Local and nonlocal defect-mediated electroweak baryogenesis

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We consider the effects of particle transport in topological defect-mediated electroweak baryogenesis scenarios. We analyze the cases of both thin and thick defects and demonstrate an enhancement of the original mechanism in both cases due to an increased effective volume in which baryogenesis occurs. This phenomenon is a result of an imperfect cancellation between the baryons and antibaryons produced on opposite faces of the defect.

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I. INTRODUCTION

Since the realization [1] that the standard model of electroweak interactions contains all the ingredients neccessary to explain the baryon asymmetry of the universe (BAU), there have been many attempts to show in detail how such an asymmetry may be dynamically generated in this context (see Refs. [2] and [3] for recent reviews).

The neccessary conditions to generate a net baryon number are [4] (1) the existence of baryon-number-violating processes, (2) C and CP violation, and (3) departure from thermal equilibrium.

Almost all presently proposed electroweak baryogenesis scenarios achieve the first requirement by using finite temperature sphaleron transitions [5,6], the second by using an extended, CP-violating Higgs sector for the standard model (C is violated maximally in the Weinberg-Salam theory), and the third by requiring that the electroweak phase transition be sufficiently strongly first order. If this final assumption holds, then the phase transition proceeds via bubble nucleation and baryogenesis takes place in the bubble walls where the Higgs fields cause the departure from thermal equilibrium.

In a recent paper [7] three of us (R.B., A.C.D., M.T.) suggested an alternative realization of the third Sakharov condition in the context of the electroweak phase transition in the presence of topological defects remaining after a previous symmetry breaking. The electroweak symmetry is restored out to some distance around these defects and, as segments of them collapse, the loss of thermal equilibrium caused by the transition from false to true vacuum allows local baryogenesis to occur at the outer edge of the defect. An estimate of the baryon to entropy ratio produced by this mechanism was performed and compared to the strength of previous mechanisms. Its strength was found to be suppressed by the ratio of the volume in defects to the total volume. For defects formed at scales close to the electroweak scale and for optimistic values of the parameters the mechanism was shown to give results consistent with the observed baryon to entropy ratio.

The advantage of such a scenario for electroweak baryogenesis is that it does not depend in any way on the order or dynamics of the electroweak phase transition. This is a significant advantage, given that the order of the phase transition is not known at present, and that in particular for a large Higgs boson mass the transition is unlikely to proceed via the nucleation of critical bubbles. A further advantage of using topological defects to seed baryogenesis is that the volume in defects decreases only as a power of time below the phase transition temperature. Therefore, as pointed out in Ref. [8], defect-mediated baryogenesis is still effective even if sphalerons are in thermal equilibrium just below the electroweak phase transition temperature. However, a potential drawback for specific implementations is the requirement that defects be formed at a scale rather close to the electroweak scale in order to avoid large volume suppression factors.

In this paper we attempt to relax this requirement by performing a more detailed analysis of the mechanism. In particular we show that the volume suppression discussed above may be relaxed when the imperfect cancellation between the baryons and antibaryons produced is considered. We also demonstrate that not only local baryogenesis [9,10] (where sphaleron transitions and CP violation take place at the same spacetime point) but also nonlocal baryogenesis [11,12] (where the sphaleron transitions and CP violation act in different regions of space: CP violation in the bubble or defect wall leads to asymmetries in quantum numbers other than baryon number which are then converted to baryon number by sphaleron processes in the larger region where the symmetry is restored) is important. These particle transport processes lead to an increase in the sphaleron transition rate (since the latter is unsuppressed in the false vacuum) and to a considerable enhancement of the local baryon number density generated and thus to an improved estimate of the net baryon asymmetry capable of being produced by this mechanism.

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The outline of this paper is as follows. In Sec. II we review our previous work and outline the details of our new approach. Section III is devoted to an analysis of local baryogenesis as considered in Ref. [7] but with the new volume enhancement factors taken into account. In Sec. IV we describe how nonlocal baryogenesis can enhance the number density of baryons produced by defects. This enhancement can take either of two forms depending on the thickness of the walls of the defects. Section V contains a discussion of the effects of different geometries (cosmic strings, domain walls, and closed and infinite defects) and types of motion (collapse and translational) on the resulting BAU. In particular, we examine several detailed examples of the scenario, one of which may be directly compared with our previous estimate. Finally, in Sec. VI, we conclude and discuss our results.

II. ENHANCEMENT OF THE BARYOGENESIS VOLUME

Let us briefly review the scenario for electroweak baryogenesis proposed in Ref. [7].

Consider a d-dimensional topological defect produced at an energy scale $\eta > \eta_{\rm EW}$, where $\eta_{\rm EW}$ is the electroweak scale. In this work we shall only be interested in one-dimensional defects, cosmic strings, and two-dimensional defects, domain walls.

Assume that there is some process whereby baryon number is violated (e.g., sphaleron-induced transitions). Further, assume that the rate Γ per unit volume for this process is given by

$$\Gamma = \begin{cases} \Gamma_B(T), & T > \eta_u, \\ 0, & T < \eta_u, \end{cases} \qquad (1)$$

where $\eta_u \leq \eta_{\rm EW}$ is the energy scale at which sphaleron transitions in the plasma external to the string become exponentially suppressed. If certain consistency conditions are satisfied (e.g., sphalerons fit inside the defect so that their rate is not significantly suppressed), then within the defect the rate of baryon number violation is $\Gamma_B(T_{\rm EW})$ after the electroweak phase transition.

Also, assume that in the Higgs sector of the electroweak theory there exist CP-odd terms (for a recent attempt to enhance CP violation in the standard model by means of a condensate of a CP-odd Z field on the bubble wall see [13]). In this case, extra CP violation not present in the standard model can take place inside the defect walls. We are interested in the relative production of baryons and antibaryons due to the motion of these topological defects (see [8,14] for other ways to use topological defects to generate the baryon to entropy ratio).

To be definite we shall assume that the CP violation is due to a CP-odd relative phase θ between the two electroweak Higgs doublets—this may be seen as the restriction that both Higgs doublets may not simultaneously take real vacuum expectation values—and that this phase changes by $\Delta\theta_{CP}$ during the transition from false to true vacuum, and by $-\Delta\theta_{CP}$ in the reverse transition [15].

In the electroweak theory baryon number is an anomalous global symmetry [7,16]. This is the origin of the baryon number violation. There are several mechanisms by which the CP-odd phase θ_{CP} can contribute to the free energy density of the theory. To be specific, we shall concentrate on a one-loop effect (see, for example, [9,17]). Tree level effects have been discussed in Refs. [10] and [18]. The one-loop contribution is

$$\mathscr{F}_B = -\frac{14}{3\pi^2 N_f}\zeta(3)\left(\frac{m}{T}\right)^2 \dot{\theta}_{CP} n_B, \qquad (2)$$

where m is the (finite temperature) mass of the particle species dominating the contribution to the anomaly and ζ is the Riemann zeta function. The coefficient of n_B in the above equation can be viewed as a chemical potential μ:

$$\mu = \frac{14}{3\pi^2 N_f}\zeta(3)\left(\frac{m}{T}\right)^2 \dot{\theta}_{CP} \qquad (3)$$

for baryon number. Thus, it is clear that if $\Delta\theta_{CP} > 0$ for a given process, then baryon number is driven positive (an excess of baryons over antibaryons is generated) and vice versa.

As a defect moves, certain regions of the background space enter the core of the defect, i.e., make the transition from true to false vacuum, while others leave the core and make the transition from false to true vacuum. In Ref. [7] it was shown that certain types of motion and evolution of defects can provide an asymmetry such that an overall baryon excess is created in the universe.

In the original work of Ref. [7] the processes responsible for the generation of the baryon asymmetry were purely local. By this we mean that the baryon-number-violating interactions and the CP violation neccessarily take place at the same spacetime point. This requirement leads to restrictions on the strength of the mechanism since an important quantity which enters the calculation is the suppression factor

$$({\rm SF}) \sim \left(\frac{V_{\rm BG}}{V}\right), \qquad (4)$$

where $V_{\rm BG}$ is the volume in which baryogenesis occurs and V is the total volume. (SF) is the factor by which defect-mediated baryogenesis is weaker than baryogenesis with bubble walls. For a collapsing topological defect, purely local baryogenesis restricts the baryogenesis volume to be the initial volume of the defect because the effects of $\dot{\theta} > 0$ on one side of the string are canceled by the effects of $\dot{\theta} < 0$ on the other. In order for (SF) not to be a prohibitively small suppression it was neccessary for us to require that the scale at which our defects are formed be close to the electroweak scale.

Our aim here is to investigate the mechanism in much more detail. In particular we wish to take into account the various types of particle interactions within the string, where the symmetry is unbroken. These interactions allow us to enhance the local effects in Ref. [7] because now the decay of antiparticles produced by the leading edge of the defect results in imperfect cancellation between the effects of the competing processes.

We shall also examine the effects of nonlocal baryogenesis which allow us to further increase the number of baryons which our mechanism produces. This may take one of the following forms.

(1) Particle reflection [11,12]: As a result of CP violation in the walls of the defect, particles with opposite chirality reflect (and transmit) differently off the wall, having as a consequence a net injected chiral flux. This flux thermalizes and diffuses into the interior of the defect where it is converted to baryons. Because of the quantum-mechanical nature of the reflection, this process is efficient only for ''thin'' walled defects.

(2) For rather thick walls two effects are known to give rise to baryogenesis: classical force [18] and nonlocal spontaneous baryogenesis [19,18]. In the former scenario, as a result of CP violation, an axial field emerges on the wall, leading to a classical force which perturbs particle densities, thus biasing baryon number. In the latter, hypercharge violating processes in the presence of an axial field on the wall are responsible for perturbing particle densities in a CP-violating manner. When the effects of particle transport are taken into account, both cases give rise to nonlocal baryogenesis.

Both these mechanisms lead to an increase in the net effective volume contributing to baryogenesis over that of local baryogenesis since we no longer rely on anomalous interactions taking place in the narrow region of the face of the defect where the changing Higgs fields provide CP violation.

The chiral asymmetry which is converted to an asymmetry in baryon number is carried by both quarks and leptons. However, since the Yukawa couplings of the top quark and the τ lepton are larger than those of the other quarks and leptons, respectively, we expect that the main contribution to the injected asymmetry will come from these particles and henceforth we ignore the effects of the other particles.

When considering nonlocal baryogenesis it is convenient to write the equation for the rate of production of baryons in the form [12]

$$\dot{B}=-\frac{N_f\Gamma_s}{2T}\sum_i \mu_i, \tag{5}$$

where the rate per unit volume for electroweak sphaleron transitions is

$$\Gamma_s=\kappa(\alpha_W T)^4, \tag{6}$$

with [20,21] $0.1\leqslant\kappa\leqslant 1$, N_f the number of families, and μ_i is the chemical potential for left-handed particles of species i. B is the number density of baryons produced locally by the process. The crucial question in applying this equation is an accurate evaluation of the chemical potentials that bias baryon number production.

III. LOCAL BARYOGENESIS

In this section we shall obtain a revised estimate for the baryon asymmetry produced by a topological defect as a consequence of local mechanisms.[1]

As a topological defect passes each point in space a number density of antibaryons is produced by local baryogenesis at the leading face of the defect and then an equal number density of baryons is produced as the trailing edge passes. Naively we would expect that these effects would cancel each other, so that any time-symmetric motion of the defect such as translation would yield no net baryon asymmetry. This is the reason that in Ref. [7] we restricted ourselves to a time-asymmetric motion, loop collapse, to generate a baryonic excess. The cancellation effects led to the suppression of the strength of our mechanism by the factor (SF) mentioned in Sec. II.

However, in this treatment we have neglected an important effect and thus underestimated the strength of the mechanism. The antibaryons produced at the leading edge of the defect at a fixed point in space spend a time interval τ inside the defect during which they may decay before the trailing edge passes by and produces baryons at the same point. The core passage time τ is given by

$$\tau=\frac{L}{v_D}, \tag{7}$$

where L is the width of the defect and v_D is its velocity.

Thus, if n_b^0 is the number density of baryons (or antibaryons) produced at either edge, we may estimate the net baryon asymmetry B produced after the defect has passed a given point once to be

$$B=n_b^0(1-e^{-\bar{\Gamma}_s\tau}), \tag{8}$$

where $\overline{\Gamma_s}$ is the rate at which antibaryons decay and may be related to the electroweak sphaleron rate by [11,12]

$$\overline{\Gamma_s}=6N_f\frac{\Gamma_s}{T^3}=6N_f\kappa\alpha_W^4 T. \tag{9}$$

The resulting average baryon number density n_b can be estimated from (3) and taking into account (8) and the volume suppression [see (4)]:

$$n_b\simeq 3N_f\frac{\Gamma_s}{T}\mu\frac{\delta}{v_D}(1-e^{-\bar{\Gamma}_s\tau})(\mathrm{SF}), \tag{10}$$

where δ is the thickness of the defect wall. The derivative of θ_{CP} and δ/v_D combine to give $\Delta\theta_{CP}$, and hence the resulting net baryon to entropy ratio becomes

$$\frac{n_b}{s}\simeq 4\kappa\alpha_W^4 g^{*-1}\left(\frac{m}{T}\right)^2\Delta\theta_{CP}\frac{V_{\mathrm{BG}}}{V}(1-e^{-\bar{\Gamma}_s\tau}), \tag{11}$$

where g^* is the number of spin degrees of freedom which enters into the equation for the entropy density

[1] There is a controversy over by how many factors of the mass and the coupling constant local baryogenesis is suppressed. We do not address this question but take the more optimistic rates. See, e.g., Dine and Thomas [27] and Cohen *et al.* [19], or Joyce *et al.* [18] for different views on this issue.

$$s=\frac{2\pi^2}{45}g^*T^3.$$

In our original estimate the total volume contributing to baryogenesis was the initial volume occupied by defects. Now, with this new effect taken into consideration, this volume is dramatically increased. All the volume swept out by the defect network participates in the effect.

We still have a volume suppression factor but its value is considerably larger than before and, as we shall see in Sec. V, in some cases it is $O(v_D)$ because the defect network can sweep out that fraction of the total volume in one Hubble expansion time.

Even if $V_{\rm BG}/V\sim 1$, we still have the suppression of this mechanism over the usual bubble wall scenarios by the factor

$$(1-e^{-\bar{\Gamma}_s L/v_D}).$$

This clearly distinguishes two cases. In the first case in which the defects are "thin," defined as $L<v_D/\bar{\Gamma}_s$, we have a suppression factor of approximately $\bar{\Gamma}_s L/v_D$. If the defects are "thick," $L>v_D/\bar{\Gamma}_s$, then there is negligible suppression due to this effect.

Let us now examine how these conditions are related to the microphysical parameters of the models. First consider nonsuperconducting defects. The electroweak symmetry is restored out to a distance [22]

$$R_s\sim\lambda^{-1/4}G^{-1/2}\eta_{\rm EW}^{-1}, \tag{12}$$

where $\eta_{\rm EW}$ is the electroweak scale, $G^2=g^2+g'^2$, and g and g' are the SU(2) and U(1) gauge couplings, respectively. The defects are considered "thin" if the Higgs self-coupling λ satisfies

$$\lambda>\left(\frac{\bar{\Gamma}_s}{v_D\eta_{\rm EW}}\right)^4\frac{1}{G^2} \tag{13}$$

and "thick" otherwise. This quantity may be estimated by evaluating $\bar{\Gamma}_s$ [see Eq. (9)] at the electroweak temperature, $G^2\sim 0.4$ and $v_D\sim 0.1$–1, resulting in the condition $\lambda>10^{-23}$–10^{-27}, an inequality which includes most of the parameter space of the theory. Thus, we may conclude that for the case of ordinary defects the suppression factor $\bar{\Gamma}_s L/v_D$ almost always applies.

Now consider the case where the defects are superconducting. If, as in Ref. [7], we estimate the current on the defects by assuming a random walk of the winding of the condensate field (assume scalar superconductivity for simplicity), then we may estimate the size of the symmetry restoration region to be [22,23]

$$R_s\sim\sqrt{\frac{1}{2\lambda}}\frac{1}{2\pi}\frac{1}{\eta_{\rm EW}}\left(\frac{\eta}{\eta_{\rm EW}}\right)^{3/4}, \tag{14}$$

where η is the scale at which the defects are formed. A similar result has been shown to hold in the two-Higgs-doublet model we are using [24]. Thus, in this case, our defects are considered "thick" if

$$\eta>\left(\frac{v_D\eta_{\rm EW}}{\bar{\Gamma}_s}\sqrt{2\lambda}2\pi\right)^{4/3}\eta_{\rm EW} \tag{15}$$

and "thin" otherwise. Using $\lambda\sim 1$ and estimating Γ from (9) we obtain $\eta>10^8$–4×10^{10} GeV.

Therefore, if the scale of the defects is in this range, then there is no additional suppression beyond the volume suppression. If the scale lies below this, then we have the factor $\Gamma L/v_D$ as in the case of ordinary defects.

The above considerations allow us to compute the asymmetry in the baryon number density at every point swept out by a topological defect of a given type. In order to make a specific prediction we need to consider a particular type of defect in a given configuration and have knowledge of the evolution of the defect network. This then enables us to make a reliable estimate for the volume suppression (SF) and hence the total baryon asymmetry. In Sec. V we shall perform this calculation in some examples. First, however, we shall examine the mechanism when the baryon production is by nonlocal means.

IV. NONLOCAL BARYOGENESIS

We now turn to the issue of baryons produced by nonlocal mechanisms within the defects. There are (at least) two distinct ways in which this can occur. One mechanism applies when the walls of the defect, where the Higgs fields are changing, are thin (in a sense which will be made precise in a moment) and the other applies when the walls are thick.

Let us first consider the case where the Higgs fields change only in a narrow region at the face of the topological defect. In analogy with the bubbles formed during a first order phase transition we refer to this as the *thin wall* case.

In this regime, effects due to local baryogenesis are heavily suppressed because CP-violating processes take place only in a very small volume in which the rate for baryon violating processes is nonzero. However, we shall see that nonlocal baryogenesis allows us to produce an appreciable baryon asymmetry due to particle transport effects [11,12].

In the rest frame of the topological defect particles within the core see a sharp potential barrier and reflect off the trailing edge in a CP-violating manner due to the gradient in the CP-odd Higgs phase in the defect wall. The same is true for particles reflecting back into the broken symmetry phase from the leading edge of the defect. It is neccessary that the walls of the defect be thin, defined as $\delta<l$ where l is the mean free path of the relevant particle species within the wall of thickness δ, in order that coherent quantum effects give unsuppressed reflection. In certain cases it may be necessary to impose a more stringent condition. Namely, if the main contribution to the reflection current comes from particles grazing on the wall, the correct condition for a free particle reflection off the wall is $\delta<\sqrt{l/T}$ [12]. In the case this condition were violated but the weaker condition $\delta<l$ still satisfied, one would expect a phase space suppression in the reflected asymmetry.

As a consequence of CP violation, there will be asymmetric reflection and transmission of particles, thus generating an injected current from both defect walls into the defect

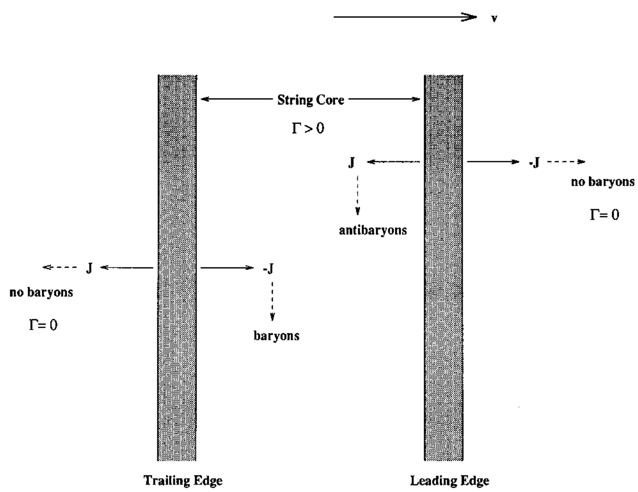

FIG. 1. Diagram of a portion of a defect, in this case a cosmic string, moving to the right through the primordial plasma. The differing decays of reflected particles within and outside the defect lead to the generation of a net baryon asymmetry.

core. As a consequence of this injected current, asymmetries in certain quantum numbers will diffuse in front of the respective face of the defect due to particle interactions and decays [11,12]. In particular, the asymmetric reflection and transmission of left- and right-handed particles will lead to a net injected chiral flux from the wall. However, there is a qualitative difference between the diffusion occurring in the two regions.

In the interior of the defect the electroweak symmetry is restored and weak sphaleron transitions are unsuppressed. This means that the chiral asymmetry carried into this region by transport of the injected particles may be converted to an asymmetry in baryon number by sphaleron transitions. In contrast, particles injected into the phase of broken symmetry may diffuse only by baryon-number-conserving scattering and decay processes since the electroweak sphaleron rate is exponentially suppressed in this region. Hence, we shall concentrate only on those particles injected into the interior of the defect. In Fig. 1 we represent the various processes on a diagram of the defect core.

Note that the currents injected into the defect core from the two walls have opposite sign. The reason for this is as follows. The asymmetry in the reflection coefficients of left- and right-handed fermions is proportional to $\dot{\theta}$, which has opposite signs at the leading and trailing edges of the defect. Nevertheless, the effect is not completely canceled. For generality we write the currents J and J' with different magnitudes. One may expect this sort of effect when one includes the plasma back reaction (frictive effects) onto the profile of $\dot{\theta}$, similarly as it was done for the dynamical profile of the phase boundary (Higgs expectation value) for an expanding bubble of true vacuum [25].

This will be unimportant since in the following we will consider baryon number production in the diffusion tail in front of the walls, and will comment on the suppression factor obtained by taking into account both walls at the end of the analysis. As we shall argue below, in the diffusion approximation the front edge does not contribute to baryogenesis.

The net baryon to entropy ratio which results via nonlocal baryogenesis in the case of thin walls can be calculated following the analyses in Refs. [11] and [12]. The baryon density produced by a single defect is given by Eq. (5) in terms of the rate of baryon-number-violating processes, in turn given by (6), and the chemical potentials μ_i for left-handed particles. These chemical potentials are a consequence of the asymmetric reflection and transmission off the walls and the resulting chiral particle asymmetry.

In the following we give a brief outline of the logic of the calculation. For details see Refs. [11] and [12]. Baryon number violation is driven by the chemical potentials for left-handed leptons or quarks. We here focus on leptons [12] (for quarks see, e.g., Ref. [11]). If there is local thermal equilibrium in front of the defect walls, as we assume, then the chemical potentials μ_i of particle species i are related to their number densities n_i by

$$n_i = \frac{T^2}{12} k_i \mu_i , \tag{16}$$

where k_i is a statistical factor which equals 1 for fermions and 2 for bosons. It is important to correctly [26] impose the constraints on quantities which are conserved in the region in front of and on the wall, but at the level of this discussion we do not need to address this point.

Using the above considerations, the chemical potential μ_L for left-handed leptons can be related to the left-handed lepton number densities L_L. These are in turn determined by particle transport. The source term in the diffusion equation is the flux J_0 resulting from the asymmetric reflection and transmission of left- and right-handed leptons off the defect wall.

The asymmetric reflection coefficients for lepton scattering is

$$\mathcal{R}_{L\to R} - \mathcal{R}_{R\to L} \simeq 2\Delta\theta_{CP} \frac{m_l^2}{m_H |p_z|}, \quad m_l < |p_z| < m_H \sim \frac{1}{\delta}, \tag{17}$$

where m_l and m_H are the lepton and Higgs boson masses, respectively, and $|p_z|$ is the momentum of the lepton perpendicular to the wall (in the wall frame). The resulting flux of left-handed leptons is

$$J_0 \simeq \frac{v_D m_l^2 m_H \Delta\theta_{CP}}{4\pi^2}, \tag{18}$$

where v_D is the defect translational velocity. Note that in order for the momentum interval in Eq. (17) to be nonvanishing, the condition $m_l \delta < 1$ needs to be satisfied.

The injected current from the wall will lead to a "diffusion tail" of particles in front of the moving wall. In the approximation when the persistence length of the injected current is much larger than the wall thickness we may to a good approximation model it as a δ-function source. In addition we assume that the decay time of leptons is much longer than the time it takes for a defect to pass so that we may neglect decays. Then the diffusion equation for a single-particle species becomes

$$D_L L_L'' + v_D L_L' = \xi^L J_0 \delta'(z), \tag{19}$$

where D_L is the diffusion constant for leptons, ξ^L is the persistence length of the current in front of the defect wall, and a prime denotes the spatial derivative in direction z perpendicular to the wall. This equation can be immediately integrated once with the integration constant specified by the left-handed lepton number at infinity, which for simplicity we set to zero. We can now easily write down the solution

$$L_L(z) = J_0 \frac{\xi_L}{D_L} e^{-\lambda_D z}, \quad z > 0, \tag{20}$$

and 0 for $z<0$, with the diffusion root $\lambda_D = v_D/D_L$. Note that in this approximation the injected current does not generate any perturbation behind the wall. This means that the appropriate current J_0 in Eq. (19) is the current injected at the *trailing edge* of the defect (J' in Fig. 1). This is true provided $L \gg \xi^L \gg \delta$ is satisfied. If the first inequality is not true, then the injected current would have to be treated as an extended source. In the case the second inequality is not satisfied, the problem becomes significantly more complex [12].

In the massless approximation the chemical potential μ_L can be related to L_L by

$$\mu_L = \frac{6}{T^2} L_L \tag{21}$$

(for details see Ref. [12]).

Inserting into Eq. (5) the sphaleron rate (6) and the above results for the chemical potential μ, and taking into account the suppression factors (4) and the analog of (8) for nonlocal baryogenesis, we obtain the final baryon to entropy ratio

$$\frac{n_b}{s} = \frac{n_b^{(0)}}{s}(1 - e^{-L\lambda_D}) \frac{V_{\rm BG}}{V}, \tag{22}$$

where L is the thickness of the defect and

$$\frac{n_b^{(0)}}{s} = \frac{1}{4\pi^2} \kappa \alpha_W^4 (g^*)^{-1} \Delta\theta_{CP} \left(\frac{m_l}{T}\right)^2 \frac{m_H}{\lambda_D} \frac{\xi^L}{D_L}. \tag{23}$$

The diffusion constant is proportional to α_W^{-2} (see Ref. [12]):

$$\frac{1}{D_L} \simeq 8 \alpha_W^2 T. \tag{24}$$

Hence, provided that sphalerons do not equilibrate in the diffusion tail,

$$\frac{n_b^{(0)}}{s} \sim 0.2 \alpha_W^2 (g^*)^{-1} \kappa \Delta\theta_{CP} \frac{1}{v_D} \left(\frac{m_l}{T}\right)^2 \frac{m_H}{T} \frac{\xi^L}{D_L}. \tag{25}$$

Since ξ^L/D_L is of the order $1/(T\delta)$, the baryon to entropy ratio obtained by nonlocal baryogenesis is proportional to α_W^2 and not α_W^4 as the result for local baryogenesis.

Now let us compare the effects of top quarks scattering off the interior of the advancing wall of the defect [11]. Several effects tend to decrease the contribution of the top quarks relative to that of τ leptons. First, their contribution is suppressed [27] since the diffusion tail is cut off in front of the wall by strong sphaleron effects. Second, the diffusion length for top quarks is smaller, thus reducing the volume in which baryogenesis takes place. Third, since quarks interact much stronger with the plasma than leptons, it is more likely that the top quark reflection will be spoiled due to the loss of quantum coherence on the wall, which is essential for the asymmetric reflection. However, there are also enhancement factors, e.g., the ratio of the squares of the masses m_t^2/m_τ^2 [see Eq. (2)].

Let us now turn briefly to the case when the walls of the defect are thick in the sense that $\delta > l$. Then there is no significant reflection of particles from the defect walls because the particles see an adiabatically changing Higgs field. One might therefore think that there is no nonlocal baryogenesis in this case.

However, as has been shown in Ref. [18] (see also Ref. [19]), in the case of thick walls there is a classical force in the equilibrium equations for baryon number that drives the equilibrium value away from zero, generating a net baryon asymmetry. We shall not consider this mechanism in detail here but shall just note that whatever the configuration or relative dimensions of the defects we always produce some contribution to the baryon asymmetry from nonlocal processes.

V. SPECIFIC GEOMETRIES AND EXAMPLES

Now that we have examined in detail the various ways in which an excess of baryons may be generated by a topological defect we can compute the total baryon asymmetry of the universe for a given type of defect with a given distribution.

Let us assume that in a volume $V = x^3$ there is a single defect of a given type with the electroweak symmetry restored out to a distance R_s. The scale x may be considered as the mean separation of defects. Then, for a single defect the volume suppression factor is

$$\frac{V_{\rm BG}}{V} \simeq \pi \frac{R^2 R_s}{x^3} \quad \text{string loop, radius } R \tag{26}$$

$$\simeq \frac{R_s}{x} v_D \text{“infinite” string} \tag{27}$$

$$\simeq v_D \text{“infinite” domain wall} \tag{28}$$

$$\simeq \frac{4\pi}{3}\left(\frac{R}{x}\right)^3 \text{ domain bubble, radius } R \tag{29}$$

$$\simeq 4\pi \left(\frac{R}{x}\right)^2 v_D \text{ translating stable bubble.} \tag{30}$$

Since we are including the total volume swept out by the defects, we also need to consider the possible additional suppression from the factor $\overline{\Gamma}_s L/v_D$ due to all the antibaryons not having time to decay before the trailing edge of the defect passes.

A. Local and nonlocal baryogenesis from cosmic strings

In our original analysis [7] we concentrated on the contribution to the baryon asymmetry from local baryogenesis in loops of cosmic string produced by a symmetry breaking at a scale $\eta > \eta_{\rm EW}$. Thus, it is interesting to compare the results of that calculation with our new predictions.

We shall consider two possibilities.

(1) The scale η is sufficiently close to the electroweak scale that the string network is in the friction-dominated epoch at the time of the electroweak phase transition.

(2) η is sufficiently greater than $\eta_{\rm EW}$ that the string network has reached a scaling solution by $t_{\rm EW}$. Note that strings with a mass per unit length μ remain in the friction-dominated era until [28] a time $t^* = (G\mu)^{-1} t_c$, where t_c is the time corresponding to the critical temperature of the phase transition.

In the first case we will make the approximation that all string loops have the same radius but in the second case we shall integrate over the loop distribution function.

First, assume that the network is in the friction-dominated epoch at $t_{\rm EW}$. Note that in this case (except in a narrow window for η) superconducting strings do not satisfy Eq. (15) and the strings are therefore (for local baryogenesis) thin enough that the additional suppression factor $\Gamma_s L/v_D$ mentioned above applies in both the ordinary and superconducting cases (for a large range of Higgs self-coupling). For nonlocal baryogenesis the suppression factor is linear in Lv_D/D. Let us further assume that we have one string loop per correlation volume at formation, via the Kibble mechanism. In one horizon volume per expansion time the total volume taking part in baryogenesis is

$$V_{\rm BG} = R_s \xi(t)^2 \left(\frac{t}{\xi(t)} \right)^3 v_D , \tag{31}$$

where we have used the largest strings with radius equal to the correlation length $\xi(t)$ and the last factor is the number of string loops per horizon volume. Thus, dividing by the horizon volume t^3 we obtain the volume suppression factor

$$({\rm SF}) = \frac{V_{\rm BG}}{V} = \frac{R_s}{\xi(t)} v_D . \tag{32}$$

The assumptions we are making about defect formation and scaling are well established (see, e.g., Ref. [29]). In particular, the initial correlation length at the time t_f when the string network freezes out (this occurs at the Ginsburg temperature T_f) is given by $\xi(t_f) \simeq \lambda^{-1} \eta^{-1}$ [30]. After t_f, the correlation length approaches its scaling value at a rate [28]

$$\xi(t) \sim \xi(t_f) \left(\frac{t}{t_f} \right)^{5/4} , \tag{33}$$

where there is some uncertainty about the prefactor (it is, however, universal in the sense that it does not depend either on the particular string model or on the string density at the time of formation). Thus, we obtain (SF) as

$$({\rm SF}) = \lambda \left(\frac{\eta_{\rm EW}}{\eta} \right)^{3/2} v_D \quad \text{ordinary strings} \tag{34}$$

$$= \lambda \left(\frac{\eta_{\rm EW}}{\eta} \right)^{3/4} v_D \quad \text{superconducting strings.} \tag{35}$$

These equations take into account only the dynamics during the first Hubble expansion time after $t_{\rm EW}$. In later expansion times, the density of strings is diluted, and hence the above results are a good approximation of the total effect of strings.

The above suppression factors are an improvement over the original ones by a factor of 1/2 in the exponent and mean that the phase transition giving rise to the neccessary defects need not lie quite so close to the electroweak scale as once imagined.

Let us now turn to the case where the defects are formed at a scale much higher than the electroweak scale so that the defect network is well described by a scaling solution at $t_{\rm EW}$. If the strings are ordinary, we expect still to have the $\Gamma_s L/v_D$ suppression but for superconducting defects we shall see that this is absent since the electroweak symmetry is restored out to such a large radius that all the antibaryons may decay before the baryons are created.

Let us again focus on string loops. The number density of string loops with radii in the range $[R, R+dR]$ is given by [31]

$$n(R,t) = \begin{cases} \nu R^{-5/2} t^{-3/2}, & \gamma t < R < t, \\ \nu \gamma^{-5/2} t^{-4}, & R < \gamma t, \end{cases} \tag{36}$$

where $\gamma \ll 1$ is a constant determined by the strength of electromagnetic radiation from the string. Loops with radius $R = \gamma t$ decay in one Hubble expansion time. In the above we are assuming that electromagnetic radiation dominates over gravitational radiation. If this is not the case, then γ must be replaced by $\gamma_g G\mu$, μ being the mass per unit length of the string ($\mu \simeq \eta^2$) and [32] $\gamma_g \sim 100$. In other words, γ is bounded from below:

$$\gamma > \gamma_g G\mu . \tag{37}$$

We can estimate the suppression factor (SF) by integrating over all the string loops present at $t_{\rm EW}$:

$$({\rm SF}) \simeq \pi \int_0^{\gamma t_{\rm EW}} dR R^2 R_s n(R, t_{\rm EW}) = \frac{\pi}{3} \nu \gamma^{1/2} \left(\frac{R_s}{t_{\rm EW}} \right) . \tag{38}$$

Without superconductivity the suppression factor for grand unified theory (GUT) strings ($\eta = 10^{16}$ GeV) is so small ($\sim 10^{-32}$) that the contribution is negligible. However, with superconducting strings we may estimate

$$({\rm SF}) \sim \nu \gamma^{1/2} \left(\frac{\eta}{m_{\rm Pl}} \right) , \tag{39}$$

so that the final baryon to entropy ratio generated by this mechanism is given by

$$\frac{n_B}{s} = \frac{n_B^0}{s} ({\rm SF}) , \tag{40}$$

with (SF) given by the above and n_B^0/s proportional to α_W^4 for local baryogenesis and to α_W^2 for nonlocal baryogenesis. Clearly, with the volume enhancement and nonlocal effects, this is an improvement over our original mechanism but unfortunately still lies below the observed value.

B. Nonlocal baryogenesis from cosmic domain walls

Let us now briefly examine what is probably the best case scenario for our mechanism.

Assume that in the early universe there is a symmetry breaking at a scale η such that cosmic domain walls are formed. One important caveat, however, is that we must assume the existence of some process by which the domain walls are removed at a later time so that they do not come to dominate the energy density of the universe. One way to eliminate the domain walls is to introduce a slight tilt in the potential which breaks the vacuum degeneracy and eventually leads to the dominance of one vacuum.

Let us first focus on domain walls formed at a scale close to the electroweak scale so that they are in the friction-dominated epoch at the time of the electroweak phase transition. We consider the effect of ‘‘infinite’’ walls in which case (SF)$\sim v_D$. Clearly, for a wide range of domain wall velocities v_D the resulting baryon to entropy ratio is comparable with what results from first order mechanisms and can agree with the observed baryon to entropy ratio for suitable choices of the parameters.

From Eq. (25) we may estimate the relevant quantities from Ref. [18] and arrive at

$$\frac{n_b^{(0)}}{s} \sim 10^{-6} \kappa \Delta \theta_{CP} y_\tau^2 v_D, \tag{41}$$

where y_τ is the Yukawa coupling for τ leptons.

Note that if the scaling solution for domain walls is maintained long after $t_{\rm EW}$, then the contributions from different Hubble time steps add up and can give an additional enhancement of the topological-defect-mediated baryogenesis scenario.

Imagine that the scaling solution for domain walls lasts for sufficiently many expansion times that an equilibrium baryon number is reached. (We will assume that the sphaleron rate in the defect core will not drop too much so that this equilibrium will indeed be reached.) This means that an equal number of baryons will be created as destroyed in the passage of a wall. If the equilibrium value for baryon number is denoted as B_0, the equilibrium balance equation, obtained by considering the trailing edge of the wall, becomes (for local baryogenesis)

$$B_0 e^{-(\bar{\Gamma}_s L/v_D)} + n_b^0 (1 - e^{-(\bar{\Gamma}_s L/v_D)}) = B_0. \tag{42}$$

The first term is the baryon density left over from what enters the leading edge of the wall, and the second term is what is created in front of the trailing edge. For nonlocal baryogenesis the balance equation reads

$$B_0 e^{-(\bar{\Gamma}_s L/v_D)} + n_b^0 (1 - e^{-(L v_D/D)}) = B_0, \tag{43}$$

which gives

$$B_0 = n_b^0 \tag{44}$$

for local baryogenesis and

$$B_0 = n_b^0 \frac{v_D^2}{\Gamma_s D} \tag{45}$$

for nonlocal baryogenesis (where we have assumed that the core is thin). Note that this result is unsuppressed for local baryogenesis and in fact enhanced for the nonlocal mechanisms (see, e.g., [12]). The interesting fact is that in the nonlocal case, the dependence on the sphaleron rate drops out of the final result completely. In order to obtain equilibrium, the number of expansion times during which the scaling solution persists must be larger than $1/\bar{\Gamma}_s L$.

Note that these examples are intended to illustrate the basic feasibility of our mechanism. Some concrete models are discussed in Ref. [33]. The baryon to entropy ratio which is generated in any model depends on the precise value of the volume suppression factor, on the precise way in which CP violation is coupled to baryon-number-violating processes during the electroweak phase transition. These are issues which should be studied in more detail. However, it seems unlikely that a model-independent analysis will be possible.

VI. DISCUSSION AND CONCLUSIONS

We have investigated in detail the production of a baryonic asymmetry at the electroweak scale where the departure from thermal equilibrium is realized by the motion of topological defects remaining after a previous phase transition. The main advantage of this scenario is that it is insensitive to the details, including the order, of the electroweak phase transition and is still effective even if sphalerons are in thermal equilibrium just below the phase transition temperature.

The electroweak symmetry is restored out to some distance around these defects and, as points in space make the transition from true to false vacuum and back again as the defect passes by, CP violation in the walls of the defect results in the production of a net baryonic excess.

Our analysis addresses the qualitatively different mechanisms of local and nonlocal baryogenesis and in each case we have evaluated the possible baryon to entropy ratio which may be generated by defects of a given dimension and distribution. In particular we have addressed the case where the defects are cosmic string loops. The key observation in this paper is that the effective volume contributing to baryogenesis is much more than was assumed at first in Ref. [7]. For the scenario with cosmic string loops this was shown to result in a less severe suppression than originally calculated.

At this point, the reader may worry that our mechanism violates the CPT theorem. Consider a moving domain wall which according to our microphysical analysis produces baryons in its wake. A CPT transformation seems to give the same string configuration moving in the same direction producing antibaryons.

The refutation of this apparent paradox is based on the following points: (1) A static string is its CPT conjugate and produces no baryons; (2) a moving string is also its CPT conjugate and produces no baryons in the absence of dissipative processes; (3) when dissipative processes (which can only have a definite sign under CPT if the system is out of thermal equilibrium) are taken into account, a moving string can create a net baryon number.

Regarding the first point, the key fact is that the CP-violating phase θ responsible for baryogenesis is CPT invariant since $\theta^{T}(x,t)=-\theta(x,-t)$, $\theta^{CP}(x,t)=-\theta(-x,t)$, and thus $\theta^{CPT}(x,t)=\theta(-x,-t)$.

The apparent paradox mentioned above is now easily refuted. Our baryogenesis mechanism depends crucially on dissipative processes. These are driven by a net chemical potential for baryon number which is generated by the motion of the strings which in turn is driven by the expanding universe. Dissipation in an expanding universe creates an arrow of time which generates the required T violation (see also [34]).

Returning to our paper, we have included in our calculation of the nonlocal mechanism the effects of particle transport. This allows us to make a detailed estimate of the total baryon asymmetry due to the imperfect cancellation of the baryons and antibaryons produced on opposite faces of the defect.

It may be useful to summarize the conditions under which our approximations in the analysis of local and nonlocal baryogenesis apply.

First, a sphaleron has to fit within the defect core (nonlocal BG) or wall (local BG). From the sphaleron rate Γ_s in the unbroken phase [see (6)], we can infer that the sphaleron radius is not larger than $(\alpha_W T)^{-1}$. In the walls, the sphalerons are rather small ($\sim m_W^{-1}$). Hence, this condition is

$$L>(\alpha_W T)^{-1}, \quad \delta>m_W^{-1}, \tag{46}$$

where L and δ are defect core and wall radii, respectively.

As discussed in Sec. III, in the case of local baryogenesis, antibaryons produced at the leading edge of the defect can annihilate with the baryons produced at the trailing edge unless the baryon density equilibrates to zero in the defect core via sphaleron processes. For this to occur, the defect must be sufficiently thick:

$$\frac{L}{v_D}>\overline{\Gamma}_s^{-1}, \tag{47}$$

where v_D is the transverse velocity of the defect. If this condition is not satisfied, there will be a suppression of the baryon to entropy ratio linear in $\overline{\Gamma}_s L/v_D$.

Last, in the case of nonlocal baryogenesis, baryon production takes place in the diffusion tail which extends in front of the trailing defect edge. In order not to get a suppression of the effect, the core must be thick compared to the diffusion tail D/v_D:

$$\frac{L v_D}{D}>1. \tag{48}$$

In calculating the effects of nonlocal baryogenesis we have used the reflection coefficients of leptons from defects calculated for planar walls. The reflection is dominated by particles with wavelength $\lambda\sim\delta$. Hence, a condition for the applicability of our calculations is $\lambda\ll R_s$, where R_s is the curvature radius of the defect wall. For domain walls, $R_s\gg L$, whereas for strings $R_s\sim L$.

In nonlocal baryogenesis mediated by a classical force [18] there are, as in the case of local baryogenesis, no further geometric suppression factors. However, for nonlocal baryogenesis by quantum reflection [11,26] there is a further condition. When averaged over phase space, the incident angle of the fermions which scatter off the defect wall is peaked at a value of

$$\frac{m_H}{2T}\simeq\frac{1}{T\delta}, \tag{49}$$

where m_H is the Higgs boson mass which determines the wall thickness. In order for the single-scattering calculations used in this paper to be valid, the typical distance d the fermions travel within the core after a reflection before hitting the wall a second time, which is

$$d\simeq\frac{2R_s}{T\delta}, \tag{50}$$

must be larger than the diffusion length $6D$, i.e.,

$$d>6D. \tag{51}$$

This condition is satisfied provided that the wall thickness is significantly smaller than the defect core size. If it is violated, then there will be a further suppression factor.

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[1] V. Kuzmin, V. Rubakov, and M. Shaposhnikov, Phys. Lett. **155B**, 36 (1985); P. Arnold and L. McLerran, Phys. Rev. D **36**, 581 (1987).
[2] N. Turok, in *Perspectives on Higgs Physics*, edited by G. Kane (World Scientific, Singapore, 1992).
[3] A. Cohen, D. Kaplan, and A. Nelson, Annu. Rev. Nucl. Part. Sci. **43**, 27 (1993).
[4] A. Sakharov, Pis'ma Zh. Éksp. Teor. Fiz. **5**, 32 (1967) [JETP Lett. **5**, 24 (1967).
[5] N. Manton, Phys. Rev. D **28**, 2019 (1983); F. Klinkhamer and N. Manton, *ibid.* **30**, 2212 (1984).
[6] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).
[7] R. Brandenberger, A.-C. Davis, and M. Trodden, Phys. Lett. B **335**, 123 (1994).

[8] R. Brandenberger, A. Davis, and M. Hindmarsh, Phys. Lett. B **263**, 239 (1991); A.-C. Davis and M. Earnshaw, Nucl. Phys. **B394**, 21 (1993).

[9] N. Turok and T. Zadrozny, Phys. Rev. Lett. **65**, 2331 (1990); Nucl. Phys. **B358**, 471 (1991); L. McLerran, M. Shaposhnikov, N. Turok, and M. Voloshin, Phys. Lett. B **256**, 451 (1991); M. Dine, P. Huet, R. Singleton, and L. Susskind, *ibid.* **257**, 351 (1991).

[10] A. Cohen, D. Kaplan, and A. Nelson, Phys. Lett. B **263**, 86 (1991).

[11] A. Cohen, D. Kaplan, and A. Nelson, Phys. Lett. B **245**, 561 (1990); Nucl. Phys. **B349**, 727 (1991); A. Nelson, D. Kaplan, and A. Cohen, *ibid.* **B373**, 453 (1992).

[12] M. Joyce, T. Prokopec, and N. Turok, Phys. Lett. B **338**, 269 (1994); Phys. Rev. D (to be published).

[13] S. Nasser and N. Turok, "Z Condensation and Standard Model Baryogenesis," Princeton Report No. PUPT-94-1456, 1994 (unpublished).

[14] R. Brandenberger and A. Davis, Phys. Lett. B **308**, 79 (1993).

[15] From the classical equations of motion one finds that $\Delta\theta_{CP}\simeq\xi-\arctan[\lambda_6\tan\xi/\lambda_5]$. Here ξ is the CP-violating phase appearing in the two-Higgs-doublet potential: $V_{CP}=\lambda_5[\mathrm{Re}(\Phi_1^\dagger\Phi_2)-\nu_1\nu_2\cos\xi]^2+\lambda_6[\mathrm{Im}(\Phi_1^\dagger\Phi_2)-\nu_1\nu_2\sin\xi]^2$. This expression becomes exact in the thick wall limit.

[16] P. Arnold, in *Testing the Standard Model*, Proceedings of the Theoretical Advanced Study Institute in Particle Physics, edited by M. Cvetic and P. Langacker (World Scientific, Singapore, 1991).

[17] S. Abel, W. Cottingham, and I. Whittingham, Nucl. Phys. **B410**, 173 (1993).

[18] M. Joyce, T. Prokopec, and N. Turok, Phys. Lett. B **336**, 41 (1994); Phys. Rev. D (to be published).

[19] A. Cohen, D. Kaplan, and A. Nelson, Phys. Lett. B **336**, 41 (1994).

[20] T. Ambjorn, M. Laursen, and M. Shaposhnikov, Phys. Lett. B **197**, 49 (1989); T. Ambjorn, T. Askgaard, H. Porter, and M. Shaposhnikov, Nucl. Phys. **B353**, 346 (1991).

[21] D. Grigoriev, M. Shaposhnikov, and N. Turok, Phys. Lett. B **275**, 395 (1992).

[22] W. Perkins and A.-C. Davis, Nucl. Phys. **B406**, 377 (1993).

[23] J. Ambjorn, N. Nielsen, and P. Olesen, Nucl. Phys. **B310**, 625 (1988).

[24] M. Trodden, Mod. Phys. Lett. A **9**, 2649 (1994).

[25] G. Moore and T. Prokopec, Phys. Rev. Lett. **75**, 777 (1995); Phys. Rev. D **52**, 7182 (1995).

[26] M. Joyce, T. Prokopec, and N. Turok, Phys. Lett. B **339**, 312 (1994).

[27] G. Giudice and M. Shaposhnikov, Phys. Lett. B **326**, 118 (1994); M. Dine and S. Thomas, *ibid.* **328**, 73 (1994).

[28] T. W. B. Kibble, Acta Phys. Pol. B **13**, 723 (1982); A. Everett, Phys. Rev. D **24**, 858 (1981); M. Hindmarsh, Ph.D. thesis, University of London, 1986.

[29] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994).

[30] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976); J. Borrill, T. W. B. Kibble, T. Vachaspati, and A. Vilenkin, Phys. Rev. D **52**, 1934 (1995); R. Brandenberger and A.-C. Davis, Phys. Lett. B **332**, 305 (1994).

[31] Ya. B. Zel'dovich, Mon. Not. R. Astron. Soc. **192**, 663 (1980); A. Vilenkin, Phys. Rev. Lett. **46**, 1169 (1981); N. Turok and R. Brandenberger, Phys. Rev. D **37**, 2075 (1986).

[32] T. Vachaspati and A. Vilenkin, Phys. Rev. D **31**, 3052 (1985); N. Turok, Nucl. Phys. **B242**, 520 (1984); C. Burden, Phys. Lett. B **164**, 277 (1985).

[33] M. Trodden, A.-C. Davis, and R. Brandenberger, Phys. Lett. B **349**, 131 (1995).

[34] A. Cohen and D. Kaplan, Phys. Lett. B **199**, 251 (1987); Nucl. Phys. **B308**, 913 (1988).