Effect of a magnetic field on the strange star

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We study the effect of a magnetic field on strange quark matter and apply the same to strange stars. We find that the strange stars become more compact in the presence of strong magnetic fields.

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There are several different scenarios for the estimation of the magnetic field strength at the surface of neutron stars. These are theoretical models of pulsar emission [1], the accretion flow in binary x-ray sources [2], and the observation of cyclotron lines [3], e.g., the spectra of pulsating x-ray sources. From a sample of more than 400 pulsars, the surface magnetic field strength lies in the interval 2×10^{10} G $\leqslant H \leqslant 2\times 10^{13}$ G [4].

Very recently, it has been argued that there are two different physical mechanisms [5,6] leading to an amplification of some initial magnetic fields in a collapsing star. The first one is due to differential rotation [5] and a dynamo action [6] is the second. In newborn neutron stars, magnetic fields might be generated as strong as $H\sim 10^{14}$–10^{16} G, or even more. In the interior of a neutron star, it probably reaches $\sim 10^{18}$ G. Therefore, it is advisable to study the effect of strong magnetic fields on compact neutron stars.

There are strong reasons for believing that hadrons are composed of quarks and the idea of quark stars has existed for about 20 years. If the neutron matter density at the core of neutron stars exceeds a few times the normal nuclear density, a deconfining phase transition to quark matter may take place. As a consequence, a normal neutron star will be converted into a hybrid star with an infinite cluster of quark matter core and a crust of neutron matter. In 1984, Witten suggested that strange matter, e.g., quark matter with strangeness per baryon of order unity, may be the true ground state [7]. The properties of strange matter at zero pressure and zero temperature were subsequently examined, and it was found that strange matter can indeed be stable for a wide range of parameters in strong interaction calculations [8]. Therefore, at the core, strange quarks will be produced through the weak decays of light quarks (u and d quarks) and ultimately a chemical equilibrium will be established among the participants. Since strange matter is energetically favorable over neutron matter, there is a possibility that the whole star may be converted into a strange star.

In this paper, we discuss the effect of a strong magnetic field on strange quark matter. We then calculate the equation of state of strange quark matter and apply the same to strange stars. According to Ref. [9], the surface magnetic fields of pulsars are very high, $H\approx 10^{12}$–10^{14} G, and in the interior layers of a star, the fields can reach values of the order $H\approx 10^{14}$–10^{18} G. Even neglecting possible twisting of the fields, which intensifies the fields by another several orders of magnitude, as a first approximation, we assume the existence of a constant uniform magnetic field in the range of $H\approx 10^{14}$–10^{18} G throughout the strange star. Such a choice is being adopted for the sake of simplicity.

For a constant magnetic field along the z axis $[\vec{A}=(Hy,0,0)]$, the single energy eigenvalue is given by [10]

$$\varepsilon_{p,n,s}=\sqrt{p_i^2+m_i^2+q_iH(2n+s+1)}, \tag{1}$$

with $n=0,1,2,\ldots$ being the principal quantum numbers for allowed Landau levels, $s=\pm 1$ refers to spin up(+) and down(−), and p_i is the component of particle (species i) momentum along the field direction. Setting $2n+s+1=2\nu$, where $\nu=0,1,2,\ldots$, we can rewrite the single particle energy eigenvalue in the form

$$\varepsilon_i=\sqrt{p_i^2+m_i^2+2\nu q_iH}. \tag{2}$$

It is very easy to see that $\nu=0$ state is singly degenerate, whereas all other states with $\nu\neq 0$ are doubly degenerate. Then the thermodynamic potential in the presence of a strong magnetic field $H(>H^{(c)}$, critical value discussed later) is given by

$$\Omega_i=-\frac{g_iq_iHT}{4\pi^2}\int d\varepsilon_i\sum_\nu \frac{dp_i}{d\varepsilon_i}\ln[1+\exp[(\mu_i-\varepsilon_i)/T]]. \tag{3}$$

Integrating by parts and substituting

$$p_i=\pm\sqrt{\varepsilon_i^2-m_i^2-2\nu q_iH}, \tag{4}$$

for all T, one finds

$$\Omega_i=-\frac{g_iq_iH}{4\pi^2}\int d\varepsilon_i\sum_\nu \frac{2\sqrt{\varepsilon_i^2-m_i^2-2\nu q_iH}}{[\exp[(\varepsilon_i-\mu_i)/T]+1]}, \tag{5}$$

where the sum over ν is restricted by the condition $\varepsilon>\sqrt{m^2+2\nu qH}$ and the factor of 2 takes into account the freedom of taking either sign in Eq. (4). For $T=0$, therefore,

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approximate the Fermi distribution by a step function and interchange the order of the summation over ν and integration over ε:

$$\Omega_i = -\frac{g_i q_i H}{2\pi^2}\sum_\nu \int_{\sqrt{m_i^2+2\nu q_i H}}^{\mu} d\varepsilon_i \sqrt{\varepsilon_i^2 - m_i^2 - 2\nu q_i H}$$

$$= -\frac{g_i q_i H}{4\pi^2}\sum_\nu \left(\mu_i\sqrt{\mu_i^2-m_i^2-2\nu q_i H} - (m_i^2+2\nu q_i H)\ln\left[\frac{\mu_i+\sqrt{\mu_i^2-m_i^2-2\nu q_i H}}{\sqrt{m_i^2+2\nu q_i H}}\right]\right). \quad (6)$$

Since the temperature $T \ll \mu$ at the core of a quark star, the presence of antiparticles can be ignored. Now instead of infinity the upper limit of the ν sum can be obtained from the relation

$$p_{Fi}^2 = \mu_i^2 - m_i^2 - 2\nu q_i H \geqslant 0, \quad (7)$$

where p_{Fi} is the Fermi momentum of the species i, which gives

$$\nu \leqslant \frac{\mu_i^2 - m_i^2}{2q_i H} = \nu_{\max}^{(i)} \quad \text{(nearest integer)}. \quad (8)$$

Therefore, the upper limit is not necessarily the same for all the components. As is well known, the energy of a charged particle changes significantly in the quantum limit if the magnetic field strength is equal to or greater than some critical value $H^{(c)} = m_i^2 c^3/(q_i\hbar)$ (in G), where m_i and q_i are, respectively, the mass and the absolute value of charge of particle i; $\hbar$ and c are the reduced Planck constant and velocity of light, respectively, both of which along with Boltzmann constant k_B are taken to be unity in our choice of units. For an electron of mass 0.5 MeV, this critical field as mentioned above is $\sim 4.4\times 10^{13}$ G, whereas for a light quark of current mass 5 MeV, this particular value becomes $\sim 4.4\times 10^{15}$ G; on the other hand, for an s quark of current mass 150 MeV, the critical field is $\sim 10^{19}$ G and hence the effect of such a huge magnetic field on the s quark has been ignored in Ref. [11]. But in our calculation, we consider the quantum-mechanical effect of quark star magnetic fields on u, d, and s quarks and electrons.

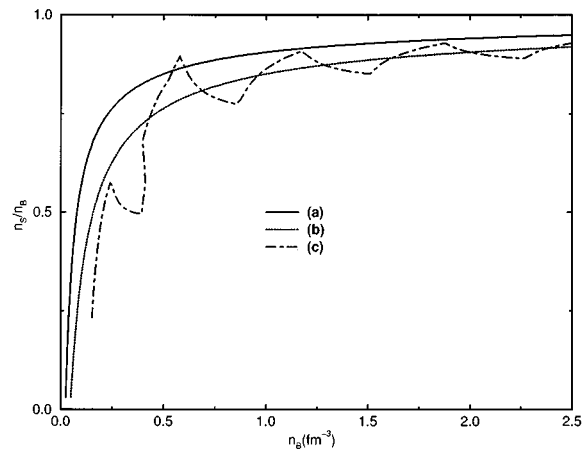

FIG. 1. Strangeness fraction plotted against the baryon density for $H=0$ [curve (a)], $H=10^{14}$ G [curve (b)], and $H=10^{18}$ G [curve (c)].

In the absence of the magnetic field, the summation over the state ν needs to be replaced by an integration. We define a variable $\theta = 2\nu q_i H$ such that

$$\sum_\nu^{\nu_{\max}} \to \int_0^{\nu_{\max}} = \lim_{H\to 0}\frac{1}{2q_i H}\int_0^{\mu_i^2-m_i^2} d\theta. \quad (9)$$

Substituting Eq. (9) into Eq. (6) and integrating over the variable θ, we finally obtain

$$\Omega_i = -\frac{g_i}{8\pi^2}\left[\frac{1}{3}\mu_i\sqrt{\mu_i^2-m_i^2}(\mu_i^2-2.5m_i^2)+\frac{1}{2}m_i^4\ln\left(\frac{\mu_i+\sqrt{\mu_i^2-m_i^2}}{m_i}\right)\right]. \quad (10)$$

This is the thermodynamical potential for the species i ($i=u,\ d,\ s$, and e) at $T=0$ in the absence of a magnetic field [8,12], where g_i is the degeneracy of the species.

In our study, we assume that strange quark matter is charge neutral and also in chemical equilibrium; then,

$$\mu_d = \mu_s = \mu = \mu_u + \mu_e, \quad (11)$$

and charge neutrality conditions give

$$2n_u - n_d - n_s - 3n_e = 0. \quad (12)$$

The baryon number density of the system is given by

$$n_B = \frac{1}{3}(n_u + n_d + n_s). \quad (13)$$

Using the above Eqs. (11), (12), and (13), one can solve numerically for the chemical potentials of all the flavors and electrons. For $T=0$, we have the number density of the species i (u, d, s, and e):

$$n_i = \frac{g_i q_i H}{2\pi^2}\sum_\nu \sqrt{\mu_i^2 - m_i^2 - 2\nu q_i H}. \quad (14)$$

The number density of the species i in the absence of a magnetic field is given by

$$n_i = \frac{g_i}{6\pi^2}(\mu_s^2 - m_s^2)^{3/2}. \quad (15)$$

In Fig. 1, we plot the strangeness fraction (n_s/n_B) as func-

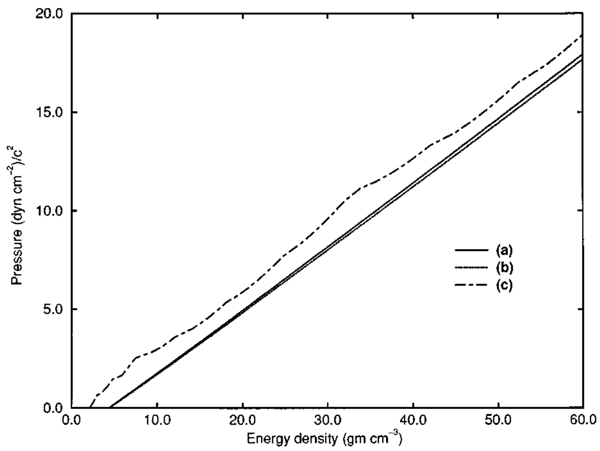

FIG. 2. Pressure plotted against energy density for three cases as mentioned in Fig. 1.

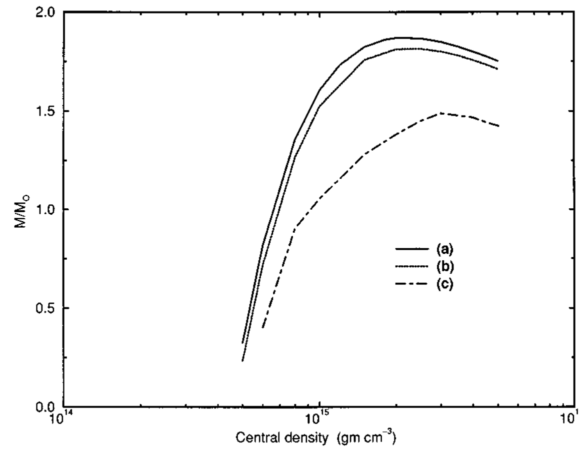

FIG. 3. Mass plotted against central density for three cases as mentioned in Fig. 1.

tion of baryon density in the strange star. Curve (a) is for zero magnetic field, whereas curves (b) and (c) correspond to a low magnetic field ($H = 10^{14}$ G) and a high magnetic field ($H = 10^{18}$ G). From this figure, we notice that the strangeness fraction changes with the magnetic field, which is low in curve (b) and shows oscillating behavior in curve (c), as consecutive Landau levels are passing the Fermi level.

The total energy density and the external pressure of strange quark matter are given, respectively, by

$$\varepsilon = \sum_i \Omega_i + B + \sum_i n_i \mu_i ,$$

$$p = -\sum_i \Omega_i - B, \tag{16}$$

where $i = u,d,s,e$. Here, we consider the conventional bag model for the sake of simplicity in the presence of magnetic fields. We are assuming that quarks are moving freely (noninteracting) within the system and as usual the current masses of both u and d quarks are extremely small, e.g., 5 MeV each, whereas for the s quark the current quark mass is to be taken as 150 MeV. We choose the bag pressure B to be 56 MeV fm^{-3}. Also, we set the low and high values of the magnetic fields to be 10^{14} G and 10^{18} G, respectively, in our calculations. Since we choose the magnetic field along the z axis, it follows from the energy-stress tensor that constant uniform magnetic fields contribute to the pressure and the energy density by $H^2/8\pi$. These are much smaller than the bag pressure, e.g., $H^2/8\pi \sim 0.25 \times 10^{-6} \ll 56$ MeV fm^{-3} for a low magnetic field (10^{14} G) and are the same order of magnitude as the bag pressure in the case of a high magnetic field (10^{18} G), e.g., $H^2/8\pi \sim 24.84$ MeV fm^{-3}.

The equations of state of strange quark matter are shown in Fig. 2. Curve (a) is for zero magnetic field, whereas curves (b) and (c) are for low and high magnetic fields, respectively. Curve (c) shows little oscillating behavior because of the high magnetic field.

TABLE I. The radius (R), mass (M), surface redshift (z), moment of inertia (I), and period of fundamental frequency (P_0) of strange stars versus central density ρ_c for three different equations of state for magnetic fields $H = 0$, 10^{14}, and 10^{18} G, respectively.

ρ_c (g cm^{-3})	R (km)	$M/M_\odot$	z	I (g cm^2)	P_0 (ms)	H (G)
0.6×10^{15}	9.23	0.82	0.16	0.61×10^{45}	0.62	
1.0×10^{15}	10.82	1.61	0.33	1.76×10^{45}	0.56	
1.5×10^{15}	10.76	1.82	0.42	1.99×10^{45}	0.52	0
2.0×10^{15}	10.49	1.87	0.45	1.94×10^{45}	0.49	
0.6×10^{15}	8.83	0.72	0.15	0.49×10^{45}	0.62	
1.0×10^{15}	10.60	1.53	0.32	1.59×10^{45}	0.56	
1.5×10^{15}	10.57	1.76	0.40	1.85×10^{45}	0.52	10^{14}
2.0×10^{15}	10.33	1.81	0.44	1.82×10^{45}	0.49	
2.5×10^{15}	10.08	1.82	0.46	1.73×10^{45}	0.47	
1.0×10^{15}	9.37	1.06	0.22	0.98×10^{45}	0.55	
1.5×10^{15}	9.64	1.28	0.28	1.24×10^{45}	0.53	
2.0×10^{15}	9.46	1.36	0.32	1.29×10^{45}	0.50	10^{18}
2.5×10^{15}	9.36	1.44	0.35	1.23×10^{45}	0.48	
3.0×10^{15}	9.32	1.50	0.38	1.21×10^{45}	0.46	

From studies of quark matter [12], it is predicted that the mass (M) of a quark star $\simeq M_{\odot}$ ($M_{\odot}$ solar mass) and radius (R) $\simeq 10$ km. These so-called quark stars have rather a different mass-radius relationship than neutron stars, but for stars of mass $\simeq 1.4 M_{\odot}$, the structure parameters of quark stars are very similar to those of neutron stars.

The mass and radius for nonrotating strange quark stars are obtained by integrating the structure equations of a relativistic spherical static star composed of a perfect fluid which is derived from Einstein's equation. These equations are given in Ref. [13]. For a given equation of state and given central density, the structure equations are integrated numerically with the boundary conditions $m(r=0)=0$, to give R and M. Though the equations of state have little oscillating behavior, this fact does not affect the characteristic structures of strange stars. The radius R is defined by the point where $p \simeq 0$. Figure 3 shows the variation of mass with central density for three equation of states as illustrated in Fig. 2. With a rise in the central density ρ_c, $M/M_{\odot}$ reaches a maximum which corresponds to the maximum mass a stable star can have. The total gravitational mass M, moment of inertia I, surface redshift z, and the period P_0 correspondingto the fundamental frequency Ω_0 are then given by $M=m(R)$, $I=I(R)$, $z=(1-2GM/Rc^2)^{-1/2}-1$, and $P_0=2\pi/\Omega_0$, respectively, where $\Omega_0=(3GM/4R^3)^{1/2}$ [14]. Table I lists the above structure parameters versus the central densities of strange stars as predicted by our equations of state (see Fig. 2) for magnetic fields $H=0$, 10^{14}, and 10^{18} G, respectively. We notice from Fig. 3 and Table I that with an increase in magnetic field strength, the star becomes more compact. The maximum mass and the corresponding radius decrease from 1.87 to 1.50 solar mass and from 10.5 to 9.3 km, respectively. Similarly, the corresponding values for surface redshift, moment of inertia, and fundamental period decrease, but the corresponding central density increases with magnetic fields.

In conclusion, we see that the presence of a strong magnetic field in strange quark matter reduces the mass and radius of strange star. We find that a high magnetic field compresses the star significantly. That means that the star becomes more compact. Also, the strangeness fraction reduces in the presence of a magnetic field and shows an oscillating behavior for a high value of the magnetic field.

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