$|V_{ub}|$ from exclusive B and D decays

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We propose a model-independent method to determine the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}|$ from exclusive B and D decays. Combining information obtainable from $B\to\rho\ell\overline{\nu_\ell}$, $B\to K^*\nu\overline{\nu}$, $D\to\rho\bar{\ell}\nu_\ell$, and $D\to K^*\bar{\ell}\nu_\ell$, a determination of $|V_{ub}|$ is possible, with an uncertainty from theory of around 10%. Theoretical uncertainties in the $B\to K^*\ell\bar{\ell}$ decay rate are discussed.

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I. INTRODUCTION

In the minimal standard model the couplings of the W bosons to the quarks are given in terms of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{ij}, which arises from diagonalizing the quark mass matrices. In the minimal standard model (i.e., one Higgs doublet), it is this matrix that is responsible for the CP nonconservation observed in kaon decay. A precise determination of the elements of the CKM matrix will play an important role in testing this picture for the origin of CP violation, and will constrain extensions of the standard model that make predictions for the form of the quark mass matrices.

The present value of the $b\to u$ element of the CKM matrix, $|V_{ub}|\simeq(0.002–0.005)$ [1] arises from a comparison of the end point region of the electron spectrum in semileptonic B decay with phenomenological models. In recent years, there has been a dramatic improvement in our understanding of the theory of inclusive semileptonic B decays [2–4]. It was shown that the electron energy spectrum $d\Gamma/dE_e$ can be predicted, including nonperturbative strong interaction effects that are parametrized by the matrix elements of local operators between B meson states. For typical values of the electron energy E_e, the lowest dimension operators are the most important and the small nonperturbative strong interaction corrections are dominated by only two matrix elements, one of which is already determined by the measured B^*-B mass splitting [3,4]. However, for the semileptonic decay rate in the end point region, $(m_B^2-m_D^2)/2m_B<E_e<(m_B^2-m_\pi^2)/2m_B$ (where low mass hadronic final states are more important), the nonperturbative strong interaction corrections are large and an infinite set of nonperturbative matrix elements are needed. It has been shown that the same matrix elements determine the rate for $B\to X_s\gamma$ in the region where the photon energy is near its maximal value [5]. In principle, experimental information on $B\to X_s\gamma$ can be used to predict the electron spectrum in the end point region of semileptonic B decay, leading to a model-independent determination of $|V_{ub}|$.

In this paper we propose a method for getting a precise model-independent value for $|V_{ub}|$, using exclusive B and D decays. Our approach gives a value of $|V_{ub}|$ that (apart from some very small factors) is valid in the limit of SU(3) flavor symmetry (on the u, d, and s quarks) or in the limit of SU(4) heavy quark spin-flavor symmetry [6] (on the c and b quarks). Consequently, the leading corrections are suppressed by factors of the small quantity $(m_s/m_c-m_s/m_b)\simeq0.1$ or $(m_s/1\text{ GeV})[\alpha_s(m_c)/\pi-\alpha_s(m_b)/\pi]\simeq0.01$, and a determination of $|V_{ub}|$ with a theoretical uncertainty of about 10% is possible.

Semileptonic $D\to K^*\bar{\ell}\nu_\ell$ decay ($\ell=e,\mu$) has been studied extensively and the form factors which characterize the hadronic $D\to K^*$ matrix element of the weak current have been determined (with some assumptions concerning their shape) from the data. In this paper we denote the form factors relevant for semileptonic transitions between a pseudoscalar meson containing a heavy quark H, and a member of the lowest-lying multiplet of vector mesons, V, by $g^{(H\to V)}$, $f^{(H\to V)}$, and $a_\pm^{(H\to V)}$, where

$$\langle V(p',\epsilon)|\overline{q}\gamma_\mu Q|H(p)\rangle = ig^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}(p+p')^\lambda(p-p')^\sigma, \quad (1a)$$

$$\langle V(p',\epsilon)|\overline{q}\gamma_\mu\gamma_5 Q|H(p)\rangle = f^{(H\to V)}\epsilon^*_\mu + a_+^{(H\to V)}(\epsilon^*\cdot p)(p+p')_\mu + a_-^{(H\to V)}(\epsilon^*\cdot p)(p-p')_\mu, \quad (1b)$$

and $\varepsilon^{0123}=-\varepsilon_{0123}=1$. The sign of g depends on this convention for the Levi-Civita tensor. We view the form factors g, f and $a_\pm$ as functions of the dimensionless variable $y=v\cdot v'$, where $p=m_H v$, $p'=m_V v'$, and $q^2=(p-p')^2=m_H^2+m_V^2-2m_H m_V y$. (Note that even though we are using the variable $v\cdot v'$, we are not treating the quarks in V as heavy.) The experimental values for the form factors for $D\to K^*\bar{\ell}\nu_\ell$ are [1]

$$f^{(D\to K^*)}(y)=\frac{1.8\text{ GeV}}{1+0.63(y-1)}, \quad (2a)$$

$$a_+^{(D\to K^*)}(y)=-\frac{0.17\text{ GeV}^{-1}}{1+0.63(y-1)}, \quad (2b)$$

$$g^{(D\to K^*)}(y)=-\frac{0.51\text{ GeV}^{-1}}{1+0.96(y-1)}. \quad (2c)$$

The form factor a_- is not measured because its contribution to the $D\to K^*\bar{\ell}\nu_\ell$ decay amplitude is proportional to the lepton mass. The minimal value of y is unity (corresponding

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to the zero recoil point where the K^* is at rest in the D rest frame) and the maximum value of y is $(m_D^2+m_{K^*}^2)/(2m_D m_{K^*})\simeq 1.3$ (corresponding to maximal K^* recoil in the D rest frame). Note that over the whole kinematic range $1<y<1.3$, f changes by less than 20%. Therefore, in the following analysis of B decays, we can extrapolate the form factors with a small uncertainty to a somewhat larger region, which in what follows we take to be $1<y<1.5$. The full kinematic region for $B\to\rho\ell\overline{\nu_\ell}$ is $1<y<3.5$.

II. SEMILEPTONIC $B\to\rho\ell\overline{\nu_\ell}$ DECAY

The differential decay rate for semileptonic B decay (neglecting the lepton mass), not summed over the lepton type ℓ, is

$$\frac{d\Gamma(B\to\rho\ell\overline{\nu_\ell})}{dy}=\frac{G_F^2|V_{ub}|^2}{48\,\pi^3}m_B^3 r^2 S(y), \tag{3}$$

where $r=m_\rho/m_B$ and $S(y)$ is the function

$$\begin{aligned}S(y)=&\sqrt{y^2-1}[|f^{(B\to\rho)}(y)|^2(2+y^2-6yr+3r^2)\\&+4\,\mathrm{Re}[a_+^{(B\to\rho)}(y)f^{(B\to\rho)}(y)]m_B^2 r(y-r)\\&\times(y^2-1)+4|a_+^{(B\to\rho)}(y)|^2 m_B^4 r^2(y^2-1)^2\\&+8|g^{(B\to\rho)}(y)|^2 m_B^4 r^2(1+r^2-2yr)(y^2-1)]\\=&|f^{(B\to\rho)}(y)|^2[1+\delta^{(B\to\rho)}(y)]\\&\times\sqrt{y^2-1}(2+y^2-6yr+3r^2).\end{aligned} \tag{4}$$

The function $\delta^{(B\to\rho)}$ depends on the ratios of form factors $a_+^{(B\to\rho)}/f^{(B\to\rho)}$ and $g^{(B\to\rho)}/f^{(B\to\rho)}$.

We can estimate $S(y)$ using combinations of heavy quark symmetry and SU(3) flavor symmetry. Heavy quark symmetry implies the relations [7]

$$f^{(B\to K^*)}(y)=\left(\frac{m_B}{m_D}\right)^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}f^{(D\to K^*)}(y), \tag{5a}$$

$$\begin{aligned}a_+^{(B\to K^*)}(y)=&\frac{1}{2}\left(\frac{m_D}{m_B}\right)^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}\\&\times\left[a_+^{(D\to K^*)}(y)\left(1+\frac{m_c}{m_b}\right)\right.\\&\left.-a_-^{(D\to K^*)}(y)\left(1-\frac{m_c}{m_b}\right)\right],\end{aligned} \tag{5b}$$

$$g^{(B\to K^*)}(y)=\left(\frac{m_D}{m_B}\right)^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}g^{(D\to K^*)}(y). \tag{5c}$$

SU(3) symmetry implies that the $\overline{B^0}\to\rho^+$ form factors are equal to the $B\to K^*$ form factors and the $B^-\to\rho^0$ form factors are equal to $1/\sqrt{2}$ times the $B\to K^*$ form factors. In the limit where the heavy quark Q has large mass, the matrix elements in Eqs. (1) depend on m_Q only through a factor of $\sqrt{m_H}$ associated with the normalization of the heavy meson states. Consequently, for large m_c, $(a_+^{(D\to K^*)}+a_-^{(D\to K^*)})/(a_+^{(D\to K^*)}-a_-^{(D\to K^*)})$ is of order $\Lambda_{\mathrm{QCD}}/m_c$, so we can set $a_-^{(D\to K^*)}=-a_+^{(D\to K^*)}$ in Eq. (5b), yielding

$$a_+^{(B\to K^*)}(y)=\left(\frac{m_D}{m_B}\right)^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}a_+^{(D\to K^*)}(y). \tag{6}$$

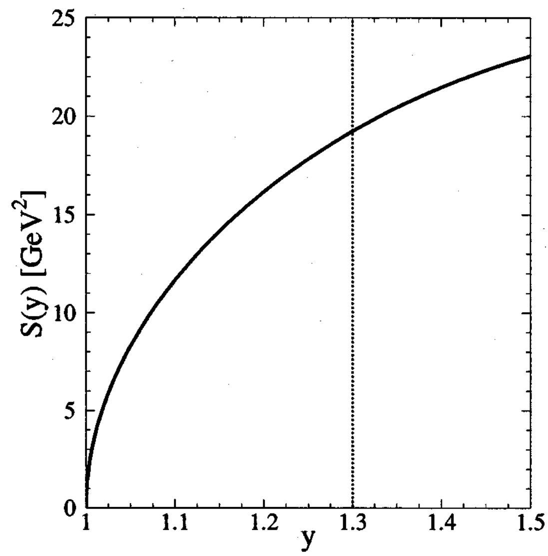

FIG. 1. The function $S(y)$ defined in Eq. (4) as a function of the kinematic variable $y=v\cdot v'$. The dotted vertical line corresponds to the kinematic limit for $D\to K^*\overline{\ell\nu_\ell}$.

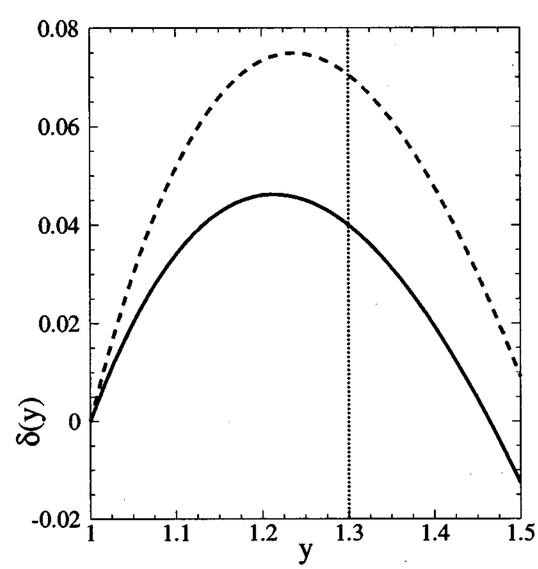

FIG. 2. The function $\delta(y)$ as a function of the kinematic variable $y=v\cdot v'$. The solid curve is $\delta^{(B\to\rho)}(y)$, the dashed curve is $\delta^{(B\to K^*)}(y)$.

Using Eqs. (5a), (5c), and (6), and SU(3) to get the $\overline{B^0}\to\rho^+\ell\overline{\nu_\ell}$ form-factors (in the region $1<y<1.5$) from those for $\overline{D}\to K^*\ell\nu_\ell$, given in Eq. (2a), gives $S(y)$ plotted in Fig. 1. We use $\alpha_s(m_b)=0.22$ and $\alpha_s(m_c)=0.39$. In Fig. 2 we plot $\delta^{(B\to\rho)}(y)$ and $\delta^{(B\to K^*)}(y)$ as functions of y. The latter function (which will be used later in this paper) is denoted by the dashed curve. Perhaps the largest uncertainty in our analysis for δ comes from setting $a_-^{(D\to K^*)}=-a_+^{(D\to K^*)}$. If $a_-^{(D\to K^*)}=-\lambda a_+^{(D\to K^*)}$, then Eq. (6) gets multiplied on its right-hand side by the factor $(1+m_D/m_B)/2+\lambda(1-m_D/m_B)/2$. In Fig. 3 we plot $\delta^{(B\to\rho)}$ and $\delta^{(B\to K^*)}$ for $\lambda=0$ and 2.

Note that δ is fairly small, indicating that $a_+^{(B\to\rho)}$ and $g^{(B\to\rho)}$ make small contributions to $S(y)$ (in the region $1<y<1.5$), so even significant corrections to Eq. (6) will not have any large impact on $S(y)$. We can use our prediction for $S(y)$ to determine $|V_{ub}|$ from the $B\to\rho\ell\overline{\nu_\ell}$ semileptonic decay rate in the region $1<y<1.5$. Our predicted $S(y)$, Fig. 1, gives a branching ratio of $5.2|V_{ub}|^2$ for $\overline{B^0}\to\rho^+\ell\overline{\nu_\ell}$ in the region $1<y<1.5$ (corresponding to $16\ \text{GeV}^2<q^2<q^2_{\max}=20\ \text{GeV}^2$, which implies $E_\ell>1.6$ GeV in the B rest frame). While such a model-independent determination of $|V_{ub}|$ may eventually be superior to a determination from a comparison of the end point of the electron spectrum with phenomenological models [8,9], there will be a sizable theoretical uncertainty associated with $|V_{ub}|$, determined in this way from order m_s SU(3) violation and order $1/m_{c,b}$ corrections to relations (5) and (6). What is needed to get a value for $|V_{ub}|$ with smaller theoretical uncertainties is an improved method for determining $|f^{(B\to\rho)}|^2(1+\delta^{(B\to\rho)})$.

Our method for determining a precise value for $|V_{ub}|$ is based on the observation that the ''Grinstein-type double ratio'' [10] $(f^{(B\to\rho)}/f^{(B\to K^*)})/(f^{(D\to\rho)}/f^{(D\to K^*)})$ is equal to unity in three separate limits of QCD (isospin violation is neglected here): (i) the limit of SU(3) flavor symmetry, $m_s\to 0$, where the strange quark mass is treated as small compared with a typical hadronic scale; (ii) the limit of SU(4) heavy quark spin-flavor symmetry, $m_{b,c}\to\infty$, where the bottom and charm quark masses are treated as large compared with a typical hadronic scale; (iii) the limit $m_c=m_b$, where the bottom and the charm quarks are related by an SU(2) flavor symmetry. Consequently,

$$f^{(B\to\rho)}=f^{(B\to K^*)}\,\frac{f^{(D\to\rho)}}{f^{(D\to K^*)}}\times\left[1+O\left(\frac{m_s}{m_c}-\frac{m_s}{m_b},\frac{m_s}{1\ \text{GeV}}\,\frac{\alpha_s(m_c)-\alpha_s(m_b)}{\pi}\right)\right]. \tag{7}$$

We propose to extract a precise value for $|f^{(B\to\rho)}|^2(1+\delta^{(B\to\rho)})$, using

$$|f^{(B\to\rho)}|^2(1+\delta^{(B\to\rho)})=|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})\left|\frac{f^{(D\to\rho)}}{f^{(D\to K^*)}}\right|^2. \tag{8}$$

Multiplying by the ratio of D decay form factors above, cancels out SU(3) violation not suppressed by factors of the heavy quark mass in the most important part of the $B\to\rho\ell\overline{\nu_\ell}$ differential decay rate, i.e., the factor of $|f^{(B\to\rho)}|^2$, leaving an uncertainty from SU(3) violation only in δ. Since, as we have argued, $|\delta|$ is likely to be less than 0.15, the effects of SU(3) violation in it can safely be neglected. The plots in Figs. 2 and 3 show the kinematic sources of SU(3) violation in δ arising from the fact that the ρ and K^* masses are not equal. There are also contributions from SU(3) violation in the ratios of the form factors a_+/f and g/f.

In principle, the form factor $f^{(D\to\rho)}$ can be obtained from experimental information on the Cabibbo suppressed decay $D\to\rho\overline{\ell}\nu_\ell$. However, at the present time, the small branching ratio [1] $\mathscr{B}(D^+\to\rho^0\overline{\mu}\nu_\mu)=(2.0^{+1.5}_{-1.3})\times 10^{-3}$ has made extraction of the form factor $f^{(D\to\rho)}$ too difficult. It may be possible in future to determine $f^{(D\to\rho)}$ from fixed target ex-

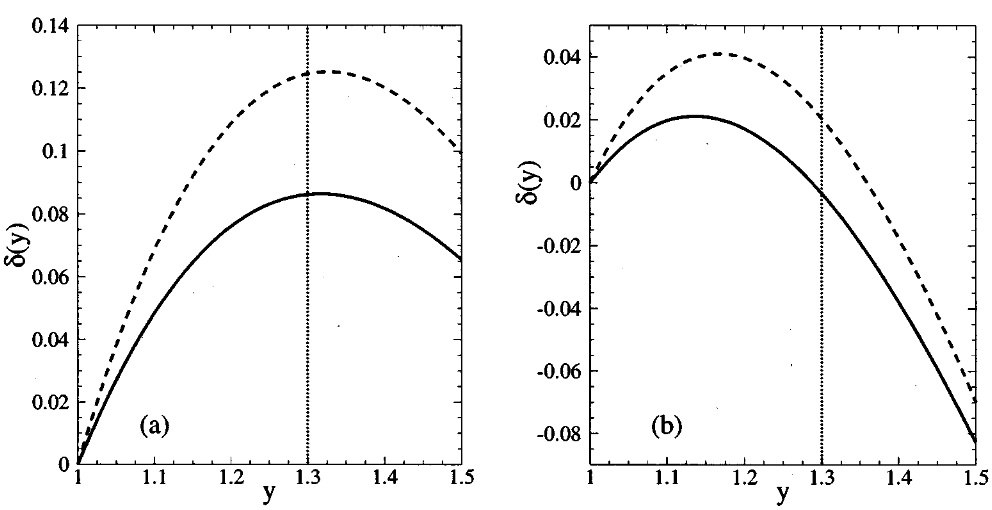

FIG. 3. The function $\delta(y)$ as a function of the kinematic variable $y=v\cdot v'$. (a) corresponds to $\lambda=0$, (b) to $\lambda=2$. The solid curves are $\delta^{(B\to\rho)}(y)$; the dashed curves are $\delta^{(B\to K^*)}(y)$.

periments or at a tau-charm factory. Assuming this can be done, the factor $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ is the remaining ingredient needed for a determination of $|f^{(B\to\rho)}|^2(1+\delta^{(B\to\rho)})$ via Eq. (8).

III. RARE *B* DECAYS

One avenue to find the factor $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ uses the exclusive rare decays $B\to K^*\ell\bar{\ell}$ or $B\to K^*\nu\bar{\nu}$, which may eventually be studied at hadron colliders, or at B factories. The effective Hamiltonian for these decays is [11–14]

$$\mathcal{H}_{\rm eff}=-\frac{4G_F}{\sqrt{2}}V_{ts}^*V_{tb}\sum C_i(\mu)O_i(\mu), \tag{9}$$

where μ is the subtraction point (hereafter we set $\mu=m_b$ and do not explicitly display the subtraction point dependence), and the operators O_i are

$$O_1=(\overline{s_{L\alpha}}\gamma_\mu b_{L\alpha})(\overline{c_{L\beta}}\gamma^\mu c_{L\beta}), \tag{10a}$$

$$O_2=(\overline{s_{L\alpha}}\gamma_\mu b_{L\beta})(\overline{c_{L\beta}}\gamma^\mu c_{L\alpha}), \tag{10b}$$

$$O_3=(\overline{s_{L\alpha}}\gamma_\mu b_{L\alpha})[(\overline{u_{L\beta}}\gamma^\mu u_{L\beta})+\cdots+(\overline{b_{L\beta}}\gamma^\mu b_{L\beta})], \tag{10c}$$

$$O_4=(\overline{s_{L\alpha}}\gamma_\mu b_{L\beta})[(\overline{u_{L\beta}}\gamma^\mu u_{L\alpha})+\cdots+(\overline{b_{L\beta}}\gamma^\mu b_{L\alpha})], \tag{10d}$$

$$O_5=(\overline{s_{L\alpha}}\gamma_\mu b_{L\alpha})[(\overline{u_{R\beta}}\gamma^\mu u_{R\beta})+\cdots+(\overline{b_{R\beta}}\gamma^\mu b_{R\beta})], \tag{10e}$$

$$O_6=(\overline{s_{L\alpha}}\gamma_\mu b_{L\beta})[(\overline{u_{R\beta}}\gamma^\mu u_{R\alpha})+\cdots+(\overline{b_{R\beta}}\gamma^\mu b_{R\alpha})], \tag{10f}$$

$$O_7=(e/16\pi^2)m_b(\overline{s_L}\sigma_{\mu\nu}b_R)F^{\mu\nu}, \tag{10g}$$

$$O_8=(g/16\pi^2)m_b(\overline{s_L}\sigma_{\mu\nu}b_R)G^{\mu\nu}, \tag{10h}$$

$$O_9=(e^2/16\pi^2)(\overline{s_L}\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\ell), \tag{10i}$$

$$O_{10}=(e^2/16\pi^2)(\overline{s_L}\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\gamma_5\ell), \tag{10j}$$

$$O_{11}=(e^2/16\pi^2\sin^2\theta_W)(\overline{s_L}\gamma_\mu b_L)[\bar{\nu}\gamma^\mu(1-\gamma_5)\nu]. \tag{10k}$$

For $B\to K^*\ell\bar{\ell}$, we need the matrix elements of O_1–O_6 and O_8 at order e^2 and to all orders in the strong interactions, and the matrix elements of O_7, O_9, and O_{10} to all orders in the strong interactions. Among the contributions to the $B\to K^*\ell\bar{\ell}$ matrix element of O_1–O_6 are the Feynman diagrams in Fig. 4, where a soft gluon (with momentum of order $k\ll\sqrt{q^2}$) connects to the $q\bar{q}$ loop. We are interested in the kinematic region $1<y<1.5$ which corresponds to a $\ell\bar{\ell}$ pair with large invariant mass squared q^2 between 14.5 GeV^2 and 19 GeV^2. In this kinematic region we have found by explicit computation that the contribution of the Feynman diagrams in Fig. 4 are suppressed by at least a factor of $k/\sqrt{q^2}$ compared, for example, to the contributions of the diagrams in Fig. 5. In the region of large q^2 (compared with the QCD

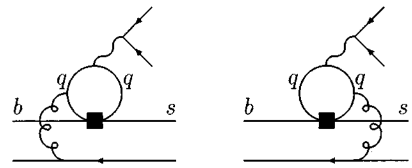

FIG. 4. Feynman diagrams whose contributions to exclusive rates are neither included in the form factors, nor in the effective Wilson coefficient $\widetilde{C}_9$. The black square represents one of the four-quark operators O_1–O_6.

scale and the mass of the quark q), the $q\bar{q}$ pair must "quickly" convert into the (color singlet) $\ell\bar{\ell}$ pair and hence the coupling of soft, long wavelength gluons to the $q\bar{q}$ pair is suppressed at all orders in QCD perturbation theory. Similar remarks hold for the matrix elements of O_8. This "factorization conjecture" implies that for $B\to K^*\ell\bar{\ell}$ at large q^2, we can take the matrix elements of O_1–O_6 and O_8 into account by adjusting the coefficients of O_7 and O_9 by a calculable short distance correction. In the next-to-leading logarithmic approximation, C_9 is replaced by an effective $\widetilde{C}_9(y)$ coupling [13]

$$\begin{aligned}\widetilde{C}_9(y)=&C_9+h(z,y)(3C_1+C_2+3C_3+C_4+3C_5+C_6)\\&-\frac{1}{2}h(0,y)(C_3+3C_4)-\frac{1}{2}h(1,y)\\&\times(4C_3+4C_4+3C_5+C_6)\\&+\frac{2}{9}(3C_3+C_4+3C_5+C_6).\end{aligned} \tag{11}$$

Here,

$$h(z,y)=-\frac{8}{9}\ln z+\frac{8}{27}+\frac{4}{9}x-\frac{2}{9}(2+x)\sqrt{|1\text{-}x|}\times\begin{cases}\left(\ln\left|\frac{\sqrt{1\text{-}x}+1}{\sqrt{1\text{-}x}-1}\right|-i\pi\right) & \text{for } x\equiv 4m_c^2/q^2<1,\\ 2\arctan(1/\sqrt{x-1}) & \text{for } x\equiv 4m_c^2/q^2>1\ ,\end{cases} \tag{12}$$

with $h(0,y)=8/27-(4/9)[\ln(q^2/m_b^2)-i\pi]$, and $z=m_c/m_b$, $r=m_{K^*}/m_B$. On the right-hand side of Eq. (12)

TABLE I. Coefficients of the O_9–O_{11} operators at the scale m_b for different values of the top quark mass. C_{10} is calculated in the leading logarithmic approximation, while C_9 and C_{11} are calculated to next-to-leading order accuracy. For C_9, in the next-to-leading logarithmic approximation, terms of order α_s are subdominant, since the leading contribution to C_9 is order $\ln(m_W^2/m_b^2)\sim 1/\alpha_s$.

	$m_t=165$ GeV	$m_t=175$ GeV	$m_t=185$ GeV
C_9	4.17	4.26	4.34
C_{10}	-4.21	-4.62	-5.04
C_{11}	1.40	1.48	1.57

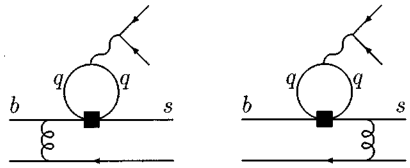

FIG. 5. Feynman diagrams whose contributions to exclusive rates are parts of the nonperturbative matrix element of $\widetilde{C}_9 O_9$.

$q^2 = m_B^2 + m_{K*}^2 - 2m_B m_{K*} y$ should be understood. Figure 5 is now part of the nonperturbative matrix element of $\widetilde{C}_9 O_9$. Note that Eq. (11) differs from Ref. [13], since the one-gluon correction to the matrix element of O_9 is viewed as a contribution to the form factors in our case.

Using $m_t = 175$ GeV, $m_b = 4.8$ GeV, $m_c = 1.4$ GeV, $\alpha_s(m_W) = 0.12$, $\alpha_s(m_b) = 0.22$, and $\sin^2\theta_W = 0.23$, the numerical values of the Wilson coefficients in the leading logarithmic approximation are $C_1 = -0.26$, $C_2 = 1.11$, $C_3 = 0.01$, $C_4 = -0.03$, $C_5 = 0.008$, $C_6 = -0.03$, $C_7 = -0.32$. The operator O_8 does not contribute at the order we are working. C_9, C_{10}, and C_{11} depend more sensitively on m_t (quadratically for $m_t \gg m_W$). In Table I we give their values for $m_t = 165$ GeV, $m_t = 175$ GeV, and $m_t = 185$ GeV.

In Eq. (11) the second term on the right-hand side, proportional to $h(z,y)$ comes from charm quark loops. Since q^2 is close to $4m_c^2$, one is not in a kinematic region where the perturbative QCD calculation of the $c\bar{c}$ loop (or factorization) can be trusted. Threshold effects, which spoil local duality, may be important. [In the kinematic region near $q^2 = 0$, the charm quarks in the loop are far off shell and Eq. (12) should be valid. However, in this region we cannot justify using Eq. (11) for the light quark loops.] Later, we examine the sensitivity of the $B \to K^* \ell \bar{\ell}$ rate in the kinematic region of interest to $c\bar{c}$ threshold effects. For slightly lower values of q^2 (or equivalently for larger values of y) than we consider, such effects are very important. The rates for $B \to K^* J/\psi \to K^* \ell \bar{\ell}$ and for $B \to K^* \psi' \to K^* \ell \bar{\ell}$ are much greater than what Eq. (11) would imply. The latter process occurs with the ψ' on mass shell at $y = 1.6$.

The hadronic matrix element of O_7 is expressed in terms of new hadronic form factors $g_\pm$ and h defined by

$$\langle V(p',\epsilon)|\overline{q}\sigma_{\mu\nu}Q|H(p)\rangle = g_+^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p+p')^\sigma + g_-^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p-p')^\sigma + h^{(H\to V)}\varepsilon_{\mu\nu\lambda\sigma}(p+p')^\lambda(p-p')^\sigma(\epsilon^*\cdot p), \tag{13a}$$

$$\langle V(p',\epsilon)|\overline{q}\sigma_{\mu\nu}\gamma_5 Q|H(p)\rangle = ig_+^{(H\to V)}[\epsilon_\nu^*(p+p')_\mu - \epsilon_\mu^*(p+p')_\nu] + ig_-^{(H\to V)}[\epsilon_\nu^*(p-p')_\mu - \epsilon_\mu^*(p-p')_\nu] + ih^{(H\to V)}[(p+p')_\nu(p-p')_\mu - (p+p')_\mu(p-p')_\nu](\epsilon^*\cdot p). \tag{13b}$$

The second relation is obtained from the first one using $\sigma^{\mu\nu} = \frac{i}{2}\varepsilon^{\mu\nu\lambda\sigma}\sigma_{\lambda\sigma}\gamma_5$. The differential decay rate for $B \to K^* \ell \bar{\ell}$ (not summed over the lepton-type ℓ) is

$$\frac{d\Gamma(B\to K^*\ell\bar{\ell})}{dy} = \frac{G_F^2|V_{ts}^*V_{tb}|^2}{24\pi^3}\left(\frac{\alpha}{4\pi}\right)^2 m_B^3 r^2[|\widetilde{C}_9(y)|^2 S'(y) + |C_{10}|^2 S(y)], \tag{14}$$

where $S(y)$ is given by the expression in Eq. (4), with the form factors replaced by those appropriate for $B \to K^*$, and $r = m_{K*}/m_B$. $S'(y)$ is obtained from $S(y)$ via the replacements

$$f^{(B\to K^*)} \to f^{(B\to K^*)} + [g_+^{(B\to K^*)}(m_B^2 - m_{K*}^2) + g_-^{(B\to K^*)} m_B^2(1+r^2-2yr)]A(y), \tag{15a}$$

$$a_+^{(B\to K^*)} \to a_+^{(B\to K^*)} + [h^{(B\to K^*)} m_B^2(1+r^2-2yr) - g_+^{(B\to K^*)}]A(y), \tag{15b}$$

$$g^{(B\to K^*)} \to g^{(B\to K^*)} - g_+^{(B\to K^*)}A(y), \tag{15c}$$

where $A(y) = 2m_b C_7/[m_B^2(1+r^2-2yr)\widetilde{C}_9(y)]$. Since C_7 is small compared to $\widetilde{C}_9$, it is convenient to rewrite the differential decay rate as

$$\frac{d\Gamma(B\to K^*\ell\bar{\ell})}{dy} = \frac{G_F^2|V_{ts}^*V_{tb}|^2}{24\pi^3}\left(\frac{\alpha}{4\pi}\right)^2 m_B^3 r^2[|\widetilde{C}_9(y)|^2 + |C_{10}|^2] \times |f^{(B\to K^*)}(y)|^2[1+\delta^{(B\to K^*)}(y)]\sqrt{y^2-1}(2+y^2-6yr+3r^2)[1+\Delta(y)], \tag{16}$$

TABLE II. Mass, width, and leptonic branching ratio of the 1^{--} $c\bar{c}$ resonances [1].

	$M_{\psi^{(n)}}$ [GeV]	$\Gamma_{\psi^{(n)}}$ [GeV]	$\mathcal{B}(\psi^{(n)}\to\ell\bar{\ell})$
$\psi^{(1)}=J/\psi$	3.097	8.8×10^{-5}	6.0×10^{-2}
$\psi^{(2)}$	3.686	2.8×10^{-4}	8.4×10^{-3}
$\psi^{(3)}$	3.77	2.4×10^{-2}	1.1×10^{-5}
$\psi^{(4)}$	4.04	5.2×10^{-2}	1.4×10^{-5}
$\psi^{(5)}$	4.16	7.8×10^{-2}	1.0×10^{-5}
$\psi^{(6)}$	4.42	4.3×10^{-2}	1.1×10^{-5}

where Δ contains the dependence of the differential decay rate on C_7.

Unitarity of the CKM matrix implies that $|V_{ts}^*V_{tb}|\simeq|V_{cs}^*V_{cb}|$ (with no more than 3% uncertainty), so that once $\Delta(y)$ is known, a value of $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ can be determined from experimental data on $B\to K^*\ell\bar{\ell}$. To find $\Delta(y)$ we use the relations between the tensor and (axial-)vector form factors derived for large m_b in Ref. [7][1]

$$g_+^{(B\to K^*)}+g_-^{(B\to K^*)}=\frac{f^{(B\to K^*)}+2\,g^{(B\to K^*)}m_Bm_{K^*}y}{m_B}, \tag{17a}$$

$$g_+^{(B\to K^*)}-g_-^{(B\to K^*)}=-2\,m_Bg^{(B\to K^*)}, \tag{17b}$$

$$h^{(B\to K^*)}=\frac{a_+^{(B\to K^*)}-a_-^{(B\to K^*)}-2\,g^{(B\to K^*)}}{2\,m_B}. \tag{17c}$$

Recent lattice QCD simulations indicate that these relations hold within 20% accuracy at the scale of the B mass [15]. In the limit where m_b is treated as heavy, $a_+^{(B\to K^*)}+a_-^{(B\to K^*)}$ is small compared with $a_+^{(B\to K^*)}-a_-^{(B\to K^*)}$, so Eq. (17c) can be simplified to

$$h^{(B\to K^*)}=\frac{a_+^{(B\to K^*)}-g^{(B\to K^*)}}{m_B}. \tag{18}$$

Note that a similar simplification for $g_+^{(B\to K^*)}+g_-^{(B\to K^*)}$ is not useful, because in Eq. (15a), $g_+^{(B\to K^*)}+g_-^{(B\to K^*)}$ is enhanced by m_B compared to $g_+^{(B\to K^*)}-g_-^{(B\to K^*)}$.

Using Eqs. (14)–(16), (17a), (17b), and (18), $\Delta(y)$ is expressed in terms of C_7, $\widetilde{C}_9$, C_{10}, $g^{(B\to K^*)}/f^{(B\to K^*)}$, and $a_+^{(B\to K^*)}/f^{(B\to K^*)}$. Using Eqs. (5) and (6) to relate ratios of $B\to K^*$ form factors to ratios of $D\to K^*$ form factors, we find that in the kinematic region $1<y<1.5$, $\Delta(y)$ changes almost linearly from $\Delta(1)\simeq-0.14$ to $\Delta(1.5)\simeq-0.18$. The value of Δ at zero recoil (using $m_b\simeq m_B$) does not depend on the ratios of form factors [16]

$$\Delta(1)=\frac{1}{|\widetilde{C}_9(1)|^2+|C_{10}|^2}\left[\frac{4\,\mathrm{Re}[C_7^*\widetilde{C}_9(1)]}{1-r}+\frac{4\,|C_7|^2}{(1-r)^2}\right]. \tag{19}$$

Even though there are $1/m_c$ corrections to Eqs. (5) and (6), they do not affect $\Delta(1)$. Furthermore, Δ is small compared with unity and has a modest y dependence. Consequently, $1/m_c$ corrections to the y dependence of Δ, and $1/m_b$ corrections to $\Delta(1)$ can only have a very small impact on the value of $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ extracted from the $B\to K^*\ell\bar{\ell}$ differential decay rate using Eq. (16).

Using the measured values of the $D\to K^*\bar{\ell}\nu_\ell$ form factors and the heavy quark symmetry relations in Eqs. (5) and (6) to get $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$, together with $|V_{cb}|=0.04$, $\tau_B=1.5$ ps, and $\alpha(m_W)=1/129$, we find that Eq. (16) gives a branching ratio of 2.9×10^{-7} for $B\to K^*\ell\bar{\ell}$ in the kinematic region $1<y<1.5$.

The largest theoretical uncertainties in using $B\to K^*\ell\bar{\ell}$ for extracting $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ come from order α_s corrections to the coefficients of the operators O_9 and O_{10} and our treatment of the $B\to K^*\ell\bar{\ell}$ matrix element of the four-quark operators. It is $h(z,y)$ that takes into account the $c\bar{c}$ loop contributions to the matrix elements of the four-quark operators.

A comparison with a phenomenological resonance saturation model [17] gives an indication of the uncertainties in the prediction for $B\to K^*\ell\bar{\ell}$ that arise from the fact that the kinematic region we focus on is not far from $D\bar{D}$ threshold. In this regard we note that using factorization to estimate the $B\to K^*\psi^{(n)}\to K^*\ell\bar{\ell}$ matrix elements of the four-quark operators ($\psi^{(n)}$ is the nth 1^{--} $c\bar{c}$ resonance), we find that in a resonance saturation model $h(z,y)$ in the second term of Eq. (11) gets replaced by[2]

$$h(z,y)\to-\kappa\frac{3\pi}{\alpha^2}\sum_n\frac{\Gamma_{\psi^{(n)}}\mathcal{B}(\psi^{(n)}\to\ell\bar{\ell})}{(q^2-M_{\psi^{(n)}}^2)/M_{\psi^{(n)}}+i\Gamma_{\psi^{(n)}}}, \tag{20}$$

where $\Gamma_{\psi^{(n)}}$ and $M_{\psi^{(n)}}$ are the width and mass of the nth 1^{--} $c\bar{c}$ resonance. Experimental values for these quantities and the branching ratios to $\ell\bar{\ell}$ are given in Table II. In Eq. (20), $\kappa=2.3e^{i\varphi_\kappa}$ is the factor that the $B\to J/\psi K^*$ amplitude,

[1]We correct some obvious factor-of-two errors in [7].

[2]For q^2 not near the resonances, there are uncertainties associated with the q^2 dependence. In Eq. (20) factors of q^2 not associated with the resonance propagator are set equal to the square of the resonance mass.

calculated using naive factorization, must be multiplied by to get the measured $B\to J/\psi K^*$ rate. Since the magnitude of κ is large, we do not assume that Eq. (20) has the same phase (i.e., $\varphi_\kappa=0$) as naive factorization would imply. Replacing $h(z,y)$ in Eq. (11) by the expression in Eq. (20), results in an effective coefficient of O_9 that we call $\widetilde{C}_9'$. A measure of the deviation of this model for the $c\bar{c}$ resonance region from the expression in Eq. (11) is given by $d(y)$, defined by

$$|\widetilde{C}_9'(y)|^2+|C_{10}|^2=[|\widetilde{C}_9(y)|^2+|C_{10}|^2][1+d(y)]. \tag{21}$$

In Fig. 6 we plot $d(y)$ for $1<y<1.5$. Note that part of $h(z,y)$ is associated with $c\bar{c}$ pairs at large virtuality, and so is reliably reproduced by QCD perturbation theory. In fact, $h(z,y)$ is scheme dependent, and so $d(y)$ is only a very crude measure of the uncertainties that arise from being near the $c\bar{c}$ threshold. The solid, dash-dotted, and dashed curves in Fig. 6 correspond, respectively, to $\varphi_\kappa=0$, $\pi/2$, and π. This analysis suggests that the uncertainty associated with the charm threshold region has on average about a 20% effect on the $B\to K^*\ell\bar{\ell}$ rate for $1<y<1.5$.

The uncertainties, involving the $D\bar{D}$ threshold region and the order α_s contributions to C_9 and C_{10}, can be avoided if the decay $B\to K^*\nu\bar{\nu}$ can be studied experimentally. While this will be difficult, the large missing energy carried by the neutrinos in the kinematic region we are interested in may help [18]. The differential decay rate for $B\to K^*\nu\bar{\nu}$ (summed over the neutrino flavors) is

$$\frac{d\Gamma(B\to K^*\nu\bar{\nu})}{dy}=\frac{G_F^2|V_{ts}^*V_{tb}|^2}{16\,\pi^3}\left(\frac{\alpha}{2\pi\sin^2\theta_W}\right)^2 m_B^3r^2|C_{11}|^2S(y)$$

$$=\frac{G_F^2|V_{ts}^*V_{tb}|^2}{16\,\pi^3}\left(\frac{\alpha}{2\pi\sin^2\theta_W}\right)^2 m_B^3r^2|C_{11}|^2|f^{(B\to K^*)}(y)|^2[1+\delta^{(B\to K^*)}(y)]\sqrt{y^2-1}(2+y^2-6yr+3r^2). \tag{22}$$

The coefficient C_{11} depends on the top quark mass (see Table I). Once the top quark mass is known more accurately, the $B\to K^*\nu\bar{\nu}$ differential decay rate provides a way to get $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ that, from a theoretical perspective, is very clean. Recall that the function $\delta^{(B\to K^*)}$ is the analogue of $\delta^{(B\to\rho)}$ that occurred in $B\to\rho\ell\bar{\nu}_\ell$ semileptonic decay, but it depends on ratios of $B\to K^*$ form factors that occur, instead of $B\to\rho$ form factors. It is plotted in Fig. 2 with the dashed curve, using Eqs. (5) and (6) to deduce the ratios of form factors $a_+^{(B\to K^*)}/f^{(B\to K^*)}$ and $g^{(B\to K^*)}/f^{(B\to K^*)}$ from the $D\to K^*\bar{\ell}\nu_\ell$ form factors. $\delta^{(B\to K^*)}(y)$ is fairly small, and so even though there is SU(3) violation in the relation between $\delta^{(B\to K^*)}$ and $\delta^{(B\to\rho)}$, this does not introduce a large uncertainty in our prediction for $|f^{(B\to\rho)}|^2(1+\delta^{(B\to\rho)})$ using Eq. (8). Using Eqs. (5) and (6) to get $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ from the measured values of the $D\to K^*\bar{\ell}\nu_\ell$ form factors, we find that Eq. (22) implies a branching ratio of 1.9×10^{-6} for $B\to K^*\nu\bar{\nu}$ in the kinematic region $1<y<1.5$.

The difference in the factor $\sqrt{y^2-1}(2+y^2-6yr+3r^2)$ for $r=m_\rho/m_B$ and $r=m_{K^*}/m_B$ divided by their sum is less than 3% for $1<y<1.5$. Therefore, it is a good approximation to rewrite Eq. (8), using Eqs. (3), (4), and (22), as

$$\frac{d\Gamma(B\to\rho\ell\bar{\nu}_\ell)}{dy}=\frac{|V_{ub}|^2}{3|V_{ts}^*V_{tb}|^2}\left(\frac{2\pi\sin^2\theta_W}{\alpha|C_{11}|}\right)^2\frac{m_\rho^2}{m_{K^*}^2}\times\frac{d\Gamma(B\to K^*\nu\bar{\nu})}{dy}\left|\frac{f^{(D\to\rho)}(y)}{f^{(D\to K^*)}(y)}\right|^2. \tag{23}$$

If SU(3) violation in the y dependence of the ratio of D decay form factors in Eq. (23) is small, then we can also compare integrated B decay rates to get a precise value for $|V_{ub}|$. Assuming that the shape of the form factors f are well approximated by simple pole forms and taking the pole mass for $f^{(D\to K^*)}$ to be 2.5 GeV (corresponding to the D_s^{**} mass) and the pole mass for $f^{(D\to\rho)}$ to be 2.4 GeV (corresponding to the D^{**} mass), we find that the ratio of D decay form factors squared in Eq. (23) varies by less than 0.5% over the range $1<y<1.5$. It may be possible to get some model-independent information on the y dependence of the ratio $f^{(D\to\rho)}/f^{(D\to K^*)}$ using the methods of Ref. [19].

The D semileptonic decay rate is almost completely saturated by the K and K^* hadronic final states. The heavy quark symmetry relations in Eqs. (5) and (6) do not imply that the rare decay mode $B\to X_s\nu\bar{\nu}$ (and also $B\to X_s\ell\bar{\ell}$ when the effects of the four-quark operators are neglected) is also saturated by these states in the kinematic region that overlaps with the D decay. For some of the D decay phase space, q^2 is small compared with m_D^2, while the scaling relations in Eqs. (5) and (6) hold for c and b quark masses treated as large with y held fixed.

IV. CONCLUDING REMARKS

In this paper we have explored the use of exclusive B and D decays to obtain a model-independent value of $|V_{ub}|$ with small theoretical uncertainties. Our method is based on the fact that the Grinstein-type double ratio of form factors $(f^{(B\to\rho)}/f^{(B\to K^*)})/(f^{(D\to\rho)}/f^{(D\to K^*)})$ is equal to unity in the SU(3) limit, and in the limit of heavy quark symmetry. A determination of $|V_{ub}|$, with an uncertainty from theory that is less than 10%, is possible using information obtainable from the decay modes $B\to\rho\ell\bar{\nu}_\ell$, $B\to K^*\nu\bar{\nu}$, $D\to\rho\bar{\ell}\nu_\ell$, and $D\to K^*\bar{\ell}\nu_\ell$. If, for $1<y<1.5$, $f^{(D\to\rho)}(y)/f^{(D\to K^*)}(y)$

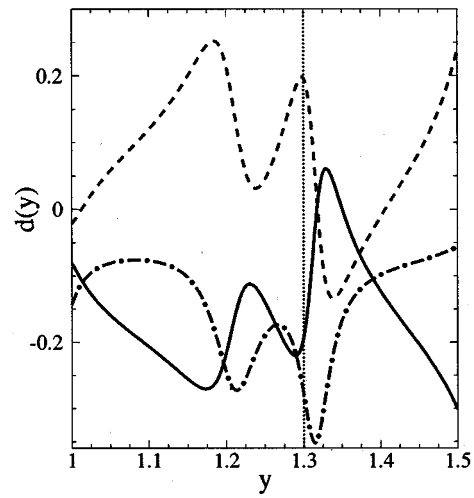

FIG. 6. The function $d(y)$ defined in Eq. (21) as a function of the kinematic variable $y=v\cdot v'$. The solid, dash-dotted, and dashed curves correspond, respectively, to $\varphi_\kappa=0$, $\pi/2$, and π.

is almost independent of y, then a precise value for $|V_{ub}|$ can be extracted from the rates for $B\to\rho\ell\bar{\nu}_\ell$ and $B\to K^*\nu\bar{\nu}$ integrated over this region in y [and $f^{(D\to\rho)}(1)/f^{(D\to K^*)}(1)$]. In a simple pole model, this ratio of D decay form factors is almost independent of y. We found that the matrix elements of the four-quark operators in the effective Hamiltonian for $B\to K^*\ell\bar{\ell}$ induce about a 20% uncertainty for the $B\to K^*\ell\bar{\ell}$ decay rate from $c\bar{c}$ threshold effects in the region $1<y<1.5$.

At the present time, the rare decays $B\to K^*\nu\bar{\nu}$ and $B\to K^*\ell\bar{\ell}$ have not been observed, and there is no information on the individual form factors for $D\to\rho\bar{\ell}\nu_\ell$. Because of this, it is difficult to give a prognosis for the ultimate utility of the ideas presented here. However, even in the absence of the complete set of information needed for a high precision determination of $|V_{ub}|$, our results may be useful. CLEO has observed about 40 $B\to\rho\ell\bar{\nu}_\ell$ events, corresponding to the branching ratio $\mathscr{B}(\bar{B}^0\to\rho^+\ell\bar{\nu}_\ell)\simeq(2\text{–}3)\times10^{-4}$ [20]. If heavy quark symmetry and SU(3) are employed to get $|f^{(B\to\rho)}|^2(1+\delta^{(B\to\rho)})$ from the measured $D\to K^*\bar{\ell}\nu_\ell$ form factors, then Eq. (3) can be used to extract $|V_{ub}|$ from the large q^2 region of the Dalitz plot for the exclusive decay $B\to\rho\ell\bar{\nu}_\ell$. We predict, with this technique, a branching ratio of $5.2|V_{ub}|^2$ for $\bar{B}^0\to\rho^+\ell\bar{\nu}_\ell$ in the region $1<y<1.5$. Lattice Monte Carlo simulations [15] (and constituent quark model calculations [21]) suggest that the violations of heavy quark symmetry and SU(3) symmetry that give corrections to the relation between $f^{(B\to\rho)}$ and $f^{(D\to K^*)}$ are not anomalously large. This method will give a value for $|V_{ub}|$ that is on a more sound theoretical footing than that which results from a comparison of the end point of the electron spectrum of inclusive semileptonic B decay with phenomenological models.

If experimental data on $B\to K^*\nu\bar{\nu}$ is available before a detailed study of semileptonic form factors for $D\to\rho\bar{\ell}\nu_\ell$ is performed, then using Eq. (22), an extraction of $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ should be possible. This gives a prediction for $|f^{(B\to\rho)}|^2(1+\delta^{(B\to\rho)})$ with correction of order m_s, but no order $1/m_c$ correction since heavy quark symmetry is not used. In this case, there is no reason to restrict our analysis to the region of phase space $1<y<1.5$. Lattice QCD results suggest that the influence of SU(3) violation on the form factors is small, and hence the value of $|V_{ub}|$ that can be extracted in this way will be fairly precise. A sizable uncertainty in the theoretical prediction for the $B\to K^*\ell\bar{\ell}$ decay rate arises from the charmonium resonance region. Without a better understanding of this, it will not be possible to extract $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ from this decay mode with high accuracy. Nonetheless, an extraction of $|f^{(B\to K^*)}|^2(1+\delta^{(B\to K^*)})$ from this mode may provide a useful determination of $|f^{(B\to\rho)}|^2(1+\delta^{(B\to\rho)})$ (and hence $|V_{ub}|$) with uncertainties now from both SU(3) violation and from the contribution of the four-quark operators to the $B\to K^*\ell\bar{\ell}$ rate.

Some improvements on the analysis in this paper are possible. Combining chiral perturbation theory for mesons containing a heavy quark with heavy vector-meson chiral perturbation theory, allows a computation of the order $m_s\ln m_s$ SU(3) violation in f [22]. Unfortunately, such an analysis cannot give a definitive result on the size of the SU(3) violations because of unknown order m_s counterterms. In this paper we have neglected the lepton masses. It is possible to include the corrections that arise from the nonzero value of the muon mass, although these are quite small.

A similar analysis to that performed in this paper can be done for the decays $B\to\pi\ell\bar{\nu}_\ell$, $B\to K\ell\bar{\ell}$, $B\to K\nu\bar{\nu}$, $D\to\pi\bar{\ell}\nu_\ell$, and $D\to K\bar{\ell}\nu_\ell$. However, in these decays there are complications because very near zero recoil ''pole contributions'' [23] spoil the simple scaling of the form factors with the heavy quark mass.

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