Can a "natural" three-generation neutrino mixing scheme satisfy everything?

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We examine the potential for a "natural" three-neutrino mixing scheme to satisfy available data and astrophysical arguments. By "natural" we mean no sterile neutrinos, and a neutrino mass hierarchy similar to that of the charged leptons. We seek to satisfy (or solve) (1) accelerator and reactor neutrino oscillation constraints, including LSND, (2) the atmospheric muon neutrino deficit problem, (3) the solar neutrino problem, (4) *r*-process nucleosynthesis in neutrino-heated supernova ejecta, and (5) cold+hot dark matter models. We argue that putative supernova *r*-process nucleosynthesis bounds on two-neutrino flavor mixing can be applied directly to three-neutrino mixing in the case where one vacuum neutrino mass eigenvalue difference is dominated by the others. We show that in this "one mass scale dominance" limit, a natural three-neutrino oscillation solution meeting all the above constraints exists only if the atmospheric neutrino data *and* the LSND data can be explained with one neutrino mass difference. In this model, an explanation for the solar neutrino data can be effected by employing the *other* independent neutrino mass difference. Such a solution is only marginally allowed by the current data, and proposed long-baseline neutrino oscillation experiments can definitively rule it out. If it were ruled out, the simultaneous solution of the above constraints by neutrino oscillations would then require sterile neutrinos and/or a neutrino mass hierarchy of a different nature than that of the charged leptons.

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I. INTRODUCTION

Neutrinos with masses and mixings are one of the simplest extensions of the standard model of elementary particles. All direct searches for neutrino oscillations using neutrino beams from accelerators and reactors have produced only upper limits, with the possible exception of the liquid scintillator neutrino detector experiment (LSND) at Los Alamos [1]. The claimed LSND signal is still somewhat controversial; in Ref. [2] the LSND data are interpreted as yielding only an upper limit. However, for the purposes of this paper we will accept the interpretation of the LSND data as a neutrino oscillation signal, except where explicitly stated otherwise.

In spite of a dearth of direct evidence, neutrino mixing is a popular explanation for a number of measured neutrino phenomena that appear to be at variance with predictions based on massless, nonmixing neutrinos. These phenomena include the so-called solar and atmospheric neutrino "deficits" ([3,4], respectively). Massive neutrinos can affect cosmological evolution [5] and large-scale structure formation [6,7]. Neutrino flavor mixing could affect supernova dynamics [8,9] and nucleosynthesis [10–12], and big bang nucleosynthesis [13,14]. These cosmological or astrophysical settings sometimes suggest stricter limits on neutrino masses and mixings than those obtainable with earth-based experiments.

For simplicity, neutrino oscillations are often analyzed in a two-flavor framework, with one neutrino mass eigenvalue difference and one mixing angle. In the wake of the LSND result, interest has grown in constructing models of neutrino mixing that accommodate "everything" [15–17]. In several of these models, results from two-flavor interpretations of various physical effects are combined to make a consistent composite model. In addition, some of these models use three independent mass differences: one each for solar neutrinos, atmospheric neutrinos, and LSND.

Two aspects of these models might be considered "unnatural." First, the use of three independent neutrino mass eigenvalue differences requires the introduction of a fourth neutrino. In light of measurements of the width of the Z_0 at the CERN e^+e^- collider LEP [18], this fourth neutrino must be taken to be "sterile" [an SU(2) singlet]. Second, these models sometimes employ an "inverted" neutrino mass hierarchy. In these inverted schemes, the neutrino mass eigenvalue most closely associated with ν_e is heavier than those associated with ν_μ or ν_τ, or the mass eigenvalue most closely associated with ν_μ is heavier than that associated with ν_τ.

One might hope that these unnatural features could be removed by employing a genuine three-neutrino mixing scheme. It is apparent that nontrivial mixing among three or more generations of neutrinos has the possibility of a richer phenomenology than models in which two-generation mixings are "stitched together." In particular, the excess of electron- over muon-induced events observed in atmospheric neutrinos could be due to the ν_μ oscillating into *both* ν_e and ν_τ, not just one or the other.

As noted earlier, several previous models have used three independent neutrino mass eigenvalue differences. However, a three-generation scheme has only two independent mass differences, which we label δm_1^2 and δm_2^2. We define δm^2 to be the difference of the squares of two vacuum neutrino mass eigenvalues, and take $\delta m^2 > 0$. In the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [19,20], matter effects can enhance or suppress neutrino mixing and may lead to flavor conversion. This mechanism is a popular explanation of the solar neutrino problem. Since a mass-level crossing is the basis of the MSW effect, we are forced to take one of the independent neutrino mass differences from a relatively nar-

row range determined by solar parameters. An MSW solution to the solar neutrino problem thus determines, for example, δm_1^2.

Note, however, that a fair range of neutrino mass differences can be employed in vacuum oscillation explanations of the LSND and atmospheric neutrino data. In a ''last resort'' effort to find a natural three-neutrino mixing scheme, we here consider the possibility of explaining both the atmospheric neutrino data and the LSND data with the other independent neutrino mass difference, δm_2^2. Unfortunately, this common neutrino mass eigenvalue difference would have to lie in the range $\delta m_2^2 \approx 0.2-0.4$ eV^2. This particular range of values for δm_2^2 would be an order of magnitude *lower* than the neutrino mass difference most commonly associated with LSND, while it would be an order of magnitude *larger* than the most popular value associated with atmospheric neutrinos. Further, this single common neutrino mass eigenvalue difference turns out to be narrowly specified: as we shall see, it is bounded from above by supernova r-process considerations (for noninverted neutrino mass hierarchies), and more strictly by the compatibility of atmospheric neutrinos with laboratory limits; and from below by the compatibility of LSND with other laboratory limits.

Is the use of the same neutrino mass difference (δm_2^2) to give a vacuum neutrino oscillation solution to *both* the LSND data and the published atmospheric neutrino results warranted? A wide range of neutrino mass differences appears to be capable of providing a neutrino oscillation explanation of the LSND data (see Fig. 3 of Ref. [1]). With one of the neutrino mass eigenvalues set to zero, the ''favored'' LSND value of $\delta m^2 \approx 6$ eV^2 [7] yields neutrino masses that are convenient from the perspective of cold+hot dark matter models. Note, however, that neutrino oscillation interpretations of the LSND data only probe mass *differences*. We can compensate for a smaller mass difference, $\delta m_2^2 \approx 0.2-0.4$ eV^2, by offsetting *all* of the neutrino mass eigenvalues from zero [15,16,21]. Such an offset would allow the *sum* of the neutrino mass eigenvalues to provide the requisite contribution of hot dark matter in the models of Ref. [7]. Furthermore, the ($\sin^2 2\theta$, δm^2) plot of the allowed LSND oscillation parameters [1] shows that compatibility with KARMEN [22], BNL E776 [23], and CCFR [24] is readily achieved for $\delta m_2^2 \approx 0.2-0.4$ eV^2 [1].

The range of δm_2^2 allowed by a neutrino oscillation solution to the atmospheric neutrino anomaly is a more subtle issue. The isotropy of the sub-GeV data makes them amenable to an oscillation solution for any $\delta m_2^2 \gtrsim 10^{-3}$ eV^2 (see, e.g., [25] and references therein). At one point it was argued that data from upward-going muons restricted δm_2^2 to values around 10^{-2} eV^2 [26]. However, this conclusion relied on calculations of *absolute* neutrino fluxes and cross sections, not just ratios of these quantities, as can be used in analyzing contained events. It has now been pointed out that calculations of absolute neutrino fluxes and cross sections different from those used in Ref. [26] permit all of the parameter space allowed by the sub-GeV contained events, in particular ''high'' ($\sim 10^{-1}$ eV^2) values of δm_2^2 [27,4].

Potentially more damaging to ''high'' δm_2^2 values in this context are the Kamiokande multi-GeV data, especially the claim that the data show zenith angle dependance [28]. The Kamiokande group's best fit to this data would imply $\delta m_2^2 \approx 10^{-2}$ eV^2, with 90% C.L. upper limits at $\delta m_2^2 \sim 0.1$ eV^2. We note that 95% C.L. contours could extend the allowed range of mass differences to $\delta m_2^2 \sim 0.3$ eV^2. In addition, the statistical significance of the Kamiokande group's best fit has been questioned [29,30].

We will now explore what would be possible if new analyses of the atmospheric neutrino data, or future data with better statistics, were to allow $\delta m_2^2 \sim 0.3$ eV^2. By assigning the other independent mass difference to be $\delta m_1^2 \sim 10^{-5}$ eV^2 to use for solar neutrinos, our scheme falls into the category that the authors of Ref. [31] call ''one mass scale dominance'' (OMSD), in which $\Delta_{32}, \Delta_{31} \gg \Delta_{21}$ (where $\Delta_{ij} \equiv |m_i^2 - m_j^2|$, and m_i and m_j are neutrino mass eigenvalues). Great simplification occurs in this limit [32]. Here we take the vacuum mass eigenstates 1, 2, and 3 to be those most closely corresponding to the electron, muon, and tau neutrino flavor eigenstates, respectively.

In Ref. [31] the OMSD limit is used for neutrinos propagating in vacuum, and it is shown that interpretations of experiments based on this scheme can easily be related to two-flavor interpretations. Additionally, Ref. [31] offers new interpretations of the available accelerator and reactor data in terms of this simple three-generation framework. In this OMSD scheme, the CP-violating phase, which is inherent in a three-neutrino mixing framework, and the mixing angle θ_{12} drop out of the problem. With this simplification, neutrino oscillation effects in vacuum can be described in terms of the two mixing angles θ_{13}, θ_{23} and one mass-squared difference, $\Delta \equiv \Delta_{32} \approx \Delta_{31}$.

Previous authors have studied the three-flavor MSW effect in the limit of well-separated mass scales [33]. For the matter density scales relevant to the solar neutrino problem, a decoupling to an effective two-flavor mixing problem occurs, allowing an MSW solution employing only θ_{12} and Δ_{12}. For density scales relevant to supernovae, a similar decoupling occurs, leading to an effective two-flavor mixing described in terms of θ_{13} and Δ. In both of these cases, the CP-violating phase does not appear in the final results.

In Sec. II we argue that, for the supernova hot-bubble–r-process environment, the survival probability $P(\nu_e \to \nu_e)$ is roughly the same in the OMSD three-neutrino mixing case as in the two-neutrino mixing case. This allows the general results of calculations of the effects of two-neutrino mixing on the supernova r process to be directly applied to OMSD three-neutrino mixing. In Sec. III we discuss the chances for obtaining a natural solution to ''everything.'' Concluding remarks, along with a discussion of the prospects of future experiments to clarify the issues discussed in this paper, are contained in Sec. IV.

II. MATTER-ENHANCED THREE-NEUTRINO MIXING IN THE OMSD LIMIT IN SUPERNOVAE

We now examine matter effects on the propagation of neutrinos in the OMSD three-neutrino mixing case in the post core-bounce supernova environment. First, we will consider only adiabatic resonant conversion. In this analysis, we will ignore neutrino-neutrino forward scattering effects as

well. We will comment on nonadiabatic neutrino state evolution and the effects of neutrino-neutrino forward scattering at the end of this section.

An important quantity for the r process in neutrino-heated supernova ejecta is $P(\nu_e \to \nu_e)$, the probability that a ν_e emitted from the neutrinosphere will still be a ν_e at the "weak freezeout radius." The weak freezeout radius is the distance from the center of the nascent neutron star at which the weak reactions freeze out of equilibrium. Above the weak freezeout radius the neutron-to-proton ratio, n/p, can be taken as essentially fixed.

The ν_e survival probability is crucial, since the average energy of the ν_e population can be altered by a resonant flavor conversion involving either the ν_μ or ν_τ populations. This follows on noting that the average energies of the ν_μ and ν_τ, $\langle E_{\nu_\mu,\nu_\tau}\rangle$, are always larger than the average energy of the ν_e, $\langle E_{\nu_e}\rangle$, in the absence of flavor conversion. Following Refs. [10,34], we can approximate the effects of neutrino flavor conversion on $\langle E_{\nu_e}\rangle$ as

$$\langle E_{\nu_e}\rangle_{\rm WFO} = P(\nu_e \to \nu_e)\langle E_{\nu_e}\rangle_{\rm NS} + [1 - P(\nu_e \to \nu_e)]\langle E_{\nu_\mu,\nu_\tau}\rangle_{\rm NS}. \quad (1)$$

Here "WFO" stands for the weak freeze-out radius, and "NS" stands for the radius of the "neutrinosphere." A more complete discussion of these issues can be found in Refs. [10,34].

As long as $P(\nu_e \to \nu_e)$ is the same for both the two- and three-neutrino mixing cases, the impact of significant resonant flavor conversion on the average energy of the electron neutrino population (and hence the n/p ratio) will be the same, since the μ and τ neutrinos have a common energy. To be able to apply the results of calculations of the effects of two-neutrino mixing to the OMSD three-neutrino case, we need to show that

$$P(\nu_e \to \nu_e)_{\text{three flavor,OMSD}} \approx P(\nu_e \to \nu_e)_{\text{two flavor}}. \quad (2)$$

The neutrino amplitude propagation equation in the mass basis is

$$i\frac{d}{dx}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \frac{1}{2E}\hat{M}^2\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}, \quad (3)$$

where E is the energy of the neutrino, and x is a time development parameter (e.g., radius). The evolution matrix is $\hat{M}^2/2E$, where we take

$$\hat{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}, \quad (4)$$

with m_1, m_2, and m_3 the neutrino mass eigenvalues. In Eq. (3), ν_i is the amplitude for the neutrino to be found in mass eigenstate i, with $i=1$, 2, or 3. Since the part of $\hat{M}^2$ proportional to the identity matrix contributes only a universal phase, we may remove it and rewrite Eq. (3) as

$$i\frac{d}{dx}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \frac{1}{6E}\begin{pmatrix} -\Delta_{21}-\Delta_{31} & 0 & 0\\ 0 & \Delta_{21}-\Delta_{32} & 0\\ 0 & 0 & \Delta_{32}+\Delta_{31}\end{pmatrix} \times \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}. \quad (5)$$

We now take the one mass scale dominance limit, and for convenience add the term $\Delta/6E \times$(identity matrix) to the evolution matrix. This will convert Eq. (5) to

$$i\frac{d}{dx}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \frac{1}{2E}\begin{pmatrix}0&0&0\\0&0&0\\0&0&\Delta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}. \quad (6)$$

Next, we switch to the flavor basis and add the effective mass term from e-ν_e forward exchange scattering. The propagation equation now becomes

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{1}{2E}\hat{\mathscr{M}}^2\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}. \quad (7)$$

Here $\hat{\mathscr{M}}^2$ is the flavor-basis effective mass matrix in matter, in which the extra contribution to the electron neutrino mass due to interactions with the background matter is denoted by A:

$$\hat{\mathscr{M}}^2 = U\begin{pmatrix}0&0&0\\0&0&0\\0&0&\Delta\end{pmatrix}U^\dagger + \begin{pmatrix}A&0&0\\0&0&0\\0&0&0\end{pmatrix}. \quad (8)$$

Here we take $A = 2\sqrt{2}G_F N_e E$, where N_e is the *net* number density of electrons. Note that in $\hat{\mathscr{M}}^2$ we have not included either the diagonal (in the flavor basis) or off-diagonal contributions to the neutrino effective mass matrix from neutrino-neutrino neutral current forward exchange scattering (cf. [8,11]). We will return to the possible effects of these neglected terms at the end of this section.

In Eq. (8), we take U to be the Cabbibo-Kobayashi-Maskawa (CKM) matrix of the Particle Data Group [35]:

$$U=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \tag{9}$$

where $c_{12}\equiv\cos\theta_{12}$, $s_{12}\equiv\sin\theta_{12}$, and so on; and δ_{13} is the CP-violating phase. We indicate the elements of U by $U_{\alpha i}$, where α is a flavor index and i is a mass eigenvalue index. In this notation, the amplitude for a neutrino to be found in flavor eigenstate α is, in terms of the amplitudes for the neutrino to be in the mass eigenstates i,

$$\nu_\alpha=\sum_i U_{\alpha i}\nu_i\,. \tag{10}$$

The matrix U is a product of three unitary matrices:

$$U=U_{\theta_{23}}U_{\theta_{13}}U_{\theta_{12}}, \tag{11a}$$

$$U_{\theta_{23}}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \tag{11b}$$

$$U_{\theta_{13}}=\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}, \tag{11c}$$

$$U_{\theta_{12}}=\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{11d}$$

We now proceed to simplify $\hat{\mathscr{M}}^2$. First, note that θ_{12} drops out:

$$U_{\theta_{12}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta \end{pmatrix}U^\dagger_{\theta_{12}}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta \end{pmatrix}. \tag{12}$$

Further, we can rotate away the angle θ_{23} by means of a unitary transformation:

$$\hat{\mathscr{M}}^2_{\rm rot}=U^\dagger_{\theta_{23}}\hat{\mathscr{M}}^2U_{\theta_{23}}. \tag{13}$$

This rotates the flavor basis, but only the ν_μ and ν_τ amplitudes are mixed. The ν_e amplitude, which we are most interested in, remains unchanged. The background matter contribution to the effective mass matrix (the matrix containing A) is seen from Eq. (8) to be unchanged as well.

After subtracting $\Delta/2\times$(identity matrix) from $\hat{\mathscr{M}}^2_{\rm rot}$, we are left with our final mass matrix in matter, $\hat{\mathscr{M}}^2_{\rm fin}$:

$$\hat{\mathscr{M}}^2_{\rm fin}=\frac{1}{2}\begin{pmatrix} 2A-\Delta\cos2\theta_{13} & 0 & \Delta\sin2\theta_{13}e^{-i\delta_{13}} \\ 0 & -\Delta & 0 \\ \Delta\sin2\theta_{13}e^{i\delta_{13}} & 0 & \Delta\cos2\theta_{13} \end{pmatrix}. \tag{14}$$

Note that the first and third rows of this matrix effectively are decoupled into a 2×2 matrix, which can be recognized as that which arises in the two-neutrino mixing case (apart from the phase factors $e^{\pm i\delta_{13}}$). The eigenvalues of this decoupled part of $\hat{\mathscr{M}}^2_{\rm fin}$ are identical to the eigenvalues in the two-neutrino mixing case.

At the neutrinosphere, the $|\nu_e\rangle$ eigenstate coincides with the heaviest mass eigenstate in matter, $|\nu_3^m\rangle$, due to the high matter density. The ν_e survival probability is thus given in the adiabatic limit by

$$P(\nu_e\to\nu_e)=|\langle\nu_e|\nu_3^m\rangle_{\rm WFO}|^2. \tag{15}$$

A similar expression is obtained in the two-mixing case, with $|\nu_2^m\rangle$ being the heavier mass eigenstate in matter. Direct comparison of the eigenvectors obtained from the decoupled 2×2 part of $\hat{\mathscr{M}}^2_{\rm fin}$ with the eigenvectors obtained in the two-neutrino mixing case reveals that

$$\langle\nu_e|\nu_3^m\rangle_{\text{three flavor,OMSD}}=e^{-i\delta_{13}}\langle\nu_e|\nu_2^m\rangle_{\text{two flavor}}\,. \tag{16}$$

Of course, the factor $e^{-i\delta_{13}}$ will disappear in the survival probability. Thus, with our assumptions and approximations we have

$$P(\nu_e\to\nu_e)_{\text{three flavor,OMSD}}=P(\nu_e\to\nu_e)_{\text{two flavor}}\,. \tag{17}$$

We now comment on nonadiabaticity. The assumption of an ''adiabatic'' resonant transformation means that a neutrino in a given mass eigenstate remains in that mass eigenstate throughout its propagation. Near a resonance, though, there is a probability that the neutrino will cross onto another mass eigenstate track (see Fig. 1 of Ref. [20]). Since in the OMSD limit of the three-neutrino mixing case there is only one resonance between two mass eigenstates, the effects of nonadiabaticity will be the same as in the two-neutrino mixing case.

Next we consider the effects of terms in the neutrino propagation matrix due to neutrino-neutrino forward scattering. The explicit calculations in Refs. [11,12] of the two-neutrino mixing case showed that one of the effects of these terms is to slightly extend the excluded region from $\delta m^2\approx4$ eV^2 down to $\delta m^2\approx1.5$ eV^2 (see Fig. 9 of Ref. [11]). This is due to the nonlinear nature of the problem: as flavor conversion takes place, the neutrino background is altered. Specifically, for small δm^2, the neutrino background evolves in such a way as to pull the resonance positions of neutrinos closer to the neutrinosphere. This behavior can bring the resonance positions associated with smaller δm^2 inside the weak freezeout radius. In turn, resonances inside the weak freezeout radius can lead to a decrease in the n/p ratio. The net result in Refs. [11,12] is that the lower limit on δm^2 in the parameter space region excluded by r-process considerations is *decreased*.

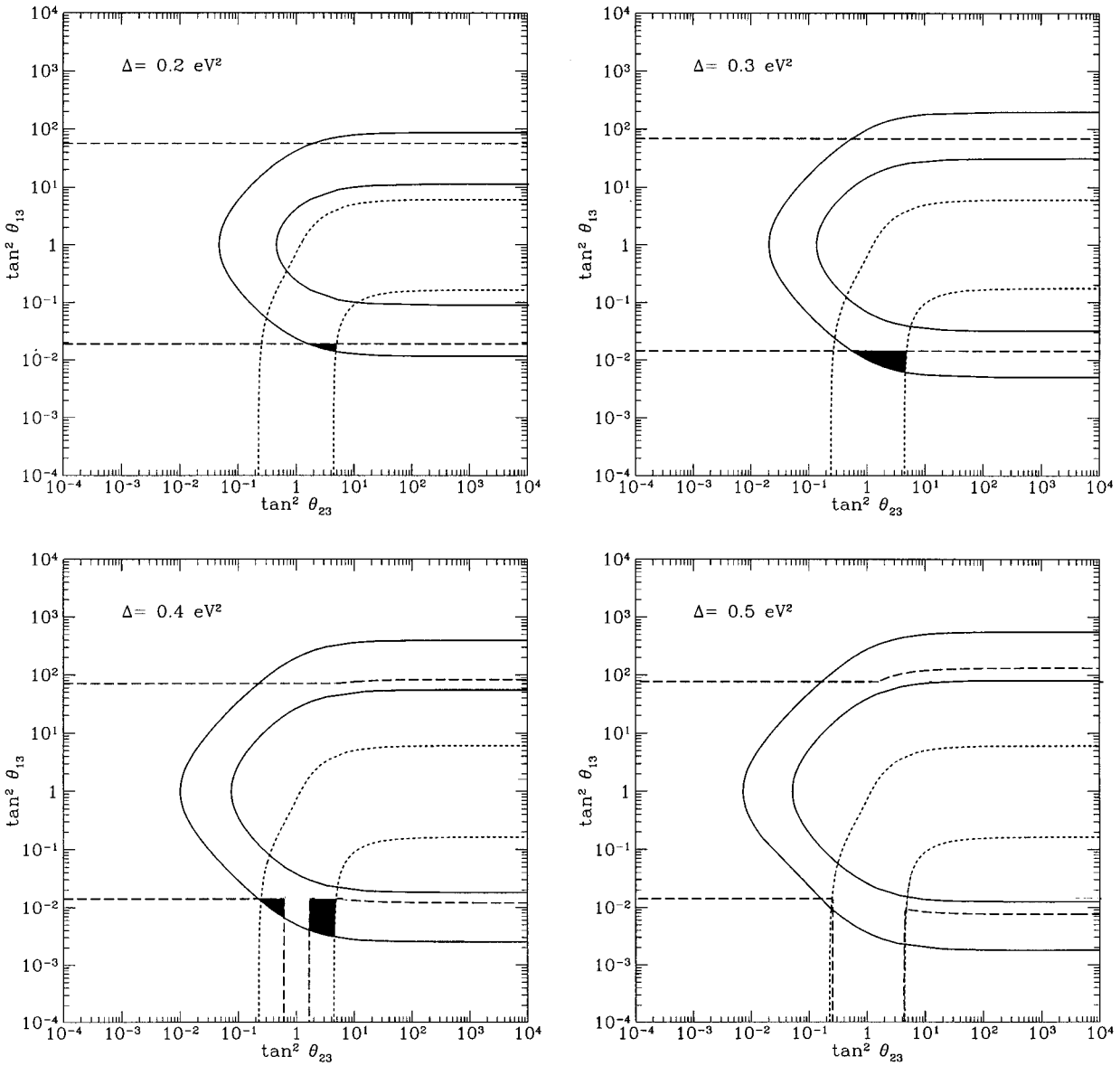

FIG. 1. Allowed values of the mixing angles θ_{13} and θ_{23} (shaded in black) are shown for four values of the dominant mass difference Δ. The region between the solid lines is the 95% C.L. detection of LSND. The region inside the long-dashed lines is excluded by accelerator or reactor 95% C.L. limits. The region enclosed by the short-dashed lines represents a solution to the sub-GeV atmospheric neutrino data, with $R=0.7$.

While we have not done explicit calculations, we argue that the effect of the neutrino background will be similar in the present case. In the OMSD limit we consider here, the ν_e can mix with two other flavors of neutrinos, but only *one* of the mass differences is large enough to cause resonant flavor conversion inside the weak freezeout radius of the supernova. Therefore, in the natural neutrino mass hierarchy considered here, we expect that allowing a third neutrino will not cause significant further evolution of the background. This is because the additional neutrino mass eigenvalue difference is too small to bring about extra flavor conversion of the ν_e [36].

In the natural scheme considered here, we can apply limits obtained in the two-neutrino mixing case, and conclude that the necessity of a neutron-rich hot-bubble environment for r-process nucleosynthesis places limits on $\sin^2 2\theta_{13}$ for $\Delta \gtrsim 1.5\ \mathrm{eV}^2$. Of course, this conclusion is valid only if the post core-bounce supernova environment is in fact the site of origin of the r-process elements.

III. AN ATTEMPT AT A THREE-NEUTRINO MIXING SOLUTION

We seek a set of neutrino masses and mixing angles consistent with the following: (1) accelerator and reactor data, including LSND; (2) atmospheric neutrinos; (3) solar neutrinos; (4) supernova r-process nucleosynthesis; (5) cold+hot dark matter models. We attempt to do this by invoking a vacuum neutrino oscillation interpretation for both LSND

and atmospheric neutrinos with $\Delta \equiv \Delta_{13} \approx \Delta_{23} \approx 0.3\ \mathrm{eV}^2$, while we solve the solar neutrino problem by employing the MSW effect with $\Delta_{12} \approx 10^{-5}\ \mathrm{eV}^2$.

First we consider the accelerator and reactor data. The authors of Ref. [31] have made a thorough reanalysis of the exisiting accelerator and reactor data, the results of which we employ here. For two-flavor vacuum neutrino oscillations, the survival probability P is given by

$$P = 1 - \sin^2 2\theta \, \sin^2\left(1.27 \frac{L\Delta}{E}\right), \tag{18}$$

where L (in m) is the path length for a neutrino initially in a flavor eigenstate at $L=0$, E the neutrino energy in MeV, and Δ is in eV^2. The two-flavor mixing angle is θ. In the OMSD limit, a reinterpretation of vacuum neutrino oscillations in terms of three-neutrino mixing is possible. In this limit, we can make the following correspondence between the two-flavor mixing angle and the three-neutrino mixing CKM matrix elements:

$$\sin^2 2\theta \Leftrightarrow 4 |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \quad \text{(appearance)}, \tag{19a}$$

$$\sin^2 2\theta \Leftrightarrow 4 |U_{\alpha 3}|^2 \, (1 - |U_{\alpha 3}|^2) \quad \text{(disappearance)}, \tag{19b}$$

for appearance and disappearance experiments, respectively.

Equation (19a) shows how the solar neutrino problem and the LSND experiment can be simultaneously explained. To solve the solar neutrino deficit, our neutrino mixing scheme requires an MSW resonant flavor conversion. In this conversion, the mass eigenstate corresponding most closely to ν_e deep in the interior of the sun corresponds more closely to ν_μ upon leaving the sun. However, a two-flavor neutrino oscillation interpretation of the LSND signal would require ν_e-ν_μ oscillation parameters inconsistent with those required for this MSW solution to the solar neutrino problem. A three-neutrino mixing interpretation provides the resolution: Eq. (19a) shows that the appearance of $\bar{\nu}_e$ in a $\bar{\nu}_\mu$ beam, as in the LSND experiment, is mediated by transitions involving the third mass eigenstate, which most closely corresponds to the ν_τ. The utility of these so-called ''indirect neutrino oscillations'' for simultaneously explaining the solar neutrino problem and certain accelerator experiments was recently noted in Ref. [37].

From Eq. (9) we have

$$|U_{e3}|^2 = \sin^2 \theta_{13}, \tag{20a}$$

$$|U_{\mu 3}|^2 = \cos^2 \theta_{13} \, \sin^2 \theta_{23}, \tag{20b}$$

$$|U_{\tau 3}|^2 = \cos^2 \theta_{13} \, \cos^2 \theta_{23}. \tag{20c}$$

In Ref. [31] the parameter space is displayed in a $(\ln \tan^2 \theta_{23}, \ln \tan^2 \theta_{13})$ plane, with a different plot necessary for each value of Δ. We shall employ this method of displaying neutrino mixing parameter space as well.

For atmospheric neutrinos, we here use only sub-GeV data (cf. our comments on multi-GeV data in Sec. I). Results of atmospheric neutrino experiments are often reported as the ''ratio of ratios'' R:

$$R = \frac{(\nu_\mu / \nu_e)_{\mathrm{data}}}{(\nu_\mu / \nu_e)_{\mathrm{Monte\ Carlo}}}. \tag{21}$$

In the sub-GeV range, for three-neutrino mixing R is given by ([25] and references therein)

$$R = \frac{P_{\mu\mu} + r P_{\mu e}}{P_{ee} + r^{-1} P_{\mu e}}. \tag{22}$$

Here r is a particular ratio of electron- to muon-type neutrinos, which we take to be $r = 0.49$ [4]. The mass scale we are interested in implies an averaging of the oscillation factors:

$$\sin^2\left(1.27 \frac{L\Delta}{E}\right) \approx \frac{1}{2}, \tag{23}$$

so that R becomes independent of Δ for large enough Δ.

Part of our putative ''solution to everything'' appears in Fig. 1. In this figure we show constraints on four $(\ln \tan^2 \theta_{23}, \ln \tan^2 \theta_{13})$ panels. Each panel corresponds to a different value of Δ ($\Delta = 0.2\ \mathrm{eV}^2$, $0.3\ \mathrm{eV}^2$, $0.4\ \mathrm{eV}^2$, $0.5\ \mathrm{eV}^2$). The accelerator and reactor data in Fig. 1 are given at 95% C.L. Except for LSND, the data are taken from the reanalysis of accelerator and reactor experiments in Ref. [31]. The LSND 95% C.L. data are taken from an early preprint of the Ref. [1] group. To delineate the band allowed by atmospheric neutrinos in Fig. 1 we have taken $R = 0.7$; the band shrinks if smaller values of R are employed. Note that $R = 0.7$ is not the central reported value. However, we have employed this high R value in the interest of discovering what the data might accommodate. This value of R is in the vicinity of the upper limits on this quantity allowed by the Kamiokande [38] and IMB [39] sub-GeV data.

The $\Delta = 0.2\ \mathrm{eV}^2$ panel of Fig. 1 shows where the lower bound on Δ originates: below $\Delta \approx 0.2\ \mathrm{eV}^2$, LSND begins to become incompatible with ν_e disappearance constraints. These ν_e disappearance limits come from reactor experiments, particularly the Bugey reactor experiment [40]. The $\Delta = 0.5\ \mathrm{eV}^2$ panel of Fig. 1 shows where the upper bound on Δ comes from: above $\Delta \approx 0.4\ \mathrm{eV}^2$, ν_μ disappearance experiments (in particular CDHSW [41]) rule out most of the atmospheric neutrino solution. The panels of Fig. 1 with $\Delta = 0.3\ \mathrm{eV}^2$ and $\Delta = 0.4\ \mathrm{eV}^2$ show the allowed regions of parameter space. We note that $\Delta \approx 0.3\ \mathrm{eV}^2$ is perhaps the safest solution. With the choice $\Delta \approx 0.4\ \mathrm{eV}^2$, the central part of the atmospheric neutrino band partially overlaps the disallowed parameter space region of CDHSW. This is significant, because the central part of the atmospheric neutrino band is typically the part that would remain if a lower value of R were to be chosen.

For solar neutrinos, we invoke an MSW solution using the parameters Δ_{12} and θ_{12}. These quantities dropped out in the above analysis of accelerator-reactor experiments, atmospheric neutrinos, and the supernova r process, due to our use of the OMSD limit. However, Δ_{12} and θ_{12} are important near the mass level crossing that occurs in the sun. This is because of the extra effective mass acquired by the ν_e. Using the decoupled two-neutrino solution obtained in Ref. [33] (valid for the well-separated mass scales which we have here), the authors of Ref. [42] obtain solutions to the solar neutrino problem. Their Figs. 4 and 6 show the allowed areas

in the (Δ_{12}, $\sin^2 2\theta_{12}/\cos 2\theta_{12}$) plane for $\sin^2\theta_{13}=0.0$ and $\sin^2\theta_{13}=0.1$, respectively. It is apparent that the "small angle" solution is essentially unchanged for these different values of $\sin^2\theta_{13}$. From our Fig. 1, we see that our putative solution has $\sin^2\theta_{13}\approx 10^{-2}$, so we must adopt the small angle solution seen in Figs. 4 and 6 of Ref. [42]: $\Delta_{12}\approx 7\times 10^{-6}$ eV^2; $\sin^2\theta_{12}\approx 2\times 10^{-3}$. The *CP*-violating phase factor only appears in terms that vanish in the approximations used in Refs. [33,42].

For massive neutrinos to be of use for cold+hot dark matter models, it is desirable to have the neutrino masses add up to about 5 eV [7]. Given our value of Δ, this implies $m_1\approx m_2\approx 1.5$ eV, and $m_3\approx 2$ eV. This may appear to put the ν_e Majorana mass in conflict with the limit $m_{\nu_e}\lesssim 0.7$ eV from neutrinoless double β decay [43]. However, this is not necessarily a serious problem. As discussed in Ref. [16], a more realistic limit on the ν_e Majorana mass may be around 1.4 eV. Also, perhaps cold+hot dark matter models could work with the sum of the neutrino masses being a little less than 5 eV. Finally, if the neutrinos have Dirac masses, the limit from neutrinoless double β decay does not apply at all.

We have found a fairly unique set of neutrino masses and mixing angles satisfying the constraints listed at the beginning of this section. We note that the value of Δ we have been led to is safe from the perspective of the *r* process. Additionally, in the absence of a significant net lepton number in the universe, neutrino oscillations with these parameters should have essentially no effect on the outcome of big bang nucleosynthesis [14].

However, this natural solution does not look very promising. The panels in Fig. 1 contain 95% C.L. data (with the exception of the atmospheric neutrino "ratio of ratios" *R*, for which confidence levels are not readily assignable). At 90% C.L., the LSND detection band shrinks, while the excluded regions from the other reactor-accelerator limits expand. This solution then exists essentially as a point in the ($\ln\tan^2\theta_{23}$, $\ln\tan^2\theta_{13}$) plane, and this only at $\Delta\approx 0.3$ eV^2. The existence of a three-neutrino mixing solution, in the OMSD limit and satisfying the five points at the beginning of this section, is therefore fragile at best.

It could also be argued that this "natural" solution is not so natural after all. For example, we have taken $\Delta_{13}\approx\Delta_{23}\gg\Delta_{12}$, in analogy with the hierarchy of mass differences of the charged leptons. For the charged leptons, this hierarchy of mass *differences* arises naturally from the hierarchy of the *masses themselves*, i.e., $m_e^2\ll m_\mu^2\ll m_\tau^2$. However, the offset of neutrino masses from zero required to make the neutrino masses sum to about 5 eV for cold+hot dark matter models causes the absolute masses to have roughly similar magnitudes, in contrast to the masses of the charged leptons.

Another unnatural feature of this putative "natural" solution is that several of the off-diagonal elements of the mixing matrix *U* have relatively large magnitudes. This is in contrast to the quark mixing case [35]. It has been recognized previously that a three-neutrino oscillation explanation of the LSND experiment requires this unusual feature [44]. Large off-diagonal terms will generally be present whenever oscillation probabilities are large, and neutrino oscillation explanations of the atmospheric neutrino anomaly and the LSND data both invoke relatively large oscillation probabilities.

Note from Fig. 1 that the LSND oscillation signal significantly restricts the "natural" solution. If the LSND data are interpreted as yielding only an upper limit as in Ref. [2], the atmospheric neutrino anomaly can be solved with any value of the mixing angle θ_{13} such that $\tan^2\theta_{13}\lesssim 10^{-2}$. The lower limit of $\Delta\approx 0.2$ eV^2 would also disappear in this case, allowing the experimentally more comfortable value of $\Delta\approx 10^{-2}$ eV^2 to be used to solve the atmospheric neutrino problem. Even without the LSND detection, however, the simultaneous solution of all the remaining constraints would still involve the unnatural features mentioned above: nearly degenerate neutrino masses and large off-diagonal elements in the neutrino mixing matrix, in contrast with the properties of the charged leptons.

IV. CONCLUSION

Some models which have sought to account for currently available clues about neutrino properties have employed neutrino oscillations with sterile neutrinos, and/or an inverted mass hierarchy [15–17]. These devices are required when two-flavor interpretations of various physical effects are "stitched together" to make a consistent composite model. Here we have sought a "natural" three-neutrino mixing scheme, without sterile neutrinos or an inverted mass hierarchy, that satisfies (1) accelerator and reactor data, including LSND, (2) atmospheric neutrinos, (3) solar neutrinos, (4) supernova *r*-process nucleosynthesis, and (5) cold+hot dark matter models. Along the way we have argued that, under some circumstances, putative supernova *r*-process nucleosynthesis bounds on two-neutrino flavor mixing can be applied directly to three-neutrino mixing in the case of one mass scale dominance.

We have found a possible "natural" solution, and it is quite restricted. The mass differences in this putative solution are $\Delta_{12}\approx 7\times 10^{-6}$ eV^2 (for an MSW solution to the solar neutrino problem), and $\Delta_{13}\approx\Delta_{23}\approx 0.3$ eV^2 (for LSND and atmospheric neutrinos). Here $\Delta_{ji}\equiv|m_j^2-m_i^2|$. To be of use to cold+hot dark matter models, we take the masses themselves to be $m_1\approx m_2\approx 1.5$ eV, and $m_3\approx 2$ eV. The mixing angles we require are $\sin^2\theta_{12}\approx 2\times 10^{-3}$, $\sin^2\theta_{13}\approx 10^{-2}$, and $\sin^2\theta_{23}\sim 0.5$ (see Figs. 1(a)–1(d) for θ_{13} and θ_{23}, and Figs. 4 and 6 of Ref. [42] for θ_{12}). While this "natural" solution exists, it is rather fragile and has some arguably unnatural features, as discussed in Sec. III.

Perhaps the main difficulty with this scheme is finding a common mass difference suitable for a vacuum neutrino oscillation solution for both LSND and the zenith-angle dependence of the Kamiokande multi-GeV atmospheric neutrino data. As the statistics for both of these experiments are not compelling at this stage, future results may shed light on the matter. Super-Kamiokande [45] will have much better statistics for the atmospheric neutrino deficit. Furthermore, the LSND experiment continues and may provide better statistics in the future. The KARMEN experiment [22], which probes regions of parameter space similar to LSND, hopefully will also report further results.

Proposed terrestrial experiments could definitively eliminate the solution presented in the last section. New reactor

experiments at San Onofre and Chooz will have increased sensitivity to δm^2, but apparently will not have increased sensitivity to the oscillation probability at the mass difference we are interested in [46]. Conversely, the CHORUS and NOMAD experiments have high sensitivity to the oscillation probability, but will probably not reach small enough δm^2 to convincingly eliminate the putative ''natural'' solution [47]. However, proposed long-baseline accelerator experiments will probe the entire region of parameter space favored by the atmospheric neutrino sub-GeV data [31,48] and could thus provide the crucial test.

Assuming the validity of the data and astrophysical arguments we have attempted to satisfy, the convincing experimental elimination of the last remaining ''natural'' solution presented here would be an important development. It would be significant evidence for the existence of sterile neutrinos and/or a neutrino mass hierarchy of a different nature than that of the charged leptons (i.e., an ''inverted'' hierarchy, in which the mass eigenvalue most closely associated with ν_e is heavier than those associated with ν_μ or ν_τ; or a hierarchy in which no one neutrino mass eigenvalue difference is dominated by the other neutrino mass eigenvalue differences).

Note added in proof. A recent work [49] reports a reanalysis of the Kamiokande multi-GeV data in which $\delta m^2_{\rm atmos} \gtrsim 0.25$ eV^2 is excluded at 90% C.L. and $\delta m^2_{\rm atmos} \gtrsim 0.47$ eV^2 is excluded at 95% C.L. Thus, $\delta^2_{\rm atmos,LSND} \approx 0.3$ eV^2 required by the ''natural'' solution is excluded at 90% C.L., but is allowed at 95% C.L.

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