

FRW-type cosmologies with adiabatic matter creation

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Some properties of cosmological models with matter creation are investigated in the framework of the Friedmann-Robertson-Walker line element. For adiabatic matter creation, as developed by Prigogine and co-workers, we derive a simple expression relating the particle number density n and energy density ρ which holds regardless of the matter creation rate. The conditions to generate inflation are discussed and by considering the natural phenomenological matter creation rate $\psi = 3\beta nH$, where β is a pure number of the order of unity and H is the Hubble parameter, a minimally modified hot big-bang model is proposed. The dynamic properties of such models can be deduced from the standard ones simply by replacing the adiabatic index γ of the equation of state by an effective parameter $\gamma_* = \gamma(1 - \beta)$. The thermodynamic behavior is determined and it is also shown that ages large enough to agree with observations are obtained even given the high values of H suggested by recent measurements.

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I. INTRODUCTION

The origin of the material content (matter plus radiation) filling the presently observed universe remains one of the most fascinating unsolved mysteries in cosmology even though many authors tried to understand the matter creation process and its effects on the evolution of the universe [1–27].

Radiation and matter constituents can quantum mechanically be produced in the context of Einstein's theory or, more generally, in any relativistic theory of gravitation. Such a process has been systematically investigated by Parker and co-workers [1] by considering the Bogoliubov mode-mixing technique in quantum field theory. This approach, roughly speaking, follows naturally from the fact that in curved space times, as well as in accelerated frames, it is usually impossible to fix *a priori* a unique vacuum state for quantum fields [2]. In particular, this means that an observer with a detector will detect at late times a nonvanishing flux of particles in a state initially set up to be empty of particles (Fulling-Unruh-Davies effect). Unfortunately, due to expected back-reaction effects, it is not so clear that such a mechanism can account for sufficient particle creation to explain either the cosmic background radiation or the matter content of the universe.

An alternative approach to matter creation, the development of which gave rise to deep physical insights for theoretical cosmology, was suggested by Tryon [3] and independently by Fomin [4]. They argued that if the net value of all

conserved quantities of the universe is zero, as for instance, the total energy (gravitational plus material), then a universe whose duration is quantumly restricted by the uncertainty relation $\Delta E \Delta t \geq \hbar/2$, could have emerged as a vacuum fluctuation. No specific scenario was proposed by these authors; however, in such an approach, the universe should be spatially closed in order to have all net charges identically zero. These ideas guided Zeldovich and other researchers to investigate the possibility that the classical space time came into existence from a quantum-gravitational tunneling process termed "spontaneous birth of the universe" [5,6].

A different but somewhat related line of development was pursued by Brout, Englert, and Gunzig [7]. They proposed a concrete scenario that provided a simultaneous generation of matter and curvature from a quantum fluctuation of the Minkowski space-time vacuum. In this model, after a first stage of creation, the universe enters a de Sitter phase by which some cosmological problems (horizon, flatness, etc.) are solved. In a subsequent period, the system finally achieves the standard Friedmann-Robertson-Walker (FRW) phase.

Many attempts to treat the matter creation process at a phenomenological macroscopic level have also long been considered in the literature based on rather disparate motivations (for a review of early literature see Ref. [8]). There have also been some claims [9–11], that particle creation during or near the Planck era could classically be modeled by bulk viscosity stress (second viscosity). This is an interesting connection since irreversible processes are believed to play a fundamental role in the problem of time asymmetry [12]. In this case, the usual thermodynamic "arrow of time," translated in this context as entropy generation due to matter creation, could provide a natural explanation of the "arrow

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of time” in the cosmological domain. The enlargement of the traditional FRW equilibrium equations to include these effects has also, at least, two additional goals: to explain the observed high entropy of the cosmic background radiation and to avoid the initial singular state existing in the standard equations [13–16].

More recently, irreversible processes have become the subject of study once again in connection with inflationary universe scenarios [17–20]. The basic idea is that bulk viscosity (matter creation) contributes at the level of the Einstein field equations (EFE’s) as a negative pressure term. It turns out that effective negative pressures are the key condition to generate inflation. Specifically, Barrow [18] introduced this idea in the framework of the new inflationary scenario. He claimed that particle creation due to nonadiabatic decay of the field driving the slow-rollover inflation can macroscopically be described by the viscous cosmological model found by Murphy [10], for which the bulk viscosity coefficient is proportional to the energy density of the fluid.

The above considerations show that since the very beginning bulk viscosity has been widely interpreted as a phenomenological description of the matter creation process in the cosmic fluid (see Refs. quoted in [21] for recent papers along this line). However, regardless of this macroscopic analogy as well as any microscopic description, it is important in itself to know how matter creation can be incorporated in the classical Einstein field equations. This question was seriously considered in the pioneering article of Prigogine and co-workers [22], who implicitly pointed out that the bulk viscosity and matter creation are not only independent processes but, in general, lead to different histories of the universe evolution. They argued that, at the expense of the gravitational field, matter creation can occur only as an irreversible process constrained by the usual requirements of nonequilibrium thermodynamics. The crucial ingredient of the new approach is the explicit use of a balance equation for the number of created particles in addition to the Einstein field equations. When properly combined with the thermodynamic second law, such an equation leads naturally to a reinterpretation of the stress tensor corresponding to an additional negative pressure term which, as should be expected, depends on the matter creation rate. This is in marked contrast to the bulk viscosity formulation, in which entropy is produced but, the number particle conservation law is taken for granted. These results were further discussed and generalized by Calvão, Lima, and Waga [23,24] through a covariant formulation allowing specific entropy variation as usually expected for nonequilibrium processes in fluids. The issue of why the processes of bulk viscosity and matter creation are not equivalent either from a dynamic or a thermodynamic point of view has been recently discussed in the literature [25,26].

The macroscopic irreversible approach to matter creation has also been applied by these authors to early universe physics. For instance, Prigogine *et al.* [22] obtained a scenario quite similar to that proposed in Ref. [7], in which the universe emerges from an initial Minkowski vacuum. Thus, to a certain extent, this work can be viewed as the macroscopic counterpart of the ideas originally proposed by Tyron [3] and Fomin [4] and further semiclassically developed in the model proposed by Brout *et al.* [7]. However, unlike the

reversible semiclassical equations considered in the latter, the phenomenological approach provides, in a natural way, the entropy burst accompanying the production of matter.

In this article we focus our attention on the “adiabatic” matter creation as originally formulated in Ref. [22] and somewhat clarified in [24]. As we shall see, unlike the standard model, to construct a definite scenario with matter creation one needs to solve a system of three coupled differential equations since the balance equation for the number density has been added to the pair of independent EFE’s. In principle, the full integration of such a system is not a trivial task because it depends on the somewhat unknown matter creation rate. However, as will be seen, it is possible to establish a simple relation between the equations for the particle number and energy density which holds regardless of the matter creation rate. This result will allow us to write the differential equation for the scale factor in terms only of the matter creation rate, thereby simplifying the analysis of the physically admissible models as well as their comparison with the bulk viscous universes. The conditions to generate inflation in this context will be generically discussed; however, unlike previous work on this subject [22–27], we are more interested in the latter stages of universe evolution. In this sense, a new class of cosmologies endowed with matter creation, leading to definite predictions in the present phase, is proposed. As argued, for all values of the curvature parameter, this is the simplest class of hot big-bang cosmologies driven by the matter creation process. As in the standard model, the thermodynamic behavior is readily computed and it is also shown that ages large enough to agree with observation can be obtained even given the high values of the Hubble parameter suggested by the recent measurements [28,29].

II. FRW EQUATIONS WITH MATTER CREATION

Let us now consider the FRW line element ($c=1$)

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2 \right), \quad (1)$$

where $k=0, \pm 1$ is the curvature parameter of the spatial section and R is the scale factor.

In that background, the nontrivial EFE for a fluid endowed with matter creation and the balance equation for the particle number density can be written as [23–25]

$$8\pi G\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \quad (2)$$

$$8\pi G(p + p_c) = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}, \quad (3)$$

$$\frac{\dot{n}}{n} + 3\frac{\dot{R}}{R} = \frac{\psi}{n}, \quad (4)$$

where an overdot means time derivative and ρ , p , n , and ψ are the energy density, thermostatic pressure, particle number density, and matter creation rate, respectively. The creation pressure p_c depends on the matter creation rate, thereby coupling Eqs. (3) and (4) to each other and, although indi-

rectly, both of them with (2) through the energy conservation law which is contained in the EFE's themselves. For "adiabatic" matter creation, this pressure assumes the form (see Ref. [24] for a more general expression)

$$p_c = \frac{\rho + p}{3nH} \psi, \quad (5)$$

where $H = \dot{R}/R$ is the Hubble parameter.

As it stands, the above system is undetermined since there are six unknowns: ρ , p , p_c , n , R , ψ , and only three equations plus the constraint (5). It thus follows that one needs to provide two more relations in order to construct a definite cosmological scenario with matter creation. The first constraint takes the form of an equation of state which is supplied by thermodynamical considerations. The one usually employed in cosmology is the so-called "gamma-law" equation of state

$$p = (\gamma - 1)\rho, \quad (6)$$

where the constant γ lies in the interval $[0,2]$. The second constraint is a specification of the exact form of the matter creation ψ (which should be determined from a more fundamental theory involving quantum processes). At this point, the procedure followed in the literature has been: (a) to integrate Eq. (4) assuming a given phenomenological law for ψ , and (b) to insert the expression of n into (5) and using (6) to obtain the evolution equation for the scale factor. Here, we consider a more general and somewhat more comprehensive approach. Firstly, we will visualize the kind of coupling existing among the balance equation (4) and the EFE. To that end, we establish the differential equation for R as a function of ψ and n . Combining Eqs. (2) and (3) with (5) and (6) it follows that

$$R\ddot{R} + \left(\frac{3\gamma - 2}{2} - \frac{\gamma\psi}{2nH} \right) \dot{R}^2 + \left(\frac{3\gamma - 2}{2} - \frac{\gamma\psi}{2nH} \right) k = 0. \quad (7)$$

This is a very enlightening expression in determining the effects of ψ on the evolution of the scale factor. Since the $3nH$ term in (4) measures the variation of n only due to the expansion of the universe, it proves convenient to introduce the dimensionless and in general time-dependent parameter

$$\beta = \frac{\psi}{3nH}, \quad (8)$$

in order to measure the effects of the matter creation rate. As expected, for $\psi = \beta = 0$, Eq. (7) reproduces the general FRW equation, thereby decoupling the subsystem formed by Eqs. (2) and (3) from (4). Indeed, unlike claims by the authors of [22], such decoupling for $\psi = 0$ happens only if the "gamma law" has been adopted (Ref. [30] examined the coupling of (2)–(4) for $\psi = 0$ and a more general equation of state). If ψ is different from zero but $\psi \ll 3nH$, that is, $\beta \ll 1$, the effects of ψ may be safely neglected. Physically, one may expect that the most interesting solutions of (7) arise during the phases in which the parameter β is of the order of unity.

To proceed further, we now establish a general expression relating n and ρ . By inserting (5) and using (6) in the energy conservation law

$$\dot{\rho} + 3(\rho + p + p_c)H = 0, \quad (9)$$

it takes the form

$$\dot{\rho} + 3\gamma\rho H = \gamma\rho \frac{\dot{\psi}}{n}. \quad (10)$$

Therefore, comparing (10) with (4) it follows that

$$\gamma \frac{\dot{n}}{n} = \frac{\dot{\rho}}{\rho}, \quad (11)$$

the solution of which is

$$n = n_0 \left(\frac{\rho}{\rho_0} \right)^{1/\gamma}, \quad (12)$$

where n_0 and ρ_0 are the values of n and ρ at a given instant (from now on the index 0 denotes the present values of the parameters). It is worth mentioning that (12) holds regardless of the specific form assumed for the matter creation rate ψ . It reduces to the right limit for a dust-filled universe since in this case $\rho = nM$, where M is the mass of the created particles. It is also easy to see that (12) does not remain valid in the more general formulation proposed in Ref. [24].

In summary, the set of Eqs. (2)–(6) has been reduced to Eq. (7) together with relation (12). Thus, taking into account relation (2) for the energy density ρ , the integration of the evolution equation for the scale factor depends only on the form of ψ . In principle, since matter creation is essentially a quantum process, the corresponding rates should be obtained from a quantum field theory in the presence of gravitational fields. However, in the absence of a well accepted model, the natural way is to investigate physically interesting solutions of (7) by adopting a phenomenological description. In the case of viscous models, for instance, the analogous step is to prescribe a form of the bulk viscosity coefficient. The one widely adopted in the literature is $\xi = \alpha\rho^\nu$, where α is a dimensional constant and ν lies in interval $[0,1]$ (see Ref. [16]).

For matter creation, Prigogine *et al.* [22] examined the consequences of assuming a rate $\psi = \alpha H^2$ (α constant) for a dust-filled FRW flat model. As one can see from (12) and (2), such a choice corresponds to $\psi = \alpha 8\pi G\rho/3$, that is, the same type of phenomenological expression first considered by Murphy [10] in the context of a cosmology with viscosity. However, as discussed in [25], the models are quite different from a physical point of view. Of course, if one chooses $\psi = \psi(n)$, Eq. (12) can always be used to rewrite the relation in terms of ρ . For instance, in the case considered in Ref. [26], ψ proportional to nH^2 implies that ψ scales with $\rho^{1+1/\gamma}$.

III. INFLATION AND MATTER CREATION

As is well known, inflation is a theory of the early universe based not solely on the possible existence of a primordial scalar field; to obtain the required dynamics, the potential of such a field needs to be able to generate, at least for a finite period, a state of negative stress, which is the key condition to realize inflation. As shown by Guth and Sher [31], a prerequisite for inflation to work is a departure from thermo-

dynamic equilibrium. In this context, it is naturally of interest to establish, at least qualitatively, how inflation can be implemented in the present irreversible matter creation theory. In order to analyze this issue, we first define an effective “adiabatic index” $\gamma_* = (1 - \psi/3nH)\gamma$, so that Eq. (7) assumes the FRW-type form

$$R\ddot{R} + \left(\frac{3\gamma_* - 2}{2}\right)\dot{R}^2 + \left(\frac{3\gamma_* - 2}{2}\right)k = 0, \quad (13)$$

which can be obtained using only the EFE (2) and (3) together with the effective equation of state $p = (\gamma_* - 1)\rho$. As usual, for early times, we will neglect the spatial curvature contributions. In this case, Eq. (9) can be rewritten as

$$\dot{H} + \frac{3\gamma_*}{2}H^2 = 0. \quad (14)$$

Hence, from the expression for γ_* we see that the condition for exponential inflation ($\dot{H} = 0$) is given by

$$\psi = 3nH, \quad (15)$$

or equivalently [from (8)], $\beta = 1$. Physically this is not a surprising fact, since the matter creation rate given by (15) has exactly the value that compensates for the dilution of particles due to expansion. As we shall see in Sec. V, exponential inflation occurs isothermally so that there is not extreme supercooling and violent subsequent reheating, as happens in all variants of inflation driven by a scalar field. As a matter of fact, with continuous matter creation, even in the power-law inflationary case, the decrease in temperature is much less than in the adiabatic case. The physical reason is quite simple. In this context, the entropy generation is concomitant with inflation, differently than what happens in the usual inflationary variants, where entropy is generated after inflation by a highly nonadiabatic process. In the new inflationary scenario, for instance, the temperature should decrease during the slow-rollover phase at least by a factor or 10^{-28} in order to maintain the radiation entropy constant. In the present scenario, the matter creation continuously reheats the medium so that the temperature need not decrease so drastically. Essentially, this is the same result arising in the framework of inflationary models driven by bulk viscosity (see for instance Ref. [19]).

Now, recalling that a violation of the strong energy condition ($\gamma_* < \frac{2}{3}$) is a sufficient condition for “power law” inflation, one can show that (15), in this case, must be replaced by

$$\psi > \left(1 - \frac{2}{3\gamma}\right)3nH, \quad (16)$$

or $\beta > 1 - 2/3\gamma$. Note that due to the matter creation process, either exponential or “power law” inflation are now compatible with the existence of usual matter described by the “ γ -law” equation of state. In fact, Eqs. (15) and (16) do not impose any constraint on the γ parameter. In this way, one may now refer, for instance, to a radiation or dust dominated “power-law” inflation, as well as to different classes of de Sitter models (see also Ref. [22]). In addition, it is easy to see that the energy density, for each value of γ , scales with

the temperature in the same fashion as happens for equilibrium states. For instance, in the era of radiation ($\gamma = 4/3$) one obtains $\rho = aT^4$. Only the time dependence of the quantities is modified [see Sec. V, specially Eq. (42)].

It should be observed that irreversible matter creation may also describe the so-called super (or pole) inflationary expansion ($\dot{H} > 0$) in the terminology of Ref. [32]. This kind of scenario appears, for instance, in string theory when taking into account the effects of the dilaton, a scalar field capable of driving superinflation. Such a model has recently been proposed as a possible alternative to standard inflation (see Ref. [33] and references therein). In the present context, as one can see from (14), the condition $\dot{H} > 0$ will be satisfied if $\gamma_* < 0$, that is, $\psi > 3nH$.

Another important point is related with the end of inflation, that is, the beginning of the FRW-type expansion. As the reader may conclude himself, the condition for inflation to come to an end can be obtained by the onset of violation of the above inequality (16). In fact, rewriting (14) as

$$\frac{\ddot{R}}{R} = \left(1 - \frac{3\gamma_*}{2}\right)H^2, \quad (17)$$

it is clear that the stage of accelerated expansion will finish ($\ddot{R} = 0$) when $\psi = (1 - 2/3\gamma)3nH$, that is, $\beta = 1 - 2/3\gamma$. For instance, for $\gamma = 4/3$ the universe evolves naturally from an exponential inflation to a FRW-type expansion provided that the matter creation decreases in the interval $3nH/2 \leq \psi \leq 3nH$. For dust ($\gamma = 1$), a slightly different interval is required since the inflationary period will finish when $\psi = nH$, that is, $\beta = 1/3$. Parenthetically, we remark that such conditions do not constrain the magnitude of the Hubble parameter to assume any specific value. As we shall see in Sec. VI, this fact is closely related with the solution of the age problem which plagues the standard model for all values of the curvature parameter.

IV. THE SIMPLEST CLASS OF MODELS

Having in mind the choices previously made for ψ we now propose a specific matter creation scenario with a slightly modified creation rate. As remarked in the introduction, we are not interested here in presenting a complete cosmological scenario with matter creation, that is, a model describing the very early universe (including inflation) and the late stages of the evolution. Our goal is a much more limited one. We try to formulate a basic scenario or equivalently, a kind of hot big-bang model minimally modified due to the matter creation, over which the inflationary mechanism, as discussed in the later section, or any other process, may be further implemented.

In our opinion, the simplest possible case, and probably the most physically appealing too, at least for times later than the Planck era, is the one for which the characteristic time scale for matter creation is the Hubble time itself. Phenomenologically, this is equivalent [using Eq. (8)] to taking

$$\psi = 3\beta nH, \quad (18)$$

where now β is a constant, which is presumably given by the particular physical model of matter creation. The above cre-

ation rate also simplifies considerably the task of solving Eq. (13), since the effective “adiabatic index” becomes $\gamma_* = \gamma(1 - \beta) = \text{const}$. In this case, it is readily seen that the generalized second-order FRW equation for $R(t)$ given by (13) can be rewritten as

$$R\ddot{R} + \Delta\dot{R}^2 + \Delta k = 0, \quad (19)$$

the first integral of which is

$$\dot{R}^2 = \frac{A}{R^{2\Delta}} - k, \quad (20)$$

where $\Delta = [3\gamma(1 - \beta) - 2]/2$ and A is a positive constant [see Eq. (2)].

Using (20) one may express the energy density, the pressures (p and p_c), and the particle number density as functions solely of the scale factor R and of the β parameter. In fact, inserting (20) into (2), one obtains

$$\rho = \rho_0 \left(\frac{R_0}{R} \right)^{3\gamma(1-\beta)}, \quad (21)$$

where $\rho_0 = 3A/8\pi GR_0^{3\gamma_*}$. The above equation shows that the densities of radiation and dust scale, respectively, as $\rho_r \sim R^{-4(1-\beta)}$ and $\rho_d \sim R^{-3(1-\beta)}$. Hence, in a model with radiation and matter, the transition from radiation to a dust dominated phase, in the course of the expansion, happens exactly as in the standard model. Note also that, although formally defined by the same expression [see (18)], the creation rates of radiation ($\gamma = 4/3$) and dust ($\gamma = 1$) are not equal, as they seem to be at first sight. For these cases, the above results are easily recovered in the usual manner, e.g., defining $n = n_r + n_d$ and similar forms for the energy density and pressure. The corresponding dominant component then will determine the final form of all physical quantities [see also comment below Eq. (24)].

Now, from (21), (5), and (6),

$$p_c = -\beta\gamma\rho_0 \left(\frac{R_0}{R} \right)^{3\gamma(1-\beta)}, \quad (22)$$

with the total pressure $P_t = p + p_c$ assuming the form

$$P_t = (\gamma_* - 1)\rho = [\gamma(1 - \beta) - 1]\rho_0 \left(\frac{R_0}{R} \right)^{3\gamma(1-\beta)}. \quad (23)$$

Finally, by integrating (4) with ψ given by (18), or more directly, using Eqs. (12) and (21), the expression for the particle number density can be written as

$$n = n_0 \left(\frac{R_0}{R} \right)^{3(1-\beta)}. \quad (24)$$

A clarifying comment about the meaning of Eqs. (21)–(24) is now in order. Firstly, we note that (24) does not depend explicitly on the γ parameter. Thus, the same scale law describes the evolution of the particle number density either for a dominant or a nondominant component. The effect of matter creation in both situations is measured by the β parameter. The situation is clearly different for the remaining equations, even though they can also be applied for the

nondominant component. In other words, for each expansion stage, the explicit time-dependent form of the energy density, equilibrium and creation pressures as well as the scale factors [see Eq. (25)], depends exclusively on the dominant component; however, the creation of the other component is not completely suppressed. Of course, this is the same kind of approximation commonly used in the standard FRW model. The only difference is that even in the radiation era, the dust component will have a nonvanishing creation pressure [see Eq. (22)], which although negligible in comparison with the creation pressure of radiation, will be responsible for the baryon production in that phase. Such considerations will be important when we discuss the thermodynamic behavior of these models (see Sec. V).

As expected, for $\beta = 0$, Eqs. (18)–(24) reduce to those of the standard FRW model for all values of the parameters k and γ . In this case, the unified solution of (19) or, equivalently (20), was found by Assad and Lima [34] in terms of hypergeometric functions. Of course, such a solution can be adapted to the present case simply by replacing the “adiabatic index” γ with the effective parameter γ_* .

For $k = 0$, the solution of (19) for all values of γ and β can be written as

$$R = R_0 \left[1 + \frac{3\gamma(1-\beta)}{2} H_0(t-t_0) \right]^{2[3\gamma(1-\beta)]}. \quad (25)$$

Note that in the limit $\gamma \rightarrow 0$ the above solution describes a de Sitter type universe for any value of β . As remarked in the previous section, such a solution can now be obtained for $\beta \rightarrow 1$ and γ assuming arbitrary values. Further, for flat models with $\gamma_* > 0$, we can choose $t_0 = 2H_0^{-1}/3\gamma(1-\beta)$, so that (25) assumes a more familiar form:

$$R(t) = R_0 \left[\frac{3\gamma(1-\beta)}{2} H_0 t \right]^{2[3\gamma(1-\beta)]}. \quad (26)$$

If $\beta = 0$, (25) and (26) reduce to the well-known expressions for the flat FRW model.

For $k \neq 0$, parametric solutions are usually more enlightening. By introducing the conformal time coordinate,

$$dt = R d\eta, \quad (27)$$

(19) can be recast as

$$RR'' + (\Delta - 1)R'^2 + \Delta k R^2 = 0, \quad (28)$$

where the primes denote conformal time derivatives.

Now, defining an auxiliary scale factor $z = R^\Delta$, it is easily seen that (28) becomes

$$z'' = 0 \quad \text{if} \quad \gamma = \frac{2}{3(1-\beta)}, \quad (29)$$

and

$$z'' + k\Delta^2 z = 0 \quad \text{if} \quad \gamma \neq \frac{2}{3(1-\beta)}, \quad (30)$$

whereas the first integral (20) is transformed into the energy conservation equation

$$\frac{1}{z}z'^2 + \frac{1}{2}k\omega^2z^2 = \frac{1}{2}\omega^2z_*^2, \tag{31}$$

where $\omega = |[3\gamma(1-\beta)-2]/2|$ and $z_* = R_*^{2\Delta}$.

As one can see by direct substitution, the general solution of (30) or equivalently (31) is

$$z = z_* \frac{\sin\left(\sqrt{k} \left| \frac{3\gamma(1-\beta)-2}{2} |(\eta + \delta) \right. \right)}{\sqrt{k}}, \tag{32}$$

where δ is an integration constant. Now, by choosing $\delta=0$ and using the inverse transformation $R = z^{1/\Delta}$, the general solution for the scale factor takes the form

$$R(t) = R_* \left[\sin\left(\sqrt{k} \left| \frac{3\gamma(1-\beta)-2}{2} |\eta \right. \right) \right]^{2/[3\gamma(1-\beta)-2]}, \tag{33}$$

and

$$t(\eta) = R_* \int d\eta \left[\sin\left(\sqrt{k} \left| \frac{3\gamma(1-\beta)-2}{\sqrt{k}} |\eta \right. \right) \right]^{2/[3\gamma(1-\beta)-2] + \text{const.}} \tag{34}$$

It should be remarked that the auxiliary scale factor $z = R^\Delta$ shows the same dynamic behavior appearing in the standard FRW model, namely it evolves as a free particle ($k=0$), a simple harmonic oscillator ($k=1$), or an “anti-oscillator” ($k=-1$). In this sense, some basic characteristics of the standard FRW models are not modified, namely that open and flat universes expand forever whereas closed geometries exhibit a turning point either when the universe expands away from the singularity ($R=0$) or starts contracting from $R=\infty$ with the models presenting a big bounce. For completeness, we observe that the physical meaning of z_* or equivalently R_* is readily obtained from the first integral (20). For instance, in the case of closed geometries with $\gamma_* \neq 2/3$, R_* is just the value of R at the turning point, that is, for which $\dot{R}(R_*)=0$. Accordingly, in the conformal time description we see from (31) that the turning point z_* ($\gamma_* \neq 2/3$) corresponds to the amplitude of the related spring-mass system of unit mass (SHO). We leave it to the reader to verify that for singular flat models $R_* = R_0$.

V. THERMODYNAMIC BEHAVIOR

The matter creation formulation considered here is a clear consequence of nonequilibrium thermodynamics in the presence of gravitational fields [22–24]. In this context, unlike other approaches to matter creation proposed in the literature (cf., for instance [35], and [36]), the explicit thermodynamic connection leads naturally to specific predictions on rates of variation of the entropy per particle and of the temperature. As shown in Ref. [24], for the case of adiabatic matter creation, these rates are

$$\dot{\sigma} = 0, \tag{35}$$

$$\frac{\dot{T}}{T} = \left(\frac{\partial p}{\partial \rho} \right)_n \frac{\dot{n}}{n}, \tag{36}$$

where σ is the dimensionless specific entropy and T is the temperature.

For pedagogical convenience we first discuss the time dependence of the temperature. Using the γ -law equation of state, the temperature evolution equation (36) takes the form

$$\frac{\dot{T}}{T} = (\gamma - 1) \frac{\dot{n}}{n}, \tag{37}$$

the integral of which is

$$n(T) = n_0 \left(\frac{T}{T_0} \right)^{1/(\gamma-1)}. \tag{38}$$

Now, replacing into (38) the deduced relation between ρ and n given by (12), the former can be written as

$$\rho(T) = \rho_0 \left(\frac{T}{T_0} \right)^{\gamma/(\gamma-1)}. \tag{39}$$

The above expressions for $n(T)$ and $\rho(T)$ are exactly the general expressions obeyed by a γ fluid in the course of adiabatic expansion [37]. As remarked in Sec. II, for the case of radiation ($\gamma=4/3$) the energy and particle number densities scale, respectively, as $\rho_r \approx T^4$ and $n_r \approx T^3$ so that the created radiation necessarily satisfies the usual equilibrium relations. This is a remarkable result. Different from other approaches for matter creation, where expressions like (38) and (39) need to be assumed (see, for instance, Ref. [36]), the equilibrium relations here follow as a consequence of the “adiabatic” condition. In fact, as discussed in detail in the literature [24,25], the condition (35) determines simultaneously the creation pressure from (5) and the temperature equation given by (36). On the other hand, combining $\rho(R)$ given by (21), with (42), we obtain the temperature law

$$T = T_0 \left(\frac{R_0}{R} \right)^{3(\gamma-1)(1-\beta)}. \tag{40}$$

The above result shows us that exponential inflation ($\beta=1$) occurs isothermally regardless of the value of γ (see Sec. II). In fact, generically, the β parameter works in the opposite sense of the expansion, that is, diminishing the cooling rate with respect to the case with no matter creation. In particular, instead of the usual result, $RT = \text{const}$, valid for radiation ($\gamma=4/3$), we find $TR^{1-\beta} = \text{const}$. Note also that by integrating (37) with $n = N/R^3$, the temperature law assumes a new form where the β parameter does not appear explicitly: namely, $N^{(1-\gamma)}TR^{3(\gamma-1)} = \text{const}$, which makes transparent the conclusion that if $N = \text{const}$, the usual evolution law is recovered. This formula does not depend on the specific creation rate assumed in the present paper. In particular, for $\gamma=4/3$ one has

$$N^{-1/3}TR = \text{const}, \tag{41}$$

as one should expect (see Ref. [24]). Now, recalling that for FRW geometries the frequency redshifts obeying, $\nu \sim R^{-1}$, the above temperature law leads inevitably to the conclusion that the usual Planckian spectrum is destroyed in the course of the evolution, in particular, after decoupling. Nevertheless, as recently shown by one of us [38], for “adiabatic” matter creation the preserved spectral distribution is given by

$$\rho_T(\nu) = \left(\frac{N(t)}{N_0}\right)^{4/3} \frac{8\pi h}{c^3} \frac{\nu^3}{\exp\left[\left(\frac{N(t)}{N_0}\right)^{1/3} \frac{h\nu}{kT}\right] - 1}, \quad (42)$$

where $N(t)$ is the comoving time dependent number of photons and N_0 is the constant value of N evaluated at some fixed epoch, say, the present time. The above distribution is a consequence of the temperature evolution law as given by (41). When there is no creation, $N(t) = N_0$, and the usual Planckian form is recovered. In addition, it is readily seen that the equilibrium relations are recovered using such a spectrum. In fact, for $\gamma = \frac{4}{3}$, it follows from (38) and (39) that $n \sim \rho^{3/4}$, and by introducing a new variable $x = (N/N_0)^{1/3} h\nu/kT$, it is easy to see that

$$\rho(T) = \int_0^\infty \rho_T(\nu) d\nu = aT^4, \quad (43)$$

where a is the usual radiation density constant. The spectrum given by (42) seems to be the most natural generalization of Planck’s radiation formula in the presence of “adiabatic” photon production. More important still, (42) cannot be distinguished from the usual blackbody spectrum at the present epoch when we take $T = T_0$ and $N(t_0) = N_0$. Therefore, models with “adiabatic” photon creation may be compatible with the present isotropy and spectral distribution of the microwave background. Of course, since photons are injected satisfying (42), which is preserved in the course of the evolution, there will be no distortions in the present relic radiation spectrum. Note that (42) is preserved precisely due to validity of the temperature law (41). In this concern, the macroscopic formulation adopted here seems to be naturally connected with some fundamental cosmological irreversible mechanism (based on microphysics), in which photons are quantum mechanically produced with the above thermal spectrum and baryons are asymmetrically created [39].

Let us now consider the entropy behavior as defined in (35). Since $\sigma = S/N$, where S and N are, respectively, the entropy of the dominant component and the corresponding number of particles, (35) can be rewritten as

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N}. \quad (44)$$

Hence, due to the matter creation processes, the universe does not expand adiabatically as happens in the standard FRW models [40]. Besides, since up to a constant factor one has $N = nR^3$, it follows from (24) that N increases as a power of R , that is,

$$N = N_0 \left(\frac{R}{R_0}\right)^{3\beta}. \quad (45)$$

Further, from Eq. (37), $S = S_0(N/N_0)$, and using the above expression one may write the photon entropy as [from now on indexes r and b refer, respectively, to radiation and baryon component (dust)]

$$S_r = S_{0r} \left(\frac{R}{R_0}\right)^{3\beta}, \quad (46)$$

where $S_{0r} \approx 10^{87}$ is the present observed radiation entropy (dimensionless). As happens in the standard model, although remaining nearly constant, the specific entropy defined by (35) does not play an important physical role. As we know, some physically meaningful information, as for instance, in nucleosynthesis studies as well as for the structure formation problem, is encoded in the specific radiation entropy per baryon. Such a quantity, defined by $\sigma_{rb} = S_r/N_b$, is proportional to the photon-baryon ratio and, up to short aniquillation period, also remains constant in the standard model. In the present context, since photons are thermally produced and baryons are continuously created, a nearly constant behavior of σ_{rb} should be expected. In fact, the net number of baryons in the comoving volume is given by [see discussion below Eq. (24)]

$$N_b = N_{0b} \left(\frac{R}{R_0}\right)^{3\beta}, \quad (47)$$

where N_{0b} is the present baryon number. It thus follows from (45) that $\sigma_{rb} = S_{0r}/N_{0b} = \sigma_0 \approx 10^{10}$, is the present photon to baryon ratio. As a kind of consistency check, we notice that if one writes the specific entropy (per baryon) in the usual form

$$\sigma_{rb} = \frac{4aT_r^3}{3n_b}, \quad (48)$$

the above result is recovered since T_r^3 and n_b evolve following the same scale law [see Eqs. (40) and (24)]. Therefore, the adiabatic formulation does not provide any explanation for the present value of photon to baryon ratio. As happens in the big-bang model, the value of σ_0 is just an initial condition. In the present model, particles are created in space time with the same temperature as the already existing ones have, otherwise the specific entropy could not remain constant, as happens in the more general formulation proposed in Ref. [24]. Naturally, from the thermodynamic point of view the model is irreversible. The burst of entropy is closely related with the creation of matter and radiation.

We would like to stress that all important thermodynamic results of the standard FRW models such as $S = S_0$, $N = N_0$, and the radiation temperature scaling $T \propto R^{-1}$ are recovered for $\beta = 0$. Finally, it is interesting to remark that the models presented here may significantly alter the standard predictions of cosmic abundances, since they alter the expansion rate and predict a new temperature law. Such results point to possible limitations on the β parameter imposed through constraints from nucleosynthesis. This issue will be addressed elsewhere.

VI. SOME OBSERVATIONAL ASPECTS

Now we illustrate some observable predictions of the models proposed in the preceding sections. Following standard lines we define the physical parameters $q = -R\ddot{R}/\dot{R}^2$ (deacceleration parameter), $H = \dot{R}/R$ (Hubble parameter), and $\Omega = \rho/\rho_c$ (density parameter), where $\rho_c = 3\dot{H}^2/8\pi G$ is the critical density.

Inserting the above quantities into Eqs. (2) and (19) we have

$$\Omega = \frac{2q}{3\gamma(1-\beta) - 2} \tag{49}$$

and

$$\frac{k}{R^2} = (\Omega - 1)H^2. \tag{50}$$

Therefore, it is clear that if $\Omega > 1$, that is, $q > [3\gamma(1-\beta) - 2]/2$ the universe is positively curved with $\rho > \rho_c$, whereas if $q \leq [3\gamma(1-\beta) - 2]/2$ it is negatively curved or flat, respectively, with $\rho \leq \rho_c$. For $\beta = 0$ the usual expression for FRW models are recovered. However, the positivity of Ω does not restrict q to be positive if $\gamma > 2/3$ as happens in the standard universe. To be more specific, in the present dust phase ($\gamma = 1$), the above expressions reduce to

$$\Omega_0 = \frac{2q_0}{1 - 3\beta}, \tag{51}$$

$$\frac{k}{R_0^2} = (\Omega_0 - 1)H_0^2. \tag{52}$$

Therefore, if $q_0 = (1 - 3\beta)/2$ we have $\Omega_0 = 1$ and from (52), the presently observed universe is flat. However, regardless of the value of k , the deacceleration parameter may be negative since the constraint $\Omega_0 > 0$ can be satisfied for $q_0 < 0$, provided that $\beta > 1/3$. As we shall see next, such a fact allows us to solve the age problem in the present context.

As we know, the age of the universe is found by integrating the generalized first integral (20). By expressing the constant A in terms of Ω_0 , R_0 , and H_0 it is straightforward to show that

$$\left(\frac{\dot{R}}{R_0}\right)^2 = H_0^2 \left[1 - \Omega_0 + \Omega_0 \left(\frac{R_0}{R}\right)^{2\Delta} \right], \tag{53}$$

the solution of which may be expressed as a formula for the time t in terms of R ,

$$t - t_* = H_0^{-1} \int_{R_*/R_0}^{R/R_0} [1 - \Omega_0 + \Omega_0 x^{-2\Delta}]^{-1/2} dx, \tag{54}$$

where t_* is the time for which $R = R_*$.

For singular models, the present age of the universe is defined by taking $R_* = t(R_*) = 0$ so that it is given by

$$t = H_0^{-1} \int_0^1 [1 - \Omega_0 + \Omega_0 x^{-2\Delta}]^{-1/2} dx, \tag{55}$$

Age of a dust-filled Universe

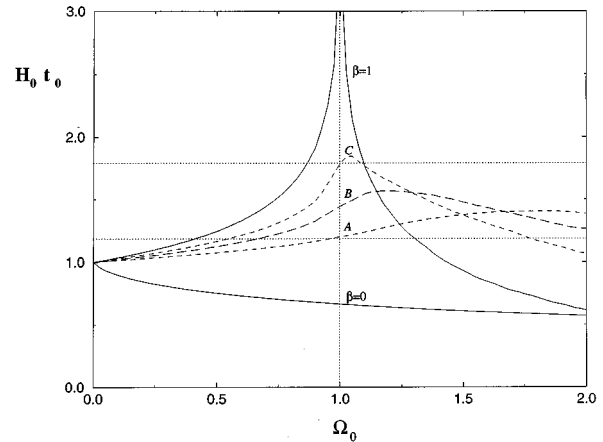


FIG. 1. The age parameter of a matter-dominated universe as a function of Ω_0 for some selected values of β . The horizontal dotted lines indicate the latest observational constraints [see Eqs. (57) and (58)]. The solid curves represent two limiting cases, the standard model ($\beta = 0$) and a de Sitter-type universe ($\beta = 1$). Curves A, B, and C, respectively, represent models with $\beta = 0.44$, $\beta = 0.54$, and $\beta = 0.63$. According to Pierce *et al.*, the standard dust model is ruled out regardless of the value of Ω_0 whereas for Friedman *et al.* only extremely open FRW models may be compatible with the observations. For $\beta > 1/3$ the matter creation process rehabilitates the flat universe, as predicted by inflation.

which for $\beta = 0$, that is, $\Delta = (3\gamma - 2)/2$, is exactly the same as in the standard model. In Figs. 1 and 2 we show the age of the universe in units of H_0^{-1} for the cases of dust and radiation dominated universes and some selected values of β . Observe that the universe can be old enough even when considering a radiation dominated phase today. Another important conclusion is that if $\beta \geq 1 - 2/3\gamma$ and $0 \leq \Omega_0 \leq 1$ the old-

Age of a radiation-filled Universe

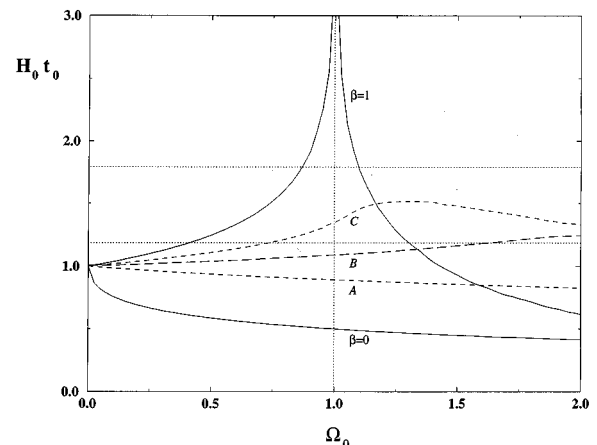


FIG. 2. The same graph of Fig. 1 for a radiation dominated universe. The solid curves now indicate the radiation-filled FRW model ($\beta = 0$) and a de Sitter-like radiation universe ($\beta = 1$). Generically, the curves are displaced downwards in comparison with the case of dust so that the models are compatible with data only for higher values of β .

est universe is the flat one ($\Omega_0=1$). In this context, our matter creation ansatz (18) changes the predictions of standard cosmology in such a way that it solves the problem of reconciling observations with the inflationary scenario. Note also that open models with a small parameter are also ruled out by recent data regardless of the value of β .

The solution of the age problem and its noticeable dependence on the β parameter can be exactly determined for flat models. In this case ($\Omega_0=1$), one can see that the age parameter reduces to

$$H_0 t_0 = 2/3 \gamma (1 - \beta). \quad (56)$$

In a matter-dominated universe we have then $H_0 t_0 = 2/3(1 - \beta)$, and in this form it is easiest to see how matter creation could solve the age problem suggested by the latest direct measurements of the Hubble constant done by Pierce *et al.* [28] and Freedman *et al.* [29] who found, respectively, $H_0 = 87 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $H_0 = 80 \pm 17 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Assuming no matter creation ($\beta=0$), these values of H_0 imply that the expansion age of a dust-filled, flat universe would be about either 7.3×10^9 years or 8.2×10^9 years, in direct contrast to the measured ages of some stars and stellar systems, believed to be at least some $(16 \pm 3) \times 10^9$ years old or even older if one adds a realistic incubation time [41]. Such measurements restrict the parameters $H_0 t_0$ to the following intervals (P and F refer to the values of H_0 given, respectively, in Refs. [28] and [29])

$$1.09 \leq H_0 t_0 \leq 1.86 \quad (\mathbf{P}) \quad (57)$$

and

$$0.85 \leq H_0 t_0 \leq 1.91 \quad (\mathbf{F}), \quad (58)$$

when in the standard flat model ($\gamma=1, \beta=0$), one would obtain exactly $2/3$. In this fashion, these recent measurements point to a serious crisis of the standard model. It would not be surprising if expectations that it could, at least marginally, be compatible with the age of the universe, will gradually be forgotten (see Figs. 1 and 2).

As can be easily seen from (56), matter creation naturally solves this problem, increasing the parameter $H_0 t_0$ while preserving the overall FRW evolution scheme. From (56), (57), and (58), the constraints on the β parameter for both cases are readily computed to be

$$0.38 \leq \beta \leq 0.64 \quad (\mathbf{P}) \quad (59)$$

and

$$0.21 \leq \beta \leq 0.64 \quad (\mathbf{F}). \quad (60)$$

Note that due to the error bar in the values of H_0 , the upper bound of β does not depend on the particular set of measurements. Naturally, a more realistic range of β probably will have an upper bound slightly smaller than 0.64 and a lower bound between 0.21 and 0.38. This will happen, for instance, if the future measurements of H_0 improve by at least one order of magnitude. Parenthetically, there are some independent indications requiring the present expansion rate to lie in the range $H_0 = 80 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see [42] and refer-

ences therein). With such precision, the β parameter will fall on the interval $0.34 \leq \beta \leq 0.60$.

As remarked earlier, the solution of the age problem is due to the possibility of $q_0 < 0$ nowadays. For instance, from (49) with $\Omega_0=1$ we can rewrite the result (56) as $H_0 t_0 = 1/(1 + q_0)$. For β in the above integral, we have for a dust-filled universe $-0.44 \leq q_0 \leq -0.1$. In connection with this we note that cosmological constant models with a reasonable value of Ω_Λ are also believed to solve easily the ‘‘age problem.’’ However, as recently shown by Maoz and Rix [43], the computed rate of gravitational lensing in such models constrain severely Ω_Λ when confronted with the existing lensing observations. Analogously, such a result seems to point out to similar limits on the β parameter, thereby leading to values $H_0 t_0$ below the range given by (57) or (58). In this concern, we remark that models with matter creation behave like scenarios driven by a decaying Λ term, instead of models with a cosmological constant [36,44]. Moreover, for these kinds of models, a lower lensing rate is usually predicted since the distance to an object with redshift z tends to be smaller than the distance to the same object for a model with Λ constant [45].

As we have discussed in the previous section, photons are always created in equilibrium with the already existing radiation. However, as we have seen, they are not dominant nowadays. In the present dust phase, the matter creation rate is given by

$$\psi_0 = 3 \beta n_0 H_0. \quad (61)$$

For $n_0 \sim 10^{-6} \text{ nucleons cm}^{-3}$ and $H_0^{-1} \sim 10^{10} \text{ yr}$ we have $\psi_0 \sim 10^{-16} \text{ nucleons cm}^{-3} \text{ yr}^{-1}$, which is nearly the same rate predicted by the steady-state universe [8,35] and also by some decaying Λ cosmologies [44]. Of course, this matter creation rate is presently far below detectable limits.

VII. CONCLUSIONS

Almost all research efforts related to the physical processes in the cosmological domain have been devoted to the standard model in its different phases. As we know, such a model is thermodynamically characterized by two different although related features; namely, entropy conservation ($S_{;\alpha}^\alpha = 0$) and number particle conservation ($N_{;\alpha}^\alpha = 0$), where S^α and N^α are, respectively, the four-vectors of entropy and number of particles.

In this paper we have investigated some cosmological consequences arising when one changes the second (and consequently the first) of the above properties. Of course this is not new as may easily be observed in the extensive literature of this subject (see [8] and references therein). The new fact justifying the present work is that we have considered a recent thermodynamic approach for which matter creation, at the expense of the gravitational field, has been properly constrained by the usual requirements of nonequilibrium thermodynamics [22–25]. In this context, thermodynamic predictions such as the temperature law and the variation rate of the entropy are computed from first principles. Such an approach points to a possible revival of interest in models with radiation and/or matter creation, which is one of the most challenging problems of theoretical physics.

For “adiabatic” matter creation a general expression relating the particle number and energy densities which holds regardless of the creation rate has been deduced and the conditions for generating inflation have also been established. It was also shown that a minimally modified big-bang model with adiabatic matter creation can be easily implemented and predicts interesting cosmological consequences. In fact, for a creation rate $\psi=3\beta nH$, the known perfect fluid solutions can be adapted by using a slightly modified “adiabatic index” $\gamma_* = \gamma(1-\beta)$. The thermodynamic behavior is also readily computed making clear how the above mentioned properties of the standard model are quantitatively changed. In particular, instead of constant entropy and $T \sim R^{-1}$, we found $T \sim R^{-(1-\beta)}$ and $S \sim R^{3\beta}$. In terms of the redshift parameter we may write $T = T_0(1+z)^{1-\beta}$ so that, for $z > 0$, the universe is cooler than the standard model. However, since the specific entropy of radiation is very high, one can show that as long as matter is in thermal contact with radiation, it will follow the same temperature law determined by the radiative component. The model is also able to harmonize a

FRW-type picture with the discrepancy existing between the latest measurements of the Hubble parameter and the age of the universe, as predicted by the standard model. Of course, in order to have a viable alternative to the standard FRW model, other well-known cosmological tests need to be investigated. Further details of our model will be published elsewhere.

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- [1] L. Parker, Phys. Rev. Lett. **21**, 562 (1968); Phys. Rev. **183**, 1057 (1969); S. A. Fulling, L. Parker, and B. I. Hu, Phys. Rev. D **10**, 3905 (1974); B. L. Hu, and L. Parker, *ibid.* **17**, 933 (1978); N. J. Papastamatiou and L. Parker, *ibid.* **19**, 2283 (1979).
- [2] N. D. Birrell and P. C. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
- [3] E. P. Tryon, Nature **246**, 396 (1973).
- [4] P. I. Fomin, Report No. ITF-73-137, Kiev, 1973 (unpublished); Dokl. Akad. Nauk. Ukr. SSR **A9**, 831 (1975).
- [5] Ya. B. Zeldovich, Sov. Astron. Lett. **7**, 322 (1981); L. P. Grishchuk and Ya. B. Zeldovich, in *Quantum Structure of Space-Time*, edited by M. Duff and C. I. Isham (Cambridge University Press, Cambridge, England, 1982).
- [6] A. Vilenken, Phys. Lett. **117B**, 25 (1982).
- [7] R. Brout, F. Englert, and E. Gunzig, Ann. Phys. (N.Y.) **115**, 78 (1979); Nucl. Phys. **B170**, 228 (1980).
- [8] J. V. Narlikar, “Nonstandard Cosmologies,” in *Vth Brazilian School of Cosmology and Gravitation*, edited by M. Novello (World Scientific, Singapore, 1987).
- [9] Ya. B. Zeldovich, JETP Lett. **12**, 307 (1970).
- [10] G. L. Murphy, Phys. Rev. D **48**, 4231 (1973).
- [11] B. L. Hu, Phys. Lett. **90A**, 375 (1982); Adv. Astrop. **1**, 23 (1983).
- [12] B. L. Hu, “Fluctuation, Dissipation, and Irreversibility in Cosmology,” in *The Physical Origin of Time-Asymmetry*, edited by J. J. Halliwell, J. Perez, and W. H. Zurek (Cambridge University Press, Cambridge, England, 1993).
- [13] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
- [14] J. D. Nightingale, Astrophys. J. **168**, 175 (1973).
- [15] Z. Klimek, Acta Cosmologica **3**, 49 (1975).
- [16] V. A. Belinski and I. M. Khalatnikov, Sov. Phys. JETP **45**, 1 (1977).
- [17] M. S. Turner, Phys. Rev. D **28**, 1243 (1983).
- [18] J. D. Barrow, Phys. Lett. **180B**, 335 (1986); Nucl. Phys. **B310**, 743 (1988).
- [19] J. A. S. Lima, R. Portugal, and I. Waga, Phys. Rev. D **37**, 2755 (1988).
- [20] N. Turok, Phys. Rev. Lett. **60**, 549 (1988).
- [21] V. B. Johi and R. Sudharsah, Astron. Lett. Comm. **28**, 217 (1992); A. Beesham, Astron. Lett. Commun. **24**, 233 (1994).
- [22] I. Prigogine, J. Geheinau, E. Gunzig, and P. Nardone, Gen. Relat. Gravit. **21**, 767 (1989).
- [23] J. A. S. Lima, M. O. Calvão, I. Waga, “Cosmology, Thermodynamics and Matter Creation,” in *Frontier Physics, Essays in Honor of Jayme Tiomno* (World Scientific, Singapore, 1990).
- [24] M. O. Calvão, J. A. S. Lima, and I. Waga, Phys. Lett. A **162**, 223 (1992).
- [25] J. A. S. Lima and A. S. M. Germano, Phys. Lett. A **170**, 373 (1992).
- [26] J. Gabriel and G. Le Denmat, Phys. Lett. A **200**, 11 (1995).
- [27] W. Zimdahl and D. Pavón, Mon. Not. R. Astron. Soc. **266**, 872 (1994).
- [28] M. J. Pierce, D. L. Welch, R. D. McClure, S. van den Bergh, R. Racine, and P. B. Stetson, Nature (London) **371**, 29 (1994).
- [29] W. L. Freedman *et al.*, Nature (London) **371**, 27 (1994).
- [30] M. O. Calvão and J. A. S. Lima, Phys. Lett. A **141**, 229 (1989).
- [31] A. Guth and M. Sher, Nature **302**, 505 (1983).
- [32] F. Lucchin and S. Matarrese, Phys. Lett. **164B**, 282 (1985).
- [33] G. Veneziano, in *PASCOS '94, Proceedings of the International Symposium on Particles, Strings, and Cosmology*, Syracuse, New York, 1994, edited by K. C. Wali (World Scientific, Singapore, 1995).
- [34] M. J. D. Assad and J. A. S. Lima, Gen. Relat. Gravit. **20**, 527 (1988).
- [35] F. Hoyle, G. Burbidge, and J. V. Narlikar, Astrophys. J. **410**, 437 (1993).
- [36] K. Freese, F. C. Adams, J. A. Friedman, and E. Mottola, Nucl. Phys. **B287**, 797 (1987); A-M. M. Abdel-Rahman, Phys. Rev. D **45**, 3497 (1992).

- [37] J. A. S. Lima and J. Santos, *Int. J. Theor. Phys.* **34**, 127 (1995).
- [38] J. A. S. Lima, "Thermodynamics of Decaying Vacuum Cosmologies," Preprint Brown-HET-1013, 1995 (unpublished).
- [39] As discussed in Ref. [38], there is a crucial test for this kind of cosmology. From (41) one can see that the temperature redshift relation is given by $T=T_0(1+z)(N/N_0)^{1/3}$. Thus, universes with "adiabatic" matter creation are, for a fixed value of z , cooler than the standard model. Such a relation may be verified observing the atomic or molecular transitions in absorbing clouds at high redshifts [46].
- [40] Note that Eq. (44) can be rewritten as $\dot{S}/S=\psi/n$. It thus follows from thermodynamic second law that $\psi\leq 0$, that is, the spacetime can only create matter, the reverse process being thermodynamically forbidden [22]. In our case, since $\psi=\beta nH$, we see that models with $H<0$ and positive values of β are ruled out from a thermodynamic point of view. For closed models with a turning point, this means that the β parameter must change sign when the universe radius increases beyond its maximum value.
- [41] D. A. VandenBergh, "The Formation and Evolution of Star Clusters," *PASP Conf. Ser.* 13, edited by K. Janes (Astron. Soc. Pacif., San Francisco, 1991), p. 183.
- [42] L. W. Krauss and M. Turner, *Gen. Rel. Gravit.* **27**, 1137 (1995).
- [43] D. Maoz and H-W. Rix, *Astrophys. J.* **416**, 425 (1993).
- [44] J. C. Carvalho, J. A. S. Lima, and I. Waga, *Phys. Rev. D* **46**, 2404 (1992); J. A. S. Lima and J. M. F. Maia, *ibid.* **49**, 5597 (1994).
- [45] L. F. Bloomfield Torres and I. Waga, Report No. astro-ph/9504101, 1995 (unpublished).
- [46] A. Songaila *et al.*, *Nature (London)* **371**, 43 (1994); D. M. Mayer, *ibid.* **371**, 13 (1994).