$SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ model with right-handed neutrinos

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We explore some more consequences of the $SU(3)_L \otimes U(1)_N$ electroweak model with right-handed neutrinos. By introducing the Z-Z' mixing angle ϕ, the *exact* physical eigenstates for neutral gauge bosons are obtained. Because of the mixing, there is a modification to the Z^1 coupling proportional to $\sin\phi$. The data from the Z decay allow us to fix the limit for ϕ as $-0.0021 \le \phi \le 0.000132$. From the neutrino neutral current scatterings, we estimate a bound for the new neutral gauge boson Z^2 mass in the range of 300 GeV, and from symmetry-breaking hierarchy a bound for the new charged and neutral (non-Hermitian) gauge bosons $Y^{\pm}, X^0$ are obtained.

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I. INTRODUCTION

In the standard model (SM) [1], each generation of fermions is anomaly free. This is true for many extensions of the SM as well, including the popular grand unified models [2]. In these models, therefore, the number of generations is completely unrestricted on theoretical grounds.

Recently, an interesting class of models has been proposed [3] in which each generation is anomalous but different generations are not exact replicas of one another, and the anomalies cancel when a number of generations are taken into account, and to be a multiple of 3. The most economical gauge group which admits such fermion representations is $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$, and it has been proposed by Pisano, Pleitez, and Frampton [4] (for further work on this model, see Refs. [5, 6]). The original model did not have right-handed neutrinos, but recently we have included them in a nontrivial way in an interesting modification of the model [7]. We have pointed out that this model is simpler than the Pisano-Pleitez-Frampton (PPF) model, since fewer Higgs multiplets are needed in order to allow the fermions to gain masses and to break the gauge symmetry.

In [8] some phenomenological aspects of the model have been considered. However, these results are based on an approximate solution for the physical eigenstates of neutral gauge bosons. The purpose of this paper is to present a further development of the model. By introducing the Z-Z' mixing angle, the exact physical eigenstates of neutral gauge bosons are obtained. Based on the data from the Z decay we fix the mixing angle; similarly, from the neutrino neutral current scattering data we estimate the Z^2 boson mass in the range of 300 GeV, which is accessible for direct searches at high energy colliders such as the Fermilab Tevatron and Next Linear Collider NLC.

This paper is organized as follows. In Sec. II we recall some features of the model and Yukawa interactions. In Sec. III we study the gauge boson sector. The charged and neutral currents are given in Sec. IV. In Sec. V the constraints on the Z-Z' mixing and masses of the new gauge bosons are obtained. Finally, our conclusions are summarized in the last section.

II. THE 331 MODEL AND YUKAWA INTERACTIONS

Like the PPF model, our model is also based on the gauge group

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_N . \quad (1)$$

This model deals with nine leptons and nine quarks. There are three left- and right-handed neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$, three charged leptons (e, μ, τ), four quarks with charge 2/3, and five quarks with charge $-1/3$.

Under the gauge symmetry (1), the three lepton generations transform as

$$f^a_L = \begin{pmatrix} \nu^a_L \\ e^a_L \\ (\nu^c_R)^a \end{pmatrix} \sim (1,3,-1/3), e^a_R \sim (1,1,-1), \quad (2)$$

where $a = 1,2,3$ is the generation index.

Two of the three quark generations transform identically and one generation (it does not matter which one) transforms in a different representation of the gauge group (1):

$$Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ d'_{iL} \end{pmatrix} \sim (3,\bar{3},0), \quad (3)$$

$$u_{iR} \sim (3,1,2/3), d_{iR} \sim (3,1,-1/3),$$

$$d'_{iR} \sim (3,1,-1/3), \; i = 1,2,$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3,3,1/3),$$

$$u_{3R} \sim (3,1,2/3), d_{3R} \sim (3,1,-1/3), T_R \sim (3,1,2/3).$$

It can easily be checked that all gauge anomalies cancel with the above choice of gauge quantum numbers. Fermion mass generation and symmetry breaking can be achieved with just three $SU(3)_L$ triplets. We define them by their Yukawa Lagrangians as:

$$\mathcal{L}^{\chi}_{\text{Yuk}} = \lambda_1 \bar{Q}_{3L} T_R \chi + \lambda_{2ij} \bar{Q}_{iL} d'_{jR} \chi^* + \text{H.c.},$$

where

$$\chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1, 3, -1/3). \tag{4}$$

If χ gets the vacuum expectation value (VEV),

$$\langle \chi \rangle^T = (0, 0, \omega/\sqrt{2}), \tag{5}$$

then the exotic 2/3 and $-1/3$ quarks gain masses and the gauge symmetry is broken to the SM gauge symmetry:

$$\begin{array}{l} SU(3)_C \otimes SU(3)_L \otimes U(1)_N \\ \quad\downarrow \quad \langle \chi \rangle \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y , \end{array} \tag{6}$$

where $Y = 2N - \sqrt{3}\lambda_8/3$ $[\lambda_8 = \text{diag}(1, 1, -2)/\sqrt{3}]$. Note that Y is identical to the standard hypercharge of the SM. Electroweak symmetry breaking and ordinary fermion mass generation are achieved with two $SU(3)_L$ triplets ρ, η which we define through their Yukawa Lagrangians as:

$$\begin{aligned} \mathcal{L}^{\eta}_{\text{Yuk}} &= \lambda_{3a} \bar{Q}_{3L} u_{aR} \eta + \lambda_{4ia} \bar{Q}_{iL} d_{aR} \eta^* + \text{H.c.}, \\ \mathcal{L}^{\rho}_{\text{Yuk}} &= \lambda_{1a} \bar{Q}_{3L} d_{aR} \rho + \lambda_{2ia} \bar{Q}_{iL} u_{aR} \rho^* + G'_{ab} \bar{f}^a_L e^b_R \rho \\ &\quad + G_{ab} \epsilon^{ijk} (\bar{f}^a_L)_i (f^b_L)^c_j (\rho^*)_k + \text{H.c.} \end{aligned} \tag{7}$$

where

$$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho'^+ \end{pmatrix} \sim (1, 3, 2/3),$$

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^0 \end{pmatrix} \sim (1, 3, -1/3). \tag{8}$$

We require the vacuum structure of ρ, η:

$$\langle \rho \rangle^T = (0, u/\sqrt{2}, 0), \ \ \langle \eta \rangle^T = (v/\sqrt{2}, 0, 0). \tag{9}$$

The last term in Eq. (7) gives the 3×3 antisymmetric mass matrix, which has eigenvalues $0, -M, M$. Hence, one of the neutrinos does not gain mass and the other two are degenerate, at least at the tree level [5]. It is easy to see that this term gives interactions which directly contribute to lepton-number violation processes such as neutrinoless double beta decay ($\beta\beta_{0\nu}$) and neutrino oscillations. The vacuum expectation value (VEV) $\langle \rho \rangle$ will generate masses for the three charged leptons, two up-type, one down-type quarks, and two of the neutrinos will gain degenerate Dirac masses with one necessarily massless, while VEV $\langle \eta \rangle$ will generate masses for the remaining quarks. The VEV's $\langle \rho \rangle$ and $\langle \eta \rangle$ also give the electroweak gauge boson masses and results in the symmetry breaking:

$$\begin{array}{l} SU(3)_C \otimes SU(3)_L \otimes U(1)_N \\ \quad\downarrow \quad \langle \chi \rangle \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ \quad\downarrow \quad \langle \rho \rangle, \langle \eta \rangle \\ SU(3)_C \otimes U(1)_Q . \end{array} \tag{10}$$

Here the electric charge is defined as

$$Q = \frac{1}{2}\lambda_3 - \frac{1}{2\sqrt{3}}\lambda_8 + N. \tag{11}$$

III. GAUGE BOSONS

The gauge bosons of this theory form an octet W^a_μ associated with $SU(3)_L$, an octet G^a_μ (gluons) with $SU(3)_C$, and a singlet B_μ associated with $U(1)_N$. It is easy to see that the massless G^a_μ gauge bosons associated with the $SU(3)_C$ group decouple from the neutral gauge boson mass matrix. For that reason, we neglect terms which contain the G^a_μ gauge bosons in the covariant derivative. The gauge boson mass matrix arises from the Higgs boson kinetic term:

$$\begin{aligned} \mathcal{L}_{kinetic} &= (D_\mu \chi)^\dagger (D^\mu \chi) + (D_\mu \rho)^\dagger (D^\mu \rho) \\ &\quad + (D_\mu \eta)^\dagger (D^\mu \eta). \end{aligned} \tag{12}$$

The covariant derivatives are

$$D_\mu = \partial_\mu + ig \sum_{a=1}^{8} W^a_\mu \frac{\lambda_a}{2} + ig_N \frac{\lambda^9}{2} N B_\mu, \tag{13}$$

where λ^a(a=1,...,8) are the $SU(3)_L$ generators, and $\lambda^9 = \sqrt{2/3}\ \text{diag}(1, 1, 1)$ are defined such that $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ and $\text{Tr}(\lambda^9 \lambda^9) = 2$, and N denotes the N charge for three Higgs multiplets.

The non-Hermitian gauge bosons $\sqrt{2}\ W^+_\mu = W^1_\mu - iW^2_\mu, \sqrt{2}\ Y^-_\mu = W^6_\mu - iW^7_\mu, \sqrt{2}\ X^0_\mu = W^4_\mu - iW^5_\mu$ have the masses [8]

$$\begin{aligned} M^2_W &= \frac{1}{4} g^2 (u^2 + v^2), \quad M^2_Y = \frac{1}{4} g^2 (v^2 + \omega^2), \\ M^2_X &= \frac{1}{4} g^2 (u^2 + \omega^2). \end{aligned} \tag{14}$$

We assume $\langle \chi \rangle \gg \langle \rho \rangle, \langle \eta \rangle$ such that $M_W \ll M_X, M_Y$. This statement is very important, because the new gauge bosons must be sufficiently heavy to keep consistency with low energy phenomenology.

As the triplet scalar χ acquires a VEV, the symmetry $SU(3)_L \otimes U(1)_N$ breaks down to $SU(2)_L \otimes U(1)_Y$. By matching the gauge coupling constants at the $SU(3)_L \otimes U(1)_N$ breaking, the coupling constant of $U(1)_Y$, g', is given by

$$\frac{1}{g'^2} = \frac{1}{3g^2} + \frac{6}{g^2_N}. \tag{15}$$

Equation (15) may be satisfied by a $3-3-1$ mixing angle θ_{3-3-1} [9]:

$$g' = \sqrt{3} g \cos\theta_{3-3-1} = \frac{1}{\sqrt{6}} g_N \sin\theta_{3-3-1}. \qquad (16)$$

As in the SM we put $g' = g\tan\theta_W$; hence, we get finally

$$\frac{g_N}{g} = \frac{3\sqrt{2}\sin\theta_W(M_{Z'})}{\sqrt{3-4\sin^2_W(M_{Z'})}}. \qquad (17)$$

In this model, $\sin^2\theta_W$ has to be smaller than $\frac{3}{4}$, while in the minimal version [4] $\sin^2\theta_W < \frac{1}{4}$.

The neutral (Hermitian) gauge bosons have the 3×3 mass matrix M^2:

$$\mathcal{L}_{\rm mass} = \frac{1}{2} V^T M^2 V, \qquad (18)$$

where

$$V^T = (W^3, W^8, B), \qquad (19)$$

and [8]

$$M^2 = \frac{1}{4} g^2 \begin{pmatrix} u^2+v^2 & -\frac{1}{\sqrt{3}}(u^2-v^2) & -\frac{2t}{3\sqrt{6}}(2u^2+v^2) \\ -\frac{1}{\sqrt{3}}(u^2-v^2) & \frac{1}{3}(u^2+v^2+4\omega^2) & \frac{2t}{9\sqrt{2}}(2u^2-v^2+2\omega^2) \\ -\frac{2t}{3\sqrt{6}}(2u^2+v^2) & \frac{2t}{9\sqrt{2}}(2u^2-v^2+2\omega^2) & \frac{2t^2}{27}(4u^2+v^2+\omega^2) \end{pmatrix}, \qquad (20)$$

with the notation $t = g_N/g$. This mass matrix can be diagonalized to obtain the eigenstate fields.

We can identify the photon field A_μ as well as the massive bosons Z and Z':

$$A_\mu = s_W W^3_\mu + c_W\left(-\frac{t_W}{\sqrt{3}} W^8_\mu + \sqrt{1-\frac{t^2_W}{3}} B_\mu\right),$$

$$Z_\mu = c_W W^3_\mu + s_W\left(-\frac{t_W}{\sqrt{3}} W^8_\mu + \sqrt{1-\frac{t^2_W}{3}} B_\mu\right),$$

$$Z'_\mu = \sqrt{1-\frac{t^2_W}{3}} W^8_\mu + \frac{t_W}{\sqrt{3}} B_\mu, \qquad (21)$$

where the mass-squared matrix for Z, Z' is given by

$$\mathcal{M}^2 = \begin{pmatrix} M^2_Z & M^2_{ZZ'} \\ M^2_{ZZ'} & M^2_{Z'} \end{pmatrix},$$

with

$$M^2_Z = \frac{g^2}{4c^2_W}(u^2+v^2) = \frac{M^2_W}{c^2_W},$$

$$M^2_{ZZ'} = \frac{g^2}{4c^2_W\sqrt{3-4s^2_W}}\left[u^2 - v^2(1-2s^2_W)\right], \qquad (22)$$

$$M^2_{Z'} = \frac{g^2}{4(3-4s^2_W)}\left[4\omega^2 + \frac{u^2}{c^2_W} + \frac{v^2(1-2s^2_W)^2}{c^2_W}\right]. \qquad (23)$$

Here we use the following notations: $s_W \equiv \sin\theta_W, c_W \equiv \cos\theta_W$, and $t_W \equiv \tan\theta_W$. From Eq. (22) we see that the limit for $M_{ZZ'}$ is

$$-\frac{(1-2s^2_W)}{\sqrt{3-4s^2_W}} M^2_Z \le M^2_{ZZ'} \le \frac{M^2_Z}{\sqrt{3-4s^2_W}}. \qquad (24)$$

Diagonalizing the mass matrix gives the mass eigenstates Z^1 and Z^2, which can be taken as mixtures:

$$Z^1 = Z\cos\phi - Z'\sin\phi,$$
$$Z^2 = Z\sin\phi + Z'\cos\phi. \qquad (25)$$

The mixing angle ϕ is given by

$$\tan^2\phi = \frac{M^2_Z - M^2_{Z^1}}{M^2_{Z^2} - M^2_Z}, \qquad (26)$$

where M_{Z^1} and M_{Z^2} are the *physical* mass eigenvalues

$$M^2_{Z^1} = \frac{1}{2}\{M^2_{Z'} + M^2_Z - [(M^2_{Z'} - M^2_Z)^2 - 4(M^2_{ZZ'})^2]^{1/2}\}, \qquad (27)$$

$$M^2_{Z^2} = \frac{1}{2}\{M^2_{Z'} + M^2_Z + [(M^2_{Z'} - M^2_Z)^2 - 4(M^2_{ZZ'})^2]^{1/2}\}. \qquad (28)$$

From Eq. (22) we see that $\phi = 0$ if $u^2 = v^2(1-2s^2_W)$. Here W, Z^1 correspond to the standard model charged and neutral gauge bosons, and there are new gauge bosons $Y^\pm, X^0$, and Z^2. A fit to precision electroweak observables gives a limit on the mixing angle (see below) of $-0.0021 \le \phi \le 0.000\,132$ and from the symmetry-breaking hierarchy $\omega \gg u, v$, Eqs. (14) and (23) give us

$$M_{Y^+} \simeq M_{X^0} \simeq \frac{\sqrt{3-4s^2_W}}{2} M_{Z^2} \simeq 0.72 M_{Z^2}. \qquad (29)$$

IV. CHARGED AND NEUTRAL CURRENTS

The interactions among the gauge bosons and fermions are read off from

$$\mathcal{L}_F = \bar{R} i\gamma^\mu(\partial_\mu + i g_N B_\mu N) R + \bar{L} i\gamma^\mu\left(\partial_\mu + i\frac{g_N}{\sqrt{6}} B_\mu N + i g\sum_{a=1}^{8} W^a_\mu \frac{\lambda_a}{2}\right) L, \qquad (30)$$

where R represents any right-handed singlet and L any left-handed triplet or antitriplet.

The interactions among the charged vector fields with leptons are

$$\mathcal{L}_l^{\rm CC} = -\frac{g}{\sqrt{2}}[\bar{\nu}_L^a\gamma^\mu e_L^a W_\mu^+ + \overline{(\nu_R^c)}^a\gamma^\mu e_L^a Y_\mu^+ + \bar{\nu}_L^a\gamma^\mu(\nu_R^c)^a X_\mu^0 + \text{H.c.}]. \quad (31)$$

For the quarks we have

$$\mathcal{L}_q^{CC} = -\frac{g}{\sqrt{2}}[(\bar{u}_{3L}\gamma^\mu d_{3L} + \bar{u}_{iL}\gamma^\mu d_{iL})W_\mu^+ + (\bar{T}_L\gamma^\mu d_{3L} + \bar{u}_{iL}\gamma^\mu d'_{iL})Y_\mu^+ + (\bar{u}_{3L}\gamma^\mu T_L - \bar{d'}_{iL}\gamma^\mu d_{iL})X_\mu^0 + \text{H.c.}]. \quad (32)$$

We can see that the interactions with the Y^+ and X^0 bosons violate the lepton number [see Eq. (31)] and the weak isospin [see Eq. (32)].

The electromagnetic current for fermions is the usual one

$$Q_f e\bar{f}\gamma^\mu f A_\mu, \quad (33)$$

where f is any fermion with $Q_f = 0, -1, 2/3, -1/3$ and the electric charge e is identified as

$$e = g\sin\theta_W. \quad (34)$$

The neutral current interactions can be written in the form

$$\mathcal{L}^{\rm NC} = \frac{g}{2c_W}\{\bar{f}\gamma^\mu[a_{1L}(f)(1-\gamma_5) + a_{1R}(f)(1+\gamma_5)]fZ_\mu^1 + \bar{f}\gamma^\mu[a_{2L}(f)(1-\gamma_5) + a_{2R}(f)(1+\gamma_5)]fZ_\mu^2\}. \quad (35)$$

The couplings of fermions with Z^1 and Z^2 bosons are given as:

$$a_{1L,R}(f) = \cos\phi\,[T^3(f_{L,R}) - s_W^2 Q(f)] + c_W^2\left[\frac{3N(f_{L,R})}{(3-4s_W^2)^{1/2}} - \frac{(3-4s_W^2)^{1/2}}{2c_W^2}Y(f_{L,R})\right]\sin\phi,$$

$$a_{2L,R}(f) = -c_W^2\left[\frac{3N(f_{L,R})}{(3-4s_W^2)^{1/2}} - \frac{(3-4s_W^2)^{1/2}}{2c_W^2}Y(f_{L,R})\right]\cos\phi + \sin\phi\,[T^3(f_{L,R}) - s_W^2 Q(f)], \quad (36)$$

where $T^3(f)$ and $Q(f)$ are, respectively, the third component of the weak isospin and the charge of the fermion f. Note that for the exotic quarks, the weak isospin is equal to zero. Equations (36) are valid for both left- and right-handed currents. Since the value of N is different for triplets and antitriplets, the Z^2 coupling to left-handed ordinary quarks is different for the third family, and thus flavor changing.

We can also express the neutral current interactions of Eq. (35) in terms of the vector and axial-vector couplings as

$$\mathcal{L}^{NC} = \frac{g}{2c_W}\{\bar{f}\gamma^\mu[g_{1V}(f) - g_{1A}(f)\gamma_5]fZ_\mu^1 + \bar{f}\gamma^\mu[g_{2V}(f) - g_{2A}(f)\gamma_5]fZ_\mu^2\}. \quad (37)$$

The values of these couplings are

$$g_{1V}(f) = \cos\phi\,[T^3(f_L) - 2s_W^2 Q(f)] + \sin\phi\left[\frac{c_W^2}{(3-4s_W^2)^{1/2}}[3N(f_L) + t_W^2 N(f_R)] - \sqrt{3-4s_W^2}\frac{Y(f_L)}{2}\right],$$

$$g_{1A}(f) = \cos\phi\,T^3(f_L) + \sin\phi\left[\frac{c_W^2}{(3-4s_W^2)^{1/2}}[3N(f_L) - t_W^2 N(f_R)] - \sqrt{3-4s_W^2}\frac{Y(f_L)}{2}\right],$$

$$g_{2V}(f) = \cos\phi\left[\sqrt{3-4s_W^2}\frac{Y(f_L)}{2} - \frac{c_W^2}{(3-4s_W^2)^{1/2}}[3N(f_L) + t_W^2 N(f_R)]\right] + \sin\phi\,[T^3(f_L) - 2s_W^2 Q(f)],$$

$$g_{2A}(f) = \cos\phi\left[\sqrt{3-4s_W^2}\frac{Y(f_L)}{2} - \frac{c_W^2}{(3-4s_W^2)^{1/2}}[3N(f_L) - t_W^2 N(f_R)]\right] + \sin\phi\,T^3(f_L).$$

The values of g_{1V}, g_{1A} and g_{2V}, g_{2A} are listed in Tables I and II, where the third generation is assumed to belong to the triplet. To get some indication as to why the top quark is so heavy, we have to treat the third generation differently from the first two as in Refs. [4] and [9].

We can realize that in the limit $\phi = 0$ the couplings to Z^1 of the ordinary leptons and quarks are the same as in the SM. Furthermore, the electric charge defined in Eq. (34) agrees with the SM. Because of this, we can test the new phenomenology beyond the SM. In this model, the exotic quarks carry electric charges 2/3 and $-1/3$, respectively, similarly to ordinary quarks. Consequently, the exotic quarks can mix with the ordinary ones. This type of mixing gives the flavor-changing neutral currents (FCNC's). These FCNC's will be induced due to breakdown of the Glashow-Iliopoulos-Maiani (GIM) mechanism. This type of situation has been discussed previously and bounds on the mixing strengths can be obtained from the nonobservation of FCNC's in the experiments beyond those predicted by the SM [10].

In the PPF model, the coupling strength of Z^2 to quarks is much stronger than that of leptons due to the factor $1/\sqrt{1-4s_W^2}$. Therefore, low-energy experiments such as neutrino-nucleus scattering and atomic parity

TABLE I. The $Z^1 \to f\bar{f}$ couplings in the 331 model with right-handed neutrinos.

f	$g_{1V}(f)$	$g_{1A}(f)$
e,μ,τ	$(-\frac{1}{2}+2s_W^2)(\cos\phi - \frac{\sin\phi}{(3-4s_W^2)^{1/2}})$	$-\frac{1}{2}(\cos\phi - \frac{\sin\phi}{(3-4s_W^2)^{1/2}})$
ν_e,ν_μ,ν_τ	$\frac{1}{2}[\cos\phi + \sin\phi(3-4s_W^2)^{1/2}]$	$\frac{1}{2}[\cos\phi + \sin\phi(3-4s_W^2)^{1/2}]$
t	$(\frac{1}{2}-\frac{4s_W^2}{3})\cos\phi + \frac{\sin\phi}{6(3-4s_W^2)^{1/2}}(3+2s_W^2)$	$\frac{1}{2}\cos\phi + \frac{\sin\phi}{(3-4s_W^2)^{1/2}}(\frac{1}{2}-s_W^2)$
b	$(-\frac{1}{2}+\frac{2s_W^2}{3})\cos\phi + \frac{(3-4s_W^2)^{1/2}}{6}\sin\phi$	$-\frac{1}{2}(\cos\phi - \frac{\sin\phi}{(3-4s_W^2)^{1/2}})$
u,c	$(\frac{1}{2}-\frac{4s_W^2}{3})(\cos\phi - \frac{\sin\phi}{(3-4s_W^2)^{1/2}})$	$\frac{1}{2}(\cos\phi - \frac{\sin\phi}{(3-4s_W^2)^{1/2}})$
d,s	$(-\frac{1}{2}+\frac{2s_W^2}{3})\cos\phi - (\frac{1}{2}-\frac{s_W^2}{3})\frac{\sin\phi}{(3-4s_W^2)^{1/2}}$	$-\frac{1}{2}\cos\phi - (\frac{1}{2}-s_W^2)\frac{\sin\phi}{(3-4s_W^2)^{1/2}}$
T	$-\frac{4}{3}s_W^2\cos\phi - (3-7s_W^2)\frac{\sin\phi}{3(3-4s_W^2)^{1/2}}$	$-c_W^2\frac{\sin\phi}{(3-4s_W^2)^{1/2}}$
d_i'	$\frac{2}{3}s_W^2\cos\phi + (3-5s_W^2)\frac{\sin\phi}{3(3-4s_W^2)^{1/2}}$	$c_W^2\frac{\sin\phi}{(3-4s_W^2)^{1/2}}$

violation measurements would be useful to further constrain the model [6]. However, from tables it is easy to see that this does not happen in our model.

In our model, the interactions with the heavy charged and neutral (non-Hermitian) vector bosons Y^+, X^0 violate the lepton number and the weak isospin. Because of the mixing, the mass eigenstate Z^1 now picks up flavor-changing couplings proportional to $\sin\phi$. However, since Z-Z' mixing is constrained to be very small, evidence of 3-3-1 FCNC's can only be probed indirectly at present via the Z^2 couplings.

V. CONSTRAINTS ON THE Z-Z' MIXING ANGLE AND THE Z^2 MASS

There are many ways to get constraints on the mixing angle ϕ and the Z^2 mass. Below we present a simple one. A constraint on the Z-Z' mixing can be followed from the Z-decay data. Hence we now calculate a Z width in this model.

A. Z decay modes

The tree-level expression for the partial width for the $Z \to f\bar{f}$ where $f = \nu_e, \ldots, e, u, d, \ldots$ is given by [11, 12]

$$\Gamma^{\rm tree}(Z\to f\bar{f}) = \frac{\rho_1 G_F}{6\sqrt{2}\pi}M_{Z^1}^3 N_C^f \times\{[g_{1V}(f)]^2 + [g_{1A}(f)]^2\}, \quad (38)$$

where N_C^f is the color factor. From Eq. (38) we get

$$\Gamma^{\rm tree}(Z\to l\bar{l}) = \frac{\rho_1 G_F}{6\sqrt{2}\pi}M_{Z^1}^3\left(\cos\phi - \frac{\sin\phi}{\sqrt{3-4s_W^2}}\right)^2 \times\left[\frac{1}{4}+\left(\frac{1}{2}-2s_W^2\right)^2\right]; \quad (39)$$

here $l = e,\mu,\tau$.

To get results consistent with experiments, the QCD and electroweak radiative corrections have to be included. The weak radiative corrections that depend upon the assumptions of the electroweak theory and on the value of the $M_{\rm top}$ and $M_{\rm Higgs}$ are accounted for by absorbing them into the coupling, which are then called the *effective* coupling $\bar{g}_{1V}$ and $\bar{g}_{1A}$. Then Eq. (38) becomes [11, 12]:

$$\Gamma(Z\to f\bar{f}) = \frac{\bar{\rho}_1 G_F}{6\sqrt{2}\pi}M_{Z^1}^3 N_C^f[(\bar{g}_{1V}(f))^2 + (\bar{g}_{1A}(f))^2] \times(1+\delta_{\rm QED})(1+\delta_{\rm QCD}), \quad (40)$$

where $\bar{\rho}_1 = 1+\Delta\rho_1$, $\delta_{\rm QED} = 3\alpha Q_f^2/4\pi$ and $\delta_{\rm QCD} = 0$ for leptons and $\delta_{\rm QCD} = (\alpha_s/\pi) + 1.409(\alpha_s/\pi)^2 - 12.805(\alpha/\pi)^3$ for quarks, α_s being the strong coupling constant at $\mu = M_Z$. Here [12] $\sqrt{\rho_1} = M_W/M_{Z^1}c_W$, and hence from Eqs. (14) and (27) we can get its explicit expression, and see that ρ_1 depends on the VEV's and s_W. In the limit $\langle\chi\rangle \gg \langle\rho\rangle, \langle\eta\rangle$, we have $\rho_1 = 1$. We will ignore the effects due to the combination of mixing and radiative corrections since both are very small: i.e.,

TABLE II. The $Z^2 \to f\bar{f}$ couplings.

f	$g_{2V}(f)$	$g_{2A}(f)$
e,μ,τ	$(-\frac{1}{2}+2s_W^2)(\sin\phi + \frac{\cos\phi}{(3-4s_W^2)^{1/2}})$	$-\frac{1}{2}(\sin\phi + \frac{\cos\phi}{(3-4s_W^2)^{1/2}})$
ν_e,ν_μ,ν_τ	$\frac{1}{2}[\sin\phi - \cos\phi(3-4s_W^2)^{1/2}]$	$\frac{1}{2}[\sin\phi - \cos\phi(3-4s_W^2)^{1/2}]$
t	$-\frac{\cos\phi}{6(3-4s_W^2)^{1/2}}(3+2s_W^2) + (\frac{1}{2}-\frac{4s_W^2}{3})\sin\phi$	$-(\frac{1}{2}-s_W^2)\frac{\cos\phi}{(3-4s_W^2)^{1/2}} + \frac{1}{2}\sin\phi$
b	$(-\frac{1}{2}+\frac{2s_W^2}{3})\sin\phi - \frac{(3-4s_W^2)^{1/2}}{6}\cos\phi$	$-\frac{1}{2}(\sin\phi + \frac{\cos\phi}{(3-4s_W^2)^{1/2}})$
u,c	$(\frac{1}{2}-\frac{4s_W^2}{3})(\sin\phi + \frac{\cos\phi}{(3-4s_W^2)^{1/2}})$	$\frac{1}{2}(\sin\phi + \frac{\cos\phi}{(3-4s_W^2)^{1/2}})$
d,s	$(-\frac{1}{2}+\frac{2s_W^2}{3})\sin\phi + (\frac{1}{2}-\frac{s_W^2}{3})\frac{\cos\phi}{(3-4s_W^2)^{1/2}}$	$-\frac{1}{2}\sin\phi + (\frac{1}{2}-s_W^2)\frac{\cos\phi}{(3-4s_W^2)^{1/2}}$
T	$-\frac{4}{3}s_W^2\sin\phi + (3-7s_W^2)\frac{\cos\phi}{3(3-4s_W^2)^{1/2}}$	$c_W^2\frac{\cos\phi}{(3-4s_W^2)^{1/2}}$
d_i'	$\frac{2}{3}s_W^2\sin\phi - (3-5s_W^2)\frac{\cos\phi}{3(3-4s_W^2)^{1/2}}$	$-c_W^2\frac{\cos\phi}{(3-4s_W^2)^{1/2}}$

$$\bar{g}_{1V}(f) = \cos\phi\ \bar{g}_V^{\rm SM}(f) + \sin\phi\ \left[\frac{\bar{c}_W^2}{(3-4\bar{s}_W^2)^{1/2}}[3N(f_L) + \bar{t}_W^2 N(f_R)] - \sqrt{3-4\bar{s}_W^2}\frac{Y(f_L)}{2}\right],$$
$$\bar{g}_{1A}(f) = \cos\phi\ \bar{g}_A^{\rm SM}(f) + \sin\phi\left[\frac{\bar{c}_W^2}{(3-4\bar{s}_W^2)^{1/2}}[3N(f_L) - \bar{t}_W^2 N(f_R)] - \sqrt{3-4\bar{s}_W^2}\frac{Y(f_L)}{2}\right].$$

The effective coupling constants depend on the fermion f and on the renormalization scheme [11, 13, 14]:

$$\bar{g}_V^{\rm SM}(f) = \sqrt{\bar{\rho}_{1f}}[T_{3L}(f) - 2Q(f)\kappa_f\bar{s}_W^2];\ \bar{g}_A^{\rm SM}(f) = \sqrt{\bar{\rho}_{1f}}T_{3L}(f).$$

For the case $f = b$, where additional vertex corrections are important, one must replace [15] $\bar{\rho}_1$ by $\bar{\rho}_b = \bar{\rho}_1(1-\frac{4}{3}\Delta\bar{\rho}_1)$ and $\bar{s}_W$ by $\bar{s}_W(1+\frac{2}{3}\Delta\bar{\rho}_1)$. Here $\bar{s}_W^2$ is the effective $\sin^2\theta_W$ [13, 16]: $\bar{s}_W^2 = (1+\Delta\kappa')s_W^2$, and s_W^2 is defined by [11, 16]:

$$s_W^2 c_W^2 = \frac{\pi\bar{\alpha}(M_Z)}{\sqrt{2}G_F M_Z^2}.$$

By assuming the masses of all the ordinary fermions except the t quark to be much lighter than the mass of the Z boson, and the masses of the exotic fermions to be much heavier than the mass of the Z boson, the total width of the Z boson is given as

$$\Gamma_{\rm total} = \Gamma(Z\to {\rm all}) = \frac{\bar{\rho}_1 G_F}{6\sqrt{2}\pi}M_{Z^1}^3\left\{\cos^2\phi\ \Delta_{\rm total}^{\rm SM} + 3\sin 2\phi\left[G + \frac{\sqrt{3-4\bar{s}_W^2}}{2} - \frac{D}{4\sqrt{3-4\bar{s}_W^2}}\right.\right.$$
$$\left.\left. +\delta_{\rm QCD}\left(G - \frac{E}{4\sqrt{3-4\bar{s}_W^2}}\right) + \frac{\alpha}{12\pi}\left(G - \frac{F}{4\sqrt{3-4\bar{s}_W^2}}\right)\right] + O(\sin^2\phi)\right\}, \tag{41}$$

where

$$D = 5 - \frac{44}{3}\bar{s}_W^2 + \frac{272}{9}\bar{s}_W^4,\quad E = 3 - \frac{20}{3}\bar{s}_W^2 + \frac{128}{9}\bar{s}_W^4,$$
$$F = 33 - \frac{332}{3}\bar{s}_W^2 + \frac{1808}{9}\bar{s}_W^4,\quad G = \frac{\sqrt{3-4\bar{s}_W^2}}{18}(3-2\bar{s}_W^2) - \frac{(3-4\bar{s}_W^2)^{3/2}}{36},$$

and

$$\Delta_{\rm total}^{\rm SM} = \sum_{f=\nu,e,u,d,s,c,b}\{[\bar{g}_V^{\rm SM}(f)]^2 + [\bar{g}_A^{\rm SM}(f)]^2\}(1+\delta_{\rm QED}^f)(1+\delta_{\rm QCD}).$$

We get then the ratio

$$R^{331} = \frac{\Gamma(Z\to l\bar{l})}{\Gamma_{\rm total}} = R_l^{\rm SM}\left\{1 - \frac{2\tan\phi}{\sqrt{3-4\bar{s}_W^2}}\left[1 + \frac{3\sqrt{3-4\bar{s}_W^2}}{\Delta_{\rm total}^{\rm SM}}\left(G + \frac{\sqrt{3-4\bar{s}_W^2}}{2} - \frac{D}{4\sqrt{3-4\bar{s}_W^2}}\right.\right.\right.$$
$$\left.\left.\left. +\delta_{\rm QCD}\left(G - \frac{E}{4\sqrt{3-4\bar{s}_W^2}}\right) + \frac{\alpha}{12\pi}\left(G - \frac{F}{4\sqrt{3-4\bar{s}_W^2}}\right)\right)\right] + O(\tan^2\phi)\right\}, \tag{42}$$

where $R_l^{\rm SM}$ denotes the SM result: $R_l^{\rm SM} = 0.033\,62$ for [11, 17] $\alpha^{-1}(M_Z) = 128.87$, $\alpha_s(M_Z) = 0.118$, and [18] $\bar{s}_W^2(M_Z) = 0.2333$. Taking the experimental result in [11] $\Gamma = (3.367 \pm 0.006)\%$, we obtain the limit for the mixing angle

$$-0.0021 \le \phi \le 0.000\,132. \tag{43}$$

As is known, recent results on left-right asymmetry A_{LR} at SLD [19] and $R_b \equiv \Gamma(Z\to b\bar{b})/\Gamma(Z\to {\rm hadrons})$ measured at the CERN e^+e^- collider LEP [20] indicate a possible disagreement at the 2 to 2.5 σ level with the SM prediction ($R_b^{\rm SM} = 0.215$ for M_t=175 GeV). If confirmed, this could indicate new physics coupled in a different way to the third generation. Therefore, it is interesting to consider R_b in this model. After some manipulations we get

$$R_b^{331} = \frac{\Gamma(Z\to b\bar{b})}{\Gamma_{\rm hadrons}} = R_b^{\rm SM}\left\{1 - 2\tan\phi\left[\frac{9-12\bar{s}_W^2+8\bar{s}_W^4}{9\sqrt{3-4\bar{s}_W^2}A_b}\right.\right.$$
$$\left.\left. + \frac{3}{(B + C_h\frac{\alpha}{12\pi})}\left(G - \frac{E}{4\sqrt{3-4\bar{s}_W^2}} + \frac{\alpha}{12\pi}\left(G - \frac{F_h}{4\sqrt{3-4\bar{s}_W^2}}\right)\right)\right] + O(\tan^2\phi)\right\}, \tag{44}$$

where $R_b^{\rm SM}$ is the SM result [11, 14]: $R_b^{\rm SM} = 0.215$ and

$$A_b = \frac{3}{2}\left(1 - \frac{4}{3}\bar{s}_W^2 + \frac{8}{9}\bar{s}_W^4\right), \quad B = \frac{15}{2} - 14\bar{s}_W^2 + \frac{44}{3}\bar{s}_W^4,$$
$$C_h = \frac{33}{2} - 38\bar{s}_W^2 + \frac{140}{3}\bar{s}_W^4,$$
$$F_h = 15 - \frac{116}{3}\bar{s}_W^2 + \frac{512}{9}\bar{s}_W^4.$$

Substituting Eq. (43) into Eq. (44) we get

$$0.214\,95 \le R_b^{331} \le 0.215\,64.$$

This result still disagrees with the recent experimental value $R_b = 0.2192 \pm 0.0018$ measured at LEP [20]. We hope, however, with the inclusion of new heavy particle loop effects like exotic quarks, Higgs scalars or of new box diagrams, this result will be improved[1] and consistent with the experimental data (for recent works on this direction see [22]).

B. Neutrino-electron scattering

The motivation for focusing on the neutrino neutral current scatterings is the following: From the theoretical point of view these reactions are basic processes free from the complications of strong interactions and can be used to determine the parameters of the theories. We emphasize that in the PPF model, these processes are almost the same as in the SM (for this purpose only neutrino-nucleus scattering and atomic parity violation, etc., are suitable). The effective four-fermion interactions relevant to ν-fermion neutral current processes, in this model, are presented as [23]

$$-\mathcal{L}_{\rm eff}^{\nu f} = \frac{\rho_1 G_F}{\sqrt{2}}\bar{\nu}\gamma_\mu(1-\gamma_5)\nu[C_L^f \bar{f}\gamma^\mu(1-\gamma_5)f + C_R^f \bar{f}\gamma^\mu(1+\gamma_5)f], \tag{45}$$

where

$$C_L^f = 2\left[[g_{1V}(\nu)+g_{1A}(\nu)][g_{1V}(f)+g_{1A}(f)] + \frac{M_{Z^1}^2}{M_{Z^2}^2}[g_{2V}(\nu)+g_{2A}(\nu)][g_{2V}(f)+g_{2A}(f)]\right],$$
$$C_R^f = 2\left[[g_{1V}(\nu)+g_{1A}(\nu)][g_{1V}(f)-g_{1A}(f)] + \frac{M_{Z^1}^2}{M_{Z^2}^2}[g_{2V}(\nu)+g_{2A}(\nu)][g_{2V}(f)-g_{2A}(f)]\right].$$

Then the total cross sections for ν_μ-e and $\bar{\nu}_\mu$-e elastic scattering processes are given, respectively, as

$$\sigma(\bar{\nu}_\mu e) = \frac{\rho_1^2 m_e E_\nu G_F^2}{6\pi}\left(1 - \frac{M_{Z^1}^2}{M_{Z^2}^2}\right)^2\left[\cos 2\phi + \frac{(1-2s_W^2)}{\sqrt{3-4s_W^2}}\sin 2\phi\right]^2 [(1-2s_W^2)^2 + 12 s_W^4]$$
$$\equiv \sigma_{\rm SM}(\bar{\nu}_\mu e)\left(1 - \frac{M_{Z^1}^2}{M_{Z^2}^2}\right)^2\left[\cos 2\phi + \frac{(1-2s_W^2)}{\sqrt{3-4s_W^2}}\sin 2\phi\right]^2, \tag{46}$$
$$\sigma(\nu_\mu e) = \frac{\rho_1^2 m_e E_\nu G_F^2}{6\pi}\left(1 - \frac{M_{Z^1}^2}{M_{Z^2}^2}\right)^2\left[\cos 2\phi + \frac{(1-2s_W^2)}{\sqrt{3-4s_W^2}}\sin 2\phi\right]^2 [3(1-2s_W^2)^2 + 4 s_W^4]$$
$$\equiv \sigma_{\rm SM}(\nu_\mu e)\left(1 - \frac{M_{Z^1}^2}{M_{Z^2}^2}\right)^2\left[\cos 2\phi + \frac{(1-2s_W^2)}{\sqrt{3-4s_W^2}}\sin 2\phi\right]^2. \tag{47}$$

In the above equations m_e is the mass of the electron and E_ν is the energy of the incident (anti)neutrino. From Eqs. (46) and (47) we get the ratio of the cross sections:

$$R = \frac{\sigma(\nu_\mu e)}{\sigma(\bar{\nu}_\mu e)} = \frac{3(1-2s_W^2)^2 + 4s_W^4}{(1-2s_W^2)^2 + 12 s_W^4} = 1.1425, \tag{48}$$

which is the same as in the SM [25]. In this model, ρ_1 is a free parameter. However, we can follow Degrassi, Fanchiotti, and Sirlin to put [12, 24] $\rho_1 = 1 + \Delta\rho_t$, where

$$\Delta\rho_t \simeq 0.0031\left(\frac{M_t}{100\ {\rm GeV}}\right)^2.$$

Taking an average value for $\phi = -0.000\,98$, $M_t = 174$ GeV [17], the running $s_W^2 = 0.21$ and putting into Eqs. (46) and (47) the experimental results on $\frac{\sigma(\bar{\nu}_\mu e)}{E_\nu} = (1.17 \pm 0.206) \times 10^{-42}$ cm^2/GeV and $\frac{\sigma(\nu_\mu e)}{E_\nu} = (1.8 \pm 0.32) \times 10^{-42}$

[1]For this purpose, the couplings to *neutral* Higgs scalars of the down-type quarks (the interaction: $\lambda_{1a}\rho^0 \bar{d}_{3L} d_{aR}$ in our model) are very important, since they lead to a contribution proportional to m_b^2 [21].

cm^2/GeV given in [25], the allowed range of the new gauge boson masses are $M_{Z^2} \geq 250$ GeV and 330 GeV, respectively. Thus Eq. (29) gives a limit for the masses of the gauge bosons $Y^{\pm}, X^0$: $M_X \geq 180$ GeV and 230 GeV, respectively. Note that these bounds weakly depend on the mixing angle ϕ, and they could be improved significantly with more precise data.

In our model, the free parameters are $\sin^2\theta_W, M_{Z^2}$, and ϕ which are constrained from experiment. M_{Z^1} is related by Eq. (26) where $M_Z = M_W/\cos\theta_W$ is the prediction for the Z mass in the absence of mixing $\phi = 0$.

VI. DISCUSSION

In this paper, we presented a further development of the 331 model with right-handed neutrinos. We have shown that this model has some advantages over the original 331 model. First, in the Higgs sector, we need only three Higgs triplets for generating fermions and gauge bosons masses as well as for breaking the gauge symmetry. Moreover in the limit $\phi = 0$, all couplings of the ordinary fermions to Z^1 boson are the same as in the SM. In this model there is no limit for the Weinberg angle $\sin^2\theta_W < \frac{3}{4}$.

In our model, the lepton number is violated both in the Higgs sector and in the heavy charged and neutral (non-Hermitian) vector bosons interactions. We also have flavor-changing neutral currents in the quark sector coupled to the new Z^2 boson. All the heavy bosons have masses depending on $\langle\chi\rangle$ and this VEV is, in principle, arbitrary.

Processes like neutrinoless double β decay and neutrino oscillations ($\nu_a \rightarrow \nu_b$; $a \neq b$), etc., are typical ones in this model.

Finally, we emphasize again that experimental data from the Z decay and $\bar{\nu}_\mu$-e, ν_μ-e scattering processes allows us to estimate the mixing angle ϕ and the new gauge boson masses. To get stronger limits we have to consider other parameters such as the left-right cross section asymmetry, N_ν, etc., and we will publish these results elsewhere [26].

To summarize, we have shown that because of the Z-Z' mixing there is a modification to the Z^1 coupling proportional to $\sin\phi$, and the Z decay gives $-0.0021 \leq \phi \leq 0.000132$. The data from neutrino neutral current elastic scatterings gives a lower bound for mass of the new neutral gauge boson M_{Z^2} in the range of 300 GeV, and from the symmetry-breaking hierarchy we get $M_{Y^+} \simeq M_{X^0} \simeq 0.72 M_{Z^2} \geq 220$ GeV. We think that new physics can arise at not too high energies.

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