Energy level inequalities in a "wrinkled" quarkonium potential

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(Received 2 June 1994)

The "concave" downward property of the standard static $q\bar{q}$ potentials leads to the energy level inequalities $E_{n+2}-E_{n+1}<E_{n+1}-E_n$ in the quarkonium mass spectrum. However, this inequality is experimentally observed to be reversed for $n=2$ in charmonium and $n=3$ in bottomonium, a fact that is inexplicable in terms of any known concave downward potential. We attempt to explain this by allowing for the violation of the concavity condition in a small interval, i.e., a "wrinkle" in some recently proposed quarkonium potentials.

PACS number(s): 12.39.Pn, 13.20.Gd, 14.40.Gx

I. INTRODUCTION

Most of the $q\bar{q}$ potentials given in the literature [1] are concave downwards: i.e., for a given potential $V(R)$,

$$\frac{\partial V}{\partial R}>0, \quad \frac{\partial^2 V}{\partial R^2}<0. \tag{1}$$

Bachas [2] has demonstrated this property on the lattice for potentials derived from gauge theories. However, the absence of a viable theory of strong interactions in the nonperturbative region and the lack of proof of confinement allows us to consider phenomenological potentials which violate this condition, at least in the region where there still exist theoretical uncertainties. Furthermore, it has been shown by Lichtenberg [3] in a recent communication that the data on the energy level spacings in heavy quarkonia show that one cannot obtain complete agreement with the experimentally observed [4] levels in charmonium and bottomonium through the use of a concave downward potential.

For a concave downward potential the adjacent energy levels follow the rule

$$E_{n+2}-E_{n+1}<E_{n+1}-E_n \tag{2}$$

while the reverse is true for a convex downward potential. Charmonium and bottomonium data [4] show that this inequality is violated for $n=2$ in charmonium and $n=3$ in bottomonium, i.e., for the J/ψ family,

$$E_4-E_3>E_3-E_2, \tag{3a}$$

and, for the Υ family,

$$E_5-E_4>E_4-E_3. \tag{3b}$$

This trend has led Lichtenberg [3] to postulate that for a description of quarkonia based on a completely static potential (with no openings for decay channels) one must have a potential that violates the concavity condition in a small interval, i.e., a potential with a "wrinkle."

II. THE WRINKLED POTENTIALS

Recently, we have proposed [5,6] two new potentials which violate the concavity condition [Eq. (1)] in a small region. We present, in this paper, an analysis of our potentials to test Lichtenberg's claim. For a better appreciation of the structure and role of the wrinkle, we give here the essential details of the potentials in question and elaborate their dependence on the various parameters used in them.

The string-inspired large distance potential in conjunction with the short distance QCD-inspired "Coulomb" potential is our zeroth-order potential:

$$V_0(R)=\begin{cases} -4\alpha_s/3R, & R<R_c, \\ K\sqrt{R^2-R_c^2}, & R>R_c, \end{cases} \tag{4}$$

where $\alpha_s(R)=12\pi/[(33-2n_f)\ln[1/(\Lambda^2R^2)]$ is the scaled strong interaction coupling constant to first order in perturbation theory and $R_c=\sqrt{\pi/6K}$.

As can be seen from the form of the "bare" potential $V_0(R)$ (Fig. 1), the long distance form is unphysical below $R=R_c$, and the short distance form is that due to perturbation theory and is dependent on the QCD cutoff Λ. By choosing Λ to be well within the experimental limits of $100<\Lambda<200$ MeV, the maximum of the Coulombic part can be made to coincide with $R=R_c$. For numerical calculations we have chosen throughout $K=0.16$ GeV^2 and $\Lambda=0.11$ GeV. This, however, leads to a discontinuity as shown in Fig. 1. Smoothing out the discontinuity at $R=R_c$ leads us to the wrinkled potential

$$V_{\rm BDKS}(R)=V_1(R)+V_2(R), \tag{5}$$

where [7]

$$V_1(R) = -\frac{4}{3}\,\alpha_s(R)\,\frac{[\ln(\Lambda R)]^2}{R\{\ln(\Lambda R)\}^2 + A\{\tanh(R-R_0)/b + \tanh R_0/b\}} \tag{6}$$

and

$$V_2(R) = K\,\tanh\left(\frac{R}{R_c}\right)\sqrt{R^2-(R_c \tanh R/R_c)^2}. \tag{7}$$

Here R_0, A, and b are in units of GeV^{-1}. The original Bambah-Dharamavir-Kaur-Sharma (BDKS) [7] potential of Ref. [5], which is closest to the roughened potential V_0, is also plotted in Fig. 1. The parameters used in this are $A=15$ GeV^{-1}, $b=R_c/60$, and $R_0=R_c-b=xR_c$ with $x=0.95$.

A few remarks about the rather complicated form of this potential are in order here.

(1) The short distance part $V_1(R)$ seeks to approximate $V_0(R)$ for $R<R_c$ while for $R>R_c$ it is negligible. $V_2(R)$ does the same for $R>R_c$. The forms of $V_1(R)$, $V_2(R)$, and $V_{\mathrm{BDKS}}(R)$ are shown in Fig. 1. Thus $V_{\mathrm{BDKS}}(R)$ is a continuous potential that retains all the features of $V_0(R)$.

(2) The parameters A and b are related to R_c and are chosen to ensure that $V_{\mathrm{BDKS}}(R)$ is equal to $V_0(R)$ in as large a region as possible and lead to a "wrinkle" in the potential. These parameters A, b, and R_0 determine the depth, width, and position of the "wrinkle," respectively.

(3) The parameter A is essential to avoid the infinity in $V_0(R)$ at $R=1/\Lambda$.

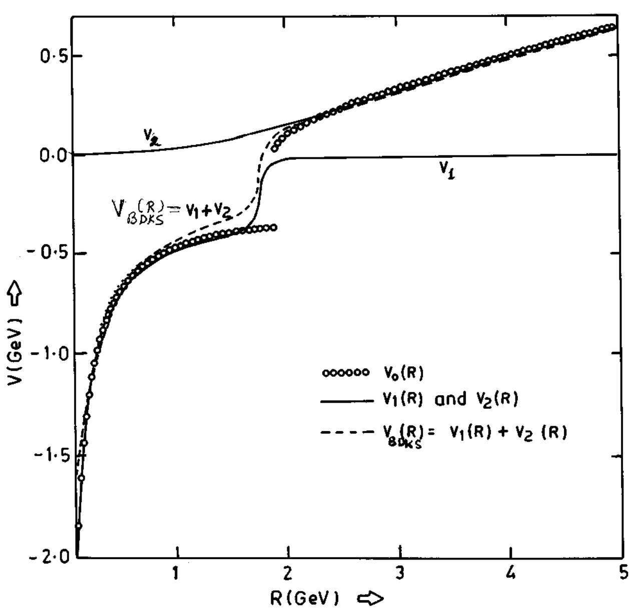

FIG. 1. The detailed structure of the BDKS potential [5] [$V_{\mathrm{BDKS}}(R)$]. ○○○ is the "bare" potential $V_0(R)$, with $\Lambda=0.11$ GeV and $K=0.16$ GeV^2; $n_f=4$ [Eq. (4)]. $V_1(R)$ [Eq. (6)] and $V_2(R)$ [Eq. (7)] constituting $V_{\mathrm{BDKS}}(R)$ are shown separately for qualitative comparison. The parameters are $A=15$ GeV^{-1}, $\Lambda=0.11$ GeV, $K=0.16$ GeV^2, and $b=R_c/60$.

(4) The parameter R_0 controls the position of the wrinkle. The combination of A and R_0 determines the shape of the potential $V_{\mathrm{BDKS}}(R)$ at the upper end of the wrinkle and its values were initially chosen to get the shape as close to the bare $V_0(R)$ as possible.

(5) The parameter b determines the width of the wrinkle. A smaller b keeps the potential $V_{\mathrm{BDKS}}(R)$ close to $V_0(R)$, while a large enough b will get rid of the wrinkle; in particular, a wrinkle-free smooth monotonic concave-downward potential is obtained for large A and large b, e.g., for $b \cong R_c/4$ and $A \sim 25$ GeV^{-1}. [Hereafter, this smooth wrinkle-free concave-downward potential is referred to as $V^{\mathrm{WF}}(R)$.] The interpolation is truly logarithmic only for this wrinkle-free case.

It should be noted that a large number of potentials, with fewer than three parameters, could have been selected for the intermediary region. However, for freedom to choose the width, depth, and position of the wrinkle, we select this particular one.

III. RESULTS AND DISCUSSION

In this section we examine Lichtenberg's comment on the effect of the wrinkle on the energy level inequalities [Eqs. (2) and (3)]. As we have seen in the earlier section, the adjustments of the parameters A and b allow us control the wrinkle. For the values of the parameters ($A=15$ GeV^{-1}, $b=0.032$ GeV^{-1}, $R_0 \cong R_c = 1.86$ GeV^{-1}) chosen in Refs. [5] and [6] these inequalities are not satisfied (Table I, columns 2 and 3). This is because the energy levels under consideration fell above the wrinkle and were governed completely by the concave part of the potential. However, if we adjusted the parameters such that the energy levels under consideration are in the region of transition then the presence of the wrinkle would alter the character of the level spacings in the desired manner.

One must observe that the potential governing the wrinkle can be visualized as a superposition of two parts (see Fig. 2): i.e.,

$$V_{\mathrm{BDKS}}(R) = V^{\mathrm{WF}}(R) + V_{\mathrm{AG}}(R), \tag{8}$$

where V_{AG} is an asymmetric Gaussian type of function (Fig. 2, curve 3) obtained by subtracting the unwrinkled potential $V^{\mathrm{WF}}(R)$ (Fig. 2, curve 1) from the wrinkled $V_{\mathrm{BDKS}}(R)$ (Fig. 2, curve 2). Thus we see that the potential $V_{\mathrm{BDKS}}(R)$ of Refs. [5] and [6] involves an asymmetric Gaussian. The "wrinkle" in the potential is confined to a very narrow region. The theoretical uncertainty in the potential is only over a distance $0.95R_c$ to $1.05R_c$ which, for $K=0.14$ GeV^2, corresponds to 1.7–1.9 GeV^{-1} (0.36–0.38 fm).

However, if we relax this stringent condition and allow the intermediary region between the "Coulombic" and the

TABLE I. Energy level differences (in GeV) of charmonium and bottomonium.

	1 Experimental [4]	2 V_{BDKS} [5]	3 V^{WF}	4 V_{BKDS} (present work)	5 $V^{\mathrm{WF}}+V$ [V as in Eq. (9)]
			Charm		
2*S*-1*S*	0.589	0.569	0.481	0.541	0.457
3*S*-2*S*	0.354	0.388	0.368	0.435	0.333
4*S*-3*S*	0.375	0.325	0.318	0.439	0.372
$M(\Psi\ 1S)$	3.097	3.044	3.074	3.134	3.074
			Bottom		
2*S*-1*S*	0.563	0.603	0.419	0.412	0.420
3*S*-2*S*	0.332	0.275	0.278	0.300	0.269
4*S*-3*S*	0.225	0.232	0.229	0.210	0.171
5*S*-4*S*	0.285	0.207	0.202	0.243	0.214
6*S*-5*S*	0.154	0.189	0.185	0.209	[a]
$M(\Psi\ 1S)$	9.460	9.421	9.438	9.431	9.439

[a]There seems to be a spurious inversion here. This needs to be investigted more carefully, theoretically as well as experimentally.

"roughened" string potential to span a distance R_c to $2R_c$, then the energy levels under consideration [$\psi(2S)$, $\psi(3S)$, $\psi(4S)$, $\Psi(3S)$, $\Psi(4S)$, and $\Psi(5S)$] fall in or around the region of the wrinkle. This happens for values of the parameters $A=0.36$ GeV^{-1}, $R_c=1.809$ GeV$^{-1}\cong R_0$, $\Lambda=0.11$ GeV, and $K=0.16$ GeV^2 (Fig. 3, curve 2). Then the energy level spacings are in reasonable agreement with experimental values and are given in Table I, column 4. We see that, qualitatively, the potential with a wide, shallow wrinkle, within the framework of the BDKS model gives us the desired inversion effect required by Eqs. (2) and (3).

In order to obtain better quantitative agreement, we can take a purely phenomenological approach, by supposing the wrinkle to be caused by a *symmetric* Gaussian (Fig. 4) of the form

$$V(R)=\nu_0\exp[-\{(R-3R_c)/R_c\}^2], \tag{9}$$

which is subtracted out of the unwrinkled potential $V^{\mathrm{WF}}(R)$. This gives a fairly good fit to the data as shown in Table I, column 5. Here $R_c=1.809$ GeV^{-1} and $V_0=0.5$ GeV. It may

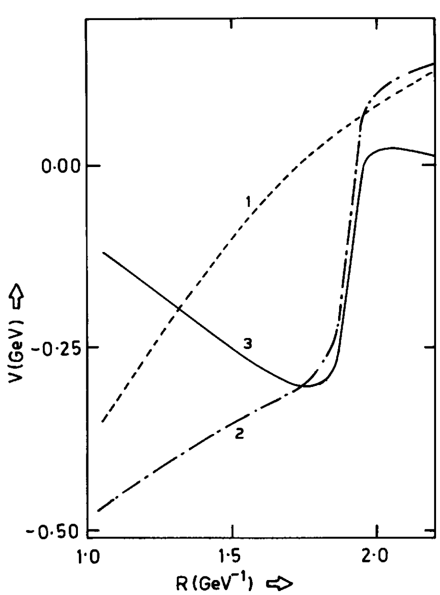

FIG. 2. The asymmetric Gaussian (V_{AG}) (curve 3) obtained by subtracting the wrinkled BDKS potential $V_{\mathrm{BDKS}}(R)$ (curve 2) from the unwrinkled one $V^{\mathrm{WF}}(R)$ (curve 1).

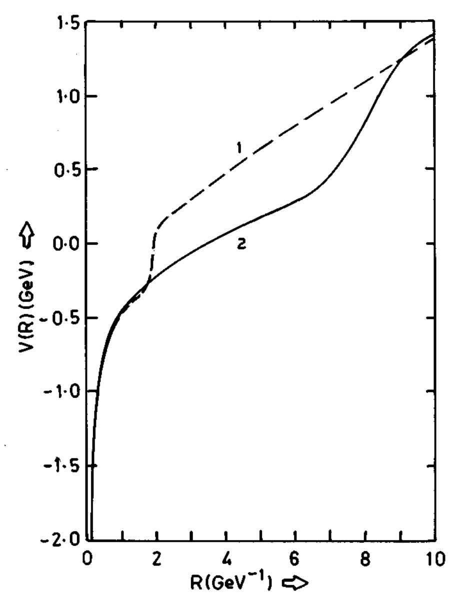

FIG. 3. The wrinkled-BDKS potential (curve 2) which produces the required energy level inversion. The parameters used are given in the text. The curve 1 is the original BDKS wrinkled potential of Ref. [5] with the same parameters as in Fig. 1.

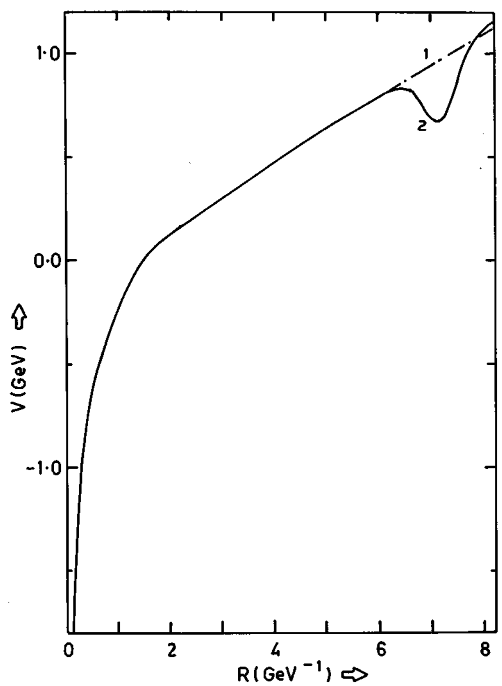

FIG. 4. The unwrinkled potential $V^{\mathrm{WF}}(R)$ (curve 1) with the symmetric Gaussian (curve 2) of Eq. (9).

be emphasized that our purpose in this communication has not been to fit the experimental data, but to demonstrate in some detail the structure of the BDKS wrinkle and its qualitative impact on the energy level differences of quarkonia.

The Gaussian wrinkle is a purely phenomenological potential for which we can claim no theoretical justification in a purely static model for heavy quarkonia. However, we can make a conjecture that if we find a way of including dynamical processes such as quark pair production [8] in the nonperturbative regime into an effective interquark potential then we may be able to produce a wrinkle. It has been shown by Tornqvist [9] that coupled channel effects may be able to produce the inversion of energy levels in the case of the Υ system. However, he has also shown that these effects cannot be absorbed into the parameters of any static quark model. We have shown that the qualitative behavior of experimentally observed mass differences can be explained by means of a potential with two different types of wrinkles. It would be interesting to see if a systematic calculation of coupled channel effects can result in an effective potential which when added to a purely concave static potential can give rise to these wrinkles. This is currently under investigation and will be reported in the near future.

ACKNOWLEDGMENTS

The authors would like to acknowledge the University Grants Commission of India for partial financial support under their various research schemes. We would especially like to thank Professor D. B. Lichtenberg for his critical comments on our earlier publications.

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[7] A note regarding the notation: In all future communications we shall follow the notations of the present paper. It may be noted that $V_{\mathrm{BDKS}}(R)$ of the present paper [Eq. (5)] is the same as $V_2(R)$ of [5] and $V_r(R)$ of [6].

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