Decay constants and semileptonic decays of heavy mesons in the relativistic quark model

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We investigate the B and D mesons in the relativistic quark model by applying the variational method with the Gaussian wave function. We calculate the Fermi momentum parameter p_F, and obtain $p_F=0.50-0.54$ GeV, which is almost independent of the input parameters α_s, m_b, m_c, and m_{sp}. We then calculate the ratio f_B/f_D, and obtain a result which is larger, by a factor of about 1.3, than $\sqrt{M_D/M_B}$ given by the naive nonrelativistic analogy. This result is in a good agreement with the recent lattice calculations. We also calculate the ratio $(M_{B*}-M_B)/(M_{D*}-M_D)$. In these calculations the wave function at the origin $\psi(0)$ is essential. We also determine p_F by comparing the theoretical prediction of the ACCMM model with the lepton energy spectrum of $B\to e\nu X$ from the recent ARGUS analysis, and find that $p_F=0.27\pm^{0.22}_{0.27}$ GeV, when we use $m_c=1.5$ GeV. However, this experimentally determined value of p_F is strongly dependent on the value of the input parameter m_c.

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I. INTRODUCTION

B meson physics is important in present high energy physics since it gives us information on the Cabibbo-Kobayaski-Maskawa (CKM) matrix elements V_{cb} and V_{ub} and it is expected to provide the CP violation phenomena. In order to extract the value of V_{ub} from the B meson decay experiments, the method of separating the $B\to X_u l\nu$ events from the $B\to X_c l\nu$ ones at the end-point region of the lepton energy spectrum has been used [1,2]. On the other hand, the method of using the hadronic invariant mass spectrum has also been suggested recently [3]. For the analysis of the inclusive semileptonic decay process, the ACCMM model [4] has been most popularly employed, where the Fermi momentum parameter p_F is introduced as the most important parameter. The value of $p_F\simeq 0.3$ GeV has been commonly used for the experimental analyses without clear experimental or theoretical support. In Ref. [5] we calculated p_F in the relativistic quark model using the variational method with the Gaussian wave function, and obtained the result $p_F=0.54$ GeV. In this paper we also study the ratios, f_B/f_D and $(M_{B*}-M_B)/(M_{D*}-M_D)$ using the same method.

When one treats the heavy-light meson in analogy with the nonrelativistic situation, one expects $f_B/f_D\simeq\sqrt{M_D/M_B}$, since the reduced masses of the light and heavy quark systems of the B and D mesons have similar values, and $f_P^2 M_P=12\,|\psi(0)|^2$ by the van Royen-Weisskopf formula for the pseudoscalar meson P [6], where $\psi(0)$ is the wave function at the origin of the relative motion of quarks. If one uses the relation $f_B/f_D\simeq\sqrt{M_D/M_B}$ with the supplementary relation $f_D/f_{D_s}\simeq\sqrt{m_d/m_s}$, one can obtain the value of f_B from that of f_{D_s}, which has the experimental results from the branching ratio of the theoretically clean $D_s^+\to\mu^+\nu_\mu$ process [7], even though further improvement of the experimental value is required. However, our calculation of f_P in the relativistic quark model, which we present in this article, shows that the above consideration with nonrelativistic analogy is deviated greatly by the relativistic motion of the light quark in the heavy-light pseudoscalar meson P. This deviation has also been exposed by the recent lattice calculations, since they give rather close values for f_B and f_D [8,9]. This situation can be understood clearly within our relativistic calculation.

The potential model has been successful for ψ and Υ families with the nonrelativistic Hamiltonian, since their heavy quarks can be treated nonrelativistically. However, for the D or B meson it has been difficult to apply the potential model because of the relativistic motion of the light quark in the D or B meson. In our calculation we work with the realistic Hamiltonian which is relativistic for the light quark and nonrelativistic for the heavy quark, and adopt the variational method. We take the Gaussian function as the trial wave function, and obtain the ground state energy (and the wave function) by minimizing the expectation value of the Hamiltonian. Using the Gaussian wave function calculated as above, we get the wave function at the origin $\psi(0)$, with which we can calculate the decay constant of the heavy-light pseudoscalar meson from the van Royen–Weisskopf formula. Through this procedure we obtain the ratio f_B/f_D. We also calculate the ratio $(M_{B*}-M_B)/(M_{D*}-M_D)$ from the chromomagnetic hyperfine splitting formula, where the information of $\psi(0)$ is essential. Finally, we compare the value

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of the Fermi momentum p_F given in our calculation with the lepton energy spectrum data of the semileptonic decay process, and find that it is just outside of one σ standard deviation.

In Sec. II, using the variational method in relativistic quark model we calculate the value of the parameter p_F, and the ratios f_B/f_D and $(M_{B*}-M_B)/(M_{D*}-M_D)$. We also extract p_F by comparing the theoretical prediction of the ACCMM model with the whole region of electron energy spectrum of $B\to e\nu X$ in Sec. III. Section IV contains the conclusions.

II. VARIATIONAL METHOD IN RELATIVISTIC QUARK MODEL, AND CALCULATIONS OF f_B/f_D AND $(M_{B*}-M_B)/(M_{D*}-M_D)$

For the B meson system we treat the b quark nonrelativistically, but we treat the u or d quark relativistically with the Hamiltonian

$$H=M+\frac{\mathbf{p}^2}{2M}+\sqrt{\mathbf{p}^2+m^2}+V(r), \tag{1}$$

where $M=m_b$ or m_c is the heavy quark mass and $m=m_{sp}$ is the u or d quark mass, i.e., the spectator quark mass in the ACCMM model. We apply the variational method to the Hamiltonian (1) with the trial wave function

$$\psi(\mathbf{r})=\left(\frac{\mu}{\sqrt{\pi}}\right)^{3/2}e^{-\mu^2r^2/2}, \tag{2}$$

where μ is the variational parameter. The Fourier transform of $\psi(\mathbf{r})$ gives the momentum space wave function $\chi(\mathbf{p})$, which is also Gaussian:

$$\chi(\mathbf{p})=\frac{1}{(\sqrt{\pi}\mu)^{3/2}}e^{-p^2/2\mu^2}. \tag{3}$$

The ground state is given by minimizing the expectation value of H,

$$\langle H\rangle=\langle\psi|H|\psi\rangle=E(\mu),\quad \frac{d}{d\mu}E(\mu)=0\ \text{ at }\ \mu=\overline{\mu}. \tag{4}$$

$\overline{E}\equiv E(\overline{\mu})$ then approximates m_B, and we get $\overline{\mu}=p_F$, the Fermi momentum parameter in the ACCMM model. The value of μ or p_F corresponds to the measure of the radius of the two-body bound state, as can be seen from the relation

$$\langle r\rangle=\frac{2}{\sqrt{\pi}}\frac{1}{\mu}\ \text{ or }\ \langle r^2\rangle^{1/2}=\frac{3}{2}\frac{1}{\mu}.$$

In Eq. (1), we take the Cornell potential, which is composed of the Coulomb and linear potentials:

$$V(r)=-\frac{\alpha_c}{r}+Kr. \tag{5}$$

For the values of the parameters $\alpha_c(\equiv\frac{4}{3}\alpha_s)$, the quark masses m_b and m_c, and K, we use the following two sets of parameters. The set (A) is that of Hagiwara *et al.* [10], which has been determined by the best fit of $(c\overline{c})$ and $(b\overline{b})$ bound state spectra. The set (B) is chosen to have the running coupling constants for the mass scales of m_B and m_D, and the quark masses m_b and m_c that were determined to give the best ψ and Υ masses for the variational ground states:

$$\text{(A)}\quad \alpha_c=0.47,\ m_b=4.75\ \text{GeV},\ m_c=1.32\ \text{GeV},\quad K=0.19\ \text{GeV}^2, \tag{6}$$

$$\text{(B)}\quad \alpha_c^B=0.32,\ m_b=4.64\ \text{GeV},\ \alpha_c^D=0.48,\quad m_c=1.33\ \text{GeV},\ K=0.19\ \text{GeV}^2. \tag{7}$$

With the Gaussian trial wave function (2) or (3), the expectation value of each term of the Hamiltonian (1) is given as

$$\left\langle\frac{\mathbf{p}^2}{2M}\right\rangle=\left\langle\chi(\mathbf{p})\left|\frac{\mathbf{p}^2}{2M}\right|\chi(\mathbf{p})\right\rangle=\frac{3}{4M}\mu^2,$$

$$\langle\sqrt{\mathbf{p}^2+m^2}\rangle=\langle\chi(\mathbf{p})|\sqrt{\mathbf{p}^2+m^2}|\chi(\mathbf{p})\rangle=\frac{4\mu}{\sqrt{\pi}}\int_0^\infty e^{-x^2}\sqrt{x^2+(m/\mu)^2}x^2dx,$$

$$\langle V(r)\rangle=\left\langle\psi(\mathbf{r})\left|-\frac{\alpha_c}{r}+Kr\right|\psi(\mathbf{r})\right\rangle=\frac{2}{\sqrt{\pi}}(-\alpha_c\mu+K/\mu). \tag{8}$$

Then we have

$$E(\mu)=\langle H\rangle=M+\frac{3}{4M}\mu^2+\frac{2}{\sqrt{\pi}}(-\alpha_c\mu+K/\mu)+\frac{4\mu}{\sqrt{\pi}}\int_0^\infty e^{-x^2}\sqrt{x^2+(m/\mu)^2}x^2dx. \tag{9}$$

For more details on this procedure of the variational method, see Ref. [5].

With the input value of $m=m_{sp}=0.15$ GeV, which is the value commonly used in experimental analyses, we minimize $E(\mu)$ of (9), and then we obtain, for the B meson,

$$p_F(B)=\overline{\mu}=0.54\ \text{GeV},\ \overline{E}(B)=5.54\ \text{GeV for (A)},\quad \overline{\mu}=0.50\ \text{GeV},\ \overline{E}(B)=5.52\ \text{GeV for (B)}. \tag{10}$$

The B meson mass is lowered from the above values if we include chromomagnetic hyperfine splitting corrections. For comparison, let us check how sensitive our calculation of p_F is by considering the case where $m=m_{sp}=0$. For $m_{sp}=0$ the integration in (9) is done easily and we obtain the following values of $\overline{\mu}=p_F$:

$$\overline{\mu}=0.53\ \text{GeV},\ \overline{E}(B)=5.52\ \text{GeV for (A)}, \tag{11}$$

$$\overline{\mu}=0.48 \text{ GeV}, \quad \overline{E(B)}=5.49 \text{ GeV for (B)}.$$

As we see in Eq. (11), the results are similar to those in (10), where $m_{sp}=0.15$ GeV. We expected this insensitivity of the value of p_F on m_{sp} because the value of m_{sp}, which should be small in any case, cannot effect the integration in (9) significantly. We also note that the theoretically determined value of $p_F(B)$ is completely independent of the input value of m_c, as can be seen from Eq. (9). Following the same procedure, we next obtain the results for the D meson with $m_{sp}=0.15$ GeV:

$$p_F(D)=\overline{\mu}=0.45 \text{ GeV}, \quad \overline{E(D)}=2.21 \text{ GeV for (A)},$$
$$\overline{\mu}=0.46 \text{ GeV}, \quad \overline{E(D)}=2.21 \text{ GeV for (B)}. \tag{12}$$

The decay constant f_P of a pseudoscalar meson P is defined by the matrix element $\langle 0|A_\mu|P(q)\rangle$:

$$\langle 0|A_\mu|P(q)\rangle = iq_\mu f_P. \tag{13}$$

By considering the low energy limit of the heavy meson annihilation, we have the relation between f_P and the ground state wave function at the origin $\psi_P(0)$ from the van Royen–Weisskopf formula including the color factor [6,7]

$$f_P^2=\frac{12}{M_P}|\psi_P(0)|^2, \tag{14}$$

where M_P is the heavy meson mass. From (14) we have the ratio of f_B and f_D:

$$\frac{f_B}{f_D}=\sqrt{\frac{M_D}{M_B}}\frac{|\psi_B(0)|}{|\psi_D(0)|}. \tag{15}$$

For the Gaussian wave function of Eq. (2), we have

$$\psi_P(0)=\left(\frac{p_F(P)}{\sqrt{\pi}}\right)^{3/2}, \tag{16}$$

then using the values of p_F in (10) and (12), we obtain

$$\frac{f_B}{f_D}=\sqrt{\frac{M_D}{M_B}}\left(\frac{p_F(B)}{p_F(D)}\right)^{3/2}=0.59\times 1.31=0.77 \text{ for (A)}$$
$$=0.59\times 1.13=0.67 \text{ for (B)}. \tag{17}$$

From (17) we see that f_B/f_D is enhanced, compared with $\sqrt{M_D/M_B}$, by the factor 1.31 for the parameter set (A), and by 1.13 for the set (B), which are given by the factor of $|\psi_B(0)/\psi_D(0)|$. Sometimes this factor has been approximated to be 1, and the relation $f_B/f_D\simeq\sqrt{M_D/M_B}$ has been used, by treating it in analogy with the nonrelativistic case [7]. However, our calculation shows that this factor is indeed important and different from 1 significantly. The factor 1.31 obtained in (17) for the parameter set (A) is in fairly good agreement with factors 1.40 of Ref. [8] and 1.39 of Ref. [9] of the recent Lattice calculations.

The mass difference between the vector meson P^* and the pseudoscalar meson P is given rise to by the chromomagnetic hyperfine splitting

$$V_{\rm hf}=\frac{2}{3m_Q\tilde{m}_q}\vec{s}_1\cdot\vec{s}_2\nabla^2\left(-\frac{\alpha_c}{r}\right), \tag{18}$$

where m_Q is the heavy quark mass that we used before (i.e., m_b or m_c), and $\tilde{m}_q$ is the constituent quark mass of the light quark, which is the effective mass for the baryon magnetic moments [11]:

$$\tilde{m}_u=\tilde{m}_d=0.33 \text{ GeV}, \quad \tilde{m}_s=0.53 \text{ GeV}. \tag{19}$$

Then the mass difference between B^* and B mesons is given by

$$M_{B^*}-M_B=\frac{8\pi\alpha_c}{3m_b\tilde{m}_u}|\psi_B(0)|^2. \tag{20}$$

Using the values of p_F in (10) and (12), we obtain the ratio of $M_{B^*}-M_B$ and $M_{D^*}-M_D$ as

$$\frac{M_{B^*}-M_B}{M_{D^*}-M_D}=\frac{m_c}{m_b}\left(\frac{|\psi_B(0)|}{|\psi_D(0)|}\right)^2=\frac{m_c}{m_b}\left(\frac{p_F(B)}{p_F(D)}\right)^3$$
$$=0.28\times 1.73=0.48 \text{ for (A)},$$
$$=0.29\times 1.28=0.37 \text{ for (B)}. \tag{21}$$

The experimental value [12] is about 0.33, which is larger than the nonrelativistic value 0.28 or 0.29, but smaller than our calculated value 0.48 or 0.37. Our calculated results are not much worse than the nonrelativistic values, even though there exists a somewhat large discrepancy compared to the experimental result. This suggests that the reason behind this discrepancy could be more subtle than the nonrelativistic consideration.

Recently, considerable progress has been made on the relation of the ACCMM model with QCD [13–15]. Bigi *et al.* [13] derived an inequality between the expectation value of the kinetic energy of the heavy quark inside the hadron and that of the chromomagnetic operator, which gives

$$\langle \mathbf{p}^2\rangle \geqslant \frac{3}{4}(M_V^2-M_P^2). \tag{22}$$

Bigi *et al.* also gave a field-theoretical derivation of this inequality [16]. The experimental values of the right-hand side of Eq. (22) are 0.366 GeV^2 for the B meson, and 0.410 GeV^2 for the D meson. These bounds correspond to $p_F\geqslant 0.49$ GeV for the B meson, and $p_F\geqslant 0.52$ GeV for the D meson, because in the ACCMM model

$$\langle \mathbf{p}^2\rangle=\int dp\; p^2\phi(p)=\frac{3}{2}p_F^2. \tag{23}$$

These lower bounds of p_F were obtained solely from the fact that the Gaussian distribution was taken in the ACCMM model, and therefore the results are independent of any input parameter values of the ACCMM model. We note that the heavy quark inside the hadron possesses more kinetic energy than the value one might expect naively from the nonrelativistic consideration. Ball and Braun [15] also calculated $\langle \mathbf{p}^2\rangle$ using the QCD sum rule approach, and obtained

$\langle \mathbf{p}^2 \rangle = 0.60 \pm 0.10$ GeV^2 for the B meson, corresponding to $p_F = 0.63 \pm 0.05$ GeV, which is similar to our results in (10).

III. DETERMINATION OF p_F FROM THE EXPERIMENTAL SPECTRUM

Until now, we have discussed the theoretical determination of p_F in the relativistic quark model using the variational method, and its implications to the heavy meson masses and the decay constants of the heavy mesons. Now we would like to determine the Fermi momentum parameter p_F by comparing the theoretical prediction with the experimental charged-lepton energy spectrum in semileptonic decays of B meson.

As discussed, the simplest model for the semileptonic B decay is the spectator model which considers the decaying b quark in the B meson as a free particle. The spectator model is usually used with the inclusion of perturbative QCD radiative corrections [17]. Then the decay width of the process $B \to X_q l \nu$ is given by

$$\Gamma_B(B \to X_q l \nu) \simeq \Gamma_b(b \to q l \nu) = |V_{bq}|^2 \left(\frac{G_F^2 m_b^5}{192\pi^3} \right) f\left(\frac{m_q}{m_b} \right) \times \left[1 - \frac{2}{3} \frac{\alpha_s}{\pi} g\left(\frac{m_q}{m_b} \right) \right], \quad (24)$$

where m_q is the mass of the final q quark decayed from the b quark. As can be seen, the decay width of the spectator model depends on m_b^5, therefore a small difference in m_b would change the decay width significantly.

Altarelli *et al.* [4] proposed for the inclusive B meson semileptonic decays their ACCMM model, which incorporates the bound-state effect by treating the b quark as a virtual-state particle, thus giving momentum dependence to the b quark mass. The virtual-state b quark mass W is given by

$$W^2(\mathbf{p}) = m_B^2 + m_{sp}^2 - 2 m_B \sqrt{\mathbf{p}^2 + m_{sp}^2} \quad (25)$$

in the B meson rest frame, where m_B is the B meson mass, $m_{\rm sp}$ is the spectator quark mass, and $\mathbf{p}$ is the momentum of the spectator quark inside B meson.

For the momentum distribution of the virtual b quark, Altarelli *et al.* considered the Fermi motion inside the B meson with the Gaussian momentum distribution,

$$\phi(\mathbf{p}; p_F) = 4\pi |\chi(\mathbf{p})|^2 = \frac{4}{\sqrt{\pi} p_F^3} e^{-\mathbf{p}^2/p_F^2}, \quad (26)$$

where the Gaussian width, p_F, is treated as a free parameter. Then the lepton energy spectrum of the B meson decay is given by

$$\frac{d\Gamma_B}{dE_l}(p_F, m_{sp}, m_q, m_B) = \int_0^{p_{\max}} dp \; p^2 \phi(\mathbf{p}; p_F) \times \frac{d\Gamma_b}{dE_l}(m_b = W, m_q, m_{sp}), \quad (27)$$

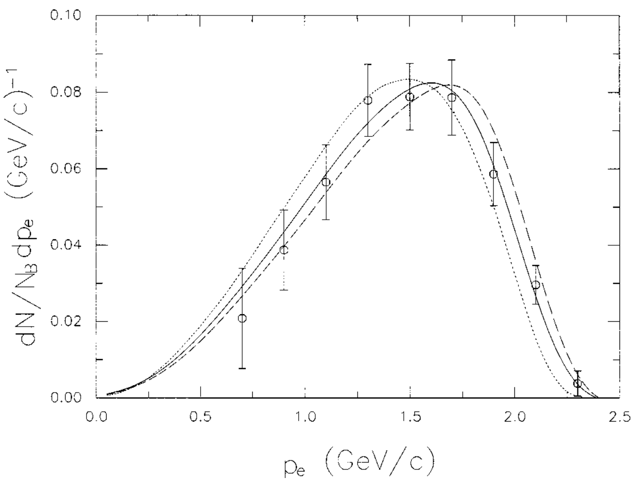

FIG. 1. The normalized lepton energy spectrum of $B \to X_c l \nu$ for the whole region of electron energy from the recent ARGUS [18] measurement. Also shown are the theoretical ACCMM model predictions [Eq. (27)] using $p_F = 0,\ 0.27,\ 0.49$ GeV, corresponding to dashed, full, and dotted lines, respectively. The minimum χ^2 equals 0.59 with $p_F = 0.27$ GeV. We fixed $m_{sp} = 0.15$ GeV and $m_q = m_c = 1.5$ GeV.

where $p_{\max}$ is the maximum kinematically allowed value of $p = |\mathbf{p}|$. The ACCMM model, therefore, introduces a new parameter p_F for the Gaussian momentum distribution of the b quark inside B meson, instead of the b quark mass of the spectator model. In this way the ACCMM model incorporates the bound state effects and reduces the strong dependence on the b quark mass in the decay width of the spectator model. The Fermi momentum parameter p_F is the most essential parameter of the ACCMM model, as we see above. However, the experimental determination of its value from the lepton energy spectrum has been very ambiguous, since various parameters of the ACCMM model, such as p_F, m_q, and m_{sp}, are fitted together from the limited region of the end-point lepton energy spectrum, and because the perturbative QCD corrections are very sensitive in the end-point region of the spectrum.

Recently, ARGUS [18] extracted the model-independent lepton energy spectrum of $B \to X_c l \nu$ for the whole region of electron energy, but with much larger uncertainties, as shown in Fig. 1. We now compare the whole region of the experimental electron energy spectrum with the theoretical prediction of the ACCMM model, [Eq. (27)], using p_F as a free parameter. We fixed $m_{sp} = 0.15$ GeV and $m_q = m_c = 1.5$ GeV, which are the values commonly used in experimental analyses. We derive the value of p_F using χ^2 analysis, and we obtain

$$p_F = 0.27 \pm^{0.22}_{0.27} \text{ GeV}. \quad (28)$$

The minimum χ^2 equals 0.59 with $p_F = 0.27$ GeV. However, the result [Eq. (28)] is found to be strongly dependent on the input value of m_c: if we instead use smaller m_c, both the best fit value of p_F and the minimum χ^2 increase, and vice versa. In Fig. 1, we also show the theoretical ACCMM model spectrums with $p_F = 0,\ 0.27,\ 0.49$ GeV, corresponding to the dashed, full, and dotted lines, respectively. The experimental data and the theoretical predictions are all normalized to the semileptonic branching ratio $B(B \to e\nu X) = 9.6\%$ following the result of ARGUS [18].

In Sec. II, we calculated theoretically the Fermi momentum parameter p_F, and obtained $p_F = 0.50 - 0.54$ GeV. We note that the theoretically calculated values are slightly outside of one σ standard deviation compared with the best-fit value of the experimental data. However, since the experimental spectrum still has large uncertainties, we cannot yet exclude the validity of a relativistic quark model for the calculation of p_F, nor can we exclude the ACCMM model itself to apply to experimental analyses for finding CKM parameters $|V_{cb}|$ and/or $|V_{ub}/V_{cb}|$. In the near future, once we get much more data from asymmetric B factories, it will be very interesting to extract the precise value of p_F once again.

In our previous work [5], we investigated the dependence of $|V_{ub}/V_{cb}|$ on p_F in the ACCMM model, and we found rather strong dependence as a function of a parameter p_F:

$$\begin{aligned} 10^2 \times |V_{ub}/V_{cb}|^2 &= 0.57 \pm 0.11 \\ &\quad (\text{ACCMM with } p_F = 0.3\ [19]), \\ &= 1.03 \pm 0.11\ (\text{ACCMM with } p_F = 0.5), \\ &= 1.02 \pm 0.20\ (\text{Isgur } et\ al.\ [20]). \end{aligned} \tag{29}$$

As can be seen, those values between the ACCMM model with $p_F = 0.3$ GeV and the Isgur *et al.* model are in large disagreement. However, if we use $p_F = 0.5$ GeV, the result of the ACCMM model becomes 1.03, and these two models are in good agreement for the value of $|V_{ub}/V_{cb}|$.

IV. CONCLUSIONS

We conclude that the value $p_F \sim 0.3$ GeV, which has been commonly used in experimental analyses, has no clear theoretical or experimental justification, even though there has recently been an assertion that the prediction of the heavy quark effective theory approach [21], far from the end-point region, gives approximately equal shape to the ACCMM model with $p_F \sim 0.3$ GeV. Therefore, it is strongly recommended to determine the value of p_F more reliably and independently, when we think of the importance of its role in experimental analyses. It is particularly important in the determination of the value of $|V_{ub}/V_{cb}|$. A better determination of p_F is also interesting theoretically since it has its own physical correspondence related to the Fermi motion inside the B meson. In this context we calculated theoretically the value of p_F in the relativistic quark model using the quantum mechanical variational method. It turns out that $p_F = 0.50 - 0.54$ GeV, which is not far from the value of p_F determined by comparing the ACCMM model prediction and the model independent lepton energy spectrum of the ARGUS measurement, $p_F = 0.27 \pm^{0.22}_{0.27}$ GeV. The theoretically determined value of p_F is almost independent of input parameters, α_s, m_b, m_c, m_{sp}, etc. On the other hand, the experimentally determined value is strongly affected by the value of the input parameter m_c.

By using the same framework, we then calculated the ratio f_B/f_D, and obtaind a result which is larger, by a factor of about 1.3, than $\sqrt{M_D/M_B}$ given by the naive nonrelativistic analogy. This result is in fairly good agreement with the recent lattice calculations. We also calculated the ratio $(M_{B*} - M_B)/(M_{D*} - M_D)$, whose results suggest the subtlety of the mechanism which gives rise to the value of this ratio.

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