Fermionic fluctuation corrections to bubble nucleation

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We determine the fermionic corrections to the nucleation rate of bubbles at the electroweak phase transition. The fermion determinant is evaluated at finite temperature, both exactly and by using the gradient expansion. The gradient expansion is found to be a reliable approximation and is used to extrapolate to the large values of $\nu_n=(2n+1)\pi T$ needed in the Matsubara sum. The contribution to the effective action is found to be negative. Only the top quark contribution is evaluated. It is smaller than the loop corrections from Higgs and W bosons and of opposite sign.

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I. INTRODUCTION

The physics of the electroweak phase transition has been discussed recently in various aspects [1]. Many subjects, as e.g., the question of baryogenesis [2,3] are still controversial [4,5]. Even the nature of the phase transition is not known at present. The temperature dependence of the effective potential has been studied in perturbation theory [6,7] as well as in lattice simulations [8,9]. If the mass of the Higgs boson is not too high (less than M_W) the phase transition is supposed to be first order [10]. In this case the transition from the symmetric vacuum with massless particles to the broken symmetry phase would proceed via bubble nucleation. This phenomenon, as well as its cosmological aspects, has been studied by various groups [11–13].

Part of the basic information needed in developing the bubble formation and expansion scenario is the determination of their nucleation rate. In the small temperature span of 1 GeV in which the phase transition takes place bubbles of various sizes can be formed. Their nucleation rate varies over several orders of magnitude. The basic rate is determined [14–16] by the exponential of the classical minimal bubble action. The semiclassical reaction rate includes, however, also preexponential factors, the fluctuation determinants, determined by the fluctuation of W boson, Higgs and fermion fields in the background of the minimal bubble profile. The bosonic fluctuations have been computed recently [17,18] and found to yield sizable suppression factors, the one-loop effective action (or equivalently the free energy divided by the temperature) being of the same order as the classical bubble action. In the high temperature theory, obtained by retaining only the Matsubara frequency 0, fermions do not contribute. However, recent determinations of the fermionic contribution to the sphaleron rate [19] let us expect that at least the top quark will influence the transition rate in an essential way. Of course we have to perform such a computation in the four-dimensional finite temperature quantum field theory, i.e., by summing over all Matsubara frequencies.

The plan of this paper is as follows. In the next section we will introduce the model and set up the basic relations for the bubble nucleation rate. The fermion determinant is defined in Sec. III. In Sec. IV we will discuss the renormalization of the leading terms, of first and second order in the external field vertex. Part of their contribution is contained already in the effective potential and should not be included again. This renders the discussion of these terms rather lengthy, the details of the calculation are given in the Appendix, therefore. In Sec. V we present the computation of the finite higher order contributions to fermionic effective action. The results of an exact numerical computation are compared with an analytic approximation based on the gradient expansion. The latter one is used, then, to obtain the actual results which are presented and discussed in Sec. VI.

II. BASIC RELATIONS

The classical action of the standard model has no solutions which describe the nucleation of bubbles. We have to include finite temperature one-loop effects in order to have a model which displays a first order phase transition and therefore bubble nucleation. The way in which this has to be done is still not established at present; various approaches are being discussed. We will use here an action obtained in the electroweak theory by evaluating at finite temperature the one-loop effective potential in the manner of Coleman and Weinberg [20]. This has been computed by various authors [21–24]; here we use the formulation of Dine *et al.* [13]. We will call this action which consists of the sum of classical action and the integral over the one-loop effective potential the ‘‘basic action.’’ This basic action displays then a first order phase transition, has bubble solutions, and will be used to compute the bubble profile.

Our aim here is to compute the fermionic part $S^F_{\rm eff}[\Phi,T]$ of the full finite temperature fermionic one-loop effective action, i.e., the logarithm of the fermion determinant in the external Higgs field configuration Φ of the bubble. Part of the fermionic effective action, the fermion contribution to the effective *potential*, is already contained in the basic action and has to be removed from $S^F_{\rm eff}$ in order to avoid double counting. While the one-loop effective potential takes into account only constant external fields, the one-loop effective action computed here will take into account the full (spherical) space dependence of the Higgs field. We will denote the fermionic correction to the basic action as $\Delta S^F_{\rm eff}$; it is ob-

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tained as the difference of the full one loop effective action and fermion contribution to the effective potential, both evaluated in the Higgs field configuration of the bubble. The same procedure should of course be performed for the gauge and Higgs field fluctuations as well in a complete computation.

We will introduce an approximation here which was found to be a very good one in [13]: we will replace in the basic action the finite temperature effective potential by its high temperature approximation. Since the fermionic contribution to the one loop effective potential will be subtracted at the end anyway in $\Delta S_{\rm eff}^F$ (and so would be the gauge field and bosonic part in a complete computation of one-loop corrections) this approximation will affect the total effective action, the sum of basic action and $\Delta S_{\rm eff}^F$, only indirectly through slightly modified bubble profiles.

The "basic" (in the sense discussed above) finite temperature action of the Higgs field from which the bubble profile is determined is then given by

$$S_{\rm FT}=\int_0^\beta d\tau\int d^3x\left[\frac{1}{2}(\partial_\mu\mathbf{\Phi})^\dagger(\partial_\mu\mathbf{\Phi})+V_{\rm HT}(\mathbf{\Phi}^\dagger\mathbf{\Phi})\right]. \tag{2.1}$$

$\mathbf{\Phi}$ is the complex doublet of Higgs fields. Here this field will always occur as a background field describing the minimal bubble. It can be parametrized then as

$$\mathbf{\Phi}(\vec{x})=v_0\Phi(\vec{x})\begin{pmatrix}0\\1\end{pmatrix}. \tag{2.2}$$

$V_{\rm HT}$ is the high temperature potential which includes the classical potential and the one-loop effective potential in the high temperature approximation:

$$V_{\rm HT}(\mathbf{\Phi}^\dagger\mathbf{\Phi})=D(T^2-T_0^2)\mathbf{\Phi}^\dagger\mathbf{\Phi}-ET(\mathbf{\Phi}^\dagger\mathbf{\Phi})^{3/2}+\frac{\lambda_T}{4}(\mathbf{\Phi}^\dagger\mathbf{\Phi})^2. \tag{2.3}$$

Its parameters are given, for $\Theta_w=0$, by

$$D=(3m_W^2+2m_t^2)/8v_0^2,$$

$$E=3g^3/32\pi,$$

$$B=3(3m_W^4-4m_t^4)/64\pi^2v_0^4,$$

$$T_0^2=(m_H^2-8v_0^2B)/4D, \tag{2.4}$$

$$\lambda_T=\lambda-3\left(3m_W^4\ln\frac{m_w^2}{a_BT^2}-4m_t^4\ln\frac{m_t^2}{a_FT^2}\right)\bigg/16\pi^2v_0^4, \tag{2.5}$$

with $\ln a_B=2\ln 4\pi-2\gamma$ and $\ln a_F=2\ln\pi-2\gamma$.

For $T>T_0$ the potential has a minimum at $|\mathbf{\Phi}|=0$ corresponding to the symmetric phase and a second minimum at

$$|\mathbf{\Phi}|=\tilde{v}(T)=\frac{3ET}{2\lambda}+\sqrt{\left(\frac{3ET}{2\lambda}\right)^2+v^2(T)}, \tag{2.6}$$

where

$$v^2(t)=\frac{2D}{\lambda_T}(T_0^2-T^2). \tag{2.7}$$

This minimum is degenerate with the one at $\mathbf{\Phi}=0$ at a temperature defined implicitly by

$$T_C=T_0/\sqrt{1-E^2/D\lambda_{T_C}}. \tag{2.8}$$

T_C marks the onset of bubble formation by thermal barrier transition.

The bare fermionic Euclidean action S_F, from which the fermion determinant is derived by integrating out the fermion field in the semiclassical approximation, can be written, for vanishing gauge fields and for Higgs field configurations of the form (2.2), in terms of four component Dirac spinors as

$$S_F=\int_0^\beta d\tau\int d^3x\left[\sum_f(\bar{\psi}^f\gamma_\mu\partial_\mu\psi^f)-\sum_{ff'}g_Y^{ff'}v_0\Phi\bar{\psi}^f\psi^{f'}\right], \tag{2.9}$$

where the Yukawa couplings are related to the fermion mass matrix via $g_Y^{ff'}v_0=m^{ff'}$. The sum over f is over flavors and colors. Here we will consider only the contribution of the top quark. This restriction applies already to the high temperature action given above, for the reason that its contribution is much larger than the one of lighter quark and lepton fields. This will also be the case for the exact one-loop action.

The process of bubble nucleation is, within the approach of Langer [14] and Coleman and Callan [15,16], followed by the work of Affleck [25], Linde [26] and others, described by the rate

$$\Gamma/V=\frac{\omega_-}{2\pi}\left(\frac{\tilde{S}}{2\pi}\right)^{3/2}\exp(-\tilde{S})\left(\frac{\mathcal{J}_F}{\mathcal{J}_B}\right)^{1/2}. \tag{2.10}$$

Here $\tilde{S}$ is the high-temperature action, Eq. (2.1), with the new rescaling, minimized by a classical τ independent minimal bubble configuration (see below). $\mathcal{J}_{F/B}$ are the fermionic and bosonic fluctuation determinants which describe the next-to-leading part of the semiclassical approach. $\mathcal{J}_F$, whose computation is the aim of this work, will be defined below; its logarithm is related to the fermionic one-loop effective action by

$$S_{\rm eff}^F=-\frac{1}{2}\ln\mathcal{J}_F. \tag{2.11}$$

Finally ω_- is the absolute value of the unstable mode frequency.

The classical bubble configuration is described by a vanishing gauge field and a real τ independent spherically symmetric Higgs field. For the bubble configuration we make the ansatz

$$\mathbf{\Phi}(\vec{x})=\tilde{v}(T)\phi(r)\begin{pmatrix}0\\1\end{pmatrix}, \tag{2.12}$$

where we have rescaled Eq. (2.2) via $v_0\Phi(r)=\tilde{v}(T)\phi(r)$. In all our numerical computations we will use the scale $[g_w\tilde{v}(T)]^{-1}$ for the coordinates. Defining the rescaled high temperature potential as

$$\tilde{V}_{\rm HT}=\frac{\lambda_T}{4g^2}\left[\phi^4-\epsilon\phi^3+\left(\frac{3}{2}\epsilon-2\right)\phi^2\right], \tag{2.13}$$

with

$$\epsilon=\frac{4ET}{\lambda_T\tilde{v}(T)}=\frac{4}{3}\left(1-\frac{v^2(T)}{\tilde{v}(T)^2}\right) \tag{2.14}$$

and the high temperature coupling

$$\tilde{g}_3^2(T)=\frac{g_wT}{\tilde{v}(T)}, \tag{2.15}$$

the bubble action is given by

$$\tilde{S}=\frac{4\pi}{\tilde{g}_3^2(T)}\int_0^\infty r^2dr\left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2+\tilde{V}_{\rm HT}(\phi)\right]. \tag{2.16}$$

It is minimized if $\phi(r)$ is a solution of the associated Euler-Lagrange equation

$$-\phi''(r)-\frac{2}{r}\phi'(r)+\frac{dV_{\rm HT}}{d\phi(r)}=0 \tag{2.17}$$

with the boundary conditions

$$\lim_{r\to\infty}\phi(r)=0 \quad \text{and} \quad \phi'(0)=0. \tag{2.18}$$

The bubble configuration varies from small thick wall bubbles to large thin wall bubbles in the narrow range ($\simeq 1$ GeV) between T_0 and T_C, both of order 100 GeV. We will use the variable

$$y=3(1-\epsilon/2), \quad 0<y<1 \tag{2.19}$$

instead of T to parametrize this range of temperatures.

III. THE FERMIONIC FLUCTUATION DETERMINANT

The bare fermionic Euclidean action S_F of quarks can be rewritten in four component Dirac notation and for time-independent background configurations as

$$S_F=\int_0^\beta d\tau\int d^3x\,\bar{\psi}[\gamma_\mu\partial_\mu-m(\vec{x})]\psi \tag{3.1}$$

$$=\int_0^\beta d\tau\int d^3x\,\psi^\dagger(\partial/\partial\tau-H)\psi, \tag{3.2}$$

where $m(\vec{x})=g_Y\tilde{v}(T)\phi(r)$ and

$$H=\gamma^0[-i\vec{\gamma}\cdot\vec{\nabla}+m(\vec{x})]. \tag{3.3}$$

The field fluctuations are subject to antiperiodic boundary conditions at $\tau=0$ and $\tau=\beta$, i.e.,

$$\psi(\vec{x},\beta)=-\psi(\vec{x},0) \tag{3.4}$$

which determines their frequencies to be the Matsubara frequencies $\nu_n=(2n+1)\pi T$ with integer n, $-\infty<n<\infty$. Integrating out the fermion field leads to the fermionic prefactor

$$\mathscr{J}_F^{1/2}=\frac{\Pi_{n\alpha}(i\nu_n+\omega_\alpha)}{\Pi_{n,\alpha}(i\nu_n+\omega_\alpha^0)}, \tag{3.5}$$

where ω_α denotes the eigenvalues of H. Using the fact that these eigenvalues occur in pairs $\pm\omega_\alpha$ we can rewrite this as

$$\mathscr{J}_F=\left(\frac{\Pi_{n\alpha}(\nu_n^2+\omega_\alpha^2)}{\Pi_{n,\alpha}[\nu_n^2+(\omega_\alpha^0)^2]}\right). \tag{3.6}$$

The fermion contribution to the effective action is therefore given by

$$S_{\rm eff}^F=-\frac{1}{2}\ln\mathscr{J}_F=-\frac{1}{2}\sum_{n=-\infty}^{\infty}\ln\det\left(\frac{\nu_n^2+\mathscr{M}}{\nu_n^2+\mathscr{M}^0}\right). \tag{3.7}$$

Here $\nu_n=(2n+1)\pi T$, the fluctuation operators $\mathscr{M}$ and $\mathscr{M}^0$ are defined as

$$\mathscr{M}=H^2=-\Delta+\mathscr{V}(\vec{x}),$$

$$\mathscr{M}^0=(H^0)^2=-\Delta, \tag{3.8}$$

and

$$\mathscr{V}=\begin{Bmatrix} m^2(\vec{x}) & -i\vec{\sigma}\vec{\nabla}m(\vec{x}) \\ i\vec{\sigma}\vec{\nabla}m(\vec{x}) & m^2(\vec{x}) \end{Bmatrix}. \tag{3.9}$$

A method for computing such fluctuation determinants numerically has been described recently [27] (see also [28–31] for earlier applications). Before we discuss the numerical part of the computation we have to ensure that the quantities we are going to compute are finite. The effective action as defined formally in Eq. (3.7) is divergent. This is easily seen be expanding it with respect to the potential $\mathscr{V}$:

$$S_{\rm eff}^F=\sum_{k=1}^\infty\frac{(-1)^k}{2k}\sum_{n=-\infty}^\infty {\rm Tr}\,[(-\Delta+\nu_n^2)^{-1}\mathscr{V}]^k=\sum_{k=1}^\infty S_{\rm eff}^{(k)}. \tag{3.10}$$

Here $(-\Delta+\nu_n^2)^{-1}$ is a formal representation of the Green function associated to that operator. The superscript (k) denotes the power of the potential $\mathscr{V}$ or equivalently the order of the associated Feynman graph. The series of Feynman graphs corresponding to (3.10) is depicted in Fig. 1. The first and second order graphs are divergent. We will see that our numerical method allows us to separate these two graphs from the remaining series:

$$S_{\rm eff}^{\overline{(3)}}=\sum_{k=3}^\infty S_{\rm eff}^{(k)} \tag{3.11}$$

which can be computed exactly and is finite. The divergences of the two leading graphs are obviously those of ordinary perturbation theory. Their divergent parts can be cancelled by the counterterms of the $T=0$ theory. This will be described in the next section, following the work of Refs. [32] and [13].

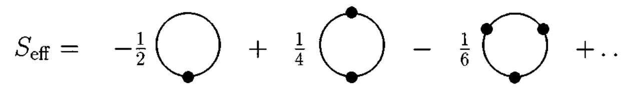

FIG. 1. The loop expansion of the effective action. The lines represent the propagators and the dots indicate the vertices $V(x)$.

IV. RENORMALIZATION

As usual in finite temperature field theory the renormalization of divergent quantities is done at zero temperature. The effective potential is renormalized in such a way that the vacuum expectation value v_0 and the Higgs boson mass, i.e., the second derivative at the broken symmetry minimum, are kept at their physical values; these are the renormalization conditions usually adopted. This fixes the counterterms in the effective action. On the other hand, we need the effective action which is expanded around the symmetric vacuum defined by $\mathbf{\Phi}=0$. The discussion of renormalization, especially at finite temperature, is therefore somewhat cumbersome.

In order to cover both kinds of expansion we rewrite the effective action as

$$S_{\rm eff}=-\frac{1}{2}\ln\mathcal{J}=-\frac{1}{2}\sum_{n=-\infty}^{+\infty}\ln\left(\frac{\det(\nu_n^2-\Delta+\mu_F^2+\mathcal{U})}{\det(\nu_n^2-\Delta+\mu_F^2)}\right), \quad (4.1)$$

where now $\mu_F=m_F=g_Y v_0$ if one expands around the $T=0$ broken symmetry vacuum and $\mu_F=0$ if one expands around the high temperature symmetric phase. The 4×4 matrix $\mathcal{U}$ is then given by

$$\mathcal{U}=\mathcal{V}-\mu_F^2=\begin{Bmatrix} m_F^2\Phi^2-\mu_F^2 & -im_F\vec{\sigma}\vec{\nabla}\Phi \\ im_F\vec{\sigma}\vec{\nabla}\Phi & m_F^2\Phi^2-\mu_F^2 \end{Bmatrix}. \quad (4.2)$$

The field Φ is normalized in such a way that it takes the value 1 if the Higgs field is at its $T=0$ vacuum expectation value.

As discussed in Sec. III we get the series of one-loop Feynman graphs depicted in Fig. 1 when we expand the logarithm in (4.1). The first two terms of this expansion, which have been labeled $S_{\rm eff}^{(1)}$ and $S_{\rm eff}^{(2)}$, are divergent and so must be renormalized. Therefore we calculate these quantities first at zero temperature with massive propagators and use the proposed renormalization conditions to fix the counterterms. Then $S_{\rm eff}^{(1)}$ and $S_{\rm eff}^{(2)}$ are calculated at finite temperature with massless propagators—i.e., in the symmetric vacuum. They can be splitted up into a zero temperature part whose divergences are canceled by the counterterms and a temperature-dependent part. In performing the calculation of $S_{\rm eff}^{(1)}$ and $S_{\rm eff}^{(2)}$ in this way one finds contributions that are already taken into account in the high temperature effective potential $V_{\rm HT}$ of Eq. (2.3). These terms must be subtracted in order to avoid double counting; we call the resulting difference $\Delta S_{\rm ren}^{(1+2)}$.

After this procedure, which is presented in detail in the Appendix, we get the following contribution to the renormalized first and second order to the finite temperature one-loop effective action:

$$\Delta S_{\rm ren}^{(1+2)}(\Phi,T)=-3\beta\frac{m_t^4}{16\pi^2}\int\frac{d^3q}{(2\pi)^3}\left(|\widetilde{\Phi^2}(q)|^2+\frac{q^2}{m_t^2}|\tilde{\Phi}(q)|^2\right)\left[\ln\left(\frac{q^2}{m_t^2}\right)-2+4\int_0^1 d\alpha\int_0^\infty\frac{dp}{E_\alpha}\frac{1}{\exp(E_\alpha/T)+1}\right]$$
$$-3\beta\frac{m_t^4}{16\pi^2}\ln\left(\frac{m_t^2}{a_F T^2}\right)\int\frac{d^3q}{(2\pi)^3}|\widetilde{\Phi^2}(q)|^2. \quad (4.3)$$

As the evalution of the renormalization parts of first and second order in $\mathcal{V}$ is completely independent of the computation of the finite part $S_{\rm eff}^{(3)}$ one may as well use different schemes of renormalization. A particular one, based on the modified minimal subtraction ($\overline{\rm MS}$) scheme has been used in [7]. If we use the $\overline{\rm MS}$ scheme we find an expression that differs from (4.3) in two places: (i) $\ln(q^2/m_t^2)$ is replaced by $\ln(q^2/\mu^2)$ where μ is the scale introduced by dimensional regularization and (ii) $\ln(m_t^2/a_F T^2)$ is replaced by $\ln(\mu^2/a_F T^2)$.

The latter modification arises because in the high temperature potential used in [7] the logarithmic corrections due to boson loops are proportional to $L_s=\ln(\mu^2/a_B T^2)$ instead of $\ln(m_W^2/a_B T^2)$ here [cf. (2.5)]. The fermionic ones, not considered in [7], would similarly contain μ instead of m_F. Of course, if the $\overline{\rm MS}$ scheme is used the couplings and the masses have to be replaced by couplings and masses defined for this scale μ. The authors of Ref. [7] use the invariance under the renormalization scale to minimize the logarithmic corrections by a special choice of μ. The fact that our first and second order corrections turn out to be small indicates that the choice $\mu\simeq m_t$ is already almost optimal. Indeed the fermionic logarithm $\ln(\mu^2/a_F T^2)$ vanishes for $\mu=\pi\exp(-\gamma)T\simeq1.76T$. For temperatures around 100 GeV this is indeed near to m_t. In [7] the scale was chosen in such a way that the bosonic logarithm L_s vanishes, requiring $\mu\simeq7T$. With such a choice our results for $\Delta S_{\rm ren}^{(1+2)}$ change appreciably. The values are given in the tables. Since both scales $\mu_F\simeq1.76T$ and $\mu_B\simeq7T$ are related to the temperature as a common scale, one could think of choosing the two scales μ_B and μ_F for the bosonic and fermionic loops, respectively. A complete discussion of such a scheme would be beyond the scope of this paper.

V. CALCULATION OF THE FINITE PART OF THE EFFECTIVE ACTION

What remains to be evaluated is the sum of all Feynman graphs of order 3 and higher. This contribution is finite, we denote it as $S_{\rm eff}^{\overline{(3)}}(\phi)$. We present an exact numerical computation, using a general theorem on functional determinants [33] and an analytic approximation based on the gradient expansion. The results of the two methods will be compared at the end of this section.

A. Numerical computation

As mentioned above the fermionic one-loop effective action at finite temperature can be written, including the color factor 3, as

$$S_{\rm eff}=-\frac{3}{2}\sum_{n=-\infty}^{\infty}\ln\det\left(\frac{-\boldsymbol{\Delta}+\nu_n^2+\mathscr{V}}{-\boldsymbol{\Delta}+\nu_n^2}\right)=-3\sum_{n=0}^{\infty}\ln\mathscr{J}(\nu_n). \tag{5.1}$$

As the background field is spherically symmetric the determinant can be decomposed into its partial wave contributions. This is readily done by introducing the usual spinors for given j and $l=j\pm 1/2$ as given in the textbook of Bjorken and Drell [34]. One finds then

$$S_{\rm eff}=-\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}6(2l+1)\ln\mathscr{J}_l(\nu_n). \tag{5.2}$$

Here the the partial wave determinants $\mathscr{J}_l(\nu)$ are defined as

$$\mathscr{J}_l(\nu)=\det\left(\frac{\mathbf{M}_l+\nu^2}{\mathbf{M}_l^0+\nu^2}\right)$$

with the partial wave fluctuation operators

$$\mathbf{M}_l=\mathbf{M}_l^0+\mathbf{V}(r),$$

$$\mathbf{M}_l^0=-\mathbf{1}\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)+\frac{1}{r^2}\begin{Bmatrix} l(l+1) & 0\\ 0 & (l+1)(l+2)\end{Bmatrix}, \tag{5.3}$$

$$\mathbf{V}(r)=\begin{Bmatrix} m^2(r) & dm(r)/dr\\ dm(r)/dr & m^2(r)\end{Bmatrix}. \tag{5.4}$$

A very fast method for computing fluctuation determinants is based on a theorem on functional determinants [33] which can be generalized to a coupled $(n\times n)$ system.

Let $\mathbf{f}(\nu,r)$ and $\mathbf{f}^0(\nu,r)$ denote the $(n\times n)$ matrices formed by n linearly independent solutions $f_i^\alpha(\nu,r)$ and $f_i^{\alpha 0}(\nu,r)$ of

$$(\mathbf{M}_l+\nu^2)_{ij}f_j^\alpha(\nu,r)=0 \tag{5.5}$$

and

$$(\mathbf{M}_l^0+\nu^2)_{ij}f_j^{\alpha 0}(\nu,r)=0, \tag{5.6}$$

respectively, with regular boundary conditions at $r=0$. Here the latin lower index denotes the n components while the different solutions are labeled by the Greek upper index. Let these solutions be normalized in such a way that

$$\lim_{r\to 0}\mathbf{f}(\nu,r)[\mathbf{f}^0(\nu,r)]^{-1}=\mathbf{1}. \tag{5.7}$$

Then the statement of the theorem is

$$\mathscr{J}_l(\nu)\equiv\frac{\det(\mathbf{M}_l+\nu^2)}{\det(\mathbf{M}_l^0+\nu^2)}=\lim_{r\to\infty}\frac{\det\mathbf{f}(\nu,r)}{\det\mathbf{f}^0(\nu,r)}. \tag{5.8}$$

Here the determinants on the left-hand side are determinants in functional space while those on the right-hand side are ordinary determinants of the $n\times n$ matrices defined above. The theorem already has been applied to the calculation of the one-loop effective action of a single scalar field on a bubble background in [27,18] and of a fermion system at temperature $T=0$ on a similar background in [35] which we refer to for more technical details.

In the numerical application the solutions f_k^α are written as [36]

$$f_k^\alpha(\nu,r)=[\delta_k^\alpha+h_k^\alpha(\nu,r)]i_{l_k}(\nu r) \tag{5.9}$$

with the boundary condition $h_k^\alpha(\nu,r)\to 0$ as $r\to 0$. Of course the value l_k depends on the channel. This way one generates a set of linearly independent solutions which near $r=0$ behave like the free solution as required by the theorem which then takes the form

$$\mathscr{J}(\nu)=\lim_{r\to\infty}\det[\delta_k^\alpha+h_k^\alpha(\nu,r)]. \tag{5.10}$$

The functions $h_k^\alpha(\nu,r)$ satisfy the differential equation

$$\frac{d^2}{dr^2}h_k^\alpha(\nu,r)+2\left(\frac{1}{r}+\nu\frac{i'_{l_k}(\nu r)}{i_{l_k}(\nu r)}\right)\frac{d}{dr}h_k^\alpha(\nu,r)$$
$$=V_{kk'}(r)[\delta_{k'}^\alpha+h_{k'}^\alpha(\nu,r)]\frac{i_{l_{k'}}(\nu r)}{i_{l_k}(\nu r)}. \tag{5.11}$$

This equation can easily be used for generating the functions h_k^α order by order in the potential V. Introducing the contribution of order k in the potential as $\mathbf{h}^{(k)}$ and defining $\mathbf{h}^{\overline{(k)}}$ via

$$\mathbf{h}^{\overline{(k)}}\equiv\sum_{j=k}^{\infty}\mathbf{h}^{(j)}$$

as in [35], the relevant contribution $S_{\rm eff}^{\overline{(3)}}$ is found to be

$$S_{\rm eff}^{\overline{(3)}}=-\sum_{n=0}^{\infty}\mathscr{K}^{\overline{(3)}}(\nu_n), \tag{5.12}$$

where

$$\overline{\mathcal{K}^{(3)}}(\nu)=\sum_{l=0}^{\infty} 6(2l+1)\lim_{r\to\infty}\{\ln\det[\mathbf{1}+\overline{\mathbf{h}^{(1)}}(\nu,r)] - \mathrm{tr}(\mathbf{h}^{(1)}(\nu,r)+\mathbf{h}^{(2)}(\nu,r)-\tfrac{1}{2}[\mathbf{h}^{(1)}(\nu,r)]^2)\}. \tag{5.13}$$

The expression $\overline{\mathcal{K}^{(3)}}(\nu)$ was evaluated by summing the partial waves computed numerically up to $l_{\max}=30$ extrapolated for higher values of l using an ansatz $al^{-5}+bl^{-6}+cl^{-7}$. The asymptotic behavior is supposed to set in at values of $l\gg\nu R$ where R is the typical radius of the bubble. Since R has typical values of 20–40, this means that for our $l_{\max}$ the extrapolation becomes unreliable already for ν of order 1. We will discuss this point again below.

B. Analytic approximation using the gradient expansion

In [35] an approximation of the gradient expansion type has been given for the one-loop effective action at zero temperature for the case of a massive fermion with Yukawa coupling to an external scalar field. The calculation to be done here is similar because the Matsubara frequency ν_n enters in the same way as a mass term and so the structure of the determinant is of the same type.

The logarithm of the determinant $\mathcal{J}(\nu)$ can be written exactly as

$$\ln\mathcal{J}(\nu)=\mathrm{Tr}\,\ln(\mathbf{1}+\mathbf{G_0}\mathcal{V}), \tag{5.14}$$

where the free Green function $\mathbf{G_0}$ is defined by

$$(\mathbf{M}+\nu^2)\mathbf{G_0}=\mathbf{1}. \tag{5.15}$$

Defining the Fourier transform of the potential $\mathcal{V}$ as

$$\tilde{\mathcal{V}}(\mathbf{q})=\int d^3x\,\mathcal{V}(\mathbf{x})\exp(-i\mathbf{qx}) \tag{5.16}$$

the fluctuation determinant can be expanded as

$$\ln\mathcal{J}(\nu)=\sum_{k=1}^{\infty}\mathrm{tr}\frac{(-1)^{k+1}}{k}\int\frac{d^3p}{(2\pi)^3}\prod_{j=1}^{k}\int\frac{d^3q_j}{(2\pi)^3}\frac{\tilde{\mathcal{V}}(\mathbf{q}_j)}{(\mathbf{p}+\mathbf{Q}_j)^2+\nu^2}(2\pi)^3\delta^{(3)}(\mathbf{Q}_k), \tag{5.17}$$

where

$$\mathbf{Q}_j=\sum_{l}^{j}\mathbf{q}_l\,.$$

Expanding the denominators including terms up to order ν^{-2k-4} we get

$$\prod_{j=1}^{k}\frac{1}{(\mathbf{p}+\mathbf{Q}_j)^2+\nu^2}\simeq\frac{1}{(\mathbf{p}^2+\nu^2)^k}\left[1-\sum_{l=1}^{k}\frac{\mathbf{Q}_l^2}{(\mathbf{p}^2+\nu^2)}+\frac{4}{3}\frac{\mathbf{p}^2}{(\mathbf{p}^2+\nu^2)^2}\left(\sum_{l>l'}\mathbf{Q}_l\mathbf{Q}_{l'}+\sum_{l=1}^{k}\mathbf{Q}_l^2\right)\right]. \tag{5.18}$$

As the potential is $\mathcal{V}=m_F^2\Phi^2+m_F\gamma\nabla\Phi$ we have

$$\mathrm{tr}\prod_{j=1}^{k}\tilde{\mathcal{V}}(\mathbf{q}_j)\simeq 4m_F^{2k}\left[\prod_{j=1}^{k}\widetilde{\Phi^2}(\mathbf{q}_j)-\frac{1}{m_F^2}\sum_{l>l'}\mathbf{q}_l\mathbf{q}_{l'}\tilde{\Phi}(\mathbf{q}_l)\tilde{\Phi}(\mathbf{q}_{l'})\prod_{j\neq l,l'}\widetilde{\Phi^2}(\mathbf{q}_j)\right]. \tag{5.19}$$

After inserting these expansions in (5.17) and transforming back to x space the remaining integrations (except one space integration) and the k summation can be done. Of course we have to omit those terms which are divergent. As we are working in four dimensions these are the terms with $k=1$ and $k=2$, they have been discussed in the previous section.

Our final result written in the form of $\mathcal{K}(\nu)=3\,\ln\mathcal{J}(\nu)$ is

$$\overline{\mathcal{K}^{(3)}}(\nu)\simeq\frac{3\nu^3}{2\pi}\int d^3x\left\{-\frac{4}{3}\left[\left(1+\frac{m_F^2\Phi^2}{\nu^2}\right)^{3/2}-1-\frac{3}{2}\frac{m_F^2\Phi^2}{\nu^2}-\frac{3}{8}\frac{m_F^4\Phi^4}{\nu^4}\right]+\frac{m_F^2(\nabla\Phi)^2}{2\nu^4}\left[\left(1-\left(1+\frac{m_F^2\Phi^2}{\nu^2}\right)^{-1/2}\right)-\frac{m_F^2\Phi^2}{3\nu^2}\left(1-\left(1+\frac{m_F^2\Phi^2}{\nu^2}\right)^{-3/2}\right)\right]\right\}. \tag{5.20}$$

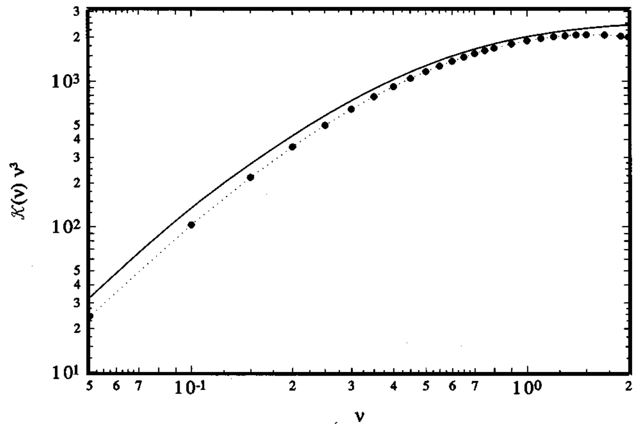

FIG. 2. Results of the numerical computation compared to the analytic approximation for $y=0.3$, $m_H=60$ GeV, and $m_t=170$ GeV. The dots interpolated by a dotted line represent the numerical results; the solid line is the analytic approximation of Eq. (5.13).

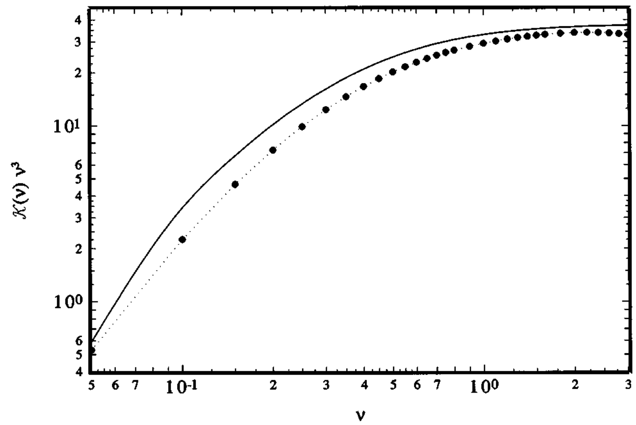

FIG. 3. The same as Fig. 2 for $y=0.6$.

This expression has to be evaluated using for Φ the numerical bubble profiles. The accuracy is only limited by the accuracy of these profiles and by that of the numerical integration. With our numerical precision the results are reliable to at least six significant digits.

The approximate results for $\overline{\mathcal{K}^{(3)}}$ can be compared with the exact numerical ones computed using (5.13). For the purpose of comparison we treat ν as a continuous parameter. We display the exact and approximate results in Figs. 2 and 3 for two typical bubble profiles, a small bubble with $y=0.6$ and a large bubble corresponding to $y=0.3$ [see (2.19) for the definition of y]. The analytic approximation is seen to describe-the trend of the exact results over the whole range. The gradient expansion is expected to converge at large ν. This expectation is substantiated by the exact numerical results in the region where they are reliable. It is seen, however, that the exact results start dropping off at values of $\nu\simeq 1$–2; as mentioned above this is related to the fact that the convergence of the partial wave summation becomes poorer with increasing ν. Since the values of ν relevant for the Matsubara frequency summation (5.12) are $\nu \geqslant \pi T \approx 9$ (in our units $g\tilde{v}$) we have to rely on the gradient expansion in computing the finite temperature effective action.

VI. RESULTS

We have computed the finite temperature fermionic effective action for Higgs boson masses of 60,70, and 80 GeV and for top quark masses $m_t=160$,170, and 180 GeV. These results are given in Tables I–III. As mentioned in Sec. II we have considered only the contribution of the top quark since lighter quarks and leptons will give negligible contributions. Their contribution has already been dismissed in the basic high temperature action (2.1) and including them would be inconsistent.

For each set of mass parameters we have determined the bubble profiles for various values of the variable y defined in (2.19); y determines the temperature and the bubble action. We give separately the renormalized first and second order contributions $\Delta S_{\rm ren}^{(1+2)}$, Eq. (4.3) determined in Sec. IV,

$$\Delta S_{\rm ren}^{(1+2)}(\Phi,T) = -3\beta\frac{m_t^4}{16\pi^2}\int\frac{d^3q}{(2\pi)^3}\left(|\widetilde{\Phi^2}(q)|^2+\frac{q^2}{m_t^2}|\tilde{\Phi}(q)|^2\right)\left[\ln\left(\frac{q^2}{m_t^2}\right)-2+4\int_0^1 d\alpha\int_0^\infty\frac{dp}{E_\alpha}\frac{1}{\exp(E_\alpha/T)+1}\right]$$
$$-3\beta\frac{m_t^4}{16\pi^2}\ln\left(\frac{m_t^2}{a_F T^2}\right)\int\frac{d^3q}{(2\pi)^3}|\widetilde{\Phi^2}(q)|^2, \tag{6.1}$$

and the finite sum of all higher order contributions $\overline{S_{\rm eff}^{(3)}}$ whose computation was discussed in the previous section. It is given by the analytic expression Eq. (5.20), inserted into the Matsubara sum (5.12):

$$\overline{S_{\rm eff}^{(3)}} = -\sum_{n=0}^{\infty}\frac{3\nu_n^3}{2\pi}\int d^3x\left\{-\frac{4}{3}\left[\left(1+\frac{m_F^2\Phi^2}{\nu_n^2}\right)^{3/2}-1-\frac{3}{2}\frac{m_F^2\Phi^2}{\nu_n^2}-\frac{3}{8}\frac{m_F^4\Phi^4}{\nu_n^4}\right]\right.$$
$$\left.+\frac{m_F^2(\nabla\Phi)^2}{2\nu_n^4}\left[\left(1-\left(1+\frac{m_F^2\Phi^2}{\nu_n^2}\right)^{-1/2}\right)-\frac{m_F^2\Phi^2}{3\nu_n^2}\left(1-\left(1+\frac{m_F^2\Phi^2}{\nu_n^2}\right)^{-3/2}\right)\right]\right\}. \tag{6.2}$$

The total one-loop effective action, reduced by the terms included already in the ‘‘basic’’ effective potential (2.3) is given, of course, by the sum

$$\Delta S_{\rm eff}^{F}=\Delta S_{\rm ren}^{(1+2)}+\overline{S_{\rm eff}^{(3)}}, \tag{6.3}$$

which is exactly the quantity that was introduced the first time in Sec. II. The numerical values presented in Tables I–III show that the fermion determinant $\Delta S_{\rm eff}^{F}$ gives a negative contribution to the effective action, which means that bubble nucleation is enhanced by this contribution [cf. Eqs. (2.10) and (2.11)]. Relative to the tree level action $\tilde{S}$ this suppression is not significant, thus supporting the semiclassical approach. Of course this is only true because a substantial part of the fermion free energy was included already in the high temperature action.

The situation changes appreciably if one uses the $\overline{\rm MS}$ scheme with the renormalization scale $\mu=4\pi\exp(-\gamma)T\simeq 7T$ as proposed in [7] in order to suppress large logarithmic corrections from boson loops. The results for this choice are also given in the tables in the column denoted as $\Delta S_{\rm ren}^{(1+2,\overline{\rm MS})}$. Of course these authors have considered only the boson sector of the SU(2) gauge theory. If one chooses a scale $\mu_F=\pi\exp(-\gamma)T\simeq 1.76T$ in order to suppress large contributions due to fermion loops then one obtains essentially our original result $\Delta S_{\rm ren}^{(1+2)}$ since this scale is almost equal to the top quark mass.

In conclusion we have found that the top quark contribution to the finite temperature one loop effective action can be described in analytic form by the expressions (6.1) and (6.2). These terms represent minor corrections to the basic action $S_{\rm FT}$. While this supports the use of the semiclassical approximation it remains a problem that the corrections due to boson loops turn out to be comparable to the basic action.

APPENDIX A

In the following we will discuss the calculation of the renormalized first and second order of the fermionic one-loop effective action both at $T=0$ with massive propagators and at finite T with massless propagators. Anticipating that we work at finite temperature we replace the space-time integration by $\beta\int d^3x$.

For the first order diagram at $T=0$ we have

$$S_{\rm eff}^{(1)}(\Phi,0)=-\frac{1}{2}4\beta\int d^3x(m_F^2|\Phi(x)|^2-\mu_F^2)\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\mu_F^2}. \tag{A1}$$

Using a four-momentum cutoff Λ we get

$$S_{\rm eff}^{(1)}(\Phi,0)=-\frac{1}{8\pi^2}\beta\int d^3x(m_F^2|\Phi(x)|^2-\mu_F^2)[\Lambda^2-\mu_F^2\ln(\Lambda^2/\mu_F^2)]. \tag{A2}$$

Before we dispose further by introducing appropriate counterterms we turn to the second order contribution at $T=0$. We have to evaluate[1]

$$S_{\rm eff}^{(2)}(\Phi,0)=\frac{1}{4}4\beta\int\frac{d^3q}{(2\pi)^3}[|(m_F^2\Phi^2-\mu_F^2)^{\sim}(q)|^2+m_F^2q^2|\tilde{\Phi}(q)|^2]\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\mu_F^2}\frac{1}{(p+q)^2+\mu_F^2}. \tag{A3}$$

Performing a standard computation, using again a four-momentum cutoff Λ, leads to the following result for the loop integral:

$$\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\mu_F^2}\frac{1}{(p+q)^2+\mu_F^2}=\frac{1}{16\pi^2}\left[\ln\left(\frac{\Lambda^2}{\mu_F^2}\right)-1-\int_0^1 d\alpha\,\ln\left(1+\frac{q^2}{\mu_F^2}\alpha(1-\alpha)\right)\right]. \tag{A4}$$

Putting this together with the first order contribution and requiring all corrections to the potential and the wave function renormalization to vanish at $q^2=0$ we find that we need a counterterm

$$S_{\rm eff}^{\rm ct}=\frac{\Lambda^2}{8\pi^2}\beta\int d^3x[m_F^2(|\Phi(x)|^2-1)]-\frac{1}{16\pi^2}\left[\ln\left(\frac{\Lambda^2}{m_F^2}\right)-1\right]\beta\int d^3x[m_F^4(|\Phi(x)|^4-1)+m_F^2|\nabla\Phi(x)|^2]$$
$$-\frac{1}{8\pi^2}m_F^4\beta\int d^3x(|\Phi(x)|^2-1). \tag{A5}$$

If the first and second order diagram are evaluated in the symmetric vacuum we find

[1] $(\cdots)^{\sim}$ denotes the Fourier transform of the expression inside the parentheses.

TABLE I. Corrections to the effective action at finite temperature for $m_H=60$ GeV. ϵ and y are defined in Eqs. (2.14) and (2.19); $\Delta S_{\rm ren}^{(1+2)}$ is defined by Eq. (4.3), $\Delta S_{\rm ren}^{(1+2,\overline{\rm MS})}$ is the corresponding quantity in the $\overline{\rm MS}$ scheme with $\mu=7T$; $S^{(3)}$ is given by Eqs. (5.12) and (5.20); $\tilde{S}$, the classical action of Eq. (2.16), is given for comparison.

m_t [GeV]	T [GeV]	ϵ	y	$\triangle S_{\rm ren}^{(1+2)}$	$\triangle S_{\rm ren}^{(1+2,\overline{\rm MS})}$	$S_{\rm eff}^{\overline{(3)}}$	$\tilde{S}$
160	94.894	1.867	0.2	-0.802	20.34	-18.923	308.02
	94.854	1.800	0.3	-0.168	8.91	-3.774	132.30
	94.682	1.600	0.6	-0.043	1.66	-0.0781	24.847
170	94.5288	1.867	0.2	+0.119	20.72	-20.418	278.47
	94.495	1.800	0.3	+0.250	9.23	-4.184	121.40
	94.347	1.600	0.6	+0.035	1.71	-0.0854	22.686
180	94.6605	1.867	0.2	+1.020	21.03	-20.929	251.60
	94.631	1.800	0.3	+0.609	9.14	-4.110	107.27
	94.5056	1.600	0.6	+0.104	1.71	-0.0855	20.245

$$S_{\rm eff}^{1+2,S}(\Phi,0)=-\frac{\Lambda^2}{8\pi^2}\beta\int d^3x m_F^2|\Phi(x)|^2+\frac{1}{16\pi^2}\beta\int\frac{d^3q}{(2\pi)^3}[m_F^4|\widetilde{\Phi^2}(q)|^2+m_F^2q^2|\tilde{\Phi}(q)|^2]\left[\ln\left(\frac{\Lambda^2}{q^2}\right)+1\right]. \quad (A6)$$

Leaving out the field-independent terms, which are due to a difference in the vacuum energy density, we get

$$S_{\rm eff}^{(1+2,S)}(\Phi,0)+S_{\rm eff}^{\rm ct}=-\frac{\beta}{16\pi^2}\int\frac{d^3q}{(2\pi)^3}[m_F^4|\widetilde{\Phi^2}(q)|^2+m_F^2q^2|\tilde{\Phi}(q)|^2]\left[\ln\left(\frac{q^2}{m_F^2}\right)-2\right]-\frac{1}{8\pi^2}m_F^2\beta\int d^3x m_F^2|\Phi(x)|^2, \quad (A7)$$

which has to be evaluated with the background profile. The last term is already contained in the high temperature potential (2.3); it occurs in the coefficient $-DT_0^2=-(m_H^2-8v_0^2B)/4$ via the top contribution to B.

Turning now to the first order diagram at finite temperature we have to evaluate

$$S_{\rm eff}^{(1)}(\Phi,T)=-\frac{1}{2}4\beta\int d^3x[m_F^2|\Phi(x)|^2-\mu_F^2]T\sum_{n=-\infty}^{+\infty}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\nu_n^2+\mu_F^2+\mathbf{p}^2}. \quad (A8)$$

In order to separate $T=0$ and finite temperature contributions we use [32]

$$T\sum_{n=-\infty}^{\infty}\frac{1}{[(2n+1)\pi T]^2+\mathbf{p}^2+\mu_F^2}=\frac{1}{2E_F}-\frac{1}{E_F}\frac{1}{\exp(E_F/T)+1}, \quad (A9)$$

with $E_F=\sqrt{\mathbf{p}^2+\mu_F^2}$. In the last line the second term vanishes as $T\to 0$, so the first one represents the $T=0$ contribution which we have considered earlier. Inserting the second part into the expression for $\mathscr{J}^{(1)}$ we find the finite temperature part

$$\Delta S_{\rm eff}^{(1)}(\Phi,T)=2\beta\int d^3x[m_F^2|\Phi(x)|^2-\mu_F^2]\int\frac{d^3p}{(2\pi)^3}\frac{1}{E_F}\frac{1}{\exp(E_F/T)+1}. \quad (A10)$$

TABLE II. Corrections to the effective action at finite temperature for $m_H=70$ GeV. Definitions as in Table I.

m_t [GeV]	T [GeV]	ϵ	y	$\triangle S_{\rm ren}^{(1+2)}$	$\triangle S_{\rm ren}^{(1+2,\overline{\rm MS})}$	$S_{\rm eff}^{\overline{(3)}}$	$\tilde{S}$
160	106.015	1.867	0.2	-1.825	14.36	-6.746	217.14
	105.980	1.800	0.3	-0.654	6.35	-1.384	94.690
	105.826	1.600	0.6	-0.127	1.18	-0.0282	17.630
170	104.7850	1.867	0.2	-1.14	15.39	-8.412	207.39
	104.7545	1.800	0.3	-0.351	6.84	-1.726	90.411
	104.6202	1.600	0.6	-0.070	1.27	-0.0352	16.830
180	104.094	1.867	0.2	-0.388	15.96	-9.602	193.72
	104.0674	1.800	0.3	-0.0515	7.09	-1.946	83.921
	103.951	1.600	0.6	-0.0119	1.32	-0.0396	15.632

TABLE III. Corrections to the effective action at finite temperature for $m_H = 80$ GeV. Definitions as in Table I.

m_t [GeV]	T [GeV]	ϵ	y	$\triangle S_{\rm ren}^{(1+2)}$	$\triangle S_{\rm ren}^{(1+24,\overline{\rm MS})}$	$S_{\rm eff}^{\overline{(3)}}$	$\tilde{S}$
160	117.541	1.867	0.2	-2.22	10.15	-2.512	156.45
	117.510	1.800	0.3	-0.849	4.52	-0.514	68.050
	117.374	1.600	0.6	-0.155	0.847	-0.0107	12.708
170	115.4848	1.867	0.2	-1.84	11.17	-3.388	153.28
	115.4573	1.800	0.3	-0.654	4.99	-0.688	66.460
	115.337	1.600	0.6	-0.118	0.93	-0.0142	12.410
180	114.0042	1.867	0.2	-1.40	11.90	-4.163	146.18
	113.9801	1.800	0.3	-0.430	5.35	-0.852	63.649
	113.874	1.600	0.6	-0.076	1.00	-0.0174	11.846

We have to evaluate this expression for a bubble in the symmetric vacuum where $\Phi \neq 0$ only locally and where $\mu_F = 0$ at spatial infinity. Then $E_F = |\mathbf{p}|$ and we find

$$\Delta S_{\rm eff}^{(1)}(\Phi,T) = \frac{T^2}{12} m_F^2 \beta \int d^3x |\Phi(x)|^2. \tag{A11}$$

This contribution is already taken into account in the T^2 term of the three-dimensional high temperature action (2.1). Therefore the finite temperature part of the first order tadpole diagram has to be omitted entirely.

As the second order contribution at finite T we have to evaluate

$$S_{\rm eff}^{(2)}(\Phi,T) = \beta \int \frac{d^3q}{(2\pi)^3} [m_F^4 |\widetilde{\Phi^2}(x)|^2 + m_F^2 q^2 |\tilde{\Phi}|^2] T \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \frac{1}{(\nu_n^2 + \mathbf{p}^2)[\nu_n^2 + (\mathbf{p}+\mathbf{q})^2]}. \tag{A12}$$

Momentum integration and Matsubara frequency summation can be carried out via

$$\begin{aligned} T \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \frac{1}{(\nu_n^2+\mathbf{p}^2)[\nu_n^2+(\mathbf{p}+\mathbf{q})^2]} &= T \sum_{n=-\infty}^{+\infty} \int_0^1 d\alpha \int \frac{d^3p}{(2\pi)^3} \frac{1}{[\nu_n^2 + \mathbf{p}^2 + \alpha(1-\alpha)\mathbf{q}^2]^2} \\ &= T \sum_{n=-\infty}^{+\infty} \int_0^1 d\alpha \int \frac{d^3p}{(2\pi)^3} \frac{-d}{d(\mathbf{p}^2)} \frac{1}{\nu_n^2 + \mathbf{p}^2 + \alpha(1-\alpha)\mathbf{q}^2} \\ &= \frac{1}{\pi^2 T} \int_0^1 d\alpha \int \frac{d^3p}{(2\pi)^3} \frac{-d}{d(\mathbf{p}^2)} \frac{\pi^2 T}{2E_\alpha} \tanh(E_\alpha/2T), \end{aligned} \tag{A13}$$

where $E_\alpha^2 = \mathbf{p}^2 + \alpha(1-\alpha)\mathbf{q}^2$. The $T=0$ part may be recovered by performing the limit $T \to 0$. Subtracting this part we find for the finite temperature supplement

$$\int_0^1 d\alpha \frac{1}{2\pi^2} \int_0^\infty dp\, p^2 \frac{-d}{2p\,dp} \frac{-1}{E_\alpha} \frac{1}{\exp(E_\alpha/T)+1} = \int_0^1 d\alpha \frac{-1}{4\pi^2} \int_0^\infty \frac{dp}{E_\alpha} \frac{1}{\exp(E_\alpha/T)+1}, \tag{A14}$$

so that

$$\Delta S_{\rm eff}^{(2)}(\Phi,T) = -\frac{1}{4\pi^2} \beta \int \frac{d^3q}{(2\pi)^3} [m_F^4 |\widetilde{\Phi^2}(q)|^2 + m_F^2 q^2 |\tilde{\Phi}(q)|^2] \int_0^1 d\alpha \int_0^\infty \frac{dp}{E_\alpha} \frac{1}{\exp(E_\alpha/T)+1}. \tag{A15}$$

Part of this term is already contained in the high temperature effective action. The momentum integral has been considered by Dolan and Jackiw [32]; it can be expanded to leading order in T, i.e., up to terms of order $1/T^2$ as

$$\int_0^\infty \frac{dp}{E_\alpha} \frac{1}{\exp(E_\alpha/T)+1} = -\frac{1}{4} \left[\ln\left(\frac{\alpha(1-\alpha)q^2}{\pi^2 T^2} \right) + 2\gamma \right] + O(q^2/T^2). \tag{A16}$$

If this is inserted into the previous equation we find

$$\Delta S_{\rm eff}^{(2)}(\Phi,T) = \frac{1}{16\pi^2} \beta \int \frac{d^3q}{(2\pi)^3} [m_F^4 |\widetilde{\Phi^2}(q)|^2 + m_F^2 q^2 |\tilde{\Phi}(q)|^2] \int_0^1 d\alpha \left[\ln\left(\frac{\alpha(1-\alpha)q^2}{\pi^2 T^2} \right) + 2\gamma \right]. \tag{A17}$$

The integration over α can then be performed, replacing the second parenthesis by

$$\left[\ln\left(\frac{q^2}{\pi^2 T^2}\right)-2+2\gamma\right]. \tag{A18}$$

If this is added to the zero temperature result (A7) the term $\ln(q^2/m_F^2)-2$ in this equation gets replaced by $\ln(T^2/m_F^2)+2\ln\pi-2\gamma$. This term appears in the high temperature potential in λ_T.

Collecting from Eqs. (A7) and (A15) the terms which have not yet been included into the high temperature potential we find the following renormalized contribution of the first and second order Feynman graphs:

$$\Delta S_{\rm ren}^{(1+2)}(\Phi,T)=-3\beta\frac{m_t^4}{16\pi^2}\int\frac{d^3q}{(2\pi)^3}\left(|\widetilde{\Phi^2}(q)|^2+\frac{q^2}{m_t^2}|\tilde{\Phi}(q)|^2\right)\left[\ln\left(\frac{q^2}{m_t^2}\right)-2+4\int_0^1 d\alpha\int_0^\infty\frac{dp}{E_\alpha}\frac{1}{\exp(E_\alpha/T)+1}\right]$$
$$-3\beta\frac{m_t^4}{16\pi^2}\ln\left(\frac{m_t^2}{a_F T^2}\right)\int\frac{d^3q}{(2\pi)^3}|\widetilde{\Phi^2}(q)|^2. \tag{A19}$$

Here we have taken into account the color factor 3 and we have replaced m_F by m_t.

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