Effective model for charmed meson semileptonic decays

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We analyze charm meson semileptonic decays using the measured ratios Γ_L/Γ_T and Γ_+/Γ_- from $D\to\overline{K^*}$ and the branching ratios for $D\to\overline{K^*}$ and $D\to\overline{K}$. First we introduce the light vector mesons in a model which combines the heavy quark effective Lagrangian and the chiral perturbation approach. We propose a method which predicts the behavior of the form factors. Using the available experimental data we determine the values of some model parameters and reproduce the observed branching ratios for $D_s\to\Phi$, $D_s\to(\eta+\eta')$, and $D\to\pi$. We make predictions for the yet unmeasured branching ratios and polarization observables. [S0556-2821(96)04509-2]

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I. INTRODUCTION

There exist several theoretical calculations aiming to describe $D\to Vl\nu$ (V is a light vector meson) or $D\to Pl\nu$ (P is a light pseudoscalar meson) semileptonic decays: relativistic or nonrelativistic quark models [1–4], lattice calculations [5], QCD sum rules [6], and a few attempts to use the heavy quark effective theory (HQET) [7–9]. On the experimental side the following quantities regarding semileptonic charm meson decays have been measured: the branching ratios $\mathcal{B}$, Γ_L/Γ_T, and Γ_+/Γ_- for $D^+\to\overline{K^{*0}}$ and the branching ratios $\mathcal{B}$ for $D^0\to K^{*-}$, $D_s^+\to\Phi$, $D^0\to K^-$, $D^+\to\overline{K^0}$, $D_s^+\to(\eta+\eta')$, $D^0\to\pi^-$, and $D^+\to\pi^0$ [10]. The purpose of this paper is to accommodate these available experimental data within a combination of HQET and the chiral perturbation theory (CHPT) description of the light meson sector. Within this framework the HQET is valid at small recoil momentum [11,12] (a zero recoil momentum is realized when the final and decaying meson states have the same velocities). HQET can give definite predictions for heavy to light ($D\to V$ or $D\to P$) semileptonic decays in the kinematic region with large momentum transfer to the lepton pair, i.e., large q^2. Unfortunately, it cannot predict the q^2 dependence of the form factors.

The experimental data for the semileptonic decays $D^0\to K^-$ [13], $D^+\to\overline{K^0}$ [14], and $D^+\to\overline{K^{*0}}$ [15,16] are unfortunately not good enough to clearly determine the q^2 dependence of the form factors. Experimentally what is known, apart from the branching ratios, are the form factors at one kinematical point, *assuming* a pole-type behavior for all the form factors. The same assumption is used also in many theoretical calculations, for example in [1,8]. This assumption seems reasonable, but within HQET the kinematic constraint on the form factors at $q^2=0$ cannot be satisfied unless a special relation is imposed between the pole masses and residues. Moreover, it was shown using QCD sum rules [6] that the form factors for axial currents exhibit a rather flat q^2 dependence.

For these reasons, we will modify the Lagrangian for heavy and light pseudoscalar and vector mesons given by the HQET and chiral symmetry [7]. Apart from the zero-recoil point we will still use the same Feynman rules for the vertices in our processes, but write down the complete propagator also for heavy mesons, instead of using the HQET propagator. In the region where the heavy meson is nearly on-shell (the region where HQET is applicable) the two prescriptions almost perfectly overlap, but due to a Feynman rule prescription for the calculation of the form factors, there are no inconsistencies at $q^2=0$. At the same time this gives a natural explanation of the pole-type form factors in the whole q^2 range and an entirely consistent picture. It enables us to determine which form factors have a pole-type or a constant behavior, confirming the results of the QCD sum rules analysis [6].

In order to show that such a simple prescription works, we will calculate the decay widths in all measured charm meson semileptonic decays. The model parameters will then be determined by the experimental data. These parameters are also important in the study of more complicated decays.

The paper is organized as follows. In Sec. II we will first write down the already known Lagrangian for heavy and light pseudoscalar and vector mesons, given by the requirements of HQET and chiral symmetry. In Sec. III we will explain the behavior of the form factors in decays $D\to V$ and $D\to P$. The free model parameters will then be determined by comparing our approach with experiment. Finally, a short summary of the results will be given in Sec. IV.

II. THE HQET AND CHPT LAGRANGIAN

A. Strong interactions

We incorporate in our Lagrangian both the heavy flavor SU(2) symmetry [11,17] with the $SU(3)_L\times SU(3)_R$ chiral

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symmetry, spontaneously broken to the diagonal $SU(3)_V$, [20] which can be used for the description of heavy and light pseudoscalar and vector mesons. A similar Lagrangian, but without the light vector octet, was first introduced by Wise [12], Burdman and Donoghue [18], and Yan *et al.* [19]. It was then generalized with the inclusion of light vector mesons in [7,9,21].

The light degrees of freedom are described by the 3×3 Hermitian matrices

$$\Pi=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta_8+\frac{\eta_0}{\sqrt{3}} \end{pmatrix} \tag{1}$$

and

$$\rho_\mu=\begin{pmatrix} \frac{\rho^0_\mu+\omega_\mu}{\sqrt{2}} & \rho^+_\mu & K^{*+}_\mu \\ \rho^-_\mu & \frac{-\rho^0_\mu+\omega_\mu}{\sqrt{2}} & K^{*0}_\mu \\ K^{*-}_\mu & \overline{K^{*0}}_\mu & \Phi_\mu \end{pmatrix} \tag{2}$$

for the pseudoscalar and vector mesons, respectively. The mass eigenstates are defined by $\eta=\eta_8\cos\theta_P-\eta_0\sin\theta_P$ and $\eta'=\eta_8\sin\theta_P+\eta_0\cos\theta_P$, where $\theta_P=(-20\pm5)^\circ$ [10] is the η-η' mixing angle. The matrices (1) and (2) are usually expressed through the combinations

$$u=\exp\left(\frac{i\Pi}{f}\right), \tag{3}$$

where f is the pseudoscalar decay constant, and

$$\hat{\rho}_\mu=i\frac{g_V}{\sqrt{2}}\rho_\mu, \tag{4}$$

where $g_V=5.9$ is given by the values of the vector masses (we consider only the case of exact vector dominance; see [21]).

Introducing the vector and axial currents $\mathscr{V}_\mu=\frac{1}{2}(u^\dagger\partial_\mu u+u\partial_\mu u^\dagger)$ and $\mathscr{A}_\mu=\frac{1}{2}(u^\dagger\partial_\mu u-u\partial_\mu u^\dagger)$ and the gauge field tensor $F_{\mu\nu}(\hat{\rho})=\partial_\mu\hat{\rho}_\nu-\partial_\nu\hat{\rho}_\mu+[\hat{\rho}_\mu,\hat{\rho}_\nu]$ the light meson part of the strong Lagrangian can be written as

$$\mathscr{L}_{\text{light}}=-\frac{f^2}{2}\{\text{tr}(\mathscr{A}_\mu\mathscr{A}^\mu)+2\,\text{tr}[(\mathscr{V}_\mu-\hat{\rho}_\mu)^2]\}+\frac{1}{2g_V^2}\text{tr}[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})]. \tag{5}$$

Both the heavy pseudoscalar and the heavy vector mesons are incorporated in the 4×4 matrix

$$H_a=\tfrac{1}{2}(1+\not{v})(D^*_{a\mu}\gamma^\mu-D_a\gamma_5), \tag{6}$$

where $a=1,2,3$ is the $SU(3)_V$ index of the light flavors, and $D^*_{a\mu}$ and D_a annihilate a spin 1 and spin 0 heavy meson $c\bar{q}_a$ of velocity v, respectively. They have a mass dimension 3/2 instead of the usual 1, so that the Lagrangian is explicitly mass independent in the heavy quark limit $m_c\to\infty$. Defining

$$\overline{H_a}=\gamma^0H_a^\dagger\gamma^0=(D^{*\dagger}_{a\mu}\gamma^\mu+D^\dagger_a\gamma_5)\tfrac{1}{2}(1+\not{v}), \tag{7}$$

we can write the leading order strong Lagrangian as

$$\begin{aligned}\mathscr{L}_{\text{even}}=\mathscr{L}_{\text{light}}&+i\,\text{Tr}[H_av_\mu(\partial^\mu+\mathscr{V}^\mu)\overline{H_a}]\\&+ig\,\text{Tr}[H_b\gamma_\mu\gamma_5(\mathscr{A}^\mu)_{ba}\overline{H_a}]\\&+i\beta\,\text{Tr}[H_bv_\mu(\mathscr{V}^\mu-\hat{\rho}^\mu)_{ba}\overline{H_a}]\\&+\frac{\beta^2}{4f^2}\text{Tr}(\overline{H_b}H_a\overline{H_a}H_b).\end{aligned} \tag{8}$$

This Lagrangian contains two unknown parameters, g and β, which are not determined by symmetry arguments, and must be determined empirically. This is the most general even-parity Lagrangian in leading order of the heavy quark mass ($m_Q\to\infty$) and chiral symmetry limit ($m_q\to0$ and the minimal number of derivatives).

We will also need the odd-parity Lagrangian for the heavy meson sector. The lowest order contribution to this Lagrangian is given by

$$\mathscr{L}_{\text{odd}}=i\lambda\,\text{Tr}[H_a\sigma_{\mu\nu}F^{\mu\nu}(\hat{\rho})_{ab}\overline{H_b}]. \tag{9}$$

The parameter λ is free, but we know that this term is of the order $1/\Lambda_\chi$ with Λ_χ being the chiral perturbation theory scale [22].

B. Weak interactions

For the semileptonic decays the weak Lagrangian is given at the quark level by the current-current Fermi interaction

$$\mathscr{L}^{\text{eff}}_{\text{SL}}(\Delta c=\Delta s=1)=-\frac{G_F}{\sqrt{2}}[(\overline{l}u_l)^\mu(\overline{s'}c)_\mu], \tag{10}$$

where G_F is the Fermi constant, $(\overline{\psi_1}\psi_2)^\mu\equiv\overline{\psi_1}\gamma^\mu(1-\gamma^5)\psi_2$, and $s'=s\cos\theta_C+d\sin\theta_C$, θ_C being the Cabibbo angle ($\sin\theta_C\approx0.222$).

At the meson level we assume that the weak current transforms as $(\overline{3}_L,1_R)$ under chiral $SU(3)_L\times SU(3)_R$ and is linear in the heavy meson field. The most general current can then be written as

$$J_\lambda=(D^{*\mu}\hat{A}_{\mu\lambda}+D\hat{B}_\lambda)u^\dagger. \tag{11}$$

The leading order in the $1/M$ expansion is obtained by demanding that the operators $\hat{A}$ and $\hat{B}$ do not act as derivatives on the heavy meson fields D and D^* [23]. We can generally expand $\hat{A}_{\mu\lambda}$ and $\hat{B}_\lambda$ in powers of the operators

$$\hat{\mathscr{O}}^{(1)}_\mu=\hat{\rho}_\mu-\mathscr{V}_\mu,\quad\hat{\mathscr{O}}^{(2)}_\mu=\mathscr{A}_\mu,\quad\hat{\mathscr{O}}^{(3)}_\mu=\partial_\mu+\mathscr{V}_\mu, \tag{12}$$

which are, together with the quark mass matrix insertion, the basic operators in our model with the correct transformation properties. Using the standard (chiral) power counting rules [24], it is easily shown that the operators (12) count as order $O(E)$, having each one derivative or one (light) vector field, while the mass matrix is of higher order, $O(E^2)$. Expanding $\hat{A}$ and $\hat{B}$ to order $O(E)$, i.e., to the linear order in powers of the operators (12), we get, recalling that $v_\mu D^{*\mu}=0$, where v is the heavy meson four-velocity,

$$\hat{A}^{\mu\lambda}=Ag^{\mu\lambda}+[B_{1,i}g^{\mu\lambda}v^{\alpha}+B_{2,i}g^{\mu\alpha}v^{\lambda}+B_{3,i}i\epsilon^{\mu\lambda\alpha\beta}v_{\beta}]\hat{\mathscr{O}}_{\alpha}^{(i)}+\cdots, \tag{13}$$

$$\hat{B}^{\lambda}=Cv^{\lambda}+[D_{1,i}g^{\lambda\alpha}+D_{2,i}v^{\lambda}v^{\alpha}]\hat{\mathscr{O}}_{\alpha}^{(i)}+\cdots. \tag{14}$$

It is clear that such an expansion is a *chiral* expansion, i.e., in powers of energy E, and need *not* be a heavy quark expansion in powers of $1/M$, as it is sometimes assumed [25]. The current (11) together with (13) and (14) is the most general one in our model to order $O(M^0)$ in the heavy quark expansion and to order $O(E^0)$ and $O(E)$ in the chiral expansion.

The $O(E^0)$ coefficients, A and C, can be expressed in terms of the heavy meson pseudoscalar and vector decay constants f_D and f_{D*}, which are equal in the heavy quark limit [11], while no such relations exist between the coefficients $B_{i,k}$ (13) and $D_{i,k}$ (14). However, due to the relation $\hat{O}_\mu^{(3)}u^\dagger=-\hat{O}_\mu^{(2)}u^\dagger$, only the operators $\hat{O}_\mu^{(1)}$ and $\hat{O}_\mu^{(2)}$ are important at order $O(E)$.

In our calculation of the D meson semileptonic decays to leading order in both $1/M$ and the chiral expansion we will need only the current proportional to D, DP, or D^* at order $O(E^0)$ and the current proportional to DV at order $O(E)$. Consequently, we can rewrite Eq. (11) with (13) and (14) as

$$\begin{aligned}J_a^\mu=&\tfrac{1}{2}\,i\alpha\mathrm{Tr}[\gamma^\mu(1-\gamma_5)H_b u_{ba}^\dagger]\\&+\alpha_1\mathrm{Tr}[\gamma_5 H_b(\hat{\rho}^\mu-\mathscr{V}^\mu)_{bc}u_{ca}^\dagger]\\&+\alpha_2\mathrm{Tr}[\gamma^\mu\gamma_5 H_b v_\alpha(\hat{\rho}^\alpha-\mathscr{V}^\alpha)_{bc}u_{ca}^\dagger]+\cdots,\end{aligned} \tag{15}$$

where $\alpha=f_D\sqrt{m_D}$ [12]. The α_1 term was first considered in [7]. We also include the α_2 term, as we must, since it is of the same order in the $1/M$ and chiral expansion as the term proportional to α_1. We will also see that the term proportional to α_2 is very important for the phenomenology of the semileptonic decays and that it cannot be neglected. In the next section we will determine α_1 and α_2 from the experimental data on semileptonic D meson decays.

III. FORM FACTORS FOR $D\to V/P$

The description of semileptonic decays is known near the zero-recoil point, but for the calculation of the branching ratios we need to extrapolate the form factors to different kinematical regions, defined by the square of the transfer momentum q^2. This has been done using the QCD sum-rule analysis [6], quark models [1–4], and lattice calculations [5]. Within a Lagrangian approach this is more difficult, since the form of the interactions is known only in the heavy meson mass limit near zero recoil. Therefore it was assumed in [1] and [8], that all form factors are pole-type functions of q^2, but with different pole masses. With the known values of form factors in the zero-recoil limit, given by the HQET, the form factors in the whole kinematic region then seem to be determined. However, this prescription possesses some shortcomings, which can be seen as follows: the $H\to V$ and $H\to P$ current matrix elements can be quite generally parametrized as

$$\begin{aligned}\langle V(\epsilon,p')|(V-A)^\mu|H(p)\rangle=&-\frac{2V(q^2)}{m_H+m_V}\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu p_\alpha p'_\beta-i\epsilon^*\cdot q\,\frac{2m_V}{q^2}q_\mu A_0(q^2)+i(m_H+m_V)\left(\epsilon^*_\mu-\frac{\epsilon^*\cdot q}{q^2}q_\mu\right)A_1(q^2)\\&-\frac{i\epsilon^*\cdot q}{m_H+m_V}\left((p+p')_\mu-\frac{m_H^2-m_V^2}{q^2}q_\mu\right)A_2(q^2)\end{aligned} \tag{16}$$

and

$$\langle P(p')|(V-A)_\mu|H(p)\rangle=\left[(p+p')_\mu-\frac{m_H^2-m_P^2}{q^2}q_\mu\right]\times F_1(q^2)+\frac{m_H^2-m_P^2}{q^2}q_\mu F_0(q^2), \tag{17}$$

where $q=p-p'$ is the exchanged momentum. In order that these matrix elements are finite at $q^2=0$, the form factors must satisfy the relations

$$A_0(0)+\frac{m_H+m_V}{2m_V}A_1(0)-\frac{m_H-m_V}{2m_V}A_2(0)=0\,, \tag{18}$$

$$F_1(0)=F_0(0). \tag{19}$$

But these equations cannot be satisfied by calculating the form factors at zero recoil where $q^2=q^2_{\max}=(m_H-m_{V(P)})^2$ and then extrapolating them to $q^2=0$ assuming a simple pole q^2 dependence, unless a relation between the model parameters is imposed. It is unreasonable to assume such a relation, since the pole masses are taken from the measured lowest-lying resonances with the correct quantum numbers and, therefore, are not free parameters.

TABLE I. The pole masses and decay constants in GeV.

H	m_H	f_H	P	m_P	f_P	V	m_V
D	1.87	0.24 ± 0.05	π	0.14	0.13	ρ	0.77
D_s	1.97	0.27 ± 0.05	K	0.50	0.16	K^*	0.89
D^*	2.01	0.24 ± 0.05	η	0.55	0.13 ± 0.008	ω	0.78
D_s^*	2.11	0.27 ± 0.05	η'	0.96	0.11 ± 0.007	Φ	1.02

The problem, therefore, is how to extrapolate the amplitude from the zero recoil point to the rest of the allowed kinematical region. We shall make a very simple, physically motivated, assumption: the vertices do not change significantly, while the propagators of the off-shell heavy mesons are given by the full propagators $1/(p^2-m^2)$ instead of the HQET propagators $1/(2mv\cdot k)$. With these assumptions we are able to incorporate the following features: (i) almost exactly the HQET prediction at the maximum q^2; (ii) a natural explanation for the pole-type form factors when appropriate; (iii) predictions of flat q^2 behavior for the form factors A_1 and A_2, which has been confirmed in the QCD sum-rule analysis of [6].

Our approach is different than in [7,8], where a pole dominance prescription for the q^2 dependence of the form factors was assumed, as in the data analysis of the semileptonic $D^0\to K^-$ [13], $D^+\to\overline{K^0}$ [14], and $D^+\to\overline{K^{*0}}$ [15,16]. In contrast, we calculate the form factors directly from our Lagrangian. For the strongly off-shell charm meson propagator we take the complete expression $1/(q^2-m_D^2)$ rather than the heavy meson limit $1/(2m_D v k)$, where $k=q-m_D v$ is the residual momentum (assuming for the moment the degeneracy of the $D-D^*$ system). The difference between the two approaches at $q^2_{\max}$ is less than 25%. Also, in our approach the pole structure $1/(1-q^2/m_D^2)$ of certain diagrams is a direct consequence of the D or D^* full propagators. From the other side, not all diagrams have intermediate D or D^* mesons and then such a q^2 dependence is absent. Consequently, we have a very simple way to determine if a particular form factor has a pole-type behavior, a constant behavior, or some combination.

Finally, we include SU(3) symmetry breaking by using the physical masses and decay constants shown in Table I. The decay constants for η and η' were taken from [26], for D_s from [27], while for the other D's theoretical predictions were used [28].

A. Decays $D\to Vl\nu_l$

There are three possible Cabibbo allowed semileptonic decays ($D^0\to K^{*-}$, $D^+\to\overline{K^{*0}}$, and $D^{s+}\to\Phi$) and four possible Cabibbo suppressed semileptonic decays ($D^0\to\rho^-$, $D^+\to\rho^0$, $D^+\to\omega$, and $D_s^+\to K^{*0}$) of the charmed mesons of the type $D\to Vl\nu_l$. The relevant form factors defined in (16), calculated in our model, are

$$\frac{1}{K_{HV}}V(q^2)=\left[2(m_H+m_V)\left(\frac{m_{H'^*}}{m_H}\right)^{1/2}\frac{m_{H'^*}}{q^2-m_{H'^*}^2}f_{H'^*}\lambda\right]\frac{g_V}{\sqrt{2}}, \tag{20}$$

TABLE II. The pole mesons and the constants K_{HV} for the $D\to V$ Cabibbo allowed and Cabibbo suppressed semileptonic decays.

H	V	H'^*	H'	K_{HV}
D^0	K^{*-}	D_s^{*+}	D_s^+	$\cos\theta_C$
D^+	$\overline{K^{*0}}$	D_s^{*+}	D_s^+	$\cos\theta_C$
D_s^+	Φ	D_s^{*+}	D_s^+	$\cos\theta_C$
D^0	ρ^-	D^{*+}	D^+	$\sin\theta_C$
D^+	ρ^0	D^{*+}	D^+	$-\frac{1}{\sqrt{2}}\sin\theta_C$
D^+	ω	D^{*+}	D^+	$\frac{1}{\sqrt{2}}\sin\theta_C$
D_s^+	K^{*0}	D^{*+}	D^+	$\sin\theta_C$

$$\frac{1}{K_{HV}}A_0(q^2)=\left[\frac{1}{m_V}\left(\frac{m_{H'}}{m_H}\right)^{1/2}\frac{q^2}{q^2-m_{H'}^2}f_{H'}\beta-\frac{\sqrt{m_H}}{m_V}\alpha_1+\frac{1}{2}\left(\frac{q^2+m_H^2-m_V^2}{m_H^2}\right)\frac{\sqrt{m_H}}{m_V}\alpha_2\right]\frac{g_V}{\sqrt{2}}, \tag{21}$$

$$\frac{1}{K_{HV}}A_1(q^2)=\left[-\frac{2\sqrt{m_H}}{m_H+m_V}\alpha_1\right]\frac{g_V}{\sqrt{2}}, \tag{22}$$

$$\frac{1}{K_{HV}}A_2(q^2)=\left[-\frac{m_H+m_V}{m_H\sqrt{m_H}}\alpha_2\right]\frac{g_V}{\sqrt{2}}, \tag{23}$$

where the pole mesons and the corresponding constants K_{HV} are given in Table II. It is convenient to introduce the helicity amplitudes for the decay $H\to Vl^+\nu_l$ as in [6]:

$$H_\pm(y)=+(m_H+m_V)A_1(y)\mp\frac{2m_H|\vec{p'}(y)|}{m_H+m_V}V(y), \tag{24}$$

$$H_0(y)=+\frac{m_H+m_V}{2m_Hm_V\sqrt{y}}[m_H^2(1-y)-m_V^2]A_1(y)-\frac{2m_H|\vec{p'}(y)|^2}{m_V(m_H+m_V)\sqrt{y}}A_2(y), \tag{25}$$

where

$$y=\frac{q^2}{m_H^2}, \tag{26}$$

and

$$|\vec{p'}(y)|^2=\frac{[m_H^2(1-y)+m_V^2]^2}{4m_H^2}-m_V^2. \tag{27}$$

In order to compare with experiment, we calculate the decay rates for polarized final light vector mesons:

$$\Gamma_a=\frac{G_F^2m_H^2}{96\pi^3}\int_0^{y_m}dyy|\vec{p'}(y)||H_a(y)|^2, \tag{28}$$

TABLE III. Four possible solutions for the model parameters as determined by the $D^+ \to \overline{K}^{*0} l^+ \nu_l$ data.

	λ [GeV^{-1}]	α_1 [GeV$^{1/2}$]	α_2 [GeV$^{1/2}$]
Set 1	-0.34 ± 0.07	-0.14 ± 0.01	-0.83 ± 0.04
Set 2	-0.34 ± 0.07	-0.14 ± 0.01	-0.10 ± 0.03
Set 3	-0.74 ± 0.14	-0.064 ± 0.007	-0.60 ± 0.03
Set 4	-0.74 ± 0.14	-0.064 ± 0.007	$+0.18\pm0.03$

where $a=+,-,0$ and

$$y_m=\left(1-\frac{m_V}{m_H}\right)^2. \tag{29}$$

The transverse, longitudinal, and total decay rates are then trivially given by

$$\Gamma_T=\Gamma_+ + \Gamma_-, \tag{30}$$

$$\Gamma_L=\Gamma_0, \tag{31}$$

$$\Gamma=\Gamma_T+\Gamma_L. \tag{32}$$

We must fit three parameters (λ, α_1, α_2) using the three measured values $\Gamma/\Gamma_{\rm tot}=0.048\pm0.004$, $\Gamma_L/\Gamma_T=1.23\pm0.13$, and $\Gamma_+/\Gamma_-=0.16\pm0.04$ for the process $D^+ \to \overline{K}^{*0} l^+ \nu_l$, taken from the Particle Data Group average of data from different experiments [10]. Unfortunately we are not able to determine the parameter β since $A_0(q^2)$ cannot be observed.

Our model parameters appear linearly in the form factors (20)–(23) and hence in the helicity amplitudes (24), (25), so the polarized decay rates (28) are quadratic functions of them. For this reason there are eight sets of solutions for the three parameters $(\lambda,\alpha_1,\alpha_2)$. It was found from the analysis of the strong decays $D^* \to D\pi$ and electromagnetic decays $D^* \to D\gamma$ [21] that the parameter λ has the same sign as the parameter λ', which describes the contribution of the magnetic moment of the heavy (charm) quark. In the heavy quark limit we have $\lambda'=-1/(6m_c)$. Assuming that the finite mass effects are not so large as to change the sign, we find that $\lambda<0$. Therefore only four solutions remain. They are shown in Table III. The errors in Table III were calculated from the experimental errors and the uncertainty in the value of f_{Ds*} (Table I).

With these experimentally determined values of the model parameters it is then straightforward to calculate the branching ratios and polarization variables for the other semileptonic decays of the type $D \to V$. The results change only slightly with different choices for the possible solutions in Table III. For this reason we will quote only the predictions for set 2. The choice of this set comes out naturally, if we assume that the experimental form factors at $q^2=0$ are numerically correct, even if they are obtained assuming an incorrect q^2 dependence. We must however stress that there is really no need for such an assumption; in principle, we could have any of the four possible sets of solutions in Table III. The results for set 2 are shown in Table IV. We see that these results are in agreement with the known experimental data. Since the experimental errors in the input parameters are small, the errors were calculated with the linear formula $\delta_x f=(\partial f/\partial x)\delta x$ and errors from different sources summed in quadrature. The correlations of the errors for λ, α_1, and α_2 were taken into account assuming that the errors in the experimental inputs were not correlated. Of course, the quoted errors do not include any systematic error related to the validity of the model. In fact, it has to be expected that corrections due to the limitations of the chiral or $1/m_c$ expansions would change the results. Expecially the $1/m_c$ corrections could be important, since m_c is far from being infinite. We estimate that the combined error due to $1/m_c$ and CHPT corrections is of the order 30%, which dominates the errors due to the experimental uncertainties in the input parameters. A more precise determination of this error would however involve an explicit calculation, which is beyond the purpose of this paper. For this reason one should not take the quoted errors in the tables too seriously, since they were calculated *assuming* the validity of the model.

TABLE IV. The branching ratios and polarization ratios for the $D \to V$ semileptonic decays. The quoted errors take into account only the experimental uncertainties in the input parameters, but not the validity of the model, as discussed in the text. Where available, the experimental data is quoted in brackets.

Decay	$\mathscr{B}$ [%]	Γ_L/Γ_T	Γ_+/Γ_-
$D^0 \to K^{*-}$	1.9 ± 0.2	1.23 ± 0.13	0.16 ± 0.04
	(2.0 ± 0.4)		
$D_s^+ \to \Phi$	1.7 ± 0.1	1.2 ± 0.1	0.16 ± 0.04
	(1.88 ± 0.29)	(0.6 ± 0.2)	
$D^0 \to \rho^-$	0.17 ± 0.02	1.4 ± 0.2	0.15 ± 0.10
$D^+ \to \rho^0$	0.22 ± 0.02	1.4 ± 0.2	0.15 ± 0.10
	(<0.37)		
$D^+ \to \omega$	0.21 ± 0.02	1.4 ± 0.2	0.16 ± 0.10
$D_s^+ \to K^{*0}$	0.17 ± 0.02	1.3 ± 0.2	0.15 ± 0.10

B. Decays $D \to P l \nu_l$

There are four Cabibbo allowed semileptonic decays ($D^0 \to K^-$, $D^+ \to \overline{K^0}$, $D_s^+ \to \eta$, and $D_s^+ \to \eta'$) and five Cabibbo suppressed semileptonic decays ($D^0 \to \pi^-$, $D^+ \to \pi^0$, $D^+ \to \eta$, $D^+ \to \eta'$, and $D_s^+ \to K^0$) of the charmed mesons of the type $D \to P l \nu_l$.

In our approach the form factors are given by

$$\frac{1}{K_{HP}}F_1(q^2)=-\frac{f_H}{2}+g f_{H'*}\frac{m_{H'*}\sqrt{m_H m_{H'*}}}{q^2-m_{H'*}^2}, \tag{33}$$

$$\frac{1}{K_{HP}}F_0(q^2)=-\frac{f_H}{2}+g f_{H'*}\sqrt{\frac{m_H}{m_{H'*}}}\left(1-2\frac{m_{H'*}^2}{m_H^2-m_P^2}\right) -\left(\frac{f_H}{2}+g f_{H'*}\sqrt{\frac{m_H}{m_{H'*}}}\right)\frac{q^2}{m_H^2-m_P^2} +2g f_{H'*}\left(1-\frac{m_{H'*}^2}{m_H^2-m_P^2}\right)\frac{m_{H'*}\sqrt{m_H m_{H'*}}}{q^2-m_{H'*}^2}, \tag{34}$$

TABLE V. The pole mesons and the constants K_{HP} for the $D \to P$ Cabibbo allowed and Cabibbo suppressed semileptonic decays. The η-η' mixing angle is θ_P and $s=\sin\theta_P$, $c=\cos\theta_P$, while θ_C is the Cabibbo angle.

H	P	H'^*	K_{HP}
D^0	K^-	D_s^{*+}	$(1/f_K)\cos\theta_C$
D^+	$\overline{K^0}$	D_s^{*+}	$(1/f_K)\cos\theta_C$
D_s^+	η	D_s^{*+}	$\frac{1}{\sqrt{8}}[(1-5c^2)/f_\eta - 5sc/f_{\eta'}]\cos\theta_C$
D_s^+	η'	D_s^{*+}	$\frac{1}{\sqrt{8}}[-5sc/f_\eta + (1-5s^2)/f_{\eta'}]\cos\theta_C$
D^0	π^-	D^{*+}	$(1/f_\pi)\sin\theta_C$
D^+	π^0	D^{*+}	$-\frac{1}{\sqrt{2}}(1/f_\pi)\sin\theta_C$
D^+	η	D^{*+}	$\frac{1}{\sqrt{8}}[(1+c^2)/f_\eta + sc/f_{\eta'}]\sin\theta_C$
D^+	η'	D^{*+}	$\frac{1}{\sqrt{8}}[sc/f_\eta + (1+s^2)/f_{\eta'}]\sin\theta_C$
D_s^+	K^0	D^{*+}	$(1/f_K)\sin\theta_C$

where the pole masses and the constants K_{HP} are given in Table V. We shall neglect the lepton mass, so the form factor F_0, which is proportional to q^μ, does not contribute to the decay width. The calculation of the decay rate is very similar to the vector case. After a trivial integration we obtain

$$\Gamma^P = \frac{G_F^2 m_H^2}{24\pi^3}\int_0^{y_m^P} dy |F_1(y)|^2 |\vec{p}\,'^P(y)|^3, \qquad (35)$$

where, similarly as in (29),

$$y_m^P = \left(1 - \frac{m_P}{m_H}\right)^2. \qquad (36)$$

The dimensionless integration variable y has been introduced with the same definition (26) as in the vector case and the three-momentum of the light pseudoscalar meson is given by (27) with m_V replaced by m_P:

$$|\vec{p}\,'^P(y)|^2 = \frac{[m_H^2(1-y)+m_P^2]^2}{4m_H^2} - m_P^2. \qquad (37)$$

Using the best known experimental branching ratio—$\mathscr{B}[D^0 \to K^- l^+ \nu_l] = (3.68 \pm 0.21)\%$ [10], we get two solutions for g:

$$(1) \quad g = 0.08 \pm 0.09, \qquad (38)$$

$$(2) \quad g = -0.90 \pm 0.19. \qquad (39)$$

The quoted errors are mainly due to the uncertainties in the value of the heavy meson decay constant f_D. Unfortunately we are not able to choose between the two possible solutions for g (38) and (39).

TABLE VI. The branching ratios for the $D \to P$ semileptonic decays, where $\mathscr{B}_1$ and $\mathscr{B}_2$ refer to the two possible solutions $g=0.08\pm0.09$ and $g=-0.90\pm0.19$, respectively. The quoted errors take into account only the experimental uncertainties in the input parameters, but not the validity of the model, as discussed in the text.

Decay	$\mathscr{B}_1$	$\mathscr{B}_2$	Expt.
$D^+ \to \overline{K^0}$	9.4 ± 0.5	9.4 ± 0.5	6.7 ± 0.8
$D_s^+ \to \eta$	3 ± 3	2 ± 2	
$D_s^+ \to \eta'$	1.6 ± 0.7	0.9 ± 0.5	
$D_s^+ \to (\eta+\eta')$	4 ± 3	3 ± 3	7.4 ± 3.2
$D^0 \to \pi^-$	0.47 ± 0.05	0.5 ± 0.5	$0.39^{+0.23}_{-0.12}$
$D^+ \to \pi^0$	0.59 ± 0.06	0.7 ± 0.6	0.57 ± 0.22
$D^+ \to \eta$	0.18 ± 0.05	0.1 ± 0.2	
$D^+ \to \eta'$	0.021 ± 0.005	0.01 ± 0.01	
$D_s^+ \to K^0$	0.4 ± 0.2	0.2 ± 0.3	

We have calculated the branching ratios for the other $D \to P$ semileptonic decays, assuming both solutions for g, which give similar results. These results are summarized in Table VI.

IV. SUMMARY

We have proposed a method to include the light vector meson resonances in the weak currents using HQET and CHPT. With the use of the weak and strong Lagrangian, we have analyzed the matrix elements of the weak currents for the decays $D^+ \to \overline{K^0}{}^* l^+ \nu_l$ and $D^0 \to K^- l^+ \nu_l$. Instead of the propagators used in HQET we have used the full propagators for the intermediate heavy meson states. In this way we obtain a pole-type behavior of the form factors for the matrix element of the vector currents, and a constant behavior of the form factors in the case of matrix elements of the axial current. The unknown parameters λ, α_1, and α_2 were determined using the experimental measurements of $\Gamma/\Gamma_{\text{tot}}$, Γ_L/Γ_T, and Γ_+/Γ_- for $D^+ \to \overline{K}{}^{*0}$, giving the four possible solutions quoted in Table III. From the $\mathscr{B}(D^0 \to K^- l^+ \nu_l)$ data the coupling g, defined in the strong Lagrangian for heavy mesons, was determined as well, but an ambiguity gives two possible solutions: $g=0.08\pm0.09$ and $g=-0.90\pm0.19$. We calculated the measured Cabibbo allowed semileptonic decays $D^+ \to \overline{K^0}$, $D_s^+ \to (\eta+\eta')$, $D^0 \to K^{*-}$, and $D_s^+ \to \Phi$, and the Cabibbo suppressed decays $D^0 \to \pi^-$, and $D^+ \to \pi^0$. The calculated branching ratios are in agreement with the experimental results. We have also predicted the other semileptonic decays that have not yet been observed.

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