Color-octet ψ' production at low $p_\perp$

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We study contributions from color-octet quarkonium formation mechanisms to $p_\perp$-integrated ψ' production cross sections in pion-nucleon reactions. The observed polarization of the ψ' is not reproduced by calculations where leading-order, leading-twist $Q\bar{Q}$ production mechanisms are combined with the color-singlet and color-octet mechanisms of bound state formation. This shows that there are other important quarkonium production mechanisms at low $p_\perp$.

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I. INTRODUCTION

The production of heavy quarkonium has been widely studied using perturbative QCD. Because of the large masses of the c and b quarks, quarkonium production amplitudes can be factorized as products of perturbative $Q\bar{Q}$ production amplitudes and nonperturbative quarkonium formation amplitudes. In the conventional model of hadroproduction [1], a color-singlet $Q\bar{Q}$ pair with the appropriate quantum numbers is produced by the leading-order perturbative mechanisms in the collision of two partons, i.e., at leading twist, and the formation process is represented by a color-singlet Schrödinger wave function.

In S-wave quarkonium production, only the value of the wave function at the origin of coordinate space, $R(0)$, is probed, because the size of the perturbatively produced $Q\bar{Q}$ pair, $O(1/m_Q)$, is small compared to the width of the wave function. On the same basis, $R(0)$ can be related to the observed leptonic width of the S-wave state: e.g.,

$$|R_{\psi'}(0)|^2=\frac{M_{\psi'}^2\Gamma(\psi'\to\ell^+\ell^-)}{4\alpha_{\rm em}^2e_c^2}. \tag{1.1}$$

Thus the normalization of hadroproduction cross sections can be expressed in terms of the leptonic widths.

The Collider Detector at Fermilab (CDF) Collaboration has recently reported several measurements of quarkonium production in $p\bar{p}$ collisions at $\sqrt{s}=$ 1.8 TeV [2,3] which contradict the predictions of the conventional model. Possible theoretical explanations of these experimental results include parton fragmentation into quarkonium, which was first studied in perturbative QCD by Braaten and Yuan [4]. Unlike the conventional parton-parton fusion mechanisms [1], fragmentation is consistent with the approximate $1/p_\perp^4$ shape of the cross sections $d\sigma/dp_\perp(p\bar{p}\to\psi(1S,2S)+X)$ at large $p_\perp$. However, if the quarkonia are treated as color-singlet $Q\bar{Q}$ systems, the normalization of the cross sections still falls short of the data by as much as a factor of 5 for $J/\psi(1S)$ production and a factor of 30 for $\psi'(2S)$ production.

A number of possible resolutions have been suggested to explain the observed ψ' surplus [5–8]. In particular, Braaten and Fleming have proposed that nonperturbative transitions between color-octet and color-singlet $c\bar{c}$ states be included in the fragmentation calculation [8]. The inclusion of color-octet matrix elements is attractive both from the theoretical and phenomenological point of view. Theoretically, octet contributions are an essential part of the perturbative picture [9,10]. Phenomenologically, octet contributions can boost the cross section of heavy quark production, not just at high $p_\perp$, but also at low $p_\perp$, where large discrepancies are also observed [11].

A systematic procedure for the factorization of quarkonium production and decay amplitudes, known as nonrelativistic QCD (NRQCD), has been formulated during the recent years [12]. NRQCD is an effective field theory which provides an expansion of quarkonium cross sections and decay widths in terms of the relative velocity v of the heavy quark and antiquark. The nonperturbative physics is factorized into an infinite number of color-singlet and color-octet matrix elements which are related to different Fock states of the quarkonium wave function. These matrix elements are free parameters of the theory, but their scaling properties as functions of m_Q and v can be predicted.

Color-octet mechanisms were originally introduced as part of a rigorous treatment of P-wave charmonium decays [10]. In the color-singlet model some of these decay widths, as well as the corresponding leading-twist P-wave production cross sections, are infrared divergent. These divergences can be absorbed into the nonperturbative color-octet matrix elements. In S-wave production and decay, such divergences do not appear. Color-octet production mechanisms nevertheless provide an attractive explanation of the wrong normalization of quarkonium production. Note that CDF has also observed a surplus of S-wave bottomonium [3] with respect to the color-singlet parton-parton fusion predictions. The bottomonium surplus appears also at $p_\perp\lesssim m_b$, where parton-parton fusion mechanisms are expected to dominate over fragmentation mechanisms.

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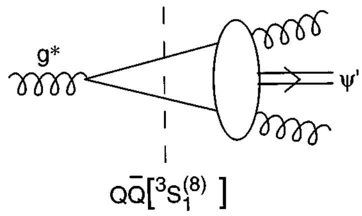

FIG. 1. Braaten and Fleming's color-octet fragmentation mechanism of ψ' production.

In the large $p_\perp$ mechanism proposed by Braaten and Fleming, the fragmenting gluon first produces a color-octet $c\bar{c}$ pair in a $^3S_1$ angular momentum state. The color-octet pair then evolves into a ψ' by a nonperturbative double chromoelectric dipole transition. This process is illustrated in Fig. 1, where the nonperturbative transition is represented by the blob. The cross section is proportional to the nonperturbative matrix element $\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle$, which is related to the weight of the $|c\bar{c}g\rangle$ Fock state in the ψ' wave function. This matrix element is suppressed by v^4 relative to the color-singlet matrix element $\langle 0|\mathcal{O}_1^{\psi'}(^3S_1)|0\rangle=(3/2\pi)|R'_\psi(0)|^2$. On the other hand, the color-octet $Q\bar{Q}$ production process $g^*\to Q\bar{Q}$ is of lower order in α_s than the color-singlet process $g^*\to Q\bar{Q}gg$. Hence the color-octet contribution to the cross section could be comparable to the color-singlet contribution. Because the color-octet matrix element is not constrained by other existing data, it can be adjusted to fit the CDF data. The value obtained in Ref. [8] is

$$\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle=0.0042\ \ (\mathrm{GeV})^3.$$

Cho and Leibovich [13], on the other hand, have combined color-octet transitions with a formula that smoothly interpolates between parton-parton fusion mechanisms and fragmentation mechanisms of $Q\bar{Q}$ production. They find that

$$\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle=0.0073\ \ (\mathrm{GeV})^3.$$

The predictions of the color-singlet parton-parton fusion model disagree with fixed-target hadroproduction data as well. We have earlier shown [11] that the model fails to reproduce the relative production rates of the J/ψ, χ_1 and χ_2 states [14] and the polarization of the J/ψ [15] and ψ' [16] observed in pion-nucleon reactions. We interpreted these discrepancies as evidence for important higher-twist mechanisms of charmonium production. It is, however, necessary to check whether the data could be reproduced within the leading-twist picture by including the color-octet transitions. At the same time, such an analysis could provide an independent determination of the color-octet matrix elements whose values have previously been extracted from CDF data.

In this paper, we study the $p_\perp$-integrated ψ' production cross section in pion-nucleon collisions. The analysis also applies to the direct component of J/ψ production, i.e., the component which is not due to the decays of higher-mass states. Total J/ψ production is a somewhat more complicated process because there are significant contributions from the decays of the χ_J and ψ' states [14].

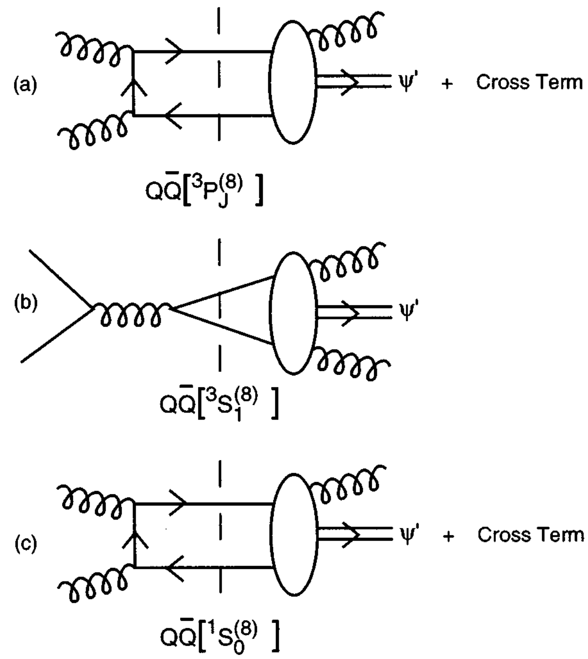

FIG. 2. The Feynman diagrams which describe the lowest-order color-octet ψ' production mechanisms. We omit diagrams that correspond to vanishing amplitudes.

The polarization of the ψ' gives important information on its production mechanisms. Measurements of polarization actually indicate that there must be other production mechanisms in addition to the leading-twist color-singlet and color-octet mechanisms.

II. COLOR-OCTET PRODUCTION MECHANISMS

At leading order in α_s and up to next-to-leading order in v^2, the subprocesses for ψ' production through color-octet intermediate states are

$$gg\to c\bar{c}\,[^3P_J^{(8)}]\to\psi'+g, \tag{2.1}$$

$$q\bar{q}\to c\bar{c}\,[^3S_1^{(8)}]\to\psi'+gg, \tag{2.2}$$

$$gg\to c\bar{c}\,[^1S_0^{(8)}]\to\psi'+g. \tag{2.3}$$

These are illustrated in Fig. 2. The process $q\bar{q}\to c\bar{c}\,[^3P_J^{(8)}]\to\psi'+g$ is of higher order in v^2 since the lowest-order nonperturbative transition (single chromoelectric dipole) is forbidden by charge conjugation. We also find that the subprocess amplitude $A(gg\to c\bar{c}\,[^3S_1^{(8)}])$vanishes in the leading order in α_s. Note that charge conjugation or Yang's theorem [17] do not require this amplitude to vanish because the $c\bar{c}$ pair is not in a color-singlet state.

The polarization of the final state ψ' is measured by the polar-angle distribution, $1+\alpha\cos^2\theta$, of its decay dileptons in their rest frame. The parameter α in the Gottfried-Jackson frame angular distribution is related to the polarized ψ' production cross sections in the following way:

$$\alpha=\frac{d\sigma(\lambda=1)-2d\sigma(\lambda=0)+d\sigma(\lambda=-1)}{d\sigma(\lambda=1)+2d\sigma(\lambda=0)+d\sigma(\lambda=-1)}, \qquad (2.4)$$

where λ is the helicity of the ψ'. If the polarized cross section is written as $\sigma(\lambda)\sim 1+a\delta_{\lambda 0}$, then $\alpha=-a/(2+a)$. Because of rotational and parity invariance, $d\sigma(\lambda=1)=d\sigma(\lambda=-1)$. Since the transverse momenta of the initial partons and the momenta of the soft gluons emitted in the nonperturbative transition are small, the helicity of the ψ' is approximately equal to the z component of its spin. It is determined by the perturbative dynamics of the subprocesses $q\bar{q},\ gg\to c\bar{c}$ and by the heavy quark spin symmetry of the nonperturbative transition. Because of the nonrelativistic nature of the $c\bar{c}$ system, the nonperturbative transition can be treated using the familiar machinery of atomic or positronium physics. In the process (2.1), the transition is a (chromo)electric dipole transition. In (2.2), it is a double electric dipole transition and in (2.3), a magnetic dipole transition. In the electric dipole transitions, the helicities of the heavy quark and antiquark are not flipped, whereas in the magnetic dipole transition one of the helicities is flipped. In processes (2.1) and (2.2), the helicity λ of the ψ' therefore equals the z component of the spin of the color-octet $c\bar{c}$ state. This is similar to the spin symmetry of heavy-flavor mesons in heavy quark effective theory [18]. In process (2.3), the total spin of the quark-antiquark system changes from $S=0$, $S_z=0$ in the color-octet state to $S(\psi')=1$, $S_z(\psi')=\lambda=\pm 1$in the final state.

In terms of the matrix elements of nonrelativistic QCD, the squares of the amplitudes for the processes (2.1), (2.2), and (2.3) are

$$\sum_J |A(gg\to c\bar{c}[{}^3P_J^{(8)}]\to\psi'(\lambda)+X)|^2=\frac{1}{24m_c}\langle 0|\mathcal{O}_8^{\psi'}({}^3P_1)|0\rangle\sum_{a,L_z}|A(gg\to c\bar{c}[(L=S=1)^{(8)};a,S_z=\lambda,L_z])|^2, \qquad (2.5)$$

$$|A(q\bar{q}\to c\bar{c}[{}^3S_1^{(8)}]\to\psi'(\lambda)+X)|^2=\frac{1}{24m_c}\langle 0|\mathcal{O}_8^{\psi'}({}^3S_1)|0\rangle\sum_a|A(q\bar{q}\to c\bar{c}[{}^3S_1^{(8)};a,S_z=\lambda])|^2, \qquad (2.6)$$

$$|A(gg\to c\bar{c}[{}^1S_0^{(8)}]\to\psi'(\lambda)+X)|^2=\frac{1}{8m_c}\langle 0|\mathcal{O}_8^{\psi'}({}^1S_0)|0\rangle\frac{1-\delta_{\lambda 0}}{2}\sum_a|A(gg\to c\bar{c}[{}^1S_0^{(8)};a])|^2, \qquad (2.7)$$

where a,S_z,L_z are the color index and the angular momentum components of the color-octet $c\bar{c}$ states. In deriving Eq. (2.5), we made use of the relation

$$\langle 0|\mathcal{O}_8^{\psi'}({}^3P_J)|0\rangle=(2J+1)\langle 0|\mathcal{O}_8^{\psi'}({}^3P_0)|0\rangle. \qquad (2.8)$$

Because the color-octet states with different J do not propagate as resonances of different masses, the sum over J on the right-hand side of Eq. (2.5) is easily done. The perturbative amplitudes are

$$A(gg\to c\bar{c}[(L=S=1)^{(8)};a,S_z,L_z]) = \sqrt{2}T^a_{AB}\epsilon^{*\alpha}(L_z)\frac{\partial}{\partial q^\alpha} \times \mathrm{Tr}[\mathcal{O}^{AB}_{gg\to c\bar{c}}(P,q)P_{1S_z}(P,q)]|_{q=0}, \qquad (2.9)$$

$$A(q\bar{q}\to c\bar{c}[{}^3S_1^{(8)};a,S_z]) = \sqrt{2}T^a_{AB}\mathrm{Tr}[\mathcal{O}^{AB}_{q\bar{q}\to c\bar{c}}(P,q)P_{1S_z}(P,q)]|_{q=0}, \qquad (2.10)$$

$$A(gg\to c\bar{c}[{}^1S_0^{(8)};a]) = \sqrt{2}T^a_{AB}\mathrm{Tr}[\mathcal{O}^{AB}_{gg\to c\bar{c}}(P,q)P_{00}(P,q)]_{q=0}, \qquad (2.11)$$

where P is the total four-momentum and $2q$ the relative four-momentum of the $c\bar{c}$ pair, ϵ is a spin-1 polarization four-vector, P_{1S_z},P_{00} are the covariant spin projection operators defined in, e.g., Ref. [13], and

$$\mathcal{O}^{AB}_{q\bar{q}\to c\bar{c}}(P,q)=\frac{4\pi\alpha_s}{(k_1+k_2)^2}\bar{v}_C(k_1,\lambda_1)\gamma_\mu T^a_{CD}u_D(k_2,\lambda_2) \times\gamma^\mu T^{AB}_a \qquad (2.12)$$

and

$$\mathcal{O}^{AB}_{gg\to c\bar{c}}(P,q)=4\pi\alpha_s\Bigg[(T_aT_b)^{AB}\not{\epsilon}_a(k_1,\lambda_1) \times\frac{(1/2)\not{P}+\not{q}-\not{k}_1+m_c}{k_1\cdot(P+2q)}\not{\epsilon}_b(k_2,\lambda_2) +(T_bT_a)^{AB}\not{\epsilon}_b(k_2,\lambda_2) \times\frac{(1/2)\not{P}+\not{q}-\not{k}_2+m_c}{k_2\cdot(P+2q)}\not{\epsilon}_a(k_1,\lambda_1)\Bigg] \qquad (2.13)$$

are the perturbative amplitudes for the production of a $c\bar{c}$ pair with color indices A,B. The momenta and helicities of the colliding partons are denoted by $k_{1,2}$ and $\lambda_{1,2}$. Taking the four-momentum of the ψ' equal to P, the ψ' production cross section is

$$\sigma(\pi N\to\psi'(\lambda)+X)=\sum_{ij}\int dx_1dx_2 f_{i/\pi}(x_1,\mu_F)\times f_{j/N}(x_2,\mu_F)\,\delta\left(1-\frac{M^2}{x_1x_2s}\right)\times\frac{\pi}{M^4}\overline{\sum}\,|A(ij\to\psi'(\lambda)+X)|^2$$
$$\equiv\sum_{ij}\Phi_{ij/\pi N}(M^2/s,\mu_F)\frac{\pi}{M^4}\times\overline{\sum}\,|A(ij\to\psi'(\lambda)+X)|^2, \tag{2.14}$$

where $f_{i/\pi}(x_1,\mu_F)$ and $f_{j/N}(x_2,\mu_F)$ are the distributions of partons of type i and j in the pion and the nucleon, μ_F is the leading-twist factorization scale, and $M=2m_c$ is the mass of the charmonium system. The contribution from the color-octet subprocesses (2.1), (2.2), and (2.3) simplifies to

$$\sigma_{\text{octet}}(\pi N\to\psi'(\lambda)+X)=O_1\langle 0|\mathcal{O}_8^{\psi'}(^3P_1)|0\rangle(3-2\delta_{\lambda 0})+O_2\langle 0|\mathcal{O}_8^{\psi'}(^1S_0)|0\rangle(1-\delta_{\lambda 0})+O_3\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle(1-\delta_{\lambda 0}), \tag{2.15}$$

where

$$O_1=\frac{5\pi^3\alpha_s^2}{9M^7}\Phi_{gg/\pi N}(M^2/s,\mu_F)=76\ \text{nb}\,(\text{GeV})^{-5}, \tag{2.16}$$

$$O_2=\frac{5\pi^3\alpha_s^2}{24M^5}\Phi_{gg/\pi N}(M^2/s,\mu_F)=390\ \text{nb}\,(\text{GeV})^{-3}, \tag{2.17}$$

$$O_3=\frac{8\pi^3\alpha_s^2}{27M^5}\sum_q\,[\Phi_{q\bar{q}/\pi N}(M^2/s,\mu_F)+\Phi_{\bar{q}q/\pi N}(M^2/s,\mu_F)]=100\ \text{nb}\,(\text{GeV})^{-3}. \tag{2.18}$$

In evaluating the numerical values, we took $\alpha_s=0.26$, $\mu_F=M=3.686$ GeV, and $E_{\text{lab}}(\pi)=300$ GeV, and used the parton distributions of the π^- and N from Ref. [19]. The octet contribution is dominantly transversely polarized; each of the three components alone corresponds to $\alpha=1/2$, $\alpha=1$, and $\alpha=1$, respectively, in the angular distribution $1+\alpha\cos^2\theta$ of the ψ' decay dileptons in the Gottfried-Jackson frame.

III. BOUNDS ON COLOR-OCTET MATRIX ELEMENTS

Together with the experimental measurement of the ψ' cross section [15,16] and our earlier determination of the leading-twist, color-singlet component [11], Eq. (2.15) can be used to set a bound on a linear combination of the three NRQCD matrix elements involved. Within the accuracy of this calculation, the experimental cross section is unpolarized ($\alpha=0.02\pm0.14$ [20]). We shall, therefore, write, for any helicity λ,

$$\sigma_{\text{expt}}(\pi^- N\to\psi'(\lambda)+X)=\frac{1}{3}\sigma_{\text{expt}}^{\text{tot}}, \tag{3.1}$$

where $\sigma_{\text{expt}}^{\text{tot}}=25\pm4$ nb at $E_{\text{lab}}(\pi)=300$ GeV [14]. The color-singlet component [11] is

$$\sigma_{\text{singlet}}(\pi N\to\psi'(\lambda)+X)=\frac{1+a\,\delta_{\lambda 0}}{3+a}\sigma_{\text{singlet}}^{\text{tot}}, \tag{3.2}$$

where $\sigma_{\text{singlet}}^{\text{tot}}=3.2$ nb and $a=-0.41$.

Since both the color-singlet and color-octet contributions are dominantly transversely polarized ($a<0$), they cannot make up all of the unpolarized ($a=0$) experimental cross section. An upper limit of σ_{octet} is obtained by assuming that the unknown component is totally longitudinally polarized:

$$\sigma_{\text{unknown}}(\pi N\to\psi'(\lambda)+X)=\delta_{\lambda 0}\sigma_{\text{unknown}}^{\text{tot}}. \tag{3.3}$$

In this case,

$$\frac{1}{3}\sigma_{\text{expt}}^{\text{tot}}=3O_1\langle^3P_1\rangle+O_2\langle^1S_0\rangle+O_3\langle^3S_1\rangle+\frac{1}{3+a}\sigma_{\text{singlet}}^{\text{tot}}+\left(-2O_1\langle^3P_1\rangle-O_2\langle^1S_0\rangle-O_3\langle^3S_1\rangle+\frac{a}{3+a}\sigma_{\text{singlet}}^{\text{tot}}+\sigma_{\text{unknown}}^{\text{tot}}\right)\delta_{\lambda 0}, \tag{3.4}$$

where we used the shorthand notation $\langle^{2S+1}L_J\rangle\equiv\langle 0|\mathcal{O}_8^{\psi'}(^{2S+1}L_J)|0\rangle$. The coefficient of $\delta_{\lambda 0}$ on the right-hand side of Eq. (3.4) should vanish, which leads to

$$\sigma_{\text{unknown}}^{\text{tot}}=2O_1\langle^3P_1\rangle+O_2\langle^1S_0\rangle+O_3\langle^3S_1\rangle-\frac{a}{3+a}\sigma_{\text{singlet}}^{\text{tot}}=\sigma_{\text{expt}}^{\text{tot}}-\sigma_{\text{singlet}}^{\text{tot}}-7O_1\langle^3P_1\rangle-2O_2\langle^1S_0\rangle-2O_3\langle^3S_1\rangle. \tag{3.5}$$

The bound on the color-octet matrix elements then is

$$3O_1\langle^3P_1\rangle+O_2\langle^1S_0\rangle+O_3\langle^3S_1\rangle\leqslant\frac{1}{3}\sigma_{\text{expt}}^{\text{tot}}-\frac{1}{3+a}\sigma_{\text{singlet}}^{\text{tot}}=7\ \text{nb}. \tag{3.6}$$

We have not assumed any K factors in the theoretical components. If we include a factor $K=2$ in each of the theoretical components, the right-hand side of Eq. (3.6) is diminished to 3 nb.

IV. DISCUSSION

We have evaluated the leading-order, leading-twist contribution to ψ' production in fixed-target πN collisions from production mechanisms which proceed through intermediate color-octet $c\bar{c}$ states. By combining this result with the experimental measurement of the ψ' cross section [14,16] and our earlier determination of the contribution from color-singlet production mechanisms [11], we have derived a bound on a linear combination of three matrix elements of nonrelativistic QCD:

$$230\ \text{nb}\ \ (\text{GeV})^{-5}\langle 0|\mathcal{O}_8^{\psi'}(^3P_1)|0\rangle + 390\ \text{nb}\ \ (\text{GeV})^{-3}\langle 0|\mathcal{O}_8^{\psi'}(^1S_0)|0\rangle + 100\ \text{nb}\ \ (\text{GeV})^{-3}\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle \leq 7\ \ \text{nb}. \tag{4.1}$$

By considering the polarization of the ψ', which follows from the perturbative-QCD dynamics of $c\bar{c}$ production and from the heavy quark spin symmetry of the nonperturbative transition, we were able to set a stronger constraint than would have been possible by considering the total cross section only. Because the leading-twist color-singlet and color-octet contributions are dominantly transversely polarized whereas the observed cross section is unpolarized, the leading-twist component could at most make up about half of the observed cross section. We therefore conclude that there exist important $c\bar{c}$ production mechanisms beyond leading twist.

Of the three NRQCD matrix elements, only the element $\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle$ has been determined from other measurements, namely the CDF charmonium production data [8,13], as discussed in the Introduction. Using the value given in Ref. [13], one obtains

$$O_3\langle 0|\mathcal{O}_8^{\psi'}(^3S_1)|0\rangle = 0.7\ \text{nb}, \tag{4.2}$$

i.e., only about 10% of the right-hand side of the constraint equation (4.1). On the one hand, this means that the color-octet explanation of the CDF ψ' surplus is not inconsistent with the existing fixed-target measurements. On the other hand, the octet mechanisms appear to be incapable of explaining the pattern of quarkonium production at low $p_\perp$ and fixed-target energies.

Note that in the color-singlet model, the ratio of the J/ψ and ψ' cross sections is predicted to be

$$\frac{\sigma(\psi')}{\sigma(J/\psi)} = \frac{M^3(J/\psi)\Gamma_{\ell\ell}(\psi')}{M^3(\psi')\Gamma_{\ell\ell}(J/\psi)} = 0.24, \tag{4.3}$$

where M and $\Gamma_{\ell\ell}$ are the masses and leptonic decay widths of the J/ψ and ψ'. The experimental photoproduction [21] and fixed-target hadroproduction [22] cross sections are consistent with Eq. (4.3). This supports our conclusion that the failure of the conventional model [1] in reproducing fixed-target data is due to neglected $c\bar{c}$ production mechanisms rather than to the neglected color-octet mechanisms of quarkonium formation. It is interesting to note that the CDF measurements are actually also consistent with Eq. (4.3).

At the final stage of this work we learned of a related study by Cho and Leibovich [23] of quarkonium production via $Q\bar{Q}[^3S_1^{(8)}]$, $Q\bar{Q}[^3P_J^{(8)}]$, and $Q\bar{Q}[^1S_0^{(8)}]$ intermediate states. By making fits to the CDF data they obtain values of the color-octet matrix elements which are consistent with NRQCD scaling rules. Analytical expressions for the total cross sections of $2\rightarrow 1$ subprocesses are also given in Ref. [23]; our results are consistent with these.

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