_B(z)$, and (3) $f(z)=f_P(z)$. The solid (dashed) curves are derived without (with) Sudakov suppression.

$$M_B = 5.279 \text{ GeV}, \quad M_b = 4.776 \text{ GeV},$$
$$K_b = 0.012 \pm 0.0026, \tag{55}$$

we obtain μ=0.90 and σ^2=0.0080±0.0017, implying that $f_B(z)$ peaks sharply around $z \approx \mu \approx 1$ and has a width of $O(\Lambda_{\rm QCD}/M_B)$. The parameters N, a, and ϵ in $f_B(z)$ can be determined using Eqs. (50)–(52) with (55) inserted, which are

$$N = 0.026\,09, \quad a = 0.9752, \quad \epsilon = 0.001\,699. \tag{56}$$

The value of a, derived from the QCD constraints (by taking finite number of moments only), is very close to unity. This is consistent with the expectation from f_P. However, we emphasize that f_P is not quite consistent with HQEFT, its first and second moment differing from Eqs. (51) and (52) by at least 45%.

The distribution functions in Eqs. (42) and (43) will serve as the initial conditions of the soft function in (39). We then derive the PQCD factorization formula for the inclusive decay $B \to X_u l \nu$, with the phenomenological inputs satisfying the QCD constraints from HQEFT-based OPE.

V. THE CHARGED LEPTON SPECTRUM

In this section we evaluate Eq. (36) numerically for various distribution functions. The charged lepton spectrum for the decay $B \to X_u l \nu$ from the naive quark model is obtained by simply choosing $f(z)=\delta(1-z)$ and ignoring the transverse momentum dependence in $J^{(0)}$ and $H^{(0)}$. A simple calculation leads to

$$\frac{1}{\Gamma_l^{(0)}} \frac{d\Gamma}{dx} = \frac{x^2}{6}(3-2x), \tag{57}$$

which corresponds to the solid curve (1) in Fig. 6. This curve does not fall off at the end point of the spectrum, contradicting the observed behavior of the inclusive semileptonic decays of B mesons. The discrepancy implies that the tree-level analysis is not appropriate, especially in the end-point region where PQCD corrections are important as discussed in Sec. III.

We then take into account Sudakov suppression from the resummation of large radiative corrections. Substituting $f(z) = \delta(1-z)$, $H^{(0)}=(x-y)(y_0-x)$ and the Fourier transform of $J^{(0)}=\delta(P_u^2)$ with $P_u^2 = M_B^2(1-y_0+y-p_\perp^2/M_B^2)$ into Eq. (36), we derive the modified quark model spectrum. This spectrum is, after integrating Eq. (36) over z and y_0, described by

$$\frac{1}{\Gamma_l^{(0)}} \frac{d\Gamma}{dx} = M_B \int_0^x dy \int_0^{1/\Lambda} db e^{-S(P_u^-, b)}(x-y)\eta \times \left[(1+y-x) J_1(\eta M_B b) - \frac{2}{M_B b}\eta J_2(\eta M_B b) + \eta^2 J_3(\eta M_B b) \right], \tag{58}$$

where $P_u^- = (1-y/x)M_B/\sqrt{2}$, $\eta = \sqrt{(x-y)(1/x-1)}$ and J_1, J_2, J_3 are the Bessel functions of order 1, 2, and 3, respectively. Note the presence of the Sudakov form factor e^{-S} and the expression in the square brackets which comes from $\tilde{J}^{(0)}$. The cutoff $1/\Lambda$ of the impact parameter b is set by the Sudakov form factor. Numerical results of Eq. (58) for Λ=0.2 GeV are shown by the dashed curve (1) in Fig. 6. Since we have neglected the $\mathbf{p}_{\nu\perp}$ dependence in J and H for simplicity, Eq. (58) is appropriate only for small and large x. Therefore, to obtain the dashed curve (1), we evaluate Eq. (58) in the regions $0 \leq x \leq 0.7$ and $0.9 \leq x \leq 1$, and then extrapolate from $x = 0.7$ to 0.9 smoothly. The dependence on Λ in our analysis

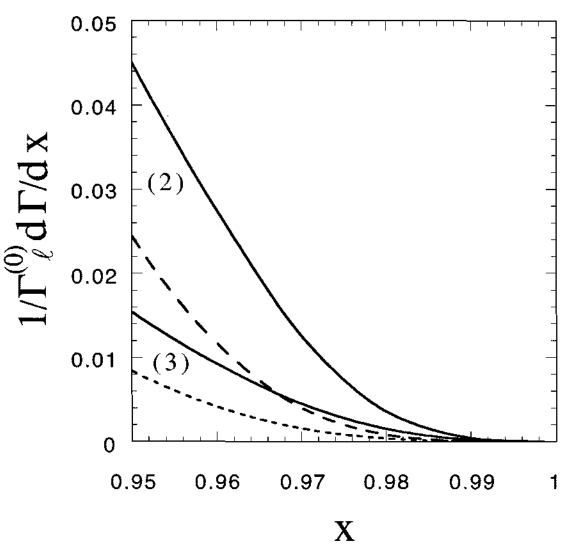

FIG. 7. Charged lepton spectra of the $B \to X_u l \nu$ decay near the end point for the use of f_B and f_P. Conventions are the same as those in Fig. 6 but with the dotted curve corresponding to the dashed curve (3).

TABLE I. The total decay rates for the quark model and for the use of the distribution functions f_B and f_P.

$\Gamma/\Gamma_l^{(0)}$	$\delta(1-x)$	f_B	f_P
Without suppression	0.0833	0.0586	0.0446
With suppression	0.0767	0.0548	0.0425
Sudakov effects	7.92%	6.48%	4.71%

is also examined, and it is found that predictions increase by only 10–20 % if Λ was set to 0.1 GeV.

One observes immediately that the Sudakov effects alone are enough to render the uprising free quark spectrum falloff at the end point. This is consistent with our expectation that the inclusion of transverse momenta and Sudakov suppression diminishes the on-shell configuration of the outgoing u-quark jet. Another important feature in Fig. 6 is that the solid and dashed curves coincide with each other in the region $x\to 0$. This indicates that the Sudakov effects almost cease to contribute away from the end point as stated in Sec. III.

The spectrum from the parton model without Sudakov suppression is obtained by adopting $H^{(0)}=(x-y)[y_0-x-(1-z)y/x]$ and $P_u^2=M_B^2[1-y_0+y-(1-z)(1-y/x)]$. With integration over y_0, we derive

$$\frac{1}{\Gamma_l^{(0)}}\frac{d\Gamma}{dx}=\int_0^x dy\int_x^1 dz\; f(z)(x-y)(y+z-x), \tag{59}$$

where $f(z)$ can be replaced by the distribution functions given in Eqs. (42) and (43). Predictions from the use of f_B and f_P are represented by the solid curves (2) and (3) in Fig. 6, respectively. Both of the spectra deviate from the quark model one slightly at small x, and vanish at the end point. Since Eq. (59) incorporates nonperturbative effects from primodial heavy-quark motion [11,13] (or soft dynamics in our formalism) through f, we conclude that these nonperturbative corrections are indeed important in the end-point region.

At last, including Sudakov suppression into Eq. (59), we arrive at the charged lepton spectrum of the $B\to X_u l\nu$ decay that takes into account both large perturbative and nonperturbative corrections:

$$\frac{1}{\Gamma_l^{(0)}}\frac{d\Gamma}{dx}=M_B\int_0^x dy\int_0^{1/\Lambda} db\int_x^1 dz\; f(z)(x-y)\xi\left[(z+y-x)J_1(\xi M_B b)-\frac{2}{M_B b}\,\xi J_2(\xi M_B b)+\xi^2 J_3(\xi M_B b)\right]e^{-S(P_u^-,b)}, \tag{60}$$

with $\xi=\sqrt{(x-y)(z/x-1)}$. Predictions from f_B and f_P are shown by the dashed curves (2) and (3), respectively. They coincide with the solid curves at small x, but descend by about 50% at $x\to 1$ as shown in Fig. 7, implying strong suppression in the end-point region. The slope of the spectrum then becomes smoother as expected.

From Fig. 6 we evaluate the total decay rate $\Gamma/\Gamma_l^{(0)}$, and results along with the Sudakov effects are displayed in Table I. We find that the overall suppression from the Sudakov effects is 8% for the quark model and less than 7% for the use of f_B and f_P. The two distribution functions lead to about 20% difference in the total decay width. Comparing to the drastic distinction between f_B and f_P with $N_P/N\sim 5$ and $\epsilon_P/\epsilon\sim 4$, our formalism is quite insensitive to the choice of different distribution functions. The overall Sudakov suppression of less than 8% indicates that PQCD corrections are actually not important for the most part of the spectrum. This is consistent with the fact that corrections from transverse momenta are an $O(1/M_Q^2)$ effect.

Note that 30% suppression on the quark model results from the distribution function f_B (the suppression from f_P is even stronger). It has been found in HQEFT that effects from nonperturbative corrections are only of $O(1/M_B^2)$, which should be less than 5% (see [22] and references therein). The small nonperturbative corrections to the total decay rate are closely related to the vanishing first moment of the residual momentum structure function f_r. This apparent discrepancy can be traced back to the fact that the B meson kinematics is employed in our formalism, while the b-quark kinematics is employed in the conventional approaches. Hence, in our quark-model analysis the b quark in fact carries the full momentum $P_b=P_B$, and thus the charged lepton energy E_l can reach the maximum $M_B/2$ $(x=1)$. This momentum configuration is allowed in factorization theorems if transverse degrees of freedom of partons were included, because its invariant $P_b^2=M_B^2-\mathbf{k}_T^2$ may still be close to the mass shell in the region without Sudakov suppression. Without $\mathbf{k}_T$, Eq. (57) should be regarded as an expression that is generated in our formalism to bear the same form as the leading-power results in HQEFT. For a free b quark with momentum $M_b v$, E_l can only reach $M_b/2$, instead of $M_B/2$. Strictly speaking, our quark-model predictions and the leading-power predictions in HQEFT have different meanings.

We stress that our results do not violate the conclusion from HQEFT, if they were interpreted in a proper way. To confirm this, we identify $P_b=(M_b^2/2P_B^-,P_B^-,\mathbf{0})$ as the momentum carried by a free b quark in factorization theorems for B meson decays, where the minus component P_b^- has been set to P_B^-. That is, the free b quark is not at rest inside the B meson. We then reexpress Eq. (59) into a form similar to that in [22]:

$$\frac{1}{\Gamma_l^{(0)}}\frac{d\Gamma}{dx}=F(x)\,\theta\left(\frac{M_b^2}{M_B^2}-x\right)+F\left(\frac{M_b^2}{M_B^2}\right)M(x), \tag{61}$$

with $F(x)=x^2(3-2x)/6$ being the quark-model prediction derived from the conventional approaches and

$$F\left(\frac{M_b^2}{M_B^2}\right)M(x)=\int_0^x dy\int_x^1 dz\ f(z)(x-y)(y+z-x)$$
$$-F(x)\theta\left(\frac{M_b^2}{M_B^2}-x\right). \tag{62}$$

The step function in Eq. (61) specifies the maximal E_l in the decay of a free b quark with the above momentum P_b. The function $M(x)$, representing nonperturbative corrections to the b quark decay, coincides with the shape function $S(x)$ defined in [22].

We shall show that the contribution from $M(x)$ to the total decay rate is indeed of $O(1/M_B^2)$. Integrating Eq. (62) over x, we obtain

$$F\left(\frac{M_b^2}{M_B^2}\right)\int_0^1 M(x)dx=\frac{1}{12}\int_0^1 dz\ z^4 f(z)-\int_0^{M_b^2/M_B^2}F(x)dx. \tag{63}$$

An arbitrary structure function f, which possesses the same moment as in Eq. (51), can be expanded in terms of δ functions:

$$f(z)=\delta(1-z)-\frac{\overline{\Lambda}}{M_B}\ \delta'(1-z)+O(\overline{\Lambda}^2/M_B^2). \tag{64}$$

Inserting Eq. (64) into (63), we justify straightforwardly that the nonperturbative correction

$$F\left(\frac{M_b}{M_B}\right)\int_0^1 M(x)dx=\int_{M_b^2/M_B^2}^1 F(x)dx-\frac{1}{12}\frac{\overline{\Lambda}}{M_B}$$
$$\times\int_0^1 dz\ z^4\delta'(1-z)+O(\overline{\Lambda}^2/M_B^2)$$
$$=\frac{1}{3}\frac{\overline{\Lambda}}{M_B}-\frac{1}{3}\frac{\overline{\Lambda}}{M_B}+O(\overline{\Lambda}^2/M_B^2) \tag{65}$$

vanishes at $O(1/M_B)$ as concluded in [22]. In summary, the quark-model contribution from the window between $x=M_b^2/M_B^2$ and $x=1$, with a width of $O(1/M_B)$, cancels the $O(1/M_B)$ correction from the structure function, such that the nonperturbative correction is of $O(1/M_B^2)$.

According to Eq. (63), the suppression from nonperturbative corrections is about 5% for f_B and more than 10% for f_P. The percentage for f_P is still large, because its first moment does not satisfy the requirement of HQEFT, and thus the cancellation at the power $1/M_B$ is not complete.

The ambiguity from the choice of distribution functions can be removed, once the spectrum of the decay $B\to X_s\gamma$ is available. We can fix the universal B meson distribution function from these data, substitute the distribution function into our formula, and predict the end-point spectrum of the decay $B\to X_u l\nu$. A model-independent extraction of the CKM matrix element $|V_{ub}|$ then becomes possible [13,16].

VI. CONCLUSION

We have studied the inclusive semileptonic $B\to X_u l\nu$ decay using the PQCD formalism. In order to separate the $B\to X_u l\nu$ signals from the $B\to X_c l\nu$ background, we must investigate the charged lepton spectrum near the end-point region within an accuracy of about 330 MeV. It has been found that there exist large perturbative corrections in this region, which are resummed into the Sudakov form factor and included into the factorization formula. The transverse degrees of freedom of the b quark diminish the on-shell configuration of the outgoing u-quark jet. The quark-model spectrum then falls off at the end point, consistent with the experimental observation. There is no ambiguity associated with the kinematic gap, because we formulate the factorization for B meson, instead of b quark, decays.

We have constructed a distribution function, whose parameters are determined by the HQEFT-based OPE, and whose width and mean are related to hadronic matrix elements of the kinematic operator. These hadronic matrix elements are then fixed by QCD sum rule results [19] and B^*-B splitting data [21]. The distribution function, absorbing important nonperturbative corrections from heavy-quark Fermi motion, can also render the quark-model spectrum vanish at the end point.

We emphasize that our formalism incorporates both large perturbative and nonperturbative corrections in the end-point region of inclusive B meson decays in a systematic way, and that it provides a natural normalization for the spectra. This enables the direct extraction of the CKM matrix element $|V_{ub}|$ from experimental data. When more data are available, our formalism can also be used to test PQCD in B meson decays.

It is an important issue that current experimental data [23] of the $B\to Xl\nu$ branching ratio suggest

$$B(B\to Xl\nu)\leqslant 11\%. \tag{66}$$

The naive quark-model prediction for this branching ratio is more than 15%. Although PQCD suppression at the end point is around 50%, the overall suppression amounts to 8% at most. With modification from the massiveness of the charm quark, our formalism can be applied equally well to the semileptonic decay $B\to X_c l\nu$. Hence, we conclude that PQCD corrections suppress the overestimated theoretical value of the semileptonic branching ratio only down to 13.8% at best. On the other hand, the distribution functions may decrease the quark-model predictions by about 30%. However, the distribution function is universal as stressed before, and it is very plausible that it gives an equal amount of suppression to nonleptonic decays. Therefore, introducing a distribution function may not be able to remove the disagreement.

Based on the above discussion, we propose three possibilities to resolve the discrepancy: (1) the distribution function suppresses semileptonic B meson decays maximally, but does nonleptonic decays minimally. This may arise from the different phase space in these two cases. (2) Factorization theorems break down in B meson decays. (3) New QCD effects or new physics appears. Blok and Mannel [24] argued that factorization theorems may still hold, and thus the confrontation between data and theoretical predictions becomes acute. To settle down the issue, a careful PQCD analysis of B meson nonleptonic decays is required. We shall discuss these subjects in a forthcoming article.

ACKNOWLEDGMENTS

We thank Dr. Xin-Heng Guo who participated in this work at the early stage. We also thank T. Mannel for communicating his works on local duality, and M. Shifman for useful discussion. This work was supported by the National Science Council of ROC under Grant Nos. NSC85-2112-M194-009 and NSC85-2112-M-001-021.

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