Quantum corrections to the Weizsäcker-Williams gluon distribution function at small x

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We compute the quantum corrections to the gluon distribution function in the background of a non-Abelian Weizsäcker-Williams field. These corrections are valid to all orders in the effective coupling $\alpha_s\mu$, where μ^2 denotes the average valence quark color charge squared per unit area. We find $\ln(1/x)$ logarithmic corrections to the classical gluon distribution function. The one-loop corrections to the classical Weizsäcker-Williams field do not contribute to these singular terms in the distribution function. Their effect is to cause the running of α_s.

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I. INTRODUCTION

In a recent work [1] we computed the Green's function in the light cone gauge $A^+ = 0$ for the small fluctuations about a background Weizsäcker-Williams gluon field. This background field is generated by the valence quarks in a large A nucleus. For small x partons, these valence quarks constitute a static and well localized source of color fields [2]. The average color charge squared per unit area of the valence quarks is denoted by μ^2 and it is of the order of $A^{1/3}$ fm^{-2}. The quantity μ is the only dimensionful parameter in the theory and as a result, the coupling constant will run as a function of it.

Previously, two of us (McL.-V.) have argued that we can compute the gluon distribution function from the light cone gauge Green's function [3]. In the present paper we use that Green's function to compute the corrections induced by quantum fluctuations on the Weizsäcker-Williams distribution function.

There are several reasons for this computation to be of interest. From the practical point of view, we hope that an analysis which provides us with a better understanding of the initial conditions for the evolution of partons in the collision of heavy nuclei to form a quark-gluon plasma will help establish a firm foundation [4] for partonic cascade models simulating such collisions [5]. For an alternative approach to the problem of initial conditions in heavy ion collisions, see Ref. [6].

From the theoretical point of view, we hope to understand the small x behavior of a nucleus starting from a QCD based approach. Let us recall that it is believed that in the small x region, the gluon distribution function computed perturbatively including leading logarithmic contributions in $\ln(1/x)$ for a single nucleon behaves like [7]

$$\frac{dN}{dx} \sim \frac{1}{x^{1+C\alpha_s}}.$$

The steep rise of the gluon distribution function for small x is sometimes referred to as the Lipatov enhancement. It is obtained by solving the Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel for the t-channel exchange of a perturbative pomeron [8]. This behavior is also exhibited in a hadron where the large x part of the hadron wave function is taken to be a heavy quark-antiquark state. By applying Hamiltonian perturbation theory to this state, it is possible to reproduce the kernel of the BFKL equation [9] for the emission of a large number of soft gluons.

However, the rapid rise of the gluon distribution function with smaller values of x is in conflict with unitarity when considering the hadron scattering total cross section at asymptotically high energies [7]. Physically, this violation of unitarity can be understood to result from ignoring effects that arise due to the large density of partons at very small values of x [10]. When the density of partons is so large that neighboring partons overlap, the t-channel picture of an independent parton cascade in x breaks down. The former signals that at very small values of x, the picture in which the partons do not interact with each other has to be modified in order to comply with the Froissart bound [11] on the growth of cross sections at asymptotically high energies. Although some work has been done in recent years to include these "higher twist" effects in describing parton evolution at high densities [10,12,13], more still remains to be done in devising a quantitative mechanism to limit this growth.

The regime of high parton density is the screening regime. This screening is presumably responsible for the shadowing phenomena observed in deep inelastic scattering experiments off nuclei at small x. It can be addressed as a collective or many body effect. It is precisely this many body problem of parton interactions that we seek to address in our work. As outlined in Refs. [2,3,14], our formalism provides us, by means of a novel weak coupling approach, with a technique to solve the many body

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problem of wee parton distributions in a large nucleus.

In this work, we will focus on one of the theoretical aspects of the problem—the nature of the small x terms in the gluon distribution function of the Weizsäcker-Williams fields generated by the valence quarks in the infinite momentum frame. We do this by computing a formula for the gluon distribution function which includes the effects of all orders in the parameter $\alpha_s\mu$. Working in the weak coupling regime $\alpha_s\mu \ll k_t$, we extract from this formula an expression for the distribution function in perturbation theory to second order in α_s. Although we approach the problem in weak coupling, we will show that after the corrections are considered, the series expansion parameter becomes $\alpha_s \ln(1/x)$ which may be large for small x's. We will argue that this forces us to devise a method to isolate and sum up the leading contributions to all orders in that effective expansion parameter in order to compute the modification to the zeroth order $1/x$ distribution function.

The outline of this work is as follows. In Sec. II, we briefly review the basic aspects of the model in which we treat the nuclear valence quarks as static sources of color charge as seen by small x partons. We also go briefly through the formalism that allows us to compute the Green's function for the small fluctuations equation and finish the section by writing the formula for this Green's function. In Sec. III, we use this Green's function and exploit its relation to the gluon density in order to compute the gluon distribution of small x gluons. We also derive from this result a formula for the leading small x terms of the distribution function in perturbation theory valid to second order in α_s. In Sec. IV, we compute the corrections to the background field induced by the fluctuation field and show that the only effect that this introduces can be absorbed into the renormalization of the background field (which is related to the renormalization of the coupling constant to one loop). Details of the calculations in Sec. III and Sec. IV are discussed in Appendix A and Appendix B, respectively. Finally we summarize our results in Sec. V.

II. THE MODEL

In QCD, a hadron is a cloud of virtual particles with a rather complicated structure. The picture gets simpler when we consider the hadron as a quantum system composed of quasireal particles (partons) with lifetimes much larger than the characteristic interaction times. This can be done in a reference frame where the hadron has a large momentum [10]. Partons with large lifetimes can produce new partons carrying smaller fractions x of the initial hadron's momentum. The small x partons will therefore densely populate the hadron and see the rest of it with its longitudinal dimension Lorentz contracted to a thin disk. In our model, we look at the small x partons in a large A nucleus ($x \ll A^{-1/3}$) where the high parton density allows us to use weak coupling techniques. The rest of the nucleus consists of the valence quarks which carry most of the nuclear momentum. They are described as a static (recoilless) source of color charges, in a reference frame in which they move with the speed of light (infinite momentum frame) [2].

The problem is well suited to be described using light cone variables [15,16]

$$y^0, y^3 \longrightarrow y^{\pm} = (y^0 \pm y^3)/\sqrt{2}. \tag{1}$$

In order to compute ground state properties of the wee partons, we define a partition function for the system. This partition function includes the sum over a large number of color configurations. To simplify the problem, we resolve the transverse space direction into cells which contain a large number of valence quarks, or equivalently, a large number of color charges. This allows us to treat the sum over color configurations classically [2]. To write the average over the color charges, we introduce a Gaussian weight by inserting into the path integral representation of the partition function the term

$$\exp\left\{-\frac{1}{2\mu^2}\int d^2x_t\ \rho^2(x)\right\}, \tag{2}$$

where ρ is the color charge density (per unit area) and the parameter μ^2 is the average color charge density squared (per unit area) in units of the coupling constant g. The introduction of the partition function, where we average over the sources of external charge, allows us to formulate the theory as a many body problem with modified propagators and vertices.

We treat the system perturbatively and the first step is to solve the classical equations of motion

$$D_\mu F^{\mu\nu} = gJ^\nu,\quad J^\nu = \delta^{+\nu}\rho(x^+, x_t)\delta(x^-), \tag{3}$$

for which (working in the light cone gauge $A_- = -A^+ = 0$) there exists a solution with $A_+ = -A^- = 0$. We require

$$A_i(x) = \theta(x^-)\alpha_i(x_t) \tag{4}$$

(hereafter, latin indices refer to transverse variables), and furthermore $F^{ij} = 0$. The latter condition implies that $\alpha(x_t)$ is a pure gauge transform of the vacuum [14]:

$$\tau\cdot\alpha_i = \frac{i}{g}U(x_t)\nabla_i U^\dagger(x_t), \tag{5}$$

where $U(x_t)$ is a SU(3) local gauge transformation whose spatial dependence is only on the two-dimensional transverse space. It is subject to the physical gauge condition

$$\nabla\cdot\left[U(x_t)\nabla U^\dagger(x_t)\right] = -ig^2\rho. \tag{6}$$

The x^+ (light cone time) dependence of the charge density is a consequence of the extended current conservation law

$$D_\mu J^\mu = 0. \tag{7}$$

The integration over the sources ρ in Eq. (2) may be written as

$$\int [dU] \exp\left[-\frac{1}{\mu^2 g^4}\mathrm{Tr}\left(\nabla_i \cdot U\frac{1}{i}\nabla_i U^\dagger\right)^2\right], \tag{8}$$

where we have ignored the Faddeev-Popov determinant. Note that the effective coupling constant for this theory is $g^2\mu$ so that the expansion parameter becomes $\alpha_s\mu/p_t$.

The Green's function can be computed from the relation

$$G_{ij}^{\alpha\beta}(x,x') = \int \frac{d\lambda}{\lambda - i\epsilon}\int \frac{d^4p}{(2\pi)^4}\delta(\lambda - p^2) \times (A_i^\alpha)_\lambda(x)(A_j^\beta)_\lambda^\dagger(x') \tag{9}$$

and the gluon distribution function can be computed from the Green's function by the relation

$$\frac{dN}{d^3k} = i\frac{2k^+}{(2\pi)^3}\sum_{\alpha,i} G_{ii}^{\alpha\alpha}(x^+,\vec{k};x^+,\vec{k}). \tag{10}$$

To relate the Green's functions to the distribution function by the above relation, the former must be averaged over the external sources of color charge.

Indeed, the distribution function, to all orders, is related by the above expression to the fully connected two point Green's function. This Green's function is given by the relation

$$\langle\langle AA\rangle\rangle_\rho = \langle\langle A_{\rm cl}\rangle\langle A_{\rm cl}\rangle + \langle A_q A_q\rangle\rangle_\rho. \tag{11}$$

In the above, $\langle A_{\rm cl}\rangle$ is the expectation value of the classical field to all orders in $\hbar$. It can be expanded as

$$\langle A_{\rm cl}\rangle = A_{\rm cl}^{(0)} + A_{\rm cl}^{(1)} + \cdots, \tag{12}$$

where $A_{\rm cl}^{(0)}$ is the solution discussed in Eqs. (4) and (5). The one-loop correction to the classical field, $A_{\rm cl}^{(1)}$, is computed in Sec. IV of this paper. The term $\langle A_q A_q\rangle$ above is the small fluctuations Green's function computed to each order in the classical field. The symbol $\langle\cdots\rangle_\rho$ indicates that we have to average over the external sources of color charge with the Gaussian weight described above.

From the above, it is clear that the zeroth order contribution is $\langle A_{\rm cl}^{(0)}A_{\rm cl}^{(0)}\rangle_\rho$. This contribution is the QCD analog of the well-known Weizsäcker-Williams distribution in classical electrodynamics. The general form of the solution is given in Ref. [14]. In the range of momenta $\alpha_s\mu \ll k_t \ll \mu$, the zeroth order solution A^μ yields a distribution function that, written in terms of $x \equiv k^+/P^+$, with P^+ the nuclear longitudinal light cone momentum, looks like

$$\frac{1}{\pi R^2}\frac{dN}{dx d^2k_t} = \frac{\alpha_s\mu^2(N_c^2-1)}{\pi^2}\frac{1}{xk_t^2}. \tag{13}$$

The above is the well-known Weizsäcker-Williams distribution scaled by $\mu^2 \approx A^{1/3}$ fm^{-2}.

With this formalism at hand, we can proceed to compute the next order contribution to the gluon distribution function. Our strategy is to compute the small fluctuations correction to the classical equation of motion. Writing the field in terms of its background and fluctuation parts

$$A^\mu(x) = B^\mu(x) + b^\mu(x), \tag{14}$$

we are able to express the equation obeyed by b^μ as [1]

$$\left[D(B)^2 g^{\mu\nu} - D^\mu(B)D^\nu(B)\right] b_\nu - 2F^{\mu\nu}b_\nu = 0, \tag{15}$$

where B is the background field which according to Eq. (4) is nonvanishing only for its transverse components. $D^\mu(B)$ is the covariant derivative with B as the gauge field (notice that $D^\pm = \partial^\pm$). As discussed in Ref. [1], the set of equations (15) can be unambiguously solved in the gauge $A^- = 0$, and by means of Eq. (9) we can compute the Green's function for the fluctuation fields in this gauge. To obtain the Green's function in the gauge $A^+ = 0$ (light cone gauge), we perform a gauge transformation on the Green's function in the $A^- = 0$ gauge and obtain finally (in the matrix representation)

$$\begin{aligned}
G_{ij}^{\alpha\beta;\alpha'\beta'}(x,y) = -\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2 - i\epsilon}\Bigg\{&\left[\delta_{ij} + \frac{p_ip_j}{p^-p^+}(2e^{ip^+(x^- - y^-)} - e^{-ip^+y^-} - e^{ip^+x^-})\right]\\
&\times\left[\theta(-x^-)\theta(-y^-)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'} + \theta(x^-)\theta(y^-)F_a^{\dagger\alpha\beta}(x_t)F_a^{\dagger\alpha'\beta'}(y_t)\right]\\
&+\theta(-x^-)\theta(y^-)\int\frac{d^2q_t}{(2\pi)^2}d^2z_t\; e^{i(q^+ - p^+)y^-}e^{i(p_t - q_t)(y_t - z_t)}\\
&\times F_a^{\dagger\alpha\beta}(z_t)F_a^{\dagger\alpha'\beta'}(y_t)\Bigg[\delta_{ij} + \frac{p_ip_j}{p^-p^+}(e^{ip^+x^-} - 1)\\
&+\frac{q_iq_j}{p^-q^+}(e^{-iq^+y^-} - 1) + \frac{p_iq_jp_t\cdot q_t}{(p^-p^+)(p^-q^+)}(e^{ip^+x^-} - 1)(e^{-iq^+y^-} - 1)\Bigg]\\
&+\theta(x^-)\theta(-y^-)\int\frac{d^2q_t}{(2\pi)^2}d^2z_t\; e^{i(p_t - q_t)(z_t - x_t)}e^{-i(q^+ - p^+)x^-}\\
&\times F_a^{\dagger\alpha\beta}(x_t)F_a^{\dagger\alpha'\beta'}(z_t)\Bigg[\delta_{ij} + \frac{p_ip_j}{p^-p^+}(e^{-ip^+y^-} - 1) + \frac{q_iq_j}{p^-q^+}(e^{iq^+x^-} - 1)\\
&+\frac{q_ip_jp_t\cdot q_t}{(p^-p^+)(p^-q^+)}(e^{-ip^+y^-} - 1)(e^{iq^+x^-} - 1)\Bigg]\Bigg\},
\end{aligned} \tag{16}$$

where $q^+ = p^+ + \frac{q_t^2 - p_t^2}{2p^-}$ and

$$F_a^{\dagger\alpha\beta}(x_t) = (U(x_t)\tau_a U^\dagger(x_t))^{\alpha\beta}. \tag{17}$$

The above result was derived in Ref. [1]. The regularization of the poles in p^- and p^+ is a rather subtle issue and was discussed at some length in the above-mentioned paper.

The Green's function $G_{ij}(x,y)$ contains the physical degrees of freedom that we need to relate to the gluon density. In the following section, the above small fluctuations propagator will be used to compute corrections to the Weizsäcker-Williams distribution function. We will show that logarithmic corrections, both in x and k_t, arise from here. In Sec. IV, we discuss the corrections that the small fluctuations induce on the classical field to one loop. These corrections, in contrast, provide no such logarithmic terms to the distribution function and their only effect is to renormalize the coupling constant and the background field.

In Appendix A, we compute the Fourier transform of the Green's function. This result is useful for the computation of the distribution functions performed in the next two sections.

III. THE GLUON DISTRIBUTION FUNCTION

In this section, we will show how one can obtain the gluon distribution function from the Green's function [Eq. (16)], corresponding to the small fluctuations about the Weizsäcker-Williams background color field. We will be concerned with the structure of the leading terms for small x and at the end of the section compute a formula for the gluon distribution function valid to α_s^2. To compute the distribution function we need to sum over all possible color configurations. This color average involves the two-dimensional gauge fields U. We start by recalling the properties of these gauge fields under our color averaging.

A. Correlation functions involving the gauge transformations U

According to Eq. (5) the gauge transformations $U(x_t)$ carry the information on the background field which enters the Green's function (16). These gauge transformations have the interesting property that the color average with the Gaussian weight [defined by Eq. (8)], of the combination

$$U^\dagger(x_t)\tau^a U(x_t) U^\dagger(y_t)\tau^a U(y_t), \tag{18}$$

can be written as [3]

$$\langle \mathrm{Tr} U^\dagger(x_t)\tau^a U(x_t) U^\dagger(y_t)\tau^a U(y_t)\rangle = \frac{(N_c^2-1)}{2}\Gamma(x_t - y_t), \tag{19}$$

where N_c is the number of colors and summation over repeated indices is implied.

As pointed out in Ref. [2], the average over the color sources yields the information about the ground state properties of the system. The average with the Gaussian weight is an artifact that simplifies the computation and we expect that as long as we resolve the nucleus on a transverse size much larger than the typical transverse quark separation, such an artifact is justified.

The function Γ factorizes the dependence on the transverse coordinates x_t, y_t and is a function of their difference. Moreover, from Eq. (19) we see that Γ is real and also

$$\Gamma(0) = 1. \tag{20}$$

Defining the Fourier transform of $\Gamma(x_t)$,

$$\gamma(p_t) = \int d^2x_t e^{-ip_t x_t}\Gamma(x_t), \tag{21}$$

we have, together with (20), the sum rule

$$\int \frac{d^2p_t}{(2\pi)^2}\gamma(p_t) = 1. \tag{22}$$

The color charge at a given transverse location will be zero on average and the only way to generate a nonzero color charge will be by fluctuations. Equation (8) can be thought of as the generator of those fluctuations and thus the function $\Gamma(x_t, y_t)$ represents the correlator of fluctuating fields at the transverse locations x_t and y_t.

In momentum space, the function $\gamma(p_t)$ can be formally computed by expanding the exponential in (8) in powers of the coupling parameter $\alpha_s\mu/p_t$ (weak coupling regime). This was done in Ref. [3] for scalars. For gluons, the result for $\alpha_s\mu \ll p_t$ is

$$\gamma(p_t) = (4\pi)^2\frac{\alpha_s^2\mu^2}{p_t^4}N_c. \tag{23}$$

We notice that the expansion is only necessary in order to analytically compute expressions of the form

$$\int d^2p_t f(p_t)\gamma(p_t), \tag{24}$$

with $f(p_t)$ a nontrivial function of p_t. However, in principle, we can perform a numerical analysis to take into account the many possible different configurations of the external field contributing to expressions such as (24). This is equivalent to considering the effect to all orders in $\alpha_s\mu/p_t$ of the different configurations of the background field. For a quantitative discussion about the properties of the distribution function, we will restrict ourselves to the weak coupling regime for which $\gamma(p_t)$ is given by Eq. (23).

B. The distribution function

With the above remarks in mind, we proceed to the computation of the gluon distribution function. We use the formula for the distribution function,

$$\frac{dN}{d^3k} = i\frac{2k^+}{(2\pi)^3}\lim_{k^+\to k'^+}\int\frac{dk^-}{2\pi}\int\frac{dk'^-}{2\pi}\langle D_{ii}^{aa}(k,k')\rangle\ , \quad (25)$$

where $\langle D_{ii}^{aa}(k,k')\rangle$ is the small fluctuations propagator in momentum space, traced over the color and Lorentz indices and averaged over the external sources of color charge. This formula follows from computing $\langle a^\dagger(p)a(p)\rangle$ as an expectation value for the gluon Fock space distribution function in the ground state generated by the external valence charges [3].

In Appendix A, we derive explicitly an expression for $\langle D_{ii}^{aa}(k,k')\rangle$. Using this result, we obtain the following integral expression for the distribution function:

$$\frac{1}{\pi R^2}\frac{dN}{d^3k} = \frac{2ik^+}{(2\pi)^3}(N_c^2-1)\lim_{k^+\to k'^+}\int\frac{dp^+d^2p_tdk^-}{(2\pi)^4}\left\{\frac{1}{p_t^2-2p^+k^--i\epsilon}\left(2+\frac{p_t^2}{k^-k^+k'^+}(2p^+-k^+-k'^+)\right)\right.$$
$$\times\left(-(2\pi)^2\delta^{(2)}(p_t-k_t)\frac{1}{p^+-k^++i\epsilon}\frac{1}{p^+-k'^+-i\epsilon}-\gamma(p_t-k_t)\frac{1}{p^+-k^+-i\epsilon}\frac{1}{p^+-k'^++i\epsilon}\right)$$
$$+\gamma(p_t-k_t)\frac{1}{k_t^2-2p^+k^--i\epsilon}\left(2-\frac{k_t^2+p_t^2}{k^-k^+}+\frac{(p_t\cdot k_t)^2}{(k^{-2}k^{+2})}\right)$$
$$\left.\times\left(\frac{1}{p^+-k^+-i\epsilon}\frac{1}{q^+-k^+-i\epsilon}+\frac{1}{p^+-k^++i\epsilon}\frac{1}{q^+-k^++i\epsilon}\right)\right\}. \quad (26)$$

In the last term of this equation, we have taken the limit that $k^+\to k'^+$ since this term has no singularity in that limit.

We now do the integration over k^-. We assume that $k^+>0$. When we do the integration, two classes of terms result. The first set of terms arise from the explicit k^- dependence in the above equation and are nonzero. The second set of terms arise from the q^+ in the last terms of the above equation. These terms result in an unrestricted integral over p^+. One can show that all the singularities of the resulting integrand are on the same side of the p^+ integration contour in the complex p^+ plane. They therefore integrate to zero. [There is a possible ambiguity in the closing of contours associated with the contour at infinity, but this term does not have any contribution proportional to $\ln(1/x)$.] Therefore, we only get the contribution from the first term, which is only nonzero for $p^+<0$:

$$\frac{1}{\pi R^2}\frac{dN}{d^3k} = (N_c^2-1)\frac{4k^+}{(2\pi)^3}\int_{-\infty}^0\frac{dp^+d^2p_t}{(2\pi)^3}\left\{\frac{1}{p^+(p^+-k^+)(k^++p^+p_t^2/k_t^2)}\left[\gamma(p_t-k_t)-(2\pi)^2\delta^{(2)}(p_t-k_t)\right]\right.$$
$$\left.\times\left[1-\frac{p^+}{k^+}\left(1+\frac{p_t^2}{k_t^2}\right)+2\left(\frac{p^+}{k^+}\right)^2\frac{(p_t\cdot k_t)^2}{k_t^4}\right]\right\}. \quad (27)$$

Now in this expression, we shall only be concerned with those terms which are proportional to $\ln(1/x)$. The terms not proportional to $\ln(1/x)$ are nonleading for small x. Moreover, we have found that within our approach, these terms are inherently ambiguous. This is due to the fact that the $\ln(1/x)$ terms can only arise by regulating the singularity in the above integral as $p^+\to\infty$. We do this by making the upper limit of integration, to be of the same order as the total momentum of a typical nucleon in the nucleus. Of course different regularization schemes will affect the nonleading terms in different ways. Presumably, the detailed longitudinal structure of the valence quark charge distribution must be known before these terms may be evaluated.

After some straightforward algebra, we find

$$\left(\frac{1}{\pi R^2}\frac{dN}{dxd^2k_t}\right)_q = \frac{8(N_c^2-1)}{(2\pi)^4}\frac{1}{x}\int\frac{d^2p_t}{(2\pi)^2}\gamma(p_t-k_t)\left[1-\frac{(p_t\cdot k_t)^2}{p_t^2k_t^2}\right]\ln\left(\frac{1}{x}\right), \quad (28)$$

where the subindex q in the left-hand side of the above equation refers to the correction to the distribution function from the small quantum fluctuation field. Equation (28) is our main result. It is normalized so that the vacuum density is zero. This can be checked by setting $U=1$ in the above equation.

The terms above can be written in the form

$$\left(\frac{1}{\pi R^2}\frac{dN}{dxd^2k_t}\right)_q = \left(\frac{N_c^2-1}{2\pi^4}\right)\frac{1}{x}\int\frac{d^2p_t}{(2\pi)^2}\gamma(p_t-k_t)\left\{\frac{p_t^2k_t^2-(p_t\cdot k_t)^2}{p_t^2k_t^2}\right\}\ln\left(\frac{1}{x}\right). \quad (29)$$

Now, shift the variable of integration $p_t \to p_t + k_t$ and expand $\gamma(p_t)$ in weak coupling. This restricts the lower limit of integration for the radial component of $\vec{p_t}$ to be $\alpha_s\mu$, which comes from the weak coupling expansion of γ. Thus the above expression becomes

$$\left(\frac{1}{\pi R^2}\frac{dN}{dxd^2k_t}\right)_q = 8N_c(N_c^2-1)\frac{\alpha_s^2\mu^2}{(\pi)^2}\frac{1}{x}\int_0^{2\pi}d\theta\int_{\alpha_s\mu}^{\infty}dp_t\frac{(1-\cos^2\theta)}{p_t[p_t^2+k_t^2+2p_tk_t\cos\theta]}\ln\left(\frac{1}{x}\right), \tag{30}$$

where θ is the angle between $\vec{p_t}$ and $\vec{k_t}$. The integral above can be performed exactly and the contribution to (10) from the $\ln(1/x)$ terms becomes

$$\left(\frac{1}{\pi R^2}\frac{dN}{dxd^2k_t}\right)_{q1} = N_c(N_c^2-1)\frac{\alpha_s^2\mu^2}{xk_t^2}\frac{C(k_t)}{\pi^3}\ln\left(\frac{1}{x}\right) \tag{31}$$

with $C(k_t)$ given by

$$C(k_t) = 2\left[\ln\left(\frac{k_t}{\alpha_s\mu}\right)+\frac{1}{2}\right]. \tag{32}$$

Notice that the above expression means that the second term in the perturbative expansion of dN/dxd^2k_t in α_s, develops the large factor $\ln(1/x)$ and that in the kinematical region of interest, the product $\alpha_s\ln(1/x)$ is not small.

Furthermore, let us impose ordering in transverse momentum. This is the statement that the main contribution to the distribution function comes from the momentum region for which the emitted gluon's transverse momentum is larger than that of the original one [10]. The effect is to restrict the integration interval for the radial component of $\vec{p_t}$ in Eq. (30) which now runs between $\alpha_s\mu$ and k_t. The reader can check that the above results in the modification of (32) which now reads like

$$C(k_t) = 2\ln\left(\frac{k_t}{\alpha_s\mu}\right). \tag{33}$$

We can now include the contribution from the background Weizsäcker-Williams field as given by Eq. (13). Thus finally, the perturbative expression for the gluon distribution function to second order in α_s becomes

$$\frac{1}{\pi R^2}\frac{dN}{dxd^2k_t} = \frac{\alpha_s\mu^2(N_c^2-1)}{\pi^2}\frac{1}{xk_t^2}\times\left\{1+\frac{\alpha_sN_c}{\pi}C(k_t)\ln\left(\frac{1}{x}\right)\right\}. \tag{34}$$

Equation (34) contains both $\ln(1/x)$ and $\ln(k_t)$ corrections to the $1/(xk_t^2)$ distribution and they represent the first order contributions to the perturbative expansion for the distribution function. In the kinematical region of validity, these corrections are large. This signals that in order to properly account for the perturbative corrections one has to devise a mechanism to isolate and sum up these leading contributions. Also notice that Eq. (28) is more general. In principle, it contains the information about the nonperturbative corrections as well. That information is in the function $\gamma(p_t)$ and it can be extracted by means of a Monte Carlo analysis for the whole k_t domain. These issues will be treated in a future work.

Diagrammatically, we can represent the background gluon field coupled to the external source (valence quarks), in momentum space, by means of Fig. 1(a). The background field (wavy line) is by itself of order $1/g$, according to Eq. (5) and the coupling to the external source (cross) can be considered to nth order in the

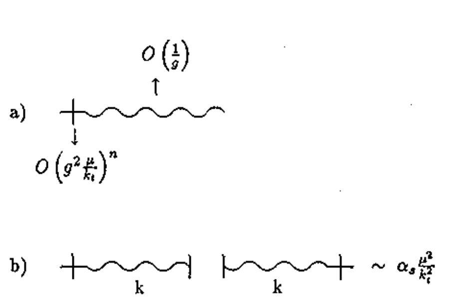

FIG. 1. (a) Coupling of the classical background field to the external source. The wavy line represents the background field which is of order $O\left(\frac{1}{g}\right)$. The external source is shown by a cross and to the lowest order in the weak coupling regime it is $O\left(\frac{\mu g^2}{k_t}\right)$. (b) Correlation of two classical background fields where the broken wavy line means that the momentum k_t is not integrated over.

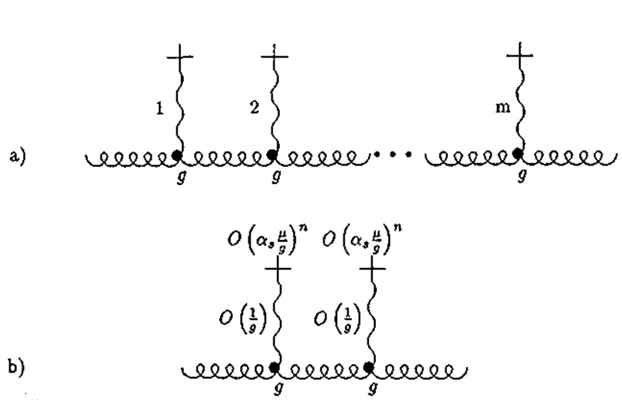

FIG. 2. (a) The perturbative expansion of the gluon propagator in the presence of the classical background field in terms of the coupling constant g. (b) The gluon propagator expanded to the second order in g.

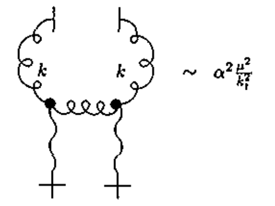

FIG. 3. The correction to the Weizsäcker-Williams distribution function to the lowest order in the weak coupling regime.

parameter $g^2\mu/k_t$ by means of the weak coupling expansion of Eq. (8). As an example, the Weizsäcker-Williams distribution is obtained through the correlation of the background field taking the average over the source to first order (n=1) in $g^2\mu/k_t$. This can be represented as in Fig. 1(b) where the broken wavy line means that the momentum k is not integrated over.

The gluon propagator in the presence of the background field can be computed perturbatively in the coupling constant g and the mth order gluon propagator can be represented as in Fig. 2(a). This is because the perturbative expansion of the gluon field involves its coupling to the background field through the covariant derivative and the background field acts as the gauge field. Notice that m has to be even since the gluon propagator is the correlator of two gluon fields and each time we couple the background field to the gluon field we introduce one power of g. In particular, the gluon propagator to second order in g can be represented as in Fig. 2(b). As suggested in this figure, the explicit dependence on α_s of a quantity such as the gluon distribution function (which involves the gluon propagator) comes about only after performing our color average through the expansion in $g^2\mu/k_t$. This is because the coupling constant dependence of the background field and the order of the perturbative expansion offset each other. As shown in Sec. III, when computing the leading small x terms for the gluon distribution function, any term for which we can use the sum rule (22) will not exhibit an explicit coupling constant dependence. This becomes the criterion to decide that such terms are vacuum contributions.

The gluon distribution function computed in Sec. III can be represented by the diagram in Fig. 3, where we expanded the coupling with the external source to first order in $g^2\mu/k_t$.

IV. LOOP CORRECTIONS TO THE CLASSICAL FIELD

Thus far, we have been concerned exclusively with the contribution of the small fluctuations propagator to the gluon distribution function. We have shown that this propagator induces large corrections proportional to $\alpha_s \ln(1/x)\ln(k_t^2)$ and $\alpha_s \ln(1/x)$ to the distribution function and have argued that the presence of these large logarithms signals the need to devise a method to sum them up to all orders in the perturbative regime. Before we do that we need to consider another contribution, to the same order, which comes from the corrections to the lowest order classical field induced by quantum fluctuations (see Fig. 4). This is apparent from Eqs. (11) and (12) where one sees that there is a contribution $\langle A_{\rm cl}^{(1)} A_{\rm cl}^{(0)}\rangle_\rho$ of the same order as $\langle\langle A_q A_q\rangle\rangle_\rho$.

In this section, we will compute the correction to the lowest order classical field induced by the quantum fluctuations. We will start by writing the total field A^μ in terms of background (classical) and fluctuation (quantum) pieces allowing for the possibility that the background field may now be different from our lowest order classical (Weizsäcker-Williams) solution. We will then write the equations of motion in terms of these new background and fluctuation fields keeping terms up to and including second order in the fluctuation fields.

Our strategy will be to consider the expectation value of the equations of motion (in the path integral sense) and to relate the correlator of two quantum fields to the gluon propagator in Eq. (16). We will show that only the + component of the equations of motion is modified and that the change could be thought of as the appearance of an induced current generated by the loop of fluctuation fields. We then proceed to explicitly compute this induced current and show that its effect is to renormalize the coupling constant g and the original background field. In other words, the modification induced by quantum fluctuations on the classical equations of motion can be cast into the standard expression for the renormalization of the coupling constant and the original background field to one loop in the light cone gauge. This result in itself is not surprising to QCD practitioners (see, for instance, Ref. [17]). What is surprising is that this result persists to all orders in the effective coupling $\alpha_S\mu$.

We start with the classical equations of motion

$$D_\mu F_a^{\mu\nu} = gJ_a^\nu \tag{35}$$

and expand the full gluon field as

$$A^\mu = B^\mu + b^\mu \tag{36}$$

where B^μ is the background (classical) field, that is $\langle A^\mu\rangle = B^\mu$ while b^μ is the fluctuation (quantum) field with $\langle b^\mu\rangle = 0$. Keeping up to quadratic terms in b^μ, the + component of the equations of motion can be written as

$$\partial_-\partial_- B_a^- + (D_i\partial_- B^i)_a = gj_a^+ + g\langle J_a^+\rangle, \tag{37}$$

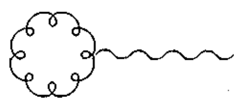

FIG. 4. Modification of the classical background field due to quantum fluctuations.

where $j_a^+(x) = f_{abc}\langle b_b^i(x)\partial^+ b_c^i(x)\rangle$. Also, D^μ is the covariant derivative with B^μ as the gauge potential. The corresponding expressions for the minus and transverse components of the equations of motion look like

$$(D_iD^iB^-)_a - (D_i\partial^-B^i)_a + (D_+\partial^+B^-)_a$$
$$+gf_{abc}\{\langle b_+^b\partial^+b_c^-\rangle + (D_i\langle b^ib^-\rangle)_{bc}$$
$$+\langle b_b^i(D^ib^-)_c\rangle - \langle b_b^i(D^-b^i)_c\rangle\} = 0, \quad (38)$$

$$(D_jD^jB^i)_a - (D_j\partial^iB^j)_a + (D_+\partial^+B^i)_a - (\partial_-D^iB^-)_a$$
$$+(\partial_-\partial^-B^i)_a + gf_{abc}\{\langle b_+^b\partial^+b_i^c\rangle + \partial_-\langle b_b^-b_c^i\rangle$$
$$+(D_j\langle b^jb^i\rangle)_{bc} - \langle b_b^j(D^ib^j)_c\rangle + \langle b_b^j(D^jb^i)_c\rangle\} = 0. \quad (39)$$

The expectation values of bilinear products of fields are related to the gluon propagator by the relation

$$\langle b_a^\mu(x)b_b^\nu(y)\rangle = -iG_{ab}^{\mu\nu}(x,y). \quad (40)$$

In the above, $a,b,c,\ldots$ are color indices and μ,ν are Lorentz indices with i,j representing the transverse components.

The reader may verify that all the terms involving bilinear products of b^μ in the minus and transverse components of the equations of motion either vanish by explicit computation, or, because they are symmetric in the color indices b and c, obviously do not contribute since they are always contracted with the totally antisymmetric structure constants f_{abc}. In other words, the minus and transverse components of the classical equations of motion are not modified by the quantum fluctuations and the set of equations reduces to

$$-\partial_-\partial^+B_a^- - (D_i\partial^+B^i)_a = gj_a^+ + g\langle J_a^+\rangle,$$

$$(D_iD^iB^-)_a - (D_i\partial^-B^i)_a + (D_+\partial^+B^-)a = 0,$$

$$(D_jD^jB^i)_a - (D_j\partial^iB^j)_a + (D_+\partial^+B^i)_a - (\partial_-D^iB^-)_a = 0. \quad (41)$$

From now on we will concentrate only on the plus component of the equation of motion given by Eq. (37). It is clear that this equation is modified by the quantum fluctuations due to the presence of the induced current. In order to understand this effect, we need to evaluate this term explicitly. For this purpose, we write it as

$$j_a^+(x) = f_{abc}\langle b_b^i(x)\partial^+b_c^i(x)\rangle$$
$$= if_{abc}\lim_{y\to x}\frac{\partial}{\partial y^-}G_{bc}^{ii}(x,y). \quad (42)$$

Diagrammatically, this term can be represented as in Fig. 4 where the wavy line is the background field and the spiral represents the loop of the fluctuation field. The loop is the vacuum polarization tensor and the component which contributes to the induced current and modifies the background field (which is purely transverse) is Π^{+i}. This allows us to represent the term $\partial^+G^{ii} \equiv D^+G^{ii}$ as $\Pi^{+i}B^i$.

We now proceed to compute the induced current explicitly. We will use our expression for the gluon propagator as given by Eq. (16). The first observation is that the terms in the Green's function with both x^- and y^- negative will be symmetric in the color indices and will not contribute. Also, it can be shown that the terms with both x^- and y^- positive yield (after we implement the limit $y\to x$ in a Lorentz-covariant way) an infinite constant (independent of the transverse loop momentum) which vanishes upon dimensional regularization [18].

However, the terms in the Green's function with opposite signs of x^- and y^- are a bit tricky. For these terms taking the partial derivative with respect to y^- followed by the limit $y\to x$ is a very delicate operation and must be performed carefully. We find it more convenient to rewrite the terms with opposite signs of x^- and y^- in the Green's function in such a way as to avoid acting with $\partial/\partial y^-$ on the terms $\theta(\pm y^-)$. To do so, we will change the two-dimensional integral over q_t to a four-dimensional integral over q. We can show that as a result, the product of θ functions of x^- and y^- will be replaced by θ functions of the light cone energy p^-. After some long but straightforward algebra we can rewrite the terms with opposite signs of x^- and y^- in the Green's function (which we call D_{ij}^{bc}) as

$$D^{bc}_{ij}(x,y) \equiv (2i)\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}(2\pi)\delta(p^- - q^-)(p^- + q^-)\frac{e^{ip(x-y)}}{(p^2-i\epsilon)(q^2-i\epsilon)}$$
$$\times\Bigg\{\theta(p^-)e^{-i(p-q)x}\mathrm{Tr}[U^\dagger(x_t)\tau^b U(x_t)\tilde{F}^c(p_t-q_t)]\Bigg[\delta_{ij} + \frac{p_ip_j}{p^-p^+}(e^{-ip^+y^-}-1)$$
$$+\frac{q_iq_j}{q^-q^+}(e^{iq^+x^-}-1) + \frac{q_ip_j(q_t\cdot p_t)}{p^-p^+q^-q^+}(e^{-ip^+y^-}-1)(e^{iq^+x^-}-1)\Bigg]$$
$$-\theta(-p^-)e^{i(p-q)y}\mathrm{Tr}[\tilde{F}^b(q_t-p_t)U^\dagger(y_t)\tau^c U(y_t)]\Bigg[\delta_{ij} + \frac{q_iq_j}{q^-q^+}(e^{-iq^+y^-}-1)$$
$$+\frac{p_ip_j}{p^-p^+}(e^{ip^+x^-}-1) + \frac{p_iq_j(q_t\cdot p_t)}{p^-p^+q^-q^+}(e^{-iq^+y^-}-1)(e^{ip^+x^-}-1)\Bigg]\Bigg\}, \qquad (43)$$

where $\tilde{F}^b(p_t)$ is the Fourier transform of $F^b(x_t) \equiv U^\dagger(x_t)\tau^b U(x_t)$. In the above expression, we are working with the color components of expression (16) (as opposed to the matrix notation), which is more suitable for the computation at hand.

According to Eq. (42), we now have to compute

$$j^+_a(x) = if_{abc}\lim_{y\to x}\frac{\partial}{\partial y^-}D^{bc}_{ii}(x,y). \qquad (44)$$

There are three distinct pieces to the computation corresponding to the different number of factors of p^- (q^-) in the denominator of each term in expression (43). For the rest of the section, we will outline the procedure for the computation of one of them, namely, the term proportional to δ_{ij} and quote the result for the other two terms. In Appendix B, we will show the detailed computation of some of the integrals necessary to fill in the intermediate steps.

There are two important details to keep in mind while computing explicitly expression (44): first, we have to implement the limiting procedure in a Lorentz covariant way. Second, when carrying out the integration over the relative and total (light cone) energies, we have to allow for a nonzero energy flow into the loop by not setting the total energy to zero, in spite of the structure of the integral which seems to require it to be so. The procedure is nothing but the well-known *point splitting method.*

In order to implement the limit $y \to x$ in a Lorentz covariant way, we first transform the induced current to momentum space and then integrate over the relative (loop) momentum. We make use of the following identity. Let $f(x,y)$ be a function of the four-dimensional variables x and y for which we want to compute the limit when $x \to y$. Fourier transform f to momentum space with respect to both x and y,

$$\tilde{f}(k,k') = \int d^4x d^4y\ f(x,y)e^{ikx}e^{ik'y}, \qquad (45)$$

and then make the change of variables

$$s = \frac{k-k'}{2},$$
$$S = \frac{k+k'}{2}, \qquad (46)$$

where s and S can be thought of as the relative and total momenta, respectively. Then

$$\tilde{f}(s,S) = \int d^4xd^4yf(x,y)e^{is(x-y)}e^{iS(x+y)}. \qquad (47)$$

Integrating over $d^4s/(2\pi)^4$ will give $\delta^4(x-y)$ which will set $y = x$ upon integrating over y. Hence,

$$\int d^4s\tilde{f}(s,S) = \int d^4x\ e^{i2Sx}f(x,x)\,, \qquad (48)$$

which is the expression for the Fourier transform of $f(x,y)$ in the limit when $y \to x$ as a function of $2S$. With the above remarks in mind, we proceed to take the partial derivative with respect to y^- in Eq. (43) and then to Fourier transform with respect to x and y to the momentum variables k and k'. Let us look at the piece proportional to δ_{ij}. Set $i = j$ and then perform the $p^\pm$, $q^\pm$, and p_t integrations to get

$$\int d^4xd^4ye^{ikx}e^{ik'y}\frac{\partial}{\partial y^-}D^{ii}_{bc}(x,y)_{(1)} = (-2\pi)\delta(k'^- + k^-)\int\frac{d^2q_t}{(2\pi)^2}\frac{(k'^- - k^-)k'^+}{(k'^-)(k^-)}\mathrm{Tr}[\tilde{F}^a(k_t+q_t)\tilde{F}^b(k'_t - q_t)]$$
$$\times\left\{\frac{\theta(k'^-)}{\left(k'^+ - \frac{(k'^2_t - i\epsilon)}{2k'^-}\right)\left(k^+ - \frac{(q_t^2-i\epsilon)}{2k^-}\right)} \frac{\theta(-k'^-)}{\left(k^+ - \frac{(k_t^2-i\epsilon)}{2k^-}\right)\left(k'^+ - \frac{(q_t^2-i\epsilon)}{2k'^-}\right)}\right\}, \qquad (49)$$

where the index 1 refers to our considering the first of the terms in Eq. (43), namely, the term proportional to δ_{ij}.

Now, perform the change of variables (46) to write the above expression in terms of relative and total momentum and integrate over the relative momentum. The expression to evaluate becomes

$$\int \frac{d^4s}{(2\pi)^4} d^4x d^4y e^{is(x-y)} e^{iS(x+y)} \frac{\partial}{\partial y^-} D^{ii}_{bc}(x,y)_{(1)}$$
$$= (-2\pi)\delta(2S^-) \int \frac{d^4s}{(2\pi)^4} \frac{d^2q_t}{(2\pi)^2} \frac{(-2s^-)(S^+ - s^+)}{(S^- - s^-)(S^- + s^-)} \mathrm{Tr}[\tilde{F}_b(s_t + S_t + q_t)\tilde{F}_c(S_t - s_t - q_t)]$$
$$\times \left\{ \frac{\theta(S^- - s^-)}{\left(S^+ - s^+ - \frac{(S_t - s_t)^2 - i\epsilon}{2(S^- - s^-)}\right)\left(S^+ + s^+ - \frac{(q_t^2 - i\epsilon)}{2(S^- + s^-)}\right)} - \frac{\theta(-S^- + s^-)}{\left(S^+ + s^+ - \frac{(S_t + s_t)^2 - i\epsilon}{2(S^- + s^-)}\right)\left(S^+ - s^+ - \frac{(q_t^2 - i\epsilon)}{2(S^- - s^-)}\right)} \right\}. \tag{50}$$

We will perform the transverse momentum integrations last. For the moment, let us concentrate on the s^+ and s^- integrals. It is more convenient to do the s^+ integration first since this can be done by contour integration. The s^+-dependent integral of the above expression, denoted by I, is

$$I = \int \frac{ds^+}{(2\pi)} \left\{ \frac{\theta(S^- - s^-)}{\left(S^+ - s^+ - \frac{(S_t - s_t)^2 - i\epsilon}{2(S^- - s^-)}\right)\left(S^+ + s^+ - \frac{(q_t^2 - i\epsilon)}{2(S^- + s^-)}\right)} - \frac{\theta(-S^- + s^-)}{\left(S^+ + s^+ - \frac{(S_t + s_t)^2 - i\epsilon}{2(S^- + s^-)}\right)\left(S^+ - s^+ - \frac{(q_t^2 - i\epsilon)}{2(S^- - s^-)}\right)} \right\} (S^+ - s^+). \tag{51}$$

This integral has a logarithmically divergent piece which can be isolated by adding and subtracting the term $(S_t - s_t)^2/2(S^- - s^-)$ to the numerator. This divergent piece can be shown to give a constant independent of the transverse loop momentum and therefore it vanishes upon dimensional regularization in the transverse direction. The remaining piece reads as

$$I = -\int \frac{ds^+}{(2\pi)} \left\{ \frac{\theta(S^- - s^-)}{\left(S^+ - s^+ - \frac{(S_t - s_t)^2 - i\epsilon}{2(S^- - s^-)}\right)\left(S^+ + s^+ - \frac{(q_t^2 - i\epsilon)}{2(S^- + s^-)}\right)} - \frac{\theta(-S^- + s^-)}{\left(S^+ + s^+ - \frac{(S_t + s_t)^2 - i\epsilon}{2(S^- + s^-)}\right)\left(S^+ - s^+ - \frac{(q_t^2 - i\epsilon)}{2(S^- - s^-)}\right)} \right\} \frac{(S_t - s_t)^2}{2(S^- - s^-)}. \tag{52}$$

Let us investigate the above expression in some detail. The integrand has two poles in the complex s^+ plane. Their location depends on the signs of $(S^- - s^-)$ and $(S^- + s^-)$. If the two poles are on the same side of the real axis, then we can close the contour of integration on the other side of the real axis and the integral vanishes. So in order to get a nonvanishing result, the two poles must be on opposite sides of the real axis. Recall that $2S$ is the total or external momentum flowing into the loop and thus it has to be kept fixed (and finite for a nontrivially zero loop integral) while working in momentum space. Thus for a given sign of S^- only one of the two terms in the integral (52) above contributes. This can be seen as follows: the first term in (52) is nonzero only if the two conditions

$$S^- - s^- > 0, \quad S^- + s^- > 0 \tag{53}$$

are satisfied, whereas the second term is nonvanishing only if

$$S^- - s^- < 0, \quad S^- + s^- < 0. \tag{54}$$

First, take S^- positive. Then only the first condition (53) gives overlapping intervals for s^-, namely $S^- > s^-$ and $s^- > -S^-$ or $-S^- < s^- < S^-$, whereas the second condition (54) does not. Therefore the second term in Eq. (52) can be disregarded and only the first one is nonvanishing. The opposite is true for S^- negative in which case only the second of the terms in Eq. (52) contributes.

The integral (50), however, turns out to be independent of the sign of S^- (as we shall describe below). The reason is the *scaling* property of the integral over s^-. This can be understood by recalling that after all, the overall expression, Eq. (50), is explicitly proportional to $\delta(2S^-)$ and any term proportional to S^- can be thrown away after scaling the integration variable s^- by S^-. With these remarks in mind, let us continue working with a definite sign of S^-, say, $S^- > 0$. Performing the integration (52) we get

$$I = \frac{-i\theta(S^- - s^-)\theta(S^- + s^-)(S^- + s^-)(S_t - s_t)^2}{S^-[4S^+S^-(1-\xi)(1+\xi) - q_t^2(1-\xi) - (S_t - s_t)^2(1+\xi)]} \tag{55}$$

in terms of which Eq. (50) is

$$\int \frac{d^4s}{(2\pi)^4} d^4x d^4y e^{is(x-y)} e^{iS(x+y)} \frac{\partial}{\partial y^-} D^{ii}_{bc}(x,y)_{(1)}$$

$$= (-2i)\delta(2S^-) \int \frac{d^2s_t}{(2\pi)^2} \frac{d^2q_t}{(2\pi)^2} \mathrm{Tr}[\tilde{F}^a(s_t + S_t + q_t)\tilde{F}^b(S_t - s_t - q_t)](S_t - s_t)^2$$
$$\times \int_{-S^-}^{S^-} \frac{s^- ds^-}{(S^- - s^-)[4S^+(S^- + s^-)(S^- - s^-) - q_t^2(S^- - s^-) - (S_t - s_t)^2(S^- + s^-)]}. \tag{56}$$

Let us now look at the integration over s^-. We scale s^- by S^-, that is we define the variable $\xi = s^-/S^-$. We notice that after scaling we can make use of the explicit factor $\delta(2S^-)$ in Eq. (50) and we can safely throw away any term which is still explicitly proportional to S^-. Thus, the term $[4S^+S^-(1-\xi)(1+\xi)]$ in the denominator of the above drops. As a result, the overall expression (50) becomes

$$\int \frac{d^4s}{(2\pi)^4} d^4x d^4y e^{is(x-y)} e^{iS(x+y)} \frac{\partial}{\partial y^-} D^{ii}_{bc}(x,y)_{(1)}$$

$$= (2i)\delta(2S^-) \int \frac{d^2s_t}{(2\pi)^2} \frac{d^2q_t}{(2\pi)^2} \mathrm{Tr}[\tilde{F}^a(s_t + S_t + q_t)\tilde{F}^b(S_t - s_t - q_t)](S_t - s_t)^2$$
$$\times \int_{-1}^{1} \frac{\xi d\xi}{(1-\xi)[q_t^2(1-\xi) + (S_t - s_t)^2(1+\xi)]}. \tag{57}$$

To proceed further, it is convenient to shift $q_t \to q_t - s_t$. Then the arguments of F'^s in the trace of the above expression become independent of s_t and can be taken outside the s_t integral which will be evaluated next. The s_t integral is a formally divergent integral and must be regulated. This is done in Appendix B using the dimensional regularization method. We find that

$$\int \frac{d^{2\omega}s_t}{(2\pi)^2} \frac{(S_t - s_t)^2}{[(q_t - s_t)^2(1-\xi) + (S_t - s_t)^2(1+\xi)]} = \frac{\Gamma(-\omega)}{16\pi} \xi(1-\xi)(S_t - q_t)^2, \tag{58}$$

which brings expression (57) to read like

$$\int \frac{d^4s}{(2\pi)^4} d^4x d^4y e^{is(x-y)} e^{iS(x+y)} \frac{\partial}{\partial y^-} D^{ii}_{bc}(x,y)_{(1)}$$

$$= (2i)\frac{\Gamma(-\omega)}{16\pi}\delta(2S^-) \int \frac{d^2q_t}{(2\pi)^2} \mathrm{Tr}[\tilde{F}^a(S_t + q_t)\tilde{F}^b(S_t - q_t)](S_t - q_t)^2 \int_{-1}^{1} \xi^2 d\xi. \tag{59}$$

The integration over ξ can now be done easily. It just gives a factor of 2/3 and the remaining transverse momentum integral can be computed by using the explicit form of $\tilde{F}$ in terms of the gauge transforms U. We also show this in Appendix B where we find that

$$\int \frac{d^2q_t}{(2\pi)^2} \mathrm{Tr}[\tilde{F}_b(S_t + q_t)\tilde{F}_c(S_t - q_t)](S_t - q_t)^2 = \frac{g^2}{2} f^{bcd} \tilde{\rho}_d(2S_t), \tag{60}$$

with

$$\tilde{\rho}_d(2S_t) = \int d^2x_t e^{2iS_t x_t} \rho_d(x_t) \tag{61}$$

being the Fourier transform of the charge density with respect to $2S_t$. Putting everything together, we get finally the result that the expression for the Fourier transform of the induced current coming from the term proportional to δ_{ij} as a function of $2S$ is

$$\tilde{j}_a^+(2S)_{(1)} \equiv i f_{abc} \int \frac{d^4s}{(2\pi)^4} d^4x d^4y e^{is(x-y)} e^{iS(x+y)} \frac{\partial}{\partial y^-} D_{bc}^{ii}(x,y)_{(1)}$$
$$= -\frac{\Gamma(-\omega)}{8\pi} g^2 \tilde{\rho}_d(2S_t)\delta(2S^-) . \qquad (62)$$

Above, we have written only the divergent part of the γ function when $\omega \to 1$ and have used that for SU(3) the product $f_{abc}f_{bcd} \equiv C_A \delta_{ad}$ with $C_A = 3$. The remaining three terms in Eq. (43) can be evaluated in a similar fashion. Here we just quote the result

$$\tilde{j}_a^+(2S)_{(2)} = \frac{\Gamma(-\omega)}{8\pi} g^2 \tilde{\rho}_d(2S_t)\delta(2S^-),$$
$$\tilde{j}_a^+(2S)_{(3)} = \frac{\Gamma(-\omega)}{8\pi} g^2 \tilde{\rho}_d(2S_t)\delta(2S^-),$$
$$\tilde{j}_a^+(2S)_{(4)} = 4\frac{\Gamma(-\omega)}{8\pi} g^2 \tilde{\rho}_d(2S_t)\delta(2S^-) . \qquad (63)$$

The final result for the Fourier transform of the induced current is obtained by adding the four terms given by Eqs. (62) and (63) and it becomes

$$\tilde{j}_a^+(p) = g^2 \Gamma(-\omega) \frac{5}{8\pi} \tilde{\rho}_a(p_t)\delta(p^-) , \qquad (64)$$

where we have renamed $2S \to p$. Thus the term that modifies the plus component of the equations of motion in momentum space just becomes $g\tilde{j}_a^+(p)$.

We proceed to argue that by rewriting this result in terms of an expression involving the components of the polarization operator, we can absorb the effect of the loop corrections on the equations of motion, into the renormalization of the coupling constant. Let us first recall that according to Eqs. (4) and (5), which are the classical solutions to the equations of motion, the expression for the zeroth order background field in momentum space can be written as

$$A_b^{i(0)}(p) \equiv \int d^4x e^{ipx} A_b^{i(0)}(x)$$
$$= (2\pi) g \left(\frac{p^i}{p_t^2 p^+}\right) \tilde{\rho}_b(p_t)\delta(p^-) . \qquad (65)$$

Therefore, notice that $g\tilde{j}_a^+(p)$ can be written as

$$g\tilde{j}_a^+(p) = \Pi_{ab}^{+i}(p) A_b^{i(0)}(p) \qquad (66)$$

with $\Pi_{ab}^{+i}(p)$ given by

$$\Pi_{ab}^{+i}(p) = g^2 p^+ p^i \left(\frac{5\Gamma(-\omega)}{16\pi^2}\right) \delta_{ab}, \qquad (67)$$

which is the standard expression for the $+i$ components of the polarization operator in light cone gauge [19]. The fact that we recover this well-known result is truly remarkable and is one indicator of the success of our formalism.

We can now take an *ansatz* for the formal solution of the system of Eqs. (41) to be

$$B^-(x) = 0,$$
$$B^i(x) = \theta(x^-)\alpha_R^i(x_t),$$
$$\tau \cdot \alpha_R^i(x) = \frac{i}{g_R} U(x_t)\nabla^i U^\dagger(x_t), \qquad (68)$$

where g_R is the renormalized coupling constant whose expression is obtained through the computation of $\Pi^{\mu\nu}$ and will be given explicitly by

$$g_R^2 = Z_3 g^2, \quad Z_3 = \left(1 + \frac{11g^2}{16\pi^2(1-\omega)}\right) . \qquad (69)$$

This in turn means, according to (68), that the field B gets renormalized by the inverse of the constant that renormalizes g:

$$B^i(x) = Z^{-1/2} A^{i(0)}(x). \qquad (70)$$

The above exercise has taught us the important lesson that the modifications to the background field introduced by the quantum fluctuations do not induce extra terms in the expression for the distribution function (28). Further, their effect can be included by replacing the coupling constant g by the renormalized coupling constant g_R.

V. SUMMARY

We have presented in this paper an expression for the quantum corrections to the Weizsäcker-Williams gluon distribution at small x valid to all orders in the parameter $\alpha_s\mu$. We used this expression to compute explicitly the leading $\ln(1/x)$ and $\ln(k_t)$ terms in the momentum regime $\alpha_s\mu \ll k_t$. We have shown that the perturbative approach introduces a series expansion parameter $\alpha_s \ln(1/x)$ which is large and thus forces us to devise a method to sum up the leading contributions to all orders in that expansion parameter. Nevertheless, the present result already signals that at small x values the gluon distribution function will be modified significantly from the $1/(xk_t^2)$ behavior.

We wish to emphasize that our central result, Eq. (28), contains in principle the information about the quantum correction to the classical distribution to all orders in the parameter $\alpha_s\mu$.

We have found that the only effect the quantum corrections have on the classical background field can be absorbed into the renormalization of the field and the running of the coupling constant.

We have not addressed the issue of summing up the perturbative series in this paper. In the weak coupling limit, this is equivalent to solving an integral equation for virtual corrections to the gluon propagator. Another issue we would like to address is whether we can relax some of the constraints in the model. In particular, we need to pay attention to the restriction set by the necessity of having a large (perhaps too large) A nucleus in order to compare our predictions to experimental data. These issues will be addressed in a future work.

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APPENDIX A: THE PROPAGATOR IN MOMENTUM SPACE

In this appendix, we shall Fourier transform the small fluctuations propagator to momentum space. This result will be useful for computing the distribution function and may also be useful for other computations. We will first consider the propagator without averaging over all possible color orientations of the source fields.

We shall derive two representations of the propagator. One representation will include an integration over the variable p^+. This representation will be useful for computing distribution functions. We will also present a second representation where we have performed this integration over p^+. Finally, we will present a result for the propagator after averaging over the colors of the external field.

We recall that in coordinate space the propagator has the form

$$\begin{aligned}
G_{ij}^{\alpha\beta;\alpha'\beta'}(x,y) = -\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\epsilon}\Bigg\{\bigg[\delta_{ij}+\frac{p_ip_j}{p^-p^+}(2e^{ip^+(x^--y^-)}-e^{-ip^+y^-}-e^{ip^+x^-})\bigg]\\
\times\Big[\theta(-x^-)\theta(-y^-)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'}+\theta(x^-)\theta(y^-)\,F_a^{\dagger\alpha\beta}(x_t)F_a^{\dagger\alpha'\beta'}(y_t)\Big]\\
+\theta(-x^-)\theta(y^-)\int\frac{d^2q_t}{(2\pi)^2}d^2z_t\; e^{i(q^+-p^+)y^-}e^{i(p_t-q_t)(y_t-z_t)}\\
\times F_a^{\dagger\alpha\beta}(z_t)F_a^{\dagger\alpha'\beta'}(y_t)\bigg[\delta_{ij}+\frac{p_ip_j}{p^-p^+}(e^{ip^+x^-}-1)\\
+\frac{q_iq_j}{p^-q^+}(e^{-iq^+y^-}-1)+\frac{p_iq_jp_t\cdot q_t}{(p^-p^+)(p^-q^+)}(e^{ip^+x^-}-1)(e^{-iq^+y^-}-1)\bigg]\\
+\theta(x^-)\theta(-y^-)\int\frac{d^2q_t}{(2\pi)^2}d^2z_t\; e^{i(p_t-q_t)(z_t-x_t)}e^{-i(q^+-p^+)x^-}\\
\times F_a^{\dagger\alpha\beta}(x_t)F_a^{\dagger\alpha'\beta'}(z_t)\bigg[\delta_{ij}+\frac{p_ip_j}{p^-p^+}(e^{-ip^+y^-}-1)+\frac{q_iq_j}{p^-q^+}(e^{iq^+x^-}-1)\\
+\frac{q_ip_jp_t\cdot q_t}{(p^-p^+)(p^-q^+)}(e^{-ip^+y^-}-1)(e^{iq^+x^-}-1)\bigg]\Bigg\},
\end{aligned} \tag{A1}$$

where $q^+=p^++\frac{q_t^2-p_t^2}{2p^-}$ and

$$F_a^{\dagger\alpha\beta}(x_t)=(U(x_t)\tau_aU^\dagger(x_t))^{\alpha\beta}. \tag{A2}$$

We now wish to define the Fourier transformed Green's function

$$G_{ij}^{\alpha\beta;\alpha'\beta'}(k,k')=\int d^4xd^4ye^{-ikx+ik'y}G_{ij}^{\alpha\beta;\alpha'\beta'}(x,y)\,, \tag{A3}$$

which we can divide into four pieces as

$$G_{ij}^{\alpha\beta;\alpha'\beta'}(k,k')=G_{--ij}^{\alpha\beta;\alpha'\beta'}(k,k')+G_{++ij}^{\alpha\beta;\alpha'\beta'}(k,k')+G_{+-ij}^{\alpha\beta;\alpha'\beta'}(k,k')+G_{-+ij}^{\alpha\beta;\alpha'\beta'}(k,k'). \tag{A4}$$

In this equation, the first index $\pm$ refers to the index of $\theta(\pm x^-)$ and the second to $\theta(\pm y^-)$ in the definition of the coordinate space Green's function.

Let us first consider the $--$ component:

$$G^{\alpha\beta;\alpha'\beta'}_{--ij}(k,k') = -\int \frac{d^4p}{(2\pi)^4} d^4x d^4y e^{i(p-k)x - i(p-k')y} \theta(-x^-)\theta(-y^-)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'} \frac{1}{p^2 - i\epsilon}$$
$$\times \left\{ \delta_{ij} + \frac{p_i p_j}{p^- p^+}(2e^{ip^+(x^- - y^-)} - e^{-ip^+y^-} - e^{ip^+x^-}) \right\} . \qquad \text{(A5)}$$

We can perform the integrations over p^-, p_t, $\vec{x}$, and $\vec{y}$ to obtain

$$G^{\alpha\beta;\alpha'\beta'}_{--ij}(k,k') = -(2\pi)\delta(k^- - k'^-)(2\pi)^2\delta^{(2)}(k_t - k'_t)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'}\Delta_{--}(k,k'), \qquad \text{(A6)}$$

where

$$\Delta_{--}(k,k') = \int dx^- dy^- \frac{dp^+}{2\pi} \frac{\theta(-x^-)\theta(-y^-)e^{-i(p^+ - k^+)x^- + i(p^+ - k'^+)y^-}}{k_t^2 - 2p^+k^- - i\epsilon}$$
$$\times \left\{ \delta_{ij} + \frac{k_i k_j}{k^- p^+}(2e^{ip^+(x^- - y^-)} - e^{-ip^+y^-} - e^{ip^+x^-}) \right\}. \qquad \text{(A7)}$$

Now let us do the x^- and y^- integrations. In performing these integrations, $i\epsilon$ factors will appear which will guarantee the convergence of the integrals at infinity. We find

$$\Delta_{--}(k,k') = \int \frac{dp^+}{2\pi} \frac{1}{k_t^2 - 2p^+k^- - i\epsilon} \frac{1}{p^+ - k^+ + i\epsilon} \frac{1}{p^+ - k'^+ - i\epsilon}$$
$$\times \left\{ \delta_{ij} + \frac{k_i k_j}{k^-(k^+ - i\epsilon)(k'^+ + i\epsilon)}(2p^+ - k^+ - k'^+) \right\} . \qquad \text{(A8)}$$

Finally, we can perform the integration over p^+ to find

$$\Delta_{--}(k,k') = \frac{i}{k'^+ - k^+ + i\epsilon} \frac{1}{k'^2 - i\epsilon} \left\{ \theta(k^-) \left(\delta_{ij} + \frac{k_i k_j}{k^-(k^+ - i\epsilon)(k'^+ + i\epsilon)}(k'^+ - k^+) \right) + k, k' \to -k', -k \right\}. \qquad \text{(A9)}$$

Now, to fully define this Green's function, we must specify the nature of the singularity at $k^- = 0$. In the last equation, we would like to use the Leibbrandt-Mandelstam prescription on $1/(k^- + i\epsilon/k^+)$ whenever we have the combination $1/k^-k^+$, and $1/(k^- + i\epsilon/k'^+)$ whenever we have the combination $1/k^-k'^+$. We would like to go further, however, and define $1/k^+k^-$ as $1/(k^-k^+ + i\epsilon)$. This can be done as follows: we use $1/(k^+ - i\epsilon) = 2\pi i\delta(k^+) + 1/(k^+ + i\epsilon)$ whenever we have the constraint that $k^- > 0$, and a similar modification when $k^- < 0$ for k'^+.

On the other hand, it is more difficult to implement the Leibbrandt-Mandelstam prescription in the expression which involves the integral over p^+. However, we will only use this result when both k^+ and k'^+ have the same sign and are nonzero. In this case the Leibbrandt-Mandelstam prescription is unambiguous.

Our results for the two representations are therefore

$$G^{\alpha\beta;\alpha'\beta'}_{--ij}(k,k') = -2\pi i\delta(k^- - k^{-\prime})(2\pi)^2\delta^{(2)}(k_t - k'_t)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'} \frac{1}{k'^2 - i\epsilon}$$
$$\times \left\{ \frac{1}{k'^+ - k^+ + i\epsilon} \left[\theta(k^-) \left(\delta_{ij} + \frac{k_i k_j}{k^- k^+ k'^+}(k'^+ - k^+) \right) + k, k' \to -k', -k \right] \right.$$
$$\left. + 2\pi i\delta(k^+)\theta(k^-)\frac{k_i k_j}{k^- k'^+} + k, k' \to -k', -k \right\}. \qquad \text{(A10)}$$

Here the k^+, k'^+, and k^- singularities are treated using the Leibbrandt-Mandelstam prescription.

For k^+ and k'^+ both nonzero and of the same sign, we have the representation

$$G^{\alpha\beta;\alpha'\beta'}_{--ij}(k,k') = -2\pi\delta(k^- - k^{-\prime})(2\pi)^2\delta^{(2)}(k_t - k'_t)\tau_a^{\alpha\beta}\tau_a^{\alpha'\beta'} \int \frac{dp^+}{2\pi} \frac{1}{k_t^2 - 2p^+k^- - i\epsilon} \frac{1}{p^+ - k^+ + i\epsilon} \frac{1}{p^+ - k'^+ - i\epsilon}$$
$$\times \left\{ \delta_{ij} + \frac{k_i k_j}{k^- k^+ k'^+}(2p^+ - k^+ - k'^+) \right\} , \qquad \text{(A11)}$$

where the k^- singularity is treated using the Leibbrandt-Mandelstam prescription.

The evaluation of the remaining contributions to the Green's functions can be done by the same methods as above. There is nothing really new in the analysis except that it is longer and more involved. The subtlety is in the treatment of the singularities in the $k^\pm$ and k'^+ variables. This has been discussed above and is treated as such. The results are

$$G^{\alpha\beta;\alpha'\beta'}_{++ij}(k,k') = -2\pi i\delta(k^- - k^{-\prime}) \int \frac{d^2p_t}{(2\pi)^2} F_a^{\dagger\alpha\beta}(p_t - k_t) F_a^{\dagger\alpha'\beta'}(k'_t - p_t)$$
$$\times \left\{ \frac{1}{k^+ - k'^+ + i\epsilon} \left[\left(\delta_{ij} + \frac{p_i p_j}{k^- k^+ k'^+}(k^+ - k'^+)\right) \frac{\theta(k^-)}{p_t^2 - 2k^-k^+ - i\epsilon} + k,k' \to -k'k \right] \right.$$
$$\left. + 2\pi i\delta(k'^+)\theta(k^-) \frac{p_i p_j}{k^- k^+} \frac{1}{p_t^2 - 2k^-k^+ - i\epsilon} + k,k' \to -k',-k \right\}. \tag{A12}$$

For k^+ and k'^+ both of the same sign and nonzero, the above is equivalent to

$$G^{\alpha\beta;\alpha'\beta'}_{++ij}(k,k') = -2\pi\delta(k^- - k^{-\prime}) \int \frac{dp^+ d^2p_t}{(2\pi)^3} F_a^{\dagger\alpha\beta}(p_t - k_t) F_a^{\dagger\alpha'\beta'}(k'_t - p_t)$$
$$\times \frac{1}{p_t^2 - 2p^+k^- - i\epsilon} \frac{1}{p^+ - k^+ - i\epsilon} \frac{1}{p^+ - k'^+ + i\epsilon} \left(\delta_{ij} + \frac{p_i p_j}{k^- k^+ k'^+}(2p^+ - k^+ - k'^+)\right). \tag{A13}$$

We finally also obtain an expression for G_{+-}. It turns out that in this expression, no restrictions on the values of k^+ and k'^+ are needed to get the singularities in $1/k^\pm$ or $1/k'^+$ into the Leibbrandt-Mandelstam form. The results are

$$G^{\alpha\beta;\alpha'\beta'}_{+-ij}(k,k') = 2\pi i\delta(k^- - k^{-\prime}) 2k^- \theta(k^-) \frac{1}{k'^2 - i\epsilon} \int \frac{d^2p_t}{(2\pi)^2} F_a^{\dagger\alpha\beta}(p_t - k_t)$$
$$\times F_a^{\dagger\alpha'\beta'}(k'_t - p_t) \frac{1}{p_t^2 - 2k^-k^+ - i\epsilon} \left\{ \delta_{ij} - \frac{k'_i k'_j}{k^- k'^+} - \frac{p_i p_j}{k^- k^+} + \frac{p_i k'_j p_t \cdot k'}{(k^- k^+)(k^- k'^+)} \right\}. \tag{A14}$$

We also have the equivalent integral representation where

$$G^{\alpha\beta;\alpha'\beta'}_{+-ij}(k,k') = 2\pi\delta(k^- - k^{-\prime}) \int \frac{dp^+ d^2p_t}{(2\pi)^3} F_a^{\dagger\alpha\beta}(p_t - k_t) F_a^{\dagger\alpha'\beta'}(k'_t - p_t)$$
$$\times \frac{1}{k_t^2 - 2k^-p^+ - i\epsilon} \frac{1}{q^+ - k^+ - i\epsilon} \frac{1}{p^+ - k'^+ - i\epsilon} \left\{ \delta_{ij} - \frac{k'_i k'_j}{k^- k'^+} - \frac{p_i p_j}{k^- k^+} + \frac{p_i k'_j p_t \cdot k'_t}{(k^- k^+)(k^- k'^+)} \right\}. \tag{A15}$$

In this equation, $q^+ = p^+ + \frac{p_t^2 - k'^2}{2k^-}$.

The expression for G_{-+} is

$$G^{\alpha\beta;\alpha'\beta'}_{+-ij}(k,k') = G^{\alpha\beta;\alpha'\beta'}_{-+ij}(-k',-k). \tag{A16}$$

We now want to convert these expressions for the propagator from the matrix basis to the component basis. To do this, we make the transformation

$$(U(x)\tau^c U^\dagger(x))(U(y)\tau^c U^\dagger(y)) \to 4(\mathrm{Tr}\,\tau^a U(x)\tau^c U^\dagger(x))(\mathrm{Tr}\,\tau^b U(y)\tau^c U^\dagger(y)). \tag{A17}$$

We then use the identity

$$\tau^c_{\alpha\beta}\tau^c_{\alpha'\beta'} = \frac{1}{2}\left(\delta_{\alpha\beta'}\delta_{\alpha'\beta} - \frac{1}{N_c}\delta_{\alpha\beta}\delta_{\alpha'\beta'}\right), \tag{A18}$$

which results in the following transformation for the definition of the propagator

$$F^{\dagger a}_{\alpha\beta}(x) F^{\dagger a}_{\alpha'\beta'}(y) \to 2\,\mathrm{Tr}\, F^a(x) F^b(y). \tag{A19}$$

We now want to proceed to derive formulas for the propagator which has been averaged over all values of the color charges of the valence quarks. To do this we define

$$\langle \mathrm{Tr}\, U^\dagger(x)\tau^a U(x) U^\dagger(y)\tau^b U(y)\rangle = \frac{1}{2}\Gamma^{ab}(x-y). \tag{A20}$$

Notice that

$$\Gamma^{ab}(0) = \delta^{ab}. \tag{A21}$$

We now define

$$\langle G^{ab}_{ij}(k,k')\rangle = (2\pi)\delta(k^- - k'^-)(2\pi)^2\delta(k_t - k'_t)D^{ab}_{ij}(k,k')\,. \tag{A22}$$

As before, we can write D^{ab}_{ij} as

$$D^{ab}_{ij} = D^{ab}_{--ij}(k,k') + D^{ab}_{++ij}(k,k') + D^{ab}_{+-ij}(k,k') + D^{ab}_{-+ij}(k,k')\,. \tag{A23}$$

Using the above substitutions we find that

$$D^{ab}_{--ij} = -\delta^{ab}\int\frac{dp^+}{2\pi}\,\frac{1}{k_t^2 - 2p^+k^- - i\epsilon}\,\frac{1}{p^+ - k^+ + i\epsilon}\,\frac{1}{p^+ - k'^+ - i\epsilon}\left[\delta_{ij} + \frac{k_ik_j}{k^-k^+k'^+}(2p^+ - k^+ - k'^+)\right], \tag{A24}$$

which is valid for k^+, k'^+ nonzero and $\mathrm{sgn}(k^+) = \mathrm{sgn}(k'^+)$. The expression with the p^+ integral completed is

$$D^{ab}_{--ij} = -i\delta^{ab}\left\{\theta(k^-)\frac{1}{k'^2}\left[\frac{1}{k'^+ - k^+ + i\epsilon}\left(\delta_{ij} + \frac{k_ik_j}{k^-k^+k'^+}(k'^+ - k^+)\right) + 2\pi i\delta(k^+)\frac{k_ik_j}{k^-k'^+}\right] + k,k' \to -k',-k\right\}. \tag{A25}$$

For D_{++} we obtain

$$D^{ab}_{++ij} = -\int\frac{dp^+d^2p_t}{(2\pi)^3}\Gamma^{ab}(p_t - k_t)\,\frac{1}{p_t^2 - 2p^+k^- - i\epsilon}$$
$$\times\frac{1}{p^+ - k^+ - i\epsilon}\,\frac{1}{p^+ - k'^+ + i\epsilon}\left[\delta_{ij} + \frac{p_ip_j}{k^-k^+k'^+}(2p^+ - k^+ - k'^+)\right], \tag{A26}$$

which is valid for k^+, k'^+ nonzero and $\mathrm{sgn}(k^+) = \mathrm{sgn}(k'^+)$. After completing the p^+ integration,

$$D^{ab}_{++ij} = \int\frac{d^2p_t}{(2\pi)^2}\Gamma^{ab}(p_t - k_t)\left\{\theta(k^-)\frac{1}{p_t^2 - 2k^-k^+ - i\epsilon}\left[\frac{1}{k^+ - k'^+}\left(\delta_{ij} + \frac{p_ip_j}{k^-k^+k'^+}(k^+ - k'^+)\right) + 2\pi i\delta(k'^+)\frac{p_ip_j}{k^-k^+}\right]\right.$$
$$\left. + k,k' \to -k',-k\right\}. \tag{A27}$$

We also have

$$D^{ab}_{+-ij} = \int\frac{dp^+d^2p_t}{(2\pi)^3}\Gamma^{ab}(p_t - k_t)\,\frac{1}{k_t^2 - 2k^-p^+ - i\epsilon}\,\frac{1}{q^+ - k^+ - i\epsilon}\,\frac{1}{p^+ - k'^+ + i\epsilon}$$
$$\times\left[\delta_{ij} - \frac{k_ik_j}{k^-k'^+} - \frac{p_ip_j}{k^-k^+} + \frac{p_ik_jp_t\cdot k_t}{(k^-k^+)(k^-k'^+)}\right], \tag{A28}$$

where, again, $q^+ = p^+ + \frac{p_t^2 - k_t^2}{2k^-}$. Doing the integration over p^+ gives

$$D^{ab}_{+-ij} = i2k^-\theta(k^-)\frac{1}{k'^2}\int\frac{d^2p_t}{(2\pi)^2}\,\Gamma^{ab}(p_t - k_t)\frac{1}{p_t^2 - 2k^-k^+ - i\epsilon}\left[\delta_{ij} - \frac{k_ik_j}{k^-k'^+} - \frac{p_ip_j}{k^-k^+} + \frac{p_ik_jp_t\cdot k_t}{(k^-k^+)(k^-k'^+)}\right]. \tag{A29}$$

Finally, the expression for D_{-+} is given by

$$D^{ab}_{-+ij}(k,k') = D^{ba}_{+-ji}(-k',-k)\,. \tag{A30}$$

APPENDIX B: COMPUTATION OF INTEGRALS

In this appendix, we will explicitly evaluate two of the integrals we use in the computation of the one-loop corrections to the classical background field. The first integral we will evaluate is

$$I_1 = \int\frac{d^2s_t}{(2\pi)^2}\,\frac{(S_t - s_t)^2}{[(q_t - s_t)^2(1-\xi) + (S_t - s_t)^2(1+\xi)]}. \tag{B1}$$

Let us first shift $s_t \to s_t + S_t$ and write the above as

$$I_1 = \int\frac{d^2s_t}{(2\pi)^2}\,\frac{s_t^2}{[(s_t + S_t - q_t)^2(1-\xi) + s_t^2(1+\xi)]}. \tag{B2}$$

The denominator in the integral can be written as

$$2s_t^2 + [2s_t\cdot(S_t - q_t) + (S_t - q_t)^2](1-\xi). \tag{B3}$$

Performing the change of variables

$$v_t = s_t + \frac{(1-\xi)}{2}(S_t - q_t),$$
$$d^2v_t = d^2s_t,$$
$$v_t^2 = s_t^2 + s_t\cdot(S_t - q_t)(1-\xi) + \frac{(1-\xi)^2}{4}(S_t-q_t)^2, \qquad \text{(B4)}$$

Eq. (B3) can be rewritten as

$$2\left[v_t^2 + \frac{(1-\xi^2)}{4}(S_t-q_t)^2\right] \qquad \text{(B5)}$$

and the integral (B2)

$$I_1 = \int \frac{d^2v_t}{(2\pi)^2}\frac{[v_t - \frac{(1-\xi)}{2}(S_t-q_t)]^2}{2[v_t^2 + \frac{(1-\xi^2)}{4}(S_t-q_t)^2]}. \qquad \text{(B6)}$$

Written in this form, we can drop the linear term in v_t and the contributing terms to (B6) will be

$$I_1 \to \int \frac{d^2v_t}{(2\pi)^2}\frac{v_t^2}{2\left[v_t^2 + \frac{(1-\xi^2)}{4}(S_t-q_t)^2\right]} + \int \frac{d^2v_t}{(2\pi)^2}\frac{(1-\xi)^2/4(S_t-q_t)^2}{2\left[v_t^2 + \frac{(1-\xi^2)}{4}(S_t-q_t)^2\right]}. \qquad \text{(B7)}$$

The first and second terms in expression (B7) are quadratically and logarithmically divergent. To regulate the divergences we compute them by dimensional regularization. We thus write (B7) as

$$I_1 = \frac{1}{2(2\pi)^2}\left(\frac{2\pi^2}{\Gamma(d/2)}\right)\left\{\int_0^\infty dv_t \frac{v_t^{d+1}}{[v_t^2 + \frac{(1-\xi^2)}{4}(S_t-q_t)^2]} + \int_0^\infty dv_t \frac{v_t^{d-1}(1-\xi)^2/4(S_t-q_t)^2}{[v_t^2 + \frac{(1-\xi^2)}{4}(S_t-q_t)^2]}\right\}. \qquad \text{(B8)}$$

To evaluate the integrals in the above expression we use the well-known formula

$$\int_0^\infty \frac{u^\beta du}{(u^2+C^2)^\alpha} = \frac{\Gamma(\frac{1}{2}(1+\beta))\Gamma(\alpha - \frac{1}{2}(1+\beta))}{2(C^2)^{\alpha-(1+\beta)/2}\Gamma(\alpha)} \qquad \text{(B9)}$$

by means of which (B8) becomes

$$I_1 = \left(\frac{d}{4}\right)\left(\frac{1}{2\pi}\right)^2 2\pi^{d/2}\Gamma(-d/2)\left(\frac{x}{1+x}\right)[(1-\xi^2)(S_t-q_t)^2/4]^{d/2}. \qquad \text{(B10)}$$

We are interested in the divergent part of this expression when $d \to 2$, thus for that we take $d=2$ everywhere except in the argument of Γ. We obtain finally

$$I_1 = \frac{\Gamma(-\frac{d}{2})}{16\pi}2\xi(1-\xi)(S_t-q_t)^2\ . \qquad \text{(B11)}$$

Next, we want to compute the integral

$$I_2 = \int \frac{d^2q_t}{(2\pi)^2}\mathrm{Tr}[F^a(S_t+q_t)F^b(S_t-q_t)][(S_t-q_t)^2 - (S_t+q_t)^2]. \qquad \text{(B12)}$$

For this purpose let us write $\tilde{F}$ in its explicit form in terms of U:

$$\tilde{F}^a(p_t) = \int d^2x_t e^{ip_tx_t}U^\dagger(x_t)\tau^a U(x_t), \qquad \text{(B13)}$$

and thus the integrand in (B12) can be written as

$$\int d^2x_t d^2y_t e^{i(S_t+q_t)x_t}e^{i(S_t-q_t)y_t}[(S_t-q_t)^2 - (S_t+q_t)^2]\mathrm{Tr}[U^\dagger(x_t)\tau^a U(x_t)U^\dagger(y_t)\tau^b U(y_t)]. \qquad \text{(B14)}$$

The factors $(S_t \pm q_t)^2$ are to be interpreted as derivatives acting on the corresponding exponential. Integrating by parts, ignoring the surface terms and with the help of Eq. (B14) we can write Eq. (B12) as

$$I_2 = -\int \frac{d^2q_t}{(2\pi)^2}d^2x_t d^2y_t e^{i(S_t+q_t)x_t}e^{i(S_t-q_t)y_t}\{\mathrm{Tr}[U^\dagger(x_t)\tau^a U(x_t)\nabla^2(U^\dagger(y_t)\tau^b U(y_t))]$$
$$-\mathrm{Tr}[\nabla^2(U^\dagger(x_t)\tau^a U(x_t))U^\dagger(y_t)\tau^b U(y_t)]\}. \qquad \text{(B15)}$$

Performing the q_t and y_t integrations, Eq. (B12) becomes

$$I_2 = -\int d^2x_t e^{2iS_t x_t}\left\{\mathrm{Tr}[U^\dagger(x_t)\tau^a U(x_t)\nabla^2(U^\dagger(x_t)\tau^b U(x_t))] - \mathrm{Tr}[\nabla^2(U^\dagger(x_t)\tau^a U(x_t))U^\dagger(x_t)\tau^b U(x_t)]\right\}. \quad \text{(B16)}$$

Since the gauge transformations U and the charge density are related by Eq. (6), one can prove the identity

$$\mathrm{Tr}[U^\dagger\tau^a U\nabla^2(U^\dagger\tau^b U) - \nabla^2(U^\dagger\tau^a U)U^\dagger\tau^b U] = -\frac{g^2}{2}\left(f^{abc}\rho_c(x_t) - f^{bac}\rho_c(x_t)\right). \quad \text{(B17)}$$

Using the antisymmetry of f^{abc} and plugging the above back into Eq. (B16) we finally obtain the result

$$I_2 = g^2 f^{abc}\int d^2x_t e^{2iS_t x_t}\rho_c(x_t)\,, \quad \text{(B18)}$$

which is the Fourier transform of the charge density with respect to $2S_t$.

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