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Axial-vector properties of the nucleon with $1/N_c$ corrections in the solitonic SU(3)-NJL model

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Expectation values of the axial-vector currents are calculated within the semibosonized SU(3)-NJL model in the next to leading order of a $1/N_c$ expansion. These $1/N_c$ corrections are shown to come from two distinctive sources: (1) the anomalous part of the Euclidean effective action related to the Wess-Zumino term of the SU(3) Skyrme model and (2) the real, nonanomalous part which in this order of $1/N_c$ has no counterpart within any local effective meson theory. The appearance of the type (2) terms is due to the *time ordering* of the collective operators entering the formulas for the axial-vector constants. They substantially improve the phenomenology of the model. The question of regularization of these quantities is discussed. The analytic symmetry-breaking terms in the strange quark mass play a minor role for $g_A^{(3)}$ and $g_A^{(0)}$. They are, however, important for $g_A^{(8)}$. Finally, the numerical values for the g_A's, $g_A^{(0)} = 0.37$, $g_A^{(3)} = 1.38$, and $g_A^{(8)} = 0.31$, reproduce reasonably well the recent data from lepton scattering ($g_A^{(0)} = 0.31$, $g_A^{(3)} = 1.26$, and $g_A^{(8)} = 0.35$).

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I. INTRODUCTION

It is a long lasting problem to determine the static properties of hadrons directly from the general theory of the strong interaction, quantum chromodynamics (QCD). Therefore one attempts to formulate an effective theory for the strong interactions which would be tractable in the low energy regime [1–5]. The quark Nambu–Jona-Lasinio (NJL) model [6,7] seems to be an excellent candidate for such a theory. Although it does not confine quarks, it shares maybe the most important features of QCD relevant for the quark bound states. These are chiral symmetry and its spontaneous breaking. In the presently investigated NJL model the nucleon consists of N_c quarks bound in a self-consistent potential. The latter is based on a nontrivial chiral field configuration constrained to the chiral circle in the Hartree approximation [8–10], which is leading in $1/N_c$ expansion.

The semibosonized NJL model is defined by a local Lagrangian density of the quark and meson fields. However if one integrates the quarks out, the resulting mesonic effective action is no longer a local one. One can, however, make a gradient expansion, which will eventually lead to a local action, although the number of terms will be infinite. Therefore it was always believed that the effective quark theories are equivalent to the effective meson theories in the sense that they correspond to some calculable local Lagrangian density, such as that of the Skyrme model, for example [11]. However, it has been shown that the hadronic matrix elements of axial-vector currents calculated within the NJL soliton model exhibit new terms, which cannot be obtained from the local mesonic theory [12–14]. This is due to two facts: (1) upon the semiclassical quantization the cranking velocities are promoted to the collective operators which do not commute with the rotation matrix itself and (2) the collective operators have to be *time ordered.* The latter can be seen within the path-integral formalism, which dictates unambiguously in which order the cranking velocity and the rotation matrix appear in the expressions for the matrix elements of the axial-vector currents. If these *nonlocal* properties of the path integral are properly taken into account, then one gets new corrections which are of order $O(N_c^0)$, whereas the leading term is $O(N_c)$. These corrections are not small and improve the phenomenological predictions of the NJL model. We will show that one can even obtain a *local limit* of these terms using the gradient expansion for the matrix element in question. On the contrary, taking the *local limit* of the present effective quark theory first (e.g., by making the gradient expansion) and then calculating the same ma-

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trix elements yields no corresponding $O(N_c^0)$ terms at all.

The example of the axial-vector constants is perhaps the most persuasive. It is well known that in the nonrelativistic quark model $g_A = (N_c + 2)/3$. This means that there are important $O(N_c^0)$ corrections to g_A, which for $N_c = 3$ amount to 60% of the leading result. In the effective meson theories the leading term for g_A scales also as N_c; however the next-to-leading corrections comes only at the $O(1/N_c)$ level in the SU(2) version of the model. This is due to the fact that effective meson Lagrangians are *even* in field derivatives. In the cranking approximation for the rotating soliton each time derivative counts as $1/N_c$. The only possible source for the contributions linear in the time derivative is the anomalous Wess-Zumino term which vanishes identically in the SU(2) case. Hence the SU(2) Skyrmion does not show any rotational $O(N_c^0)$ corrections[1] in a contradiction to the nonrelativistic quark model. In the NJL model, however, the $O(N_c^0)$ terms appear in a natural way due to the *time-ordering* discussed above. Such terms are also present in the SU(3) version of the model. However, apart from them in the SU(3) case, both in the NJL and in the Skyrme model, the anomalous $O(N_c^0)$ terms appear. In the NJL model in the leading order of the gradient expansion these terms correspond to the Witten's anomalous current [15] of a local mesonic theory.

In the previous paper [13] we have calculated the three SU(3) axial decay constants $g_A^{(3)}$, $g_A^{(8)}$, and $g_A^{(0)}$ in the chiral limit within the semibosonized NJL model. This model reproduces the hyperon spectra [16] and also the isospin splittings within baryon multiplets[2] [17]. So far the properties of the axial currents have been investigated in the NJL model only for the case of SU(2) [18,10]. Recently the rotational corrections for SU(2) have been roughly estimated in [12], neglecting the sea contribution, and to full extent in [14]. Beyond this there are only calculations of the axial coupling constants within the pseudoscalar SU(3) Skyrme model [19] and the pseudoscalar vector meson SU(3) Skyrme model [20].

In contrast with our previous paper [13], where we ignored the regularization of the new terms, now we implement the *time-ordering* within the framework of the regularized effective Euclidean action (EEA). This treatment allows us to make a clear distinction between the terms which emerge from the real or from the imaginary (anomalous) part of the EEA. Actually it turns out that the new *time-ordered* terms emerge from the real part of the EEA and therefore have to be regularized. In the present approach the regularization prescription is unique and the regularization function is derived in a well-defined manner.

In the present paper we furthermore extend our previous calculations and calculate the m_s corrections to the axial-vector coupling constants. In addition to the theoretical interest due to the new *time-ordered* corrections, the axial constants are of utmost phenomenological importance as far as the comparison with the recent measurements of the polarized proton and neutron structure functions is concerned. So the main phenomenological concern of this paper will be a comparison of the model predictions with the experimental data for these quantities. We will especially concentrate on the role of the *time-ordered* corrections and furthermore on the so-called *anomalous* quantities, which are dominated by the valence contributions [21,16,22].

The organization of the paper is as follows. In Sec. II we review the basic features of the NJL model with special emphasis on the solitonic description. In Sec. III we describe the quantization procedure and summarize the results on the hyperon splittings. We use the mass splittings to fix the constituent mass M. In Sec. IV we derive expressions for the expectation values of currents in the NJL model. Special emphasis is put on the new contributions from the time ordering and their regularization. In Sec. V general formulas of Sec. IV are applied to calculate the axial currents expectation values in the chiral limit. Then in Sec. VI we discuss mass corrections to the axial currents. Our numerical results are presented in Sec. VII. Section VIII contains a brief comparison with the results of the Skyrme model. We present our conclusions in Sec. IX.

In the Appendices we present a derivation of the regularization functions (Appendix A) and a gradient expansion for normal and time-ordered quantities (Appendix B).

II. THE SU(3) NAMBU–JONA-LASINIO MODEL: SOLITONS

The quark Nambu–Jona-Lasinio model [6,7] can be written in the four-fermion formulation as

$$\mathcal{L}_{\rm NJL} = \bar{q}(x)(i\partial\!\!\!/ - m)q(x) - 2G[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2] \,. \tag{1}$$

Here the summation over the λ^a matrices is implicit (with $\lambda_0 = \sqrt{2/3}$) and m is the current quark mass matrix. In the chiral limit the Lagrangian has the desired $\mathrm{SU}(3)_R \otimes \mathrm{SU}(3)_L$ symmetry in addition to the $\mathrm{U}(1)_V \otimes \mathrm{U}(1)_A$, where the $\mathrm{U}(1)_A$ is the symmetry which is not shared by QCD. In principle, one could introduce the 't Hooft term into the Lagrangian, which breaks the $\mathrm{U}(1)_A$ explicitly. It could then serve as a source for the η' mass, which otherwise would be a Goldstone boson. This was recently done by Kato *et al.* [23] and the resulting profile for the SU(2) soliton was very similar to the solutions that are restricted to the chiral circle (i.e., nonlinear case) and which are used here. So we believe that the effects of the 't Hooft determinant on the solitonic observables of the present calculations are small.

Performing the bosonization procedure of Eguchi [24] one arrives at a new classical Lagrangian $\mathcal{L}_{\rm NJL}^{\rm bos}$ (whose

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[1]One should, however, remember a possible contribution from the anomalous baryon current.

[2]The free parameters are M, the constituent quark mass and cutoff parameters, and to some extent m_s, the strange quark mass.

explicit form is not needed here), which contains in addition to the quark fields scalar and pseudoscalar meson fields. Because the quark fields appear only quadratically, one can integrate over the fermionic functional integral to obtain the following effective Euclidean action (EEA), corresponding to the leading contribution in an $1/N_c$ expansion:

$$S_{\text{eff}}[U(x)] = -\text{Sp}\ln[-i\partial\!\!\!/ + P_R\mathcal{M}(x) + P_L\mathcal{M}^\dagger(x) + m] + \frac{1}{4G}\text{tr}_\lambda(\mathcal{M}\mathcal{M}^\dagger) \ . \quad (2)$$

Here Sp is the functional trace in color, flavor, and momentum space and $\mathcal{M} = g(\sigma^a\lambda^a + i\pi^a\lambda^a)$ is the chiral matrix in terms of σ- and π-meson fields. $P_{R(L)} = \frac{1}{2}(1\pm\gamma_5)$ are the right (left) helicity projection operators. Alternatively one can write $\mathcal{M} = \xi_L^\dagger M\xi_R$ in terms of the unitary matrices ξ_L, ξ_R and a Hermitian matrix M. Furthermore the unitary gauge $\xi_R = \xi_L^\dagger = \xi$ can be chosen to eliminate the redundant degrees of freedom. In order to obtain stable solitonic solutions [25–27] it is important that the scalar degrees of freedom in M are frozen, such that M is just the constituent quark mass matrix and ξ is related to the chiral field by $U = \xi^2$. The latter can be parametrized as $\exp(i\pi^a\lambda^2/f_\pi)$. This parametrization corresponds in SU(2) to the chiral circle condition $\sigma_{(2)}^2 + \vec{\pi}^2 = \text{const}$, where $\sigma_{(2)}$ is the SU(2) isoscalar σ field.

From Eq. (2) the parameters of the model can be fixed by requiring experimental values for the pion decay constant $f_\pi = 93$ MeV, the pion and the kaon mass, $m_\pi = 139$ MeV and $m_K = 496$ MeV (see Ref. [16] for details). As a result the constituent quark mass is the only free parameter of the model, which can, e.g., be used to fix baryonic properties [16,17].

Solitonic solutions of Eq. (2) can be found by making a time-independent *hedgehog* ansatz for the chiral field $U_0(\vec{x}) = \exp[i\vec{\tau}\cdot\vec{r}\theta(r)]$ and writing the SU(2) action in the chiral limit in terms of a single particle intrinsic Hamiltonian:

$$H = -i\gamma_4[-i\gamma_i\partial_i + MU(\vec{x},t)] \ . \quad (3)$$

The gamma matrices in Eq. (93) are taken as anti-Hermitian in Euclidean space. We will specify the SU(3) extension of the SU(2) ansatz in the next section. In the proper time regularization [28], the effective action becomes, in this case,

$$S_{\text{eff}} = \text{Sp}\int\frac{du}{u}\phi(u,\Lambda_1,\Lambda_2)\exp[-u(D_0^\dagger D_0)] \ , \quad (4)$$

where $D_0 = \partial_\tau + H$ and where $\phi = c\theta(u - 1/\Lambda_1^2) + (1-c)\theta(u - 1/\Lambda_2^2)$ is the regularization function of Ref. [16], which reproduces common values for the current quark masses and quark condensates in the vacuum. It is implicitly assumed that the vacuum contribution is subtracted from Eq. (4). The classical equations of motion can be solved self-consistently for the chiral field $U = U_0$, resulting in a localized soliton with unit winding number. The energy spectrum of the Hamiltonian operator H [Eq. (3)] for the baryon number one sector contains a discrete valence level inside a mass gap of the size $2M$ [9,8,29]. Then the classical energy of the soliton can be written as [29]

$$M_{\text{cl}} = N_cE_{\text{val}} + N_cE_{\text{sea}} \ , \quad (5)$$

where E_{val} is the energy of the valence level and E_{sea} resembles the polarization the Dirac sea as a sum over the whole spectrum of the Hamiltonian operator H.

III. QUANTIZATION OF ZERO MODES AND MASS SPLITTINGS

The purpose of this section is to apply the semiclassical quantization method to the solitons [9,8,30] of Sec. II, which result from the classical and time-independent equations of motion. The idea hereby is the following. In order to quantize the system one can perform a time-dependent transformation [31] either in the direction of the symmetry or orthogonal to it. If the symmetry is at least an approximate one, then the excitations in the symmetry direction should be the dominant contribution to the low lying resonances of the model. In order to check this numerically in the present model, in addition to the usual expansion of the EEA up to the second order in the rotational velocity, we also *consistently* consider the quadratic corrections from the strange symmetry breaking terms.

Therefore, following the treatment of Ref. [31] we quantize the soliton by introducing time-dependent SU(3) rotations and impose canonical quantization conditions for the collective coordinates of the rotation matrix. This will allow for the definition of the SU(3) generators and of the corresponding baryon states.

First we make use of the trivial embedding of Witten [32] of the SU(2) chiral field $U_0(\vec{x}) = (\sigma_{(2)} + i\gamma_5\vec{\tau}\vec{\pi})/f_\pi$ into the isospin subgroup of SU(3) according to

$$U(\vec{x}) = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix} \ . \quad (6)$$

The soliton solutions of SU(2) are also solutions for SU(3). In the quantization procedure the embedding (6) generates the correct quantum numbers for baryons [32].

Next one introduces a time-dependent rotation $U(\vec{x},t) = A(t)U(x)A(t)^\dagger$. This rotation can be undone by rotating the quarks fields: $\tilde{q} = A(t)^\dagger q$ and $\bar{\tilde{q}} = \bar{q}A(t)$. Then one has to replace D_0 in the effective action Eq. (4) by $D = D_0 + A^\dagger\dot{A} - i\gamma_4A^\dagger mA$, where

$$A^\dagger\dot{A} = i\Omega_E = \frac{i}{2}\lambda_a\Omega_E^a \quad (7)$$

and the relation between Euclidean and Minkowski velocities holds: $i\Omega_E \Leftrightarrow \Omega_M$ and $\Omega_E^\dagger = \Omega_E$.

Expanding $S_{\text{eff}}^{\text{rot}} = -\text{Sp}\log D$ of Ref. [16] up to the quadratic order in Ω_E and assuming the time independence of $\Omega_E(t)$ one gets (in Minkowski metric and in the chiral limit)

$$L_0 = -M_{\rm cl} + \frac{1}{2}\Omega_a I_{ab}\Omega_b - \frac{N_c}{2\sqrt{3}}\Omega_8 \ , \tag{8}$$

where tensor of inertia $I_{ab} = \mathrm{diag}(I_1, I_1, I_1, I_2, I_2, I_2, I_2, 0)$ can be found in Ref. [16].

The original path integral $\int \mathcal{D}U(x,t)$ will be in the following approximated by the integral over the rotation matrices $A(t)$ only, neglecting translations and other fluctuations [29]. This is known as the quantization of the rotational zero modes [31]. Functional integration over all $A(t)$, is equivalent in the Hamiltonian operator formulation to [33]

$$\int \mathcal{D}A(t)\exp\left(-\int_{-T/2}^{T/2} dt L_0\right)$$
$$= \langle A(T/2)|\exp(-TH^{(0)})|A(-T/2)\rangle \ , \tag{9}$$

where $H^{(0)}$ is a collective rotational Hamiltonian corresponding to the Lagrangian of Eq. (8). In this way the path-integral can be evaluated in terms of the eigenstates of the collective Hamiltonian. This is discussed at length in Sec. IV.

Before we calculate the axial properties we shall concentrate on the mass splittings. This allows to study the perturbation expansion in m_s and to fix the remaining parameter of the model—the constituent quark mass M.

To calculate the mass splittings one has to expand the effective action in powers of the current quark mass matrix $m = \mu_0\lambda_0 - \mu_8\lambda_8 - \mu_3\lambda_3$ with

$$\mu_0 = \frac{1}{\sqrt{6}}(m_u + m_d + m_s),$$
$$\mu_8 = \frac{1}{\sqrt{12}}(2m_s - m_u m_d),$$
$$\mu_3 = \frac{1}{2}m_d - m_u \ . \tag{10}$$

We expand the effective action up to terms of the order of m_s, m_s^2, $m_s\Omega$, and Ω^2 (expansion in Ω corresponds to the expansion in $1/N_c$) [34]:

$$L_m = -\sigma m_s + \sigma m_s D_{88} \ , \tag{11}$$

$$L_{m\Omega} = -\frac{2}{\sqrt{3}} m_s D_{8a} K_{ab}\Omega_b \ , \tag{12}$$

$$L_{m^2} = \frac{2}{9}m_s^2 (N_0(1-D_{88})^2 + 3N_{ab}D_{8a}D_{8b}) \ , \tag{13}$$

where the constant σ is related to the sigma term $\Sigma = 3/2(m_u + m_d)\sigma$ and $D_{ab} = \frac{1}{2}\mathrm{Tr}(A^\dagger\lambda_a A\lambda_b)$. We define in this order $L^{\rm tot} = L_0 + L_m + L_{m\Omega} + L_{m^2}$. The mass spectrum obtained with the help of $L_0 + L_m + L_{m\Omega}$ was discussed in Refs. [16,17]; there one can also find explicit formulas for $K_{ab} = \mathrm{diag}(K_1, K_1, K_1, K_2, K_2, K_2, 0)$. Let us here only mention that the *anomalous* moments of inertia K_i are nearly entirely given by the valence part, whereas the contribution of the valence level to I_i amounts to approximately 60%. The new feature of the present calculation is the presence of the moments of inertia $N_{ab} = \mathrm{diag}(N_1, N_1, N_1, N_2, N_2, N_2, N_2, N_0/3)$ in L_{m^2} defined as

$$N_{ab} = \frac{N_c}{4}\sum_{n,m}\langle m|\lambda_a\gamma_0|n\rangle\langle n|\lambda_b\gamma_0|m\rangle \mathcal{R}_\beta(E_n, E_m) \ , \tag{14}$$

where $\mathcal{R}_\beta(E_n, E_m)$ is given by

$$\mathcal{R}_\beta(E_n, E_m)$$
$$= \frac{1}{2\sqrt{\pi}}\int \frac{du}{\sqrt{u}}\phi(u)\left[\frac{E_n e^{-uE_n^2} - E_m e^{-uE_m^2}}{E_n - E_m}\right] \tag{15}$$

and differs from the regularization function for the usual moment of inertia $\mathcal{R}_I(E_n, E_m)$ [16] because of the different Hermiticity behavior of the mass term and the Coriolis term (Ω) in $S_{\rm eff}^{\rm rot}$.

The value of $N_{0,1,2}$ together with the values of $I_{1,2}$, $K_{1,2}$, and Σ for $M = 423$ MeV are given by (in fm) $N_0 = 0.668$, $N_1 = 0.438$, $N_2 = 0.370$ fm, $I_1 = 1.178$, $I_2 = 0.569$ fm, $K_1 = 0.369$, $K_2 = 0.255$, and $\Sigma = 56.14$ MeV. The numerical values for other constituent masses can be found, e.g., in Ref. [34].

The Lagrangian of Eq. (8) and Eqs. (11) and (12) resembles the Skyrmion Lagrangian with vector mesons (cf. Ref. [20]). The quantization proceeds as in the Skyrme model; one defines the quantities (see, e.g., [35–38] for details)

$$J_a = -R_a = I_{ab}\Omega_b - \mu_i D_{ib}K_{ba} - \delta_{a8}\frac{N_c}{2\sqrt{3}} \tag{16}$$

($i = 3$ and 8, $a, b = 1, \ldots, 8$) which are promoted to the spin operators $\hat{J}_a = -\hat{R}_a$. The flavor operators read: $\hat{T}_a = -D_{ab}\hat{J}_b$. Note that despite the fact that $\hat{J}_a$ fulfill the SU(3) algebra, only $\hat{J}_{1,2,3}$ have the meaning of the symmetry generators. That is due to the structure of the SU(3) hedgehog ansatz and is reflected in the fact that $\hat{J}_8 = -N_c/\sqrt{12}$ generates a constraint. Therefore the wave function of the baryon state $B = Y, T, T_3, J, J_3$ belonging to the SU(3) representation $\mathcal{R}$ reads

$$|\mathcal{R}, B\rangle = \sqrt{\mathrm{dim}\mathcal{R}}\langle Y, I, I_3|D^{(\mathcal{R})}(A)| - Y', J, -J_3\rangle^* \ , \tag{17}$$

where the right hypercharge Y' is in fact constrained to be -1. The lowest SU(3) representations which contain states with $Y = 1$ are $\mathcal{R} = 8$ and $\mathcal{R} = 10$. The quantized collective Hamiltonian $H^{\rm tot}$ from

$$H^{\rm tot} = \sum_a \Omega_a \frac{\partial L^{\rm tot}}{\partial\Omega_a} - L^{\rm tot} \tag{18}$$

reads in the chiral limit ($\mu_i = 0$)

$$H^{(0)} = M_{\rm cl} + H_{\rm SU(2)} + H_{\rm SU(3)} \ , \tag{19}$$

$$H_{SU(2)} = \frac{1}{2I_1} C_2(SU(2)),$$
$$H_{SU(3)} = \frac{1}{2I_2} \left[C_2(SU(3)) - C_2(SU(2)) - \frac{N_c^2}{12} \right] .$$

Here C_2 denote the Casimir operators of the spin SU(2) and flavor SU(3). M_{cl} is the classical soliton mass. In this paper we will concentrate on the mass splittings. These are determined in the present model by analytic strange mass contributions of the order O_{m_s} and $O(m_s^2)$. Whereas linear terms are given by Eqs. (11) and (12), the various contributions from quadratic m_s corrections can be classified as follows.

Quadratic terms from the expansion of the EEA, corresponding to the Lagrangian in Eq. (13). This will be referred to as the *dynamical* corrections.

Quadratic terms from replacing the rotational velocities in Eq. (16) by the generators J_a by using Eq. (18). This will be referred to as the *kinematical* corrections.

Quadratic corrections the collective wave-function, as will be discussed in the following. This will be referred to as the *wave-function* corrections.

Then the Hamiltonian from Eq. (18) up to terms linear and quadratic in m_s reads

$$H^{(1)} = \left\{ \sigma - r_2 Y - (\sigma - r_2) D_{88} + \frac{2}{\sqrt{3}} (r_1 - r_2) \sum_{A=1}^{3} D_{8A} J_A \right\} m_s ,$$
$$H^{(2)}_{\rm kin} = \frac{2}{3} \left\{ r_2 K_2 (1 - D_{88}^2) + (r_1 K_1 - r_2 K_2) \sum_{A=1}^{3} D_{8A}^2 \right\} m_s^2 ,$$
$$H^{(2)}_{\rm dyn} = -\frac{2}{9} \left\{ (N_0 + 3N_2) - 2N_0 D_{88} + (N_0 - 3N_2) D_{88}^2 + 3(N_1 - N_2) \sum_{A=1}^{3} D_{8A}^2 \right\} m_s^2 , \tag{20}$$

where $r_i = K_i/I_i$. According to the classification given above we have split the $O(m_s^2)$ Hamiltonian into the *kinematical* part $H^{(2)}_{\rm kin}$, and the *dynamical* part $H^{(2)}_{\rm dyn}$.

The Hamiltonian $H^{(1)}$ mixes states of different SU(3) representations; therefore the wave functions is no longer a pure octet but rather a mixture:

$$|B\rangle = |\mathbf{8}, B\rangle + c^{B}_{\overline{10}} |\overline{\mathbf{10}}, B\rangle + c^{B}_{27} |\mathbf{27}, B\rangle , \tag{21}$$

where $B = N, \Lambda, \Sigma, \Xi$. The coefficients $c^B_{\mathcal{R}}$ depend linearly on m_s, therefore with this accuracy there is no need to change the normalization of the wave function. In the following we will need their explicit form only for the octetlike states:

$$c^{B}_{\overline{10}} = \frac{\sqrt{5}}{15} (\sigma - r_1) \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} I_2 m_s,$$

$$c^B_{27} = \frac{1}{75} (3\sigma + r_1 - 4r_2) \begin{bmatrix} \sqrt{6} \\ 3 \\ 2 \\ \sqrt{6} \end{bmatrix} I_2 m_s \tag{22}$$

in the basis $[N, \Lambda, \Sigma, \Xi]$. The corresponding $O(m_s^2)$ contribution to the energy are explicitly given in Ref. [34].

In order to fix the value of the constituent mass M we adopt the following procedure: first for given M we find the optimal m_s which reproduces **10-8** splitting $\Delta_{10\text{-}8}$. This dependence can be read out from Fig. 1 of Ref. [34]. Next we plot the m_s dependence of the deviations *theory-experiment* for each hyperon (Fig. 2). One should remember that for each m_s the optimal constituent quark mass M was used, so that $\Delta_{10\text{-}8}$ was automatically reproduced for each m_s. The smallest deviations ± 7 MeV for all splittings correspond to $m_s \approx 180$ MeV and to $M \approx 423$ MeV respectively.

It is important to note that all three quadratic m_s contributions, i.e., kinematical, dynamical, and wave function corrections are equally important [34]. It is therefore inconsistent to calculate only the wave function corrections, as it is sometimes done (the Yabu-Ando diagonalization) and neglect the other ones.

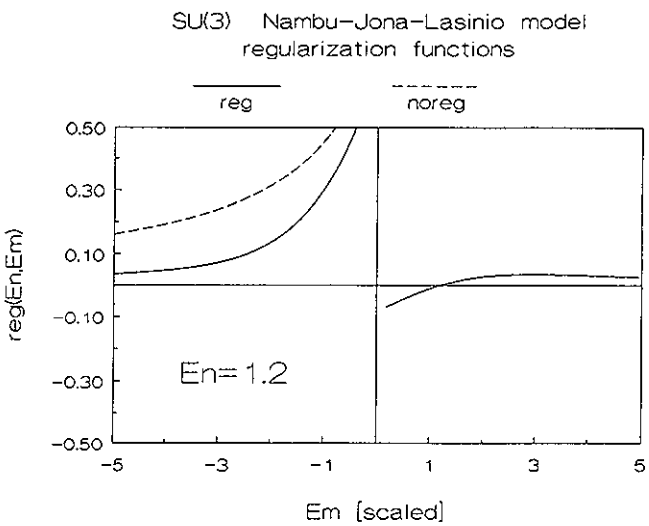

FIG. 1. The regularization function $\mathcal{R}_Q(E_n, E_m)$ for the time-ordered expressions (reg) for fixed E_n and $M = 400$ MeV in dependence of E_m. This is compared to the function (noreg), which is obtained in the infinite cutoff limit.

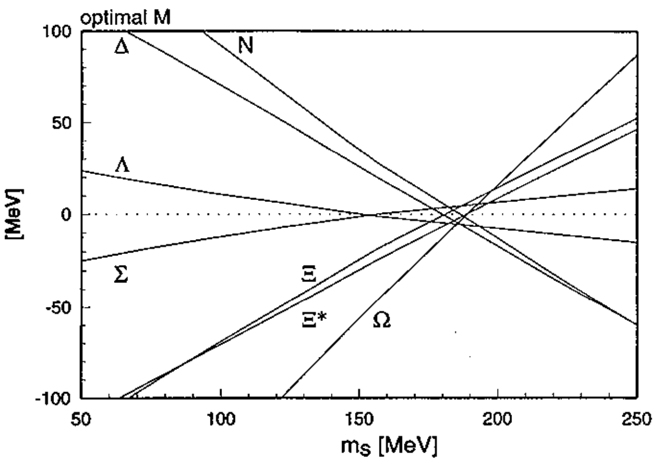

FIG. 2. The model deviations (i.e., *theory experiment*) for octet and decuplet baryons as functions of m_s for the optimal constituent quark mass M.

IV. CURRENTS EXPECTATION VALUES IN THE NJL MODEL

In order to calculate observables like the axial vector currents, one has to consider the path integral expectation value of these operators. This should be done by considering the quark and baryon correlation functions which were investigated for solitonic configurations in Refs. [29,10]. In this section we will show in a pedagogical way how the *time ordering* within a quark loop together with the collective quantization brings up corrections linear in the rotational velocity Ω [12–14]. It will become clear that the fundamental features of the *nonlocality* of the NJL effective action and the path integral relation (9) to the time-ordered product of the operators are of the utmost importance. In particular, it will be clearly shown that there is no arbitrariness in deriving these corrections. The approach presented here will be different from the one of Ref. [13], where these new corrections were calculated from the unregularized expressions.

The correlator under consideration in the presence of $\mathcal{L}_{\rm NJL}^{\rm bos}$ discussed in Sec. II is defined by [29]

$$\begin{aligned}\langle \mathcal{A}_\mu^a(x)\rangle &= \langle J_B(\vec{y}_0,T/2)\bar{q}(x)\gamma_\mu\gamma_5\lambda^a q(x) J_B^\dagger(\vec{z}_0,-T/2)\rangle \\ &= \int \mathcal{L}\bar{q}\mathcal{D}q\mathcal{D}A J_B(\vec{y}_0,T/2)\bar{q}(x)\gamma_\mu\gamma_5\lambda^a q(x) \\ &\quad \times J_B^\dagger(\vec{z}_0,-T/2)\exp\left(-\int_{-T/2}^{T/2} dt\int d^3x\mathcal{L}_{\rm NJL}^{\rm bos}\right) ,\end{aligned} \tag{23}$$

where the integration over the meson fields is restricted to the rotational zero modes. The Ioffe currents $J_B(\vec{x},t)$ can be defined in terms of the quark field operators [39]

$$J_B(\vec{x},t) = \frac{1}{N_c!}\epsilon_{c_1\cdots c_{N_c}}\Gamma^{f_1\cdots f_{N_c}} q_{f_1,c_1}(x)\cdots q_{f_{N_c},c_{N_c}}(x), \tag{24}$$

where $\Gamma_{f_1\cdots f_{N_c}}$ is a symmetric matrix in flavor and spin space and the $\epsilon_{c_1\cdots c_{N_c}}$ accounts for the antisymmetry in color indices. Integrating out the quark fields the sea part of the correlator can be written as[3]

$$\begin{aligned}\langle \mathcal{A}_\mu^a(\vec{x},t)\rangle_{\rm sea} = \mathcal{N}\bar{\Gamma}^{f_1\cdots f_{N_c}}\Gamma^{g_1\cdots g_{N_c}} \int \mathcal{D}A\exp\left(-\int_t^{T/2} dt' L^{\rm tot}(\Omega(t'))\right) \frac{\delta}{\delta s^a(\vec{x},t)}{\rm Sp}\log D[s] \\ \times\exp\left(-\int_{-T/2}^{t} dt' L^{\rm tot}(\Omega(t'))\right)\sum_{n={\rm val}}\prod_{i=1}^{N_c} A(T/2)\langle y_0|n\rangle_{f_i}\langle n|z_0\rangle_{g_i}A(-T/2) ,\end{aligned} \tag{25}$$

where

$$D[s] = \partial_t + H + A^\dagger \dot{A} - i\gamma_4 A^\dagger m A + is^b\gamma_4\gamma_i\gamma_5 A^\dagger I_b A .$$

In order to obtain the axial vector coupling constants $g_A^{(a)}$ for Eq. (25) the following definitions for I_b hold:

$$g_A^{(0)} : I_b = 1,\quad g_A^{(3)} : I_b = \lambda_3,\quad g_A^{(8)} : I_b = \lambda_8 . \tag{26}$$

One should note that the correlator (25) contains the nonlocal object $1/D[s]$, which is expanded in $D[s=0] - D_0$, i.e., in terms of the rotational velocity and the current quark mass:

$$\frac{\delta}{\delta s^a(\vec{x},t)}{\rm Sp}\ln D[s] = {\rm Sp}\frac{1}{D_0}\frac{\delta D[s]}{\delta s^a(\vec{x},t)} - {\rm Sp}\frac{1}{D_0}(D[s=0]-D_0)\frac{1}{D_0}\frac{\delta D[s]}{\delta s^a(\vec{x},t)}\cdots . \tag{27}$$

The first term gives the $O(N_c)$ contribution to the axial current [29] and has a counterpart in local mesonic theories. The second term, which is of the order $O(N_c^0)$, needs careful analysis. One should note that the propagator

[3]The valence part will be considered at the end of the section.

$$G(\vec{x}_1,t_1,\vec{x}_2,t_2) = \langle \vec{x}_1,t_1|\frac{1}{D_0}|\vec{x}_2,t_2\rangle$$
$$= \theta(t_1-t_2)\sum_{n>0} e^{-E_n(t_1-t_2)}\langle \vec{x}_1,t_1|n\rangle\langle n|\vec{x}_2,t_2\rangle - \theta(t_2-t_1)\sum_{n<0} e^{-E_n(t_1-t_2)}\langle \vec{x}_1,t_1|n\rangle\langle n|\vec{x}_2,t_2\rangle \quad (28)$$

which appears in Eq. (27) after inserting the complete set of eigenstates is nonlocal in time and space. Using the definition $O_1(A(t)) = D[s=0] - D_0$ and $O_2(A(t)) = \delta D[s]/\delta s^a(x)$ the second term on the right-hand side (RHS) of Eq. (27) becomes[4]

$$-\mathrm{Tr}_{c\gamma\lambda}\int d^3\vec{x}_1 dt_1 G(\vec{x},t,\vec{x}_1,t_1) \times O_1(t_1)G(\vec{x}_1,t_1,\vec{x},t)O_2(t) \ , \quad (29)$$

where $\mathrm{Tr}_{c\gamma\lambda}$ denotes the trace over color, Dirac, and flavor indices. The product $O_1(t_1)O_2(t)$ in Eq. (29) has to be time-ordered due to the presence of the θ functions in Eq. (28). Simultaneously the c numbers O_i have to be replaced by the operators $\hat{O}_i$ according to the prescription of the semiclassical quantization. Making use of Eq. (9) one can express the path integral over $A(t)$ by a time-ordered product:

$$\langle A(-T/2)|\mathcal{T}(\hat{O}_1(A(x_1))\hat{O}_2(A(x_2)))|A(T/2)\rangle = \int_{A=A(-T/2)}^{A=A(T/2)} \mathcal{D}AO_1(A(x_1))O_2(A(x_2)) \times\exp\left(-\int_{T/2}^{T/2} dt'\mathcal{L}^{\mathrm{tot}}\right) \ . \quad (30)$$

This leads to the useful definition of a time-ordered trace

$$\mathrm{Sp}_{(\mathrm{to})}\hat{O}_1\hat{O}_2 = \mathrm{Tr}_{c\gamma\lambda}\int d^4x\langle x|\mathcal{T}(\hat{O}_1\hat{O}_2)|x\rangle \ , \quad (31)$$

where

$$\mathcal{T}((\hat{O}_1(t_1)\hat{O}_2(t_2))) = \theta(t_1-t_2)\hat{O}_1(t_1)\hat{O}_2(t_2) + \theta(t_2-t_1)\hat{O}_2(t_2)\hat{O}_1(t_1) \ .$$

The advantage of retaining the trace in this form is the straightforward applicability of the regularization procedure. Let us split the effective action into real and imaginary parts. Then the expectation value of the axial current from Eq. (25) reduces to an ordinary integral over the collective coordinates ξ_A of SU(3):

$$\langle A_i^a(\vec{x},t)\rangle_{\mathrm{sea}} = \int d\xi_A(t)\langle B(\xi_A)|\frac{\delta}{\delta s^a(\vec{x},t)}[\mathrm{Re\,Sp}_{(\mathrm{to})}\ln D + i\,\mathrm{ImSp}_{(\mathrm{to})}\ln D]|_{s=0}|B(\xi_A)\rangle \ ,$$
$$= \int d\xi_A(t)\langle B(\xi_A)|\frac{\delta}{\delta s^a(\vec{x},t)}\frac{1}{2}\left[\mathrm{Sp}_{(\mathrm{to})}\ln D^\dagger D + \mathrm{Sp}_{(\mathrm{tot})}\ln\frac{D}{D^\dagger}\right]\Bigg|_{s=0}|B(\xi_A)\rangle \ , \quad (32)$$

where $|B(\xi_A)\rangle$ is the baryon wave function of Eq. (21) and $D = D[s]$. It is important to consider $s^a = s^a(\vec{x},t)$ as explicitly time dependent (see Appendix A). Preserving vector gauge invariance [40] by using the proper time regularization [28] we regularize the real part of the effective action similarly to Eq. (4):

$$\mathrm{Sp}_{(\mathrm{to})}\ln D^\dagger D \to (-)\mathrm{Sp}_{(\mathrm{to})}\int\frac{du}{u}\phi(u,\Lambda_1,\Lambda_2) \times\exp(-uD^\dagger D) \ , \quad (33)$$

where $\phi(u)$ is given in Sec. II. Note that for the *symmetric* contributions [13], i.e., when the operators $\hat{O}_1$ and $\hat{O}_2$ commute,[5] the time ordering has no influence.

[4]In our case O_1 corresponds either to the mass correction proportional to D_{ab} or to the rotational correction proportional to Ω_a, i.e., to the generator $\hat{J}_a$, and O_2 corresponds to the current insertion proportional to the D function.

[5]In our case this happens if the index of a generator $\hat{J}_a$ is such that it commutes with the D_{bc} function.

The valence part can be obtained from the sea part (32) by calculating the full correlation function [29]. Alternatively by introducing $D' = D-\mu$, where μ is a chemical potential with $0 < \mu < E_{\mathrm{val}}$ [9], $S_{\mathrm{eff}}^{\mathrm{tot}} = S_{\mathrm{eff}}^{\mathrm{val}} + S_{\mathrm{eff}}^{\mathrm{sea}}$ with the definitions $S_{\mathrm{eff}}^{\mathrm{val}} = S_{\mathrm{eff}}[D'] - S_{\mathrm{eff}}[D]$ and $S_{\mathrm{eff}}^{\mathrm{sea}} = S_{\mathrm{eff}}[D]$. The subtraction of the vacuum contributions is implicitly understood.

Although in this section we have concentrated on the axial currents, it is clear that the above results hold for any kind of current in the NJL model.

V. AXIAL CURRENTS IN THE CHIRAL LIMIT

A. The lowest order contribution $\sim \Omega^0$

The axial vector coupling constants $g_A^{(a)}$, defined as the corresponding form factor in the limit $q^2 = 0$, can be calculated from Eq. (32). By comparison with Eq. (32) one can define a collective operator $\hat{g}_A^{(a)}$ such that

$$g_A^{(a)} = \int d^3x \langle A^a_{i=3}(\vec{x},t)\rangle$$
$$= \int d\xi_A \langle B(\xi_A)|\hat{g}_A^{(a)}|B(\xi_A)\rangle \ . \qquad (34)$$

$\hat{g}_A^{(a)}$ is obtained by expanding $\langle A_i^a(\vec{x},t)\rangle$ in Eq. (32) in terms of the rotational frequency and the strange quark mass but without performing the final $d\xi_A$ integration. The rotational velocity is then replaced by the generators (16) yielding after the integration over d^3x the collective operator $\hat{g}_A^{(a)}$ expressed in terms of $\hat{J}_a$ and Wigner functions D_{ab}.

The lowest-order result in Ω (i.e., Ω^0) comes from the proper-time regularized real part of the EEA (33). One obtains for $a = 3$ and 8 (see Appendix A for details)

$$\hat{g}_A^{(a)}(\Omega^0, m_s^0) = M_3 D_{a3} \quad \text{for } a = 3 \text{ and } 8 \ , \qquad (35)$$

where $A^\dagger I_a A = D_{ab}\lambda_b$ and $A^\dagger I_a A = 1$ for $a = 0$. At this level $\hat{g}_A^{(0)} \equiv 0$ [41]. The quantity $M_3 = M_3^{\rm val} + M_3^{\rm sea}$ comes from the real part of the action and is given by

$$M_3^{\rm val} = N_c \langle {\rm val}|\sigma_3\lambda_3|{\rm val}\rangle \qquad (36)$$

and

$$M_3^{\rm sea} = -\frac{N_c}{2}\sum_{\text{all } n}\langle n|\sigma_3\lambda_3|n\rangle {\rm sgn}(E_n)\mathcal{R}_\Sigma(E_n) \ , \qquad (37)$$

where the regularization function reads [21]

$$\mathcal{R}_\Sigma(E_n) = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{du}{\sqrt{u}} e^{-u}\phi\left(\frac{u}{E_n^2}\right) \ . \qquad (38)$$

The values of M_3 for the constituent mass $M \approx 423$ MeV read $M_3^{\rm val} = -2.209$, $M_3^{\rm sea} = -0.316$, $M_3^{\rm tot} = -2.525$.

B. Anomalous $1/N_c$ corrections from the imaginary part of the EEA

As explained in the beginning of Sec. IV $1/N_c$ corrections (i.e., terms linear in the rotational velocity Ω) to Eq. (35) can be unambiguously separated into *local* quantities which emerge from the imaginary part of the effective Euclidean action, and *nonlocal* quantities emerging from the real part due to the explicit time ordering of the collective operators. The former quantities are related to Witten's anomalous axial current [32], whereas the latter ones have no counterpart in mesonic effective theories. It will turn out that in a certain sense they renormalize the leading contribution of the axial current given by Eq. (35) (compare with Refs. [42,43]).

In the chiral limit the anomalous corrections linear in Ω can be written as

$$\hat{g}_A^{(a)} = -M_{bc} i\{D_{ab}, \Omega_E^c\} = -2M_{bc}D_{ab}\Omega_M^c \qquad (39)$$

with

$$M_{bc} = \frac{N_c}{4}\sum_{n,m}\langle n|\sigma_3\lambda_b|m\rangle\langle m|\lambda_c|n\rangle \mathcal{R}_\mathcal{M}(E_n,E_m) \qquad (40)$$

and the cutoff independent regularization function $\mathcal{R}_\mathcal{M}$

$$\mathcal{R}_\mathcal{M}(E_n,E_m) = \frac{1}{2}\frac{{\rm sgn}(E_n-\mu) - {\rm sgn}(E_m-\mu)}{E_n - E_m} \ . \qquad (41)$$

As already noted the chemical potential μ is chosen in such a way that it always lies between the *valence level* and the positive continuum of states. In this way the quantities M_{bc} correspond to the full baryon number one contribution and therefore contain the sum of the valence and the sea part. Additionally we define for later use $\bar{M}_{8a} = \sqrt{3}M_{8a}$ and $\bar{M}_{a8} = \sqrt{3}M_{a8}$. The only nonvanishing contributions in Eq. (39) are M_{83} and $M_{44} = M_{55} = -M_{66} = -M_{77}$. Using the symmetries of the hedgehog states one can write for $a = 3$ and 8:

$$\hat{g}_A^{(a)} = -\frac{2\bar{M}_{83}}{\sqrt{3}}D_{a8}\Omega_3 - 4M_{44}d_{3bb}D_{ab}\Omega_b \ , \qquad (42)$$

where the sum over $b = 4,\ldots,7$ is understood. The values of M_{44} and $\bar{M}_{83}$ entering Eq. (42) read (in fm) $M_{44}^{\rm val} = -0.288$, $M_{44}^{\rm sea} = -0.012$, $M_{44}^{\rm tot} = -0.301$ and $\bar{M}_{83}^{\rm val} = -0.422$, $\bar{M}_{83}^{\rm sea} = -0.016$, $\bar{M}_{83}^{\rm tot} = -0.438$ for $M \approx 423$ MeV. It is clear from the form of Eq. (42) that the anomalous corrections linear in Ω vanish in the SU(2) case.

C. Nonlocal $1/N_c$ corrections from the real part of the EEA

Let us now turn to the main objective of the present paper, namely to the nonlocal $1/N_c$ corrections which are due to the explicit time ordering of the collective operators [12,14,13]. Straightforward application of Eq. (28) into Eq. (29) leads to a double sum over the opposite sign energy levels of the intrinsic Dirac Hamiltonian (3), where the operators O_1 and O_2 are time ordered. Making use of the symmetry properties of the matrix elements of the intrinsic operators:

$$\langle n|\lambda^c|m\rangle = -\langle m_G|\lambda^c|n_G\rangle,$$
$$\langle n|\sigma_3\lambda^b|m\rangle = \langle m_G|\sigma_3\lambda^b|n_g\rangle \ , \qquad (43)$$

where n_G denotes a G-parity conjugated state, one eventually arrives at

$$\hat{g}_A^{(a)} = \frac{N_c}{4} i[\Omega_E^c, D_{ab}]\sum_{m,n}\langle n|\lambda^c|m\rangle\langle m|\sigma_3\lambda^b|n\rangle$$
$$\times \mathcal{R}_\mathcal{Q}(E_n,E_m) \ , \qquad (44)$$

where the rather complicated regularization function $\mathcal{R}_\mathcal{Q}$ is given by

$$\mathcal{R}_\mathcal{Q}(E_n,E_m) = \int_0^1 \frac{d\alpha}{2\pi}\frac{\alpha E_n - (1-\alpha)E_m}{\sqrt{\alpha(1-\alpha)}}$$
$$\times c_i \frac{\exp\{-[\alpha E_n^2 + (1-\alpha)E_m^2]/\Lambda_i^2\}}{\alpha E_n^2 + (1-\alpha)E_m^2} \ . \qquad (45)$$

Here the proper-time u integration for our steplike functions $\phi(u)$ has been already performed (see Appendix A for a general expression). In the limit $\Lambda_i \to \infty$ Eq. (45) immediately reduces to Eq. (A13) of Appendix A and coincides therefore with our former prescription in Ref. [13]. However, as we will see later, with the regularization properly taken into account, the physical values will come out much better.

Using the quantization condition for Ω and making use of the commutator $[\hat{J}_c, D_{ab}] = i f_{cbd} D_{ad}$ [35], Eq. (44) can be written as

$$\hat{g}_A^{(a)} = \frac{-i f_{cdb} D_{ad}}{I_{cc}} Q_{bc} = -\left(\frac{2iQ_{12}}{I_1} + \frac{2iQ_{45}}{I_2}\right) D_{a3} \ , \quad (46)$$

where the quantities Q_{bc} coming from the real part of the EEA are given by $Q_{bc} = Q_{bc}^{\rm val} + Q_{bc}^{\rm sea}$. Explicitly the valence part reads

$$Q_{bc}^{\rm val} = \frac{N_c}{2} \sum_n \frac{\langle n|\sigma_3\lambda_b|{\rm val}\rangle\langle{\rm val}|\lambda_c|n\rangle}{E_n - E_{\rm val}} {\rm sgn} E_n \quad (47)$$

and the sea part:

$$Q_{bc}^{\rm sea} = \frac{N_c}{4} \sum_{m,n} \langle n|\sigma_3\lambda_b|m\rangle\langle m|\lambda_c|n\rangle \mathcal{R}_Q(E_n, E_m) \ . \quad (48)$$

The numerical values for the Q_{bc} in the real representation $Q_{12} = -2iQ_{-+}$ and $Q_{45} = i\bar{Q}_{45}$ read (in fm): $Q_{-+}^{\rm val} = 0.279$, $Q_{-+}^{\rm val} = 0.019$, $Q_{-+}^{\rm tot} = 0.298$ and $\bar{Q}_{45}^{\rm val} = -0.279$, $\bar{Q}_{45}^{\rm sea} = -0.018$, $\bar{Q}_{45}^{\rm tot} = -0.297$ for $M \approx 423$ MeV.

The valence contribution $Q_{bc}^{\rm val}$ differs from the formula given in Ref. [12], where the existence of such corrections was claimed for the first time. The correct path-integral formula is given by Eqs. (47) and (48). Numerically however the difference between our expression for $Q_{bc}^{\rm val}$ and the expression of Ref. [12] is quite small. Note also that in Ref. [12] the sea contribution to Q_{bc} was erroneously claimed to be identically zero. Again numerically $Q_{bc}^{\rm sea}$ is rather small.

Putting all these corrections together one obtains:

$$\hat{g}_A^{(a)}(\Omega^0 + \Omega^1, m_s^0) = \left[M_3 - \frac{4Q_{-+}}{I_1} + \frac{2\bar{Q}_{45}}{I_2}\right] D_{a3} - \frac{2\bar{M}_{83}}{\sqrt{3}I_1} D_{a8}\hat{J}_3 - \frac{4M_{44}}{I_2} d_{3bb} D_{ab}\hat{J}_b \quad \text{for } a = 3, 8 \quad (49)$$

(b runs over $4, \ldots, 7$). Note that all the quantities M_3, Q_{bc}, M_{bc} and also I_1, I_2 are of the order $O(N_c)$, such that the Q_{bc} terms in the brackets indeed correspond to $1/N_c$ corrections to the lowest order result. In other words if one neglects the anomalous, purely SU(3) contribution in Eq. (49), the ratio of different $\hat{g}_A^{(a)}$'s has no $1/N_c$ correction [43].

D. The anomalous singlet axial-vector current

The singlet axial-vector current was already given in [44] and it gets only anomalous contribution linear in Ω:

$$\hat{g}_A^{(0)}(\Omega^1, m_s^0) = -\frac{2\bar{M}_{83}}{I_1} \hat{J}_3 \ . \quad (50)$$

Note that Eq. (50), given here in the context of SU(3), coincides exactly with the SU(2) result of Ref. [10]. This is because only spin eigenvalues (J_3) enter here, whereas the other $\hat{g}_A^{(a)}$'s always contain D functions, whose matrix elements depend crucially on the SU($N_{\rm flavor}$) algebra used.

E. The axial currents in the leading order of gradient expansion

All g_A's consist of the valence quark and the sea quark contribution. In the limit of the large soliton size the valence contribution dies out and only the sea quarks contribute. In this limit the gradient approximation [45,46,9] holds. It corresponds to a local effective meson theory. We have used the gradient expansion to check the numerical results. Even or the realistic soliton sizes (of about 1 fm) the gradient expanded expressions can be used as a good approximation to the sea quark contribution and the total g_A is obtained by adding the valence quark terms. The lowest order result for SU(2) is given in Ref. [18] and it coincides with the expressions from the Skyrme model.

Terms linear in Ω can be also gradient expanded. In SU(3) one gets the anomalous contribution coinciding with the Wess-Zumino-Witten term in the Skyrme model. Using the results of Appendix B one can also calculate the gradient expansion of the *time-ordered* terms.[6] Neglecting the anomalous terms one obtains altogether (for $a = 3, 8$)

$$\hat{g}_A^{(a)} = \int dr\, r^2 \left(\theta' + \frac{\sin 2\theta}{r}\right) \times \left[\frac{8\pi}{3} f_\pi^2 + \frac{N_c}{3}\frac{M}{4I_1} + \frac{N_c}{3}\frac{M}{8I_2}\right] D_{a3} + \cdots \ . \quad (51)$$

It is once again clear from Eq. (51) that the time-ordered contributions (i.e., from Q_{bc}) led to the renormalization of g_A in the sense that they are also proportional to D_{a3} and to the same integral over the hedgehog profile function θ. In SU(2), where the anomalous terms for the axial current [dots in Eq. (51)] vanish, the ratios of $g_A^{(a)}$'s for different baryons have no $1/N_c$ corrections. This was also found by Dashen and Manohar from large N_c QCD [42,43]. Furthermore the SU(2) result resembles very much the odd nonrelativistic quark model prediction for the $1/N_c$ correction, which is given by $g_A = (N_c + 2)/3$. Using the value $I_1 = 1.178$ fm for $M \approx 423$ MeV, in the SU(2) case without I_2 the zeroth order result of Eq. (51) gets an approximately 25% correction from the $1/N_c$

[6] The moments Q_{bc} are evaluated here in the infinite cutoff limit in order to get a simple and not explicitly cutoff dependent result.

TABLE I. The axial-vector coupling constant $g_A^{(3)}$ for the SU(3) Nambu–Jona-Lasinio model in dependence on the constituent quark mass M. The strange current quark mass is chosen as $m_s = 180$ MeV. The final model predictions are given by $g_A^{(3)}(\Omega^1)$ in SU(2) and $g_A^{(3)}(\Omega^1, m_s)$ in SU(3). The experimental value is given by $g_A^{(3),\text{expt}} = 1.26$.

	SU(2)		SU(3)		
M (MeV)	$g_A^{(3)}(\Omega^0)$	$g_A^{(3)}(\Omega^1)$	$g_A^{(3)}(\Omega^0, m_s^0)$	$g_A^{(3)}(\Omega^1, m_s^0)$	$g_A^{(3)}(\Omega^1, m_s^1)$
363	0.920	1.302	0.644	1.482	1.603
395	0.873	1.224	0.611	1.381	1.473
419	0.841	1.179	0.589	1.328	1.407
423	0.837	1.173	0.585	1.314	1.380
465	0.792	1.109	0.554	1.250	1.308

term; substantially less than the full solitonic $1/N_c$ contribution, which amounts to $\simeq 40\%$ (see Table I). In the SU(3) case however, there are additional corrections from the third term in the square brackets of Eq. (51) ($\sim 1/I_2$) *and* from the anomalous terms [dots in Eq. (51)], such that the simple rescaling factor does not exist any more.

VI. STRANGE MASS CORRECTIONS FOR g_A

In this section we will evaluate the symmetry breaking corrections to the axial currents due to the nonvanishing strange quark mass. These arise from the term $A^\dagger m A = \mu_0 - \mu_8 \lambda^a D_{8a}$. In the linear order in m_s and in the zeroth order in Ω neither the contributions from the imaginary part nor from the explicit time ordering (because D functions always commute with each other) exist. Therefore the entire symmetry breaking contribution comes from the real part of the EEA. Performing the expansion of the real part of the EEA in m_s one gets, for $a = 3, 8$,

$$\hat{g}_A^{(a)}(\Omega^0, m_s^1) = -\frac{4m_s}{\sqrt{3}} R_{38} D_{a3}(1 - D_{88}) + \frac{4m_s}{\sqrt{3}} R_{83} D_{a8} D_{83} + \frac{8m_s}{\sqrt{3}} d_{3bb} R_{44} D_{ab} D_{8b} \quad (52)$$

with $b = 4, \ldots, 7$. The proper time regularized quantities $R_{bc} = R_{bc}^{\text{val}} + R_{bc}^{\text{sea}}$ are given by

$$R_{bc}^{\text{val}} = \frac{N_c}{2} \sum_n \frac{\langle n|\sigma_3 \lambda_b|\text{val}\rangle\langle\text{val}|\lambda_c \gamma_0|n\rangle}{E_n - E_v} \quad (53)$$

and

$$R_{bc}^{\text{sea}} = \frac{N_c}{4} \sum_{n,m} \langle n|\sigma_3 \lambda_b|m\rangle\langle m|\lambda_c \gamma_0|n\rangle \mathcal{R}_\beta(E_n, E_m) \quad (54)$$

with

$$\mathcal{R}_\beta(E_n, E_m) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{du}{\sqrt{u}} \phi(u) \left[\frac{E_n e^{-uE_n^2} - E_m e^{-uE_m^2}}{E_n - E_m}\right] . \quad (55)$$

For future use we also define $\bar{R}_{83} = \sqrt{3} R_{88}$ and $\bar{R}_{38} = \sqrt{3} R_{38}$. Note that $\mathcal{R}_\beta(E_n, E_m)$ is different from the regularization functions $\mathcal{R}_\mathcal{I}(E_n, E_m)$ and $\mathcal{R}_\mathcal{Q}(E_n, E_m)$. The origin of this difference, which however survives only in the finite cutoff case, is the different Hermiticity behavior of the current and the mass term on the one hand and the Coriolis term $i\Omega_E$ in the Dirac operator $D[s]$ (25) on the other hand. The latter one turns out to be anti-Hermitian in Euclidean space, whereas the former ones are Hermitian. Because the proper time regularization rests on building $D_E^\dagger D_E$ from the very beginning, different signs emerge and lead to the different regularization functions. Their substantial different behavior can be seen in Fig. 1 of Ref. [44] and in Fig. 1 of the present paper. The numerical values of R_{ab} for $M \approx 423$ MeV read (in fm) $\bar{R}_{83}^{\text{val}} = -0.095$, $\bar{R}_{83}^{\text{sea}} = -0.091$, $\bar{R}_{83}^{\text{tot}} = -0.186$ $\bar{R}_{44}^{\text{val}} = -0.148$, $\bar{R}_{44}^{\text{sea}} = -0.030$, $\bar{R}_{44}^{\text{tot}} = -0.179$ and $\bar{R}_{38}^{\text{val}} = 0.086$, $\bar{R}_{38}^{\text{sea}} = -0.073$, $\bar{R}_{38}^{\text{tot}} = 0.012$.

Apart from these *dynamical* terms originating from the action we have in addition the *kinematical* $O(m_s)$ terms arising from the quantization condition (16). Together with the chiral limit result of Eq. (49) we obtain, up to the linear order in the symmetry breaking and in the rotational frequency (for $a = 3$ and 8),

$$\hat{g}_A^{(a)} = \left[M_3 - \frac{4Q_{-+}}{I_1} + \frac{2\bar{Q}_{45}}{I_2}\right] D_{a3} - \frac{4M_{44}}{I_2} d_{3bb} D_{ab} \hat{J}_b - \frac{2\bar{M}_{83}}{\sqrt{3} I_1} \left(1 + \frac{4m_s}{\sqrt{3}} \frac{K_1}{I_1} D_{83}\right) D_{a8} \hat{J}_3 + \frac{4m_s}{\sqrt{3}} R_{83} D_{a8} D_{83} + \frac{8m_s}{\sqrt{3}} \left(R_{44} - M_{44} \frac{K_2}{I_2}\right) d_{3bb} D_{ab} D_{8b} - \frac{4m_s}{\sqrt{3}} R_{38} D_{a3}(1 - D_{88}) , \quad (56)$$

where, as usually, the index b in d_{3bb} runs over $4, \ldots, 7$.

The quantities R_{83} and M_{83} are already known from the expression of the flavor singlet axial constant [44]. We found there,[7] in the same order,

$$\hat{g}_A^{(0)} = -\frac{2\bar{M}_{83}}{I_1}\hat{J}_3 - \frac{4m_s}{\sqrt{3}}D_{83}\left(\frac{K_1}{I_1}\bar{M}_{83} - \bar{R}_{83}\right) . \quad (57)$$

The main difference between Eqs. (56) and (57) is that the lowest order term for $g_A^{(0)}$ is purely anomalous, whereas the corresponding term for the $g_A^{(3,8)}$ is nonanomalous. We come back to this point, when we make the comparison with the Skyrme model in Sec. VIII.

Up to now we have considered only the linear m_s corrections resulting from the expansion of the effective action. However, there is also another kind of m_s correction, resulting from the exact wave function of the collective Hamiltonian given in Eq. (21). When the $O(m_s^0)\hat{g}_A$ operator (49), (50) is sandwiched between the $O(m_s^1)$ wave functions (WF's) we get the additional contribution linear in m_s which reads, in the basis $[N, \Lambda, \Sigma, \Xi]$,

$$g_A^{(3)}(\mathrm{WF}) = -\frac{2I_2 m_s}{45}\left\{\left(a + b + \frac{\sqrt{3}}{2}c\right)(\sigma - r_1)\begin{bmatrix}2\\0\\1\\0\end{bmatrix} + \frac{2}{75}\left(a + 2b - \frac{3\sqrt{3}}{2}c\right)(3\sigma + r_1 - 4r_2)\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}\right\} T_3 J_3 ,$$

$$g_A^{(8)}(\mathrm{WF}) = \frac{2\sqrt{3}I_2 m_s}{45}\left\{\left(a + b + \frac{\sqrt{3}}{2}c\right)(\sigma - r_1)\begin{bmatrix}2\\0\\1\\0\end{bmatrix} - \frac{1}{75}\left(a + 2b - \frac{3\sqrt{3}}{2}c\right)(3\sigma + r_1 - 4r_2)\begin{bmatrix}6\\9\\4\\6\end{bmatrix}\right\} J_3 ,$$

$$g_A^{(0)}(\mathrm{WF}) = 0 , \quad (58)$$

where constants a, b, c are defined through the form of the axial current operator in the chiral limit [see Eqs. (49), (50)]:

$$\hat{g}_A^{(a)} = aD_{a3} + bd_{3bb}D_{ab}\hat{J}_b + cD_{a8}\hat{J}_3 .$$

T_3 and J_3 in Eqs. (58) stand for the eigenvalues of the respective isospin and spin operators.

VII. NUMERICAL RESULTS FOR AXIAL CURRENTS

The three different measurements of the spin asymmetry in the polarized lepton-nucleon deep inelastic scattering [47–50] have been recently reexamined by Ellis and Karliner [51,52]. The message of their work is that, whereas the Bjorken sum rule [53] is in agreement with the data, the Ellis-Jaffe sum rule [54,55] is violated and the results read finally

$$g_A^{(0),\mathrm{expt}} = 0.31 \pm 0.07, \quad g_A^{(8),\mathrm{expt}} = 0.35 \pm 0.04,$$
$$g_A^{(3),\mathrm{expt}} = 1.26 . \quad (59)$$

In this section we discuss our numerical results for the three axial decay constants $g_A^{(3)}$, $g_A^{(8)}$, and $g_A^{(0)}$ including the strange mass corrections (see Fig. 3). They are summarized in Tables I, II, and III. The collective matrix elements used in the calculation are given in Table IV. Our final values for a constituent quark mass $M \approx 423$ MeV are given by

$$g_A^{(0)} = 0.37, \quad g_A^{(8)} = 0.31, \quad \text{and } g_A^{(3)} = 1.38 . \quad (60)$$

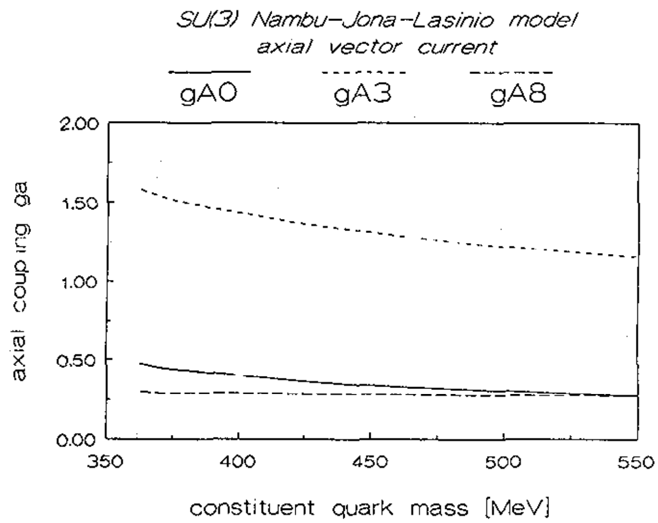

FIG. 3. The $g_A^{(0)}$, $g_A^{(3)}$, and $g_A^{(8)}$ are shown for self-consistent chiral fields in dependence of the constituent quark mass. The strange current quark mass is chosen as $m_s = 180$ MeV, according to a best fit to the hyperon spectra.

[7] Comparison with Ref. [44] can be done by identifying $\bar{M}_{83} = \beta_1$ and $\bar{R}_{83} = \beta_2$. Note also that the sign in Eq. (5) in Ref. [44] is misprinted.

TABLE II. The axial vector coupling constant $g_A^{(0)}$ and $g_A^{(8)}$ for the SU(3) Nambu–Jona-Lasinio model in dependence of the constituent quark mass M. The strange current quark mass is chosen as $m_s = 180$ MeV. The final model predictions are given by $g_A^{(0)}(\Omega^1, m_s)$ and $g_A^{(8)}(\Omega^1, m_s)$. The experimental values are given by $g_A^{(8),\text{expt}} = 0.35\pm0.04$ and $g_A^{(0),\text{expt}} = 0.31\pm0.07$ (Ellis and Karliner [52]).

M (MeV)	$g_A^{(8)}(\Omega^0, m_s^0)$	$g_A^{(8)}(\Omega^1, m_s^0)$	$g_A^{(8)}(\Omega^1, m_s^1)$	$g_A^{(0)}(\Omega^1, m_s^0)$	$g_A^{(0)}(\Omega^1, m_s^1)$
363	0.159	0.443	0.328	0.462	0.475
395	0.151	0.408	0.316	0.401	0.409
419	0.145	0.389	0.309	0.371	0.377
423	0.144	0.385	0.308	0.364	0.370
465	0.137	0.363	0.301	0.328	0.331

A. Axial vector coupling constants

In Table I the difference between SU(2) and SU(3) results for $g_A^{(3)}$ can be seen in each order of the $1/N_c$ expansion. Obviously the lowest order contribution (Ω^0) in SU(3) is significantly smaller than in SU(2) due to the fact that the SU(3) expectation value of the corresponding D function D_{33} is only 70% of the SU(2) value. The anomalous contribution of Eq. (42) linear in Ω which is nonzero only in the SU(3) case acts as a substitute for this group-theoretical reduction. Indeed, it leads to an almost exact readjustment of the SU(3) value to the SU(2) one. For our preferred value of $M \approx 423$ MeV from the hyperon spectra and $m_s \approx 180$ MeV,[8] it pushes the leading order SU(3) result up to $g_A^{(3)} \simeq 0.84$. These two values of the model parameters M and m_s will be used in the following discussion of the numerical results. Due to the presence of the quantities Q_{bc} from the explicit time-ordering, the SU(2), as well as the SU(3) results, have corrections linear in the rotational velocity. These conceptually new terms have no counterparts in the ordinary Skyrme model. Similarly to the old nonrelativistic quark model estimates of the $1/N_c$ correction, i.e., $g_A^{(3)} = N_c/3 + 2/3$, these new terms turn out to be of the order of 50% of the leading term. For SU(2) they push $g_A^{(3)}$ from 0.84 to 1.15 and in SU(3) they give the final value of 1.31. Note that the latter value is obtained with regularized *time-ordered* quantities Q_{bc}. In Ref. [13], where the regularization was neglected as the first approximation, the sea part of the quantities Q_{bc} made a $\simeq 30\%$ contribution to the total value of the Q_{bc}. Here, with the regularized sea, its contribution amounts to less than 3%.

Various contributions from the strange quark mass (kinematical, dynamical, and wave function) increase the value of $g_A^{(3)}$ of about $\simeq 5\%$ up to $g_A^{(3)} = 1.38$, such that the experimental value $g_A^{(3),\text{exp}} = 1.26$ is overestimated only by $\simeq 10\%$.

It has to be stressed that this is in contrast to all calculations within the purely pseudoscalar [19] or pseudoscalar and vector Skyrme model [20,56], in which $g_A^{(3)}$ is underestimated by $\simeq 30\%$. That this significant difference is due to the presence of the new terms from the time ordering of the functional trace is most clearly evident from Table III. There the flavor contributions to the axial current are given for the Skyrme model and the NJL model without and with the time-ordered (T) corrections. Without the new corrections the NJL model resembles very much the numerical results of the SU(3) Skyrme model with vector mesons. This was already noted at the level of the collective Hamiltonian for the mass splittings in Ref. [16] and here again can be seen numerically for the axial currents with high accuracy.

B. Spin properties

Apart from $g_A^{(3)}$ in Table II we list also the values for $g_A^{(0)}$, partially given already in Ref. [44], and for $g_A^{(8)}$. Neglecting the $U_A(1)$ anomaly for the present calculations, the spin of the proton, which is carried by the quarks and which is equal to the matrix element of the flavor singlet axial vector current, has no contribution in the order Ω^0, but gets the first nonvanishing contribution in the linear order of Ω. This, as can be seen from Table II, is also a dominating contribution, which gets only a very small strange mass correction. For $M = 423$ MeV, the theoretical value of $g_A^{(0)} \simeq 0.37$ is a little bit above the experimental error bars. Nevertheless one has to keep in mind that the analysis of the experimental data is still under debate [51,52] and the inclusion of higher twist effects might even push $g_A^{(0),\text{exp}}$ toward our present value

TABLE III. Various contributions to the axial vector current of the proton in terms of u, d, and s quarks for $M = 423$ MeV and $m_s = 180$ MeV. A comparison is made between the Skyrme model with vector mesons [20] (Skyrme, vector), the NJL model without (NJL, scalar) and with the *time-ordered* corrections of this paper (NJL, scalar, T). In the last column experimental values from Ellis and Karliner [52] are given.

	Skyrme (vector)	NJL (scalar)	NJL (scalar,T)	"Experiment"
Δu	0.63	0.64	0.902	0.83±0.03
Δd	−0.31	−0.24	−0.478	−0.43±0.03
Δs	−0.03	−0.02	−0.054	−0.10±0.03

[8]Note that $m_s = 150$ MeV would correspond to physical value of $m_K = 496$ MeV, whereas $m_s = 180$ MeV corresponds to the slightly bigger $m_K \simeq 540$ MeV.

TABLE IV. Matrix elements of the operators for $g_A^{(a)}$ in the proton state with spin up, where the index i is always running from 1 to 3 and b from 4 to 7.

D_{33}	D_{38}	D_{88}	$d_{3bb}D_{3b}\hat{J}_b$	$d_{3bb}D_{8b}\hat{J}b$	$d_{3bb}D_{3b}D_{8b}$	$d_{3bb}D_{8b}D_{8b}$
$-7/30$	$\sqrt{3}/30$	$3/10$	$7/60$	$1/(20\sqrt{3})$	$-11/(90\sqrt{3})$	$1/30$
$D_{38}D_{83}$	$D_{33}D_{88}$	D_{83}	$D_{83}D_{88}$	$D_{3i}R_i$	$D_{3i}D_{8i}$	$D_{3b}D_{8b}$
$-1/45$	$-4/45$	$-\sqrt{3}/30$	0	$7/20$	$\sqrt{3}/45$	$-\sqrt{3}/45$

[52]. One should note that our result for $g_A^{(0)}$ is clearly dominated by the valence contribution, which could be denoted as the *connected* part of the full correlation function. The sea contribution, i.e., the so-called *disconnected* part, has the same sign but is negligibly small. This is different from a very recent lattice calculation [57], where the disconnected part screens the connected part and contributes half of the value but with a different sign. However the self-consistent pion profile contains connected and disconnected contributions from the polarization of the Dirac sea, so that in the present model a clear separation of both contributions in the strict sense is not possible.

Experimental extraction of $g_A^{(8)}$ from the hyperon semileptonic decays depends on how the strange quark mass corrections to the SU(3) symmetric result are taken into account. Therefore the experimental error bars on this quantity may be at present too small. In the present calculation we obtain $g_A^{(8)} = 0.31$ to be compared with the "experimental" number of [52] $g_A^{(8),\mathrm{exp}} = 0.35 \pm 0.04$.

So from our calculations one can conclude that for the "fixed" mass of $M = 423$ MeV and $m_s = 180$ MeV all three axial vector coupling constants are quite close to the experimental values of [52]. From Fig. 3 it can be seen that for larger mass of $M \simeq 550$ MeV, $g_A^{(3)}$ and $g_A^{(0)}$ almost coincide with the experimental values, whereas $g_A^{(8)}$, having relative large negative strange quark mass correction, deviates from the central value of 0.35. One should however keep in mind that the large m_s corrections are usually ignored in the analysis of the hyperon semileptonic decays, which influences the value of $g_A^{(8)}$ as well as $g_A^{(0)}$.

VIII. COMPARISON WITH THE SKYRME MODEL

Now we want to compare our results with the Skyrme model, which can be regarded as a large constituent quark mass limit of the NJL model [9,46]. We will focus here on the Skyrme model, in which vector mesons and in addition kaon fluctuations and the gauged Wess-Zumino term are added. Then the collective operator has the structure [20]

$$\hat{g}_A^{(3)} = a_1 D_{33} + a_2 d_{3aa} D_{3a} R_a + a_3 D_{38} + a_4 d_{3aa} D_{3a} D_{8a} + a_5 D_{33}(1 - D_{88}) + a_6 D_{38} D_{83} \quad (61)$$

which corresponds effectively to the expression for the NJL model. Although the origin of the various coefficients is quite different in the NJL and Skyrme model, both approaches give effectively the same operator structure for g_A.

However one should stress here two important differences: first of all the new corrections linear in Ω which arise due to the time ordering within the fermion loop vanish in the Skyrme model identically. The Skyrme model is based upon local Lagrangian density which, apart from the Wess-Zumino term, is even in time derivatives and therefore the spatial components of the axial currents are also even not allowing for terms linear in Ω. Second, even if one restricts oneself to the terms not including the corrections due to the time-ordering (local limit) the contribution of the valence quarks in the present model makes our results qualitatively different from the ones of the Skyrme model. The coefficient a_2, e.g., is in the Skyrme model with purely pseudoscalar mesons dominated by the induced kaon fluctuations [19], which are neglected in the present NJL model. If the vector mesons are included in the Skyrme approach the situation does not change qualitatively [20]. In the Skyrme model a_2 gives only a 10% contribution to g_A [19], whereas the dominating valence contribution to M_{44}

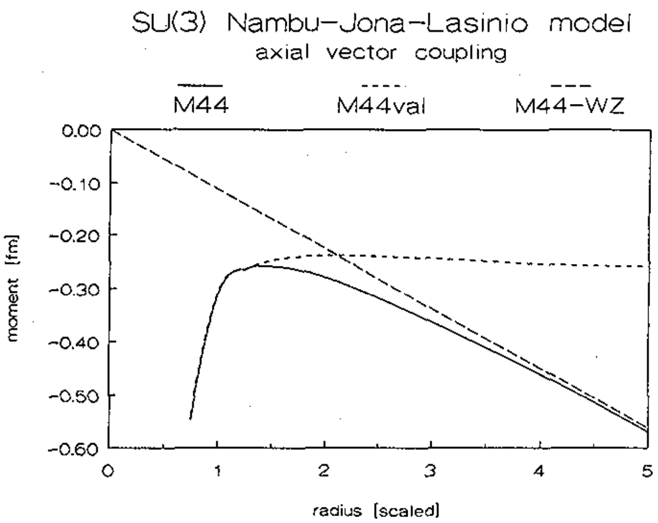

FIG. 4. The anomalous quantity M_{44} compared with the leading term of a gradient expansion, which comes exactly from the Wess-Zumino action for SU(3) pseudoscalar fields. This is done for a fixed linear profile in dependence of the radius R for $M = 372$ MeV.

(see Fig. 4) in the NJL model gives almost a 30% contribution to g_A if the terms due to the time ordering are neglected. The fact that the total values for g_A in the local limit of the present approach and in the vector meson pseudoscalar Skyrme model in Ref. [19] roughly coincide hinges on the rescaling procedure for the parameter e in Ref. [19], which tend to increase g_A. The Wess-Zumino contributions to g_A in the Skyrme approach play a minor role; in the NJL model the anomalous part of the action, containing the WZ term in lowest order of the gradient expansion, gives $\simeq 1/4$ of the total amount.

IX. SUMMARY AND DISCUSSION

In this paper we have extended our recent analysis of the corrections to the axial currents which appear due to the time ordering of the quark loop and semiclassical quantization [13] to the case of the regularized effective action. Moreover, we have investigated the strange current mass corrections to the axial currents of the semibosonized SU(3) Nambu–Jona-Laninio model.

In the semibosonized NJL model baryons are understood as solitonic solutions of the classical equations of motion. However the solitons do not carry proper quantum numbers and the semiclassical quantization procedure has to be applied in order to describe the mass splittings within the strange baryon multiplets. This treatment is based on introducing time-dependent rotations in the direction of the zero modes, followed by the canonical quantization of the collective coordinates of the rotation matrix. Since these zero modes contribute significantly to the mass splittings [16], it was a challenging task to look at the axial currents, which can be related to the recent measurements of the spin structure functions [51,52,48,49,47].

First the constituent mass M was fixed by looking at the hyperon mass splittings up to the terms quadratic in m_s. These are reproduced with unexpectedly high accuracy and point towards a constituent quark mass of $M \approx 423$ MeV. We have also explicitly shown that the wave function corrections and the corrections due to the expansion of the effective action are comparable and therefore it is inconsistent to perform only the Yabu-Ando diagonalization of the first order Hamiltonian $H^{(1)}$.

Second, we considered the axial vector currents with the inclusion of the corrections linear in the rotational velocity. The new contributions which appear due to the time ordering of the quark loop and semiclassical quantization have been shown explicitly to come from the real part of the effective action. If a regularization is implemented, they are dominated by the contribution of the valence quarks. In the SU(3) model there are also other contributions linear in Ω which come from the imaginary part of the effective action and as such do not require regularization. They are also dominated by the valence contribution. This concerns the leading term of $g_A^{(0)}$, which vanishes in the pure pseudoscalar Skyrme model [41], whereas it is nonvanishing (however small) in the present model, in rough agreement with experiment.

The expression for $g_A^{(3)}$ has a $\simeq 25\%$ rotational contribution from the imaginary part of the effective Euclidean action, which vanishes in the SU(2) case and which can be related to Witten's formula for the axial vector current from the Wess-Zumino effective action. Moreover it has a $\simeq$ 30% contribution due to the explicit time ordering (Q_{bc}) of the collective operators. These terms are not present in the local theories like the Skyrme model. In the present model they are entirely due to the nonlocality of the fermion determinant. Performing the gradient expansion of these quantities, it can be shown that these terms have the same mesonic structure as the lowest order term [see Eq. (51)]. This is similar to recent findings of Dashen and Manohar [43] within large-N_c QCD and to the nonrelativistic quark model result of $g_A^{(3)} = (N_c + 2)/3$. *Quantitatively, despite the fact that* the lowest order SU(3) result is reduced by a group theoretical factor of 0.7 with respect to the SU(2) case, the new time-ordering and anomalous contributions push the total value of $g_A^{(3)}$ upwards.

In addition, we have considered the corrections linear in the strange quark mass. They are consistently derived from the expansion of the effective action, the quantization condition as well as from the higher representations of the wave function in the spirit of the Yabu-Ando diagonalization. However the effect on $g_A^{(3)}$ is not large and finally one ends up with $g_A^{(3)} = 1.38$ for $M = 423$ MeV, which is only $\simeq$ 10% above the experimental value of $g_A^{(3),\exp} = 1.26$. Here it should be stressed that such nice agreement was never obtained within the pseudoscalar or pseudoscalar and vector Skyrme model [19,20]. This qualitative and quantitative difference comes from the new nonlocal $1/N_c$ corrections present in the NJL model.

The $g_A^{(0)}$ exists already in SU(2) and the only effect in SU(3) is a small shift due to the finite symmetry breaking m_s. This is in contrast to $g_A^{(8)}$, which vanishes in the SU(2) case, and which in SU(3) gets the entire contribution from the rotation and from the strange quark mass. In the chiral limit the values for $g_A^{(8)}$ and $g_A^{(0)}$ are quite close to each other, as suggested in [58], however the strange mass corrections reduce the value of $g_A^{(8)}$ by $\simeq 25\%$, whereas the explicit symmetry breaking has almost no influence on $g_A^{(0)}$. This holds at least if we take all linear m_s corrections into account and even the m_s^2 corrections, which can be calculated in this framework from the nonsymmetric wave functions [17]. The final values $g_A^{(0)} \simeq 0.37$ and $g_A^{(8)} \simeq 0.31$ for $M = 423$ MeV and $m_s = 180$ MeV are to be compared with the experimental data extracted from the recent analysis of Ellis and Karliner [52]; i.e., $g_A^{(0),\exp} \simeq 0.31 \pm 0.07$ and $g_A^{(8),\exp} \simeq 0.35 \pm 0.04$. Apparently the theoretical values are only slightly outside the experimental values.

Qualitatively the following can be said: $g_A^{(0)}$ which represents the part of the proton spin carried by the quarks, gets a nonvanishing expectation value entirely due to the anomalous part of the EEA. In a nonrelativistic constituent quark model, when the total spin of the proton is entirely carried by three quarks, $g_A^{(0)}$ equals *one*. The present model gives values close to experiment probably since the proton is treated entirely as a many-body system rotating in the spin-isospin space. Thus one cannot attribute the spin of the proton to single elementary particles but only to the whole system. Moreover the angular momentum of the quarks is explicitly taken into account by solving the Dirac equation in the grand-spin basis [10]. Therefore the fact that, in agreement with experiments, only a fraction of about 20–30 % of the nucleon spin is carried by the quark spins is not surprising at all in the present model.

Altogether the picture which emerges is quite satisfactory. Mass splittings are accurately reproduced and axial currents are in good agreement with the experimental data if rotational $1/N_c$ corrections are taken into account. In particular the spin of the proton originates in this model to about 35% from the spin of the quarks, a number being in reasonable agreement to the world data reported by the Spin Muon Collaboration (SMC) [49]. Together with the numerical results for the Gottfried sum [59,60] the model provides a good reproduction of the experimental data.

There is still one important point which should be discussed, namely the PCAC (partial conservation of axial-vector current) relation in the present approach. One can show that for a time independent hedgehog ansatz the PCAC relation is fulfilled. However, for the rotating soliton, i.e., for the time dependent hedgehog ansatz it is no longer true and PCAC is violated. The reason for this apparent puzzle is that the rotating hedgehog is not a solution of the time dependent equations of motion. It should be viewed as a variational Ansatz and as such it violates PCAC.

On the purely theoretical side the presence of the new terms linear in Ω calculated in this paper poses a serious problem to effective meson theories like the Skyrme model, where such terms vanish identically. Another theoretical question which deserves a comment is the convergence of the expansion in Ω. The large size of the corrections calculated in this paper raises the question whether the first order corrections in Ω are sufficient. One way to tackle this problem would be to calculate the Ω^2 corrections to the axial currents. This is the highest power of Ω one should consider, since the collective Hamiltonian itself is truncated in that order. Despite the technical difficulties in calculating these terms, the preliminary estimates indicate that they are not negligible.[9] Therefore one has to conclude that the expansion in Ω is slowly convergent. Moreover the formalism of the collective quantization has to be revised if one wants to include terms higher than Ω^2. These questions are certainly beyond the scope of this paper, where we had to content ourselves with the linear corrections alone. In addition, despite the fact that mass splittings are well reproduced, the absolute energies provide still some problems which are associated with the zero-point corrections [61,16] and boson fluctuations.

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APPENDIX A: DERIVATION OF THE REGULARIZATION FUNCTIONS FOR THE AXIAL CURRENT

Here we want to give an explicit derivation of the Ω^0 and Ω^1 contributions to the axial current. We emphasize the method of regularization of nonanomalous quantities from the explicit time ordering of the collective operators within the proper-time regularization scheme [28]. Then the real part can be written as

$$\mathrm{Re}\hat{A}_i^a(x) = -\frac{1}{2}\frac{\delta}{\delta s_i^a(x)}\mathrm{Sp}_{(\mathrm{to})}\int\frac{du}{u}\phi(u)\exp(-uD^\dagger D)\ , \tag{A1}$$

where

$$D = \partial_t + H + i\Omega_E - is_i^a\gamma_4\gamma_i\gamma_5 A^\dagger\lambda^a A \tag{A2}$$

and

$$D^\dagger = -\partial_t + H - i\Omega_E - is_i^a\gamma_4\gamma_i\gamma_5 A^\dagger\lambda^a A\ , \tag{A3}$$

such that

$$\begin{aligned} D^\dagger D = &-\partial^2 + H^2 + \Omega_E^2 - i[\Omega_E, H] - i\{\Omega_E, \partial_t\} - is_i^a\{\gamma_4\gamma_i\gamma_5 A^\dagger\lambda_a A, H\} \\ &+ s_i^a\gamma_4\gamma_i\gamma_5[A^\dagger\lambda_a A, \Omega_E] - i\gamma_4\gamma_i\gamma_5[s_i^a A^\dagger\lambda_a A, \partial_t]\ . \end{aligned} \tag{A4}$$

[9] M. Wakamatsu (private communication).

Then one has to expand $D^\dagger D$ around $D_0^\dagger D_0$ with $D_0 = \partial_t + H$, i.e., one expands in terms of s_i^a and Ω_E. This is done by using the Schwinger-Dyson formula

$$\exp(-uD^\dagger D) = \exp(-uD_0^\dagger D_0) - u\int_0^1 d\alpha \exp(-u\alpha D_0^\dagger D_0)[D^\dagger D - D_0^\dagger D_0]\exp[-u(1-\alpha)D_0^\dagger D_0]$$
$$+u^2\int_0^1 d\beta\int_0^{1-\beta} d\alpha e^{-u\alpha D_0^\dagger D_0}[D^\dagger D - D_0^\dagger D_0]e^{-u\beta D_0^\dagger D_0}[D^\dagger D - D_0^\dagger D_0]e^{-u(1-\alpha-\beta)D_0^\dagger D_0} + \cdots \; . \quad \text{(A5)}$$

In the lowest order Ω^0, one obtains

$$\mathrm{Re}\hat{A}_i^a(x) = -\frac{1}{2}\frac{\delta}{\delta s_i^a(x)}\mathrm{Sp}_{(t_0)}\int\frac{du}{u}\phi(u)\int d\alpha\exp(-u\alpha D_0^\dagger D_0)(uis_i^a\{\gamma_4\gamma_i\gamma_5 A^\dagger\lambda_a A, H\})\exp[-u(1-\alpha)D_0^\dagger D_0]\;, \quad \text{(A6)}$$

which after some simple manipulations gives Eq. (35) (see Ref. [18]).

Now we want to consider the Ω_E^1 corrections to the current. Let us define $V_1 = -i[\Omega_E, H] - i\{\Omega_E, \partial_t\}$, $V_2 = -is_i^a\{\gamma_4\gamma_i\gamma_5 A^\dagger\lambda_a A, H\}$, $V_3 = s_i^a\gamma_4\gamma_i\gamma_5[A^\dagger\lambda_a A, \Omega_E]$, and $V_4 = -i\gamma_4\gamma_i\gamma_5[s_i^a A^\dagger\lambda_a A, \partial_t]$. Consistently in the order Ω_E^1 one has to consider combinations of V_1 and V_2 as well as the single sum $V_3 + V_4$. Note that it is important to retain s_i^a as time dependent in Eq. (A4), because otherwise the two terms $V_3 + V_4$ cancel. This can be seen using $[A^\dagger\lambda_a A, \partial_t] = i[\Omega_E, A^\dagger\lambda_a A]$. After some lengthy algebra the operator $\hat{g}_A^{(a)}$ defined in Eq. (34) can be written as

$$\hat{g}_A^{(a)} = -\frac{N_c}{4}\int dt'\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}(2\omega E_n + 2\omega' E_m)\exp[i(\omega-\omega')(t'-t)]$$
$$\times\int_0^1 d\alpha\int_0^\infty du u\phi(u)\exp\{-u[\alpha(\omega^2+E_n^2)+(1-\alpha)(\omega^2+E_m^2)]\}$$
$$\times\langle n|\lambda^c|m\rangle\langle m|\sigma_3\lambda^b|n\rangle\mathcal{T}(\Omega_E^c(t')D_{ab}(t))\;. \quad \text{(A7)}$$

where t is the fixed and arbitrary time of $A_\mu(\vec{x}, t)$. Performing now the dt integration with special care to the time-ordered product $\mathcal{T}(\Omega_E^c(t)D_{ab}(t_0))$ one gets the relation

$$\int dt'\exp[i(\omega-\omega')(t'-t)]\mathcal{T}[\Omega_E^c(t')D_{ab}(t)]$$

$$= \frac{1}{i}\left[P\frac{1}{\omega-\omega'} + i\pi\delta(\omega-\omega')\right]D_{ab}(t)\Omega_E^c - \frac{1}{i}\left[P\frac{1}{\omega-\omega'} - i\pi\delta(\omega-\omega')\right]\Omega_E^c D_{ab}(t)$$

$$= \frac{1}{i}P\frac{1}{\omega-\omega'}[D_{ab}(t), \Omega_E^c] + \pi\delta(\omega-\omega')\{D_{ab}(t_0), \Omega_E^c\}\;. \quad \text{(A8)}$$

Note that after the time-ordering the angular velocities are again assumed to be time independent in order to perform the $\int dt'$ integration. The last term in Eq. (A8) vanishes because the δ function makes the integral in Eq. (A7) odd in ω. Therefore if the indices of Ω_E^c and D_{ab} are such that $[D_{ab}, \Omega_E^c] = 0$ then Eq. (A8) is identically zero. Evaluating the ω, ω' integration finally gives

$$\hat{g}_A^{(a)} = -\frac{N_c}{4}\frac{if_{edb}D_{ad}}{I_{ec}}\sum_{m,n}\langle n|\lambda^c|m\rangle\langle m|\sigma_3\lambda^b|n\rangle\mathcal{R}_Q(E_n, E_m)\;, \quad \text{(A9)}$$

where the regularization function is given by

$$\mathcal{R}_Q(E_n, E_m) = \frac{1}{2\pi}\int du\phi(u)\int_0^1 d\alpha\frac{\alpha E_n - (1-\alpha)E_m}{\sqrt{\alpha(1-\alpha)}}\exp\{-u[\alpha E_n^2 + (1-\alpha)E_m^2]\}\;, \quad \text{(A10)}$$

which in contrast to the regularization function for the usual moment of inertia $\mathcal{R}_\mathcal{I}(E_n, E_m)$ or $\mathcal{R}_\beta(E_n, E_m)$ is antisymmetric with respect to E_m and E_n. The du integration can be performed analytically in the case of steplike regularization functions $\phi(u) = c_i\theta(u - 1/\Lambda_i^2)$ and gives

$$\mathcal{R}_Q(E_n, E_m) = c_i\int_0^1\frac{d\alpha}{2\pi}\frac{\alpha E_n - (1-\alpha)E_m}{\sqrt{\alpha(1-\alpha)}}\frac{\exp\{-[\alpha E_n^2 + (1-\alpha)E_m^2]/\Lambda_i^2\}}{\alpha E_n^2 + (1-\alpha)E_m^2}\;. \quad \text{(A11)}$$

Using the formula

$$\int_0^1 \frac{d\alpha}{\sqrt{\alpha(1-\alpha)}}\frac{1}{q-\alpha p} = \frac{\pi}{\sqrt{q(p-q)}}, \quad 0 < p < q \,, \tag{A12}$$

the infinite cutoff limit of Eq. (45) is given by (p. 219 of Ref. [62]):

$$\mathcal{R}_Q(E_n, E_m) = \frac{1}{|E_n - E_m|}\frac{\mathrm{sgn}E_n - \mathrm{sgn}E_m}{2} \tag{A13}$$

and was used in Ref. [13] to calculate the $1/N_c$ corrections. Defining

$$Q_{bc} = \frac{N_c}{4}\sum_{m,n}\langle n|\lambda^c|m\rangle\langle m|\sigma_3\lambda^b|n\rangle\mathcal{R}_Q(E_n,E_m) \tag{A14}$$

the operator $\hat{g}_A^{(a)}$ for these contributions can be rewritten as

$$\hat{g}^a_{A,(\mathrm{to})} = -\frac{2iQ_{12}}{I_1}D_{a3} - \frac{2iQ_{45}}{I_2}D_{a3} \,. \tag{A15}$$

Equation (A15) follows directly from the *real* part of the Euclidean effective action given by Eq. (A1). Therefore these $1/N_c$ corrections described above have no counterpart in the Wess-Zumino term which follows from the imaginary part of the Euclidean action. As such they vanish identically in any local mesonic theory like the Skyrme model for instance.

APPENDIX B: COMPARISON WITH THE GRADIENT EXPANSION

In order to check the results of the numerical diagonalization one should always consult the long wavelength expansion of the coefficients appearing in the expressions for the observables. This technique is described at length in Ref. [45]. It also clarifies the question of whether the exact numerical value can be approximated by the gradient expanded quantities, or, in other words, whether the local mesonic theory, like the Skyrme model, for instance, is a good approximation to the NJL model.

1. The lowest order result from the real part of the EEA

For the lowest order (Ω^0) only the quantity M_3, which already exists in SU(2), contributes to $g_A^{(3)}$. Its gradient expansion can be found by expanding $D_0^\dagger D_0 = -\partial^2 + M^2 + iM\gamma_i\partial_i U(x)$ in Eq. (A6) in terms of the gradients $\partial_i U(x)$. The result is

$$M_3^{\mathrm{grd}} = \frac{2}{3}\int d^3x(\sigma\partial_i\pi_i - \pi_i\partial_i\sigma) \,. \tag{B1}$$

Equation (B1) can be rewritten in terms of the chiral angle θ and for π and σ on the chiral circle $\sigma(r) = \cos\theta(r)$ and $\pi(r) = \sin\theta(r)$:

$$M_3^{\mathrm{grd}} = \frac{8\pi}{3}f_\pi^2\int dr\, r^2\left(\theta' + \frac{2\sin\theta\cos\theta}{r}\right) \,. \tag{B2}$$

For the simplest case of a linear profile $\theta(r) = \pi(1 - r/2R)$, it reduces to

$$M_3^{\mathrm{grad}} = -\frac{32\pi^2}{9}f_\pi^2 R^2\left(1 - \frac{3}{2\pi^2}\right) \,. \tag{B3}$$

This quadratic behavior of M_3^{grd} is explicitly checked by using a large R profile function θ as an input for the quark wave functions of the exact formula for M_3.

2. The anomalous terms from the imaginary part of the EEA

The axial vector current also gets a contribution from the imaginary part of the EEA, which is nonvanishing only in the SU(3) case. In a local mesonic theory it can be derived from the Wess-Zumino term [32]. Here we want to show shortly how to derive this contribution from the nonlocal EEA of the present NJL model. Consider the operator quantity

$$\mathrm{Im}\hat{A}_i^a(x) = \frac{1}{2}\frac{\delta}{\delta s^a(x)}\mathrm{Sp}_{(\mathrm{to})}\left[\frac{1}{D} - \frac{1}{D^\dagger}\right] \times i\gamma_4\gamma_i\gamma_5\lambda^b D_{ab}s^a(x) \,. \tag{B4}$$

with $D = \partial_t + H + i\Omega_E$. Writing the denominators as $D^\dagger D$ one can write

$$\mathrm{Im}\hat{A}_i^a(x) = \frac{1}{2}\frac{\delta}{\delta s^a(x)}\mathrm{Sp}_{(\mathrm{to})}\frac{1}{D^\dagger D} \times[D^\dagger i\gamma_4\gamma_i\gamma_5\lambda^b D_{ab} - i\gamma_4\gamma_i\gamma_5\lambda^b D_{ab}D]s^a(x) \tag{B5}$$

Expanding $D^\dagger D$ again in terms of the gradients and after some laborious algebra leads to

$$M_{44}^{\mathrm{grd}} = \frac{N_c}{16\pi^2}\frac{1}{f_\pi^3}\epsilon_{0\mu\nu3}\epsilon_{3ab}\int d^3x\partial_\mu\pi^a(x)\partial_\nu\pi^b(x)\sigma(x) \,. \tag{B6}$$

In the case of the hedgehog ansatz, i.e., $\sigma(r) = \cos\theta(r)$ and $\pi(r) = \sin\theta(r)$, Eq. (B6) reduces to

$$M_{44}^{\mathrm{grd}} = -\frac{N_c}{6\pi}\frac{1}{f_\pi^3}\int dr\, r\partial_r\sigma(r)\pi(r)^2 = \frac{N_c}{6\pi}\int dr\, r\theta'(r)\sin^3\theta(r) \,. \tag{B7}$$

For the linear profile $\theta(r) = \pi(1 - r/2R)$ we obtain a compact expression:

$$M_{44}^{\mathrm{grd}} = -\frac{2}{3\pi}R \,. \tag{B8}$$

This linear behavior for large size chiral fields is numerically checked in Fig. 4.

3. The Ω^1 terms for the real part of the EEA

In this section we derive the gradient expansion for the nonlocal terms (Q_{ab}). For this part of the axial vector current one obtains

$$\mathrm{Re}\hat{A}_i^a(x) = -\frac{1}{2}\mathrm{Tr}_{\gamma,\tau,c}\int dt' \left\langle \vec{x},t\left|\frac{1}{\partial_t+H}\right|t'\right\rangle \lambda_c \left\langle t'\left|\frac{1}{\partial_t+H}\gamma_0\gamma_i\gamma_5\lambda_b\right|\vec{x},t\right\rangle \mathcal{T}[\Omega_c(t')D_{ab}(t)] \,, \tag{B9}$$

where the regularization is neglected here for simplicity. Inserting eigenstates of ∂_t and H and using Eq. (A8) we can define

$$\hat{g}_A^{(a)} = \left[\frac{X_{12}^3}{I_1}+\frac{X_{45}^3}{I_2}\right]D_{a3}\,, \tag{B10}$$

where the X quantities can be calculated from

$$X_{bc}^i = \mathrm{Tr}_{\gamma,\tau,c}\int d^3x \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}P\frac{1}{\omega-\omega'}\left\langle\vec{x}\left|\frac{1}{-i\omega+H}\lambda_c\frac{1}{-i\omega'+H}\gamma_0\gamma_i\gamma_5\lambda_b\right|\vec{x}\right\rangle . \tag{B11}$$

Then the recipe is to multiply denominators and numerators by the Hermitian conjugate of the denominators and recover $H^2 = -\partial_i^2 + M^2 + iM\gamma_i\partial_i U(x)$ in denominators, which can be expanded in terms of the gradients. Then these expressions can be straightforwardly simplified to the pure SU(2) quantity

$$\begin{aligned} X_{12}^3 = -2iQ_{12}^{\mathrm{grd}} &= -\frac{N_cM}{144\pi}\int d^3x\frac{1}{f_\pi^2}(\pi^i\partial_i\sigma+\sigma\partial_i\pi^i) + \frac{N_cM}{48\pi}\int d^3x\frac{1}{f_\pi^2}(\sigma\partial_i\pi^i-\pi^i\partial_i\sigma) \\ &= -\frac{N_cM}{12}\int dr r^2\left(\theta'+\frac{\sin 2\theta}{r}\right) - \frac{N_cM}{36}\int dr r^2\left(\theta'\cos 2\theta+\frac{\sin 2\theta}{r}\right) \end{aligned} \tag{B12}$$

and the pure SU(3) quantity

$$\begin{aligned} X_{45}^3 = -2iQ_{45}^{\mathrm{grd}} &= \frac{N_cM}{96\pi}\int d^3x\frac{1}{f_\pi^2}(\sigma\partial_i\pi^i-\pi^i\partial_i\sigma) \\ &= -\frac{N_cM}{24}\int dr\, r^2\left(\theta'+\frac{\sin 2\theta}{r}\right)\,, \end{aligned} \tag{B13}$$

where the first line for X_{12}^3 is a total divergence and vanishes for chiral fields, which vanish at least as $1/r^2$ for $r\to\infty$. Assuming physical profiles, which vanish exponentially with the pion mass, the axial vector current operator can be written as in Eq. (51). Note that $I_1, I_2 \sim N_c$, $M\sim N_c^0$, and $f_\pi^2 \sim N_c$, such that the last two terms in Eq. (51) represent a $1/N_c$ correction. Therefore Eq. (51) resembles very much the result of Dashen and Manohar [43,42], which states that the $1/N_c$ corrections to the axial current lead only to a renormalization of g_A. Or in other words, the ratio of different coupling constants has *no* $1/N_c$ correction.

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