B decay into light gluinos

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Flavor-changing interactions of the gluino allow the b quark to decay into the strange quark plus a gluino pair if the gluino is in the ultralow mass window below 1 GeV. In this case the enhancement of the nonleptonic b decay could explain the anomalous semileptonic branching ratio.

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In the last few years it has been noted by many authors that a light gluino would help to explain several anomalies at the Z scale and other discrepancies between experiment and theory [1–3]. Also, the impact of the light gluino on the branching ratio $b\to s\gamma$ has been recently investigated in [4]. Surprisingly enough, the region of gluino mass below 0.7 GeV is poorly constrained by experiment [5]. Here, assuming that the gluino is in this low mass window, we propose that the decay $b\to s\tilde{g}\tilde{g}$ might contribute considerably to the b total width thus reducing the theoretical prediction of the B semileptonic branching ratio.

Currently, the experimental value for the semileptonic branching ratio is $\mathscr{B}_{\rm SL}(B)|_{\rm expt}=(10.43\pm0.24)\%$ [6], while the theoretical prediction gives a lower bound of 12.5% [7]. The last number also includes perturbative QCD corrections. In general, nonperturbative QCD corrections are not expected to increase the inclusive nonleptonic widths of B mesons significantly as was argued in [7]. However, the possibility that the width $\Gamma(b\to c\bar{c}s)$ might be enhanced because of larger than expected nonperturbative corrections in the $b\to c\bar{c}s$ channel and thus reduce the semileptonic branching ratio of b, was discussed in [7–9]. It was concluded therein that such an enhancement would increase simultaneously the charm multiplicity in B decays up to about 1.3, which would be more than 15% higher than the value from current experimental data. Unless future measurements of the charm multiplicity lead to this expected value, it could be plausible that there are new contributions to charmless b decays, that were unaccounted for in the theoretical prediction of $\mathscr{B}_{\rm SL}(B)$. Although some authors [9] still prefer rather conservative explanations of the B semileptonic branching ratio puzzle, they do not exclude the possibility of scenarios from beyond the standard model contributing to the solution of the problem. In any case, having the current experimental data in mind, the discrepancy between theory and experiment, which is at least 14%, still has to be explained.

In the following, we show that the b decay to the s quark and a light gluino pair can easily increase the nonleptonic branching ratio in certain regions of its parameter space by an amount high enough to saturate the discrepancy.

We also show that this process contributes to the total width of b by a considerably larger amount than that by the process $b\to sg$ if the gluino is below 1 GeV.

The process we are dealing with is a tree-level, flavor-changing, neutral-current process with a down-type squark in the intermediate state. The decay rate is calculated using the quark-squark-gluino Lagrangian [4], given by

$$\mathscr{L}_{q\tilde{q}\tilde{g}}=i\sqrt{2}g_s\tilde{q}_i^{\dagger a}\bar{\tilde{q}}_\alpha(\lambda_\alpha/2)_{ab}\left[\Gamma_L^{ip}\frac{1-\gamma_5}{2}+\Gamma_R^{ip}\frac{1+\gamma_5}{2}\right]q_p^b, \tag{1}$$

where p stands for the quark generation (in our case $p=b$ or s) and i labels the squark states ($i=b_L,b_R,s_L,s_R,d_L,d_R$). The λ_α are the eight generators of color SU(3). The matrices Γ_L and Γ_R are (6×3) matrices given by

$$\Gamma_L=\tilde{U}^\dagger\begin{pmatrix}I\\0\end{pmatrix},\quad \Gamma_R=\tilde{U}^\dagger\begin{pmatrix}0\\I\end{pmatrix}, \tag{2}$$

where $\tilde{U}$ is the matrix that diagonalizes the down-type squark mass matrix squared, $M_{\tilde{d}}^2$. Adopting the notation of [4] , $M_{\tilde{d}}^2$ is written as

$$M_{\tilde{d}}^2=\begin{pmatrix}m_{0L}^2I+\hat{M}_d^2+cK^\dagger\hat{M}_u^2K & Am_0\hat{M}_d\\ Am_0\hat{M}_d & m_{0R}^2I+\hat{M}_d^2\end{pmatrix}, \tag{3}$$

in a basis where the 3×3 down-type quark mass matrix is diagonal. The matrices $\hat{M}_u$ and $\hat{M}_d$ are diagonal up- and down-type quark mass matrices, respectively, and K is the Cabibbo-Kobayashi-Maskawa mixing matrix. For simplicity, we take $m_{0L}=m_{0R}$ and equal to the universal scalar mass m_0. In order to simplify the process of analytic diagonalization of $M_{\tilde{d}}^2$, we take the trilinear scalar coupling A equal to zero, which does not affect the result significantly. The c parameter, which is responsible for flavor-violating interactions, plays an important role in our numerical estimates of the $b\to s\tilde{g}\tilde{g}$ branching ratio. Some authors [10] take c of order 0.01 or even lower, while others [4] suggest that c can be somewhat larger in magnitude. As regards the sign of the c parameter, $c<0$ is preferred in the minimal supersymmetric standard model (MSSM). In this paper, we treat c as a phenomenological parameter to be experimentally constrained.

In the case $A=0$, $M_{\tilde{d}}^2$ is block diagonal and only the upper-left block needs to be diagonalized. The upper-left block can be written in the form

$$M^2_{\tilde{d}(3\times3)}=m_0^2\left[I+\begin{pmatrix}0&0&0\\0&0&0\\0&0&b\end{pmatrix}+c'\begin{pmatrix}0&0&0\\0&0&\epsilon\\0&\epsilon&1\end{pmatrix}\right], \quad (4)$$

where we have neglected the masses of the d and s quarks with respect to the mass of the b quark, and similarly, m_u and m_c with respect to m_t. The modified parameter c' is equal to cm_t^2/m_0^2 and b to m_b^2/m_0^2. Only the two leading terms were kept in the product $K^\dagger \hat{M}_u^2 K$, namely, those proportional to $|K_{tb}|^2$ and $K^*_{ts}K_{tb}$, the first being taken equal to unity and the latter being denoted by ϵ. The matrix that diagonalizes $M^2_{\tilde{d}(3\times3)}$ is found to be

$$\tilde{U}_{(3\times3)}=\frac{1}{(2f)^{1/2}}\begin{pmatrix}(2f)^{1/2}&0&0\\0&(f+b+c')^{1/2}&(f-b-c')^{1/2}\\0&-2\epsilon c'(f+b+c')^{-1/2}&2\epsilon c'(f-b-c')^{-1/2}\end{pmatrix}, \quad (5)$$

where f is a function of the variables b, c', and ϵ defined by

$$f(b,c',\epsilon)=\sqrt{(b+c')^2+4\epsilon^2c'^2}. \quad (6)$$

The complete diagonalizing matrix $\tilde{U}$ is given by

$$\tilde{U}=\begin{pmatrix}\tilde{U}_{(3\times3)}&0\\0&I\end{pmatrix}, \quad (7)$$

where I is the (3×3) identity matrix. The evaluation of $\tilde{U}^\dagger M^2_{\tilde{d}}\tilde{U}$ gives a diagonal matrix with the squark masses squared on the diagonal. For the left-handed mass-eigenstates, one gets

$$m^2_{\tilde{d}_L}=m_0^2, \quad (8)$$

$$m^2_{\tilde{s}_L}=m_0^2\left(1+\frac{b}{2}+\frac{c'}{2}-\frac{1}{2}f(b,c',\epsilon)\right), \quad (9)$$

$$m^2_{\tilde{b}_L}=m_0^2\left(1+\frac{b}{2}+\frac{c'}{2}+\frac{1}{2}f(b,c',\epsilon)\right), \quad (10)$$

while the right-handed ones get masses

$$m^2_{\tilde{d}_R}=m_0^2+m_d^2,\quad m^2_{\tilde{s}_R}=m_0^2+m_s^2,\quad m^2_{\tilde{b}_R}=m_0^2+m_b^2. \quad (11)$$

The matrix Γ_L needed for calculation of the decay rate $\Gamma(b\to s\tilde{g}\tilde{g})$ can be written as

$$\Gamma_L=\tilde{U}^\dagger\begin{pmatrix}I\\0\end{pmatrix}=\begin{pmatrix}\tilde{U}^\dagger_{(3\times3)}\\0\end{pmatrix}. \quad (12)$$

Note that the matrix $\tilde{U}^\dagger_{(3\times3)}$ reduces to the identity matrix in the limit $c\to0$, as it should. The matrix Γ_R is trivially found to be

$$\Gamma_R=\begin{pmatrix}0\\I\end{pmatrix}. \quad (13)$$

Having found the exact form of the matrices Γ_L and Γ_R, the calculation of $\Gamma(b\to s\tilde{g}\tilde{g})$ can be completed analytically. The invariant matrix element $\mathscr{M}$ consists of terms corresponding to the exchange of $\tilde{b}_L$, $\tilde{b}_R$, $\tilde{s}_L$, and $\tilde{s}_R$. We have neglected much smaller terms with $\tilde{d}_L$ or $\tilde{d}_R$ exchange. After performing the three-body Lorentz-invariant phase space integration, the decay rate becomes

$$\Gamma(b\to s\tilde{g}\tilde{g})=2\frac{\alpha_S^2m_b^5}{54\pi}I\left(\frac{m_s}{m_b}\right)\sum_{i,j}\frac{1}{m_i^2m_j^2}\times(\Gamma_L^{ib}\Gamma_L^{\dagger bj}+\Gamma_R^{ib}\Gamma_R^{\dagger bj})(\Gamma_L^{\dagger si}\Gamma_L^{js}+\Gamma_R^{\dagger si}\Gamma_R^{js}), \quad (14)$$

for $i,j=\tilde{b}_L,\tilde{b}_R,\tilde{s}_L,\tilde{s}_R$. The function $I(x)$ is given by

$$I(x)=1-8x^2+24x^4\ln x+8x^6-x^8. \quad (15)$$

The overall multiplicative factor of 2 in Eq. (14) is because of the Majorana nature of the external gluinos. Using the "diagonal" character of Γ_R, this can be futher reduced to

$$\Gamma(b\to s\tilde{g}\tilde{g})=\frac{\alpha_S^2m_b^5}{27\pi}I\left(\frac{m_s}{m_b}\right)\sum_{i,j}\frac{1}{m_i^2m_j^2}\Gamma_L^{ib}\Gamma_L^{\dagger bj}\Gamma_L^{\dagger si}\Gamma_L^{js}. \quad (16)$$

The sum can be written in terms of the squark masses

$$\sum_{i,j}\frac{1}{m_i^2m_j^2}\Gamma_L^{ib}\Gamma_L^{\dagger bj}\Gamma_L^{\dagger si}\Gamma_L^{js}=\frac{\epsilon^2c'^2}{f^2}\frac{(m^2_{\tilde{b}_L}-m^2_{\tilde{s}_L})^2}{m^4_{\tilde{b}_L}m^4_{\tilde{s}_L}}. \quad (17)$$

Using the expressions for the squark masses obtained as the eigenvalues of $M^2_{\tilde{d}}$, we can write the result for $\Gamma(b\to s\tilde{g}\tilde{g})$ in the form

$$\Gamma(b\to s\tilde{g}\tilde{g})=\frac{\alpha_S^2m_b^5}{27\pi}I\left(\frac{m_s}{m_b}\right)\frac{m_t^4}{m_0^8}|K^*_{ts}|^2\times\left(\frac{c}{1+\frac{m_b^2}{m_0^2}+c\frac{m_t^2}{m_0^2}-c^2|K^*_{ts}|^2\frac{m_t^4}{m_0^4}}\right)^2. \quad (18)$$

Both the terms m_b^2/m_0^2 and $c^2|K^*_{ts}|^2(m_t^4/m_0^4)$ in the denominator can be neglected with respect to cm_t^2/m_0^2, if m_0 is larger than 80 GeV. Note also that $\Gamma(b\to s\tilde{g}\tilde{g})$ cannot de-

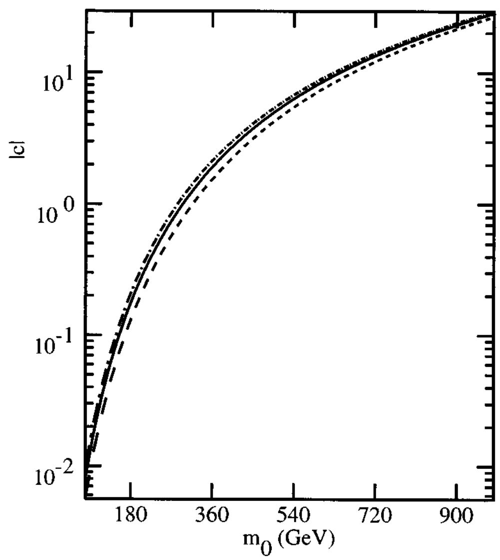

FIG. 1. The absolute value of c (c is assumed to be negative) is plotted as a function of the universal scalar mass m_0 for $R_{s\tilde{g}\tilde{g}}=$ 0.3 (dot-dashed line), $=0.2$ (solid line), and $=0.1$ (dashed line). The mass of the t quark was taken to be 170 GeV.

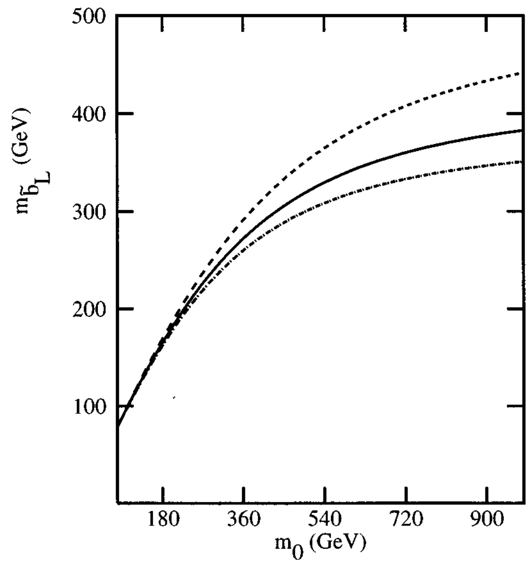

FIG. 2. The mass of the b_L squark is plotted as a function of the universal scalar mass m_0 for $R_{s\tilde{g}\tilde{g}}=$ 0.3 (dot-dashed line), $=0.2$ (solid line), and $=0.1$ (dashed line). The mass of the t quark was taken to be 170 GeV.

velop a pole because of the experimental lower limit on the masses of squarks in the intermediate state. For example, according to [2] we can require in the light gluino case that $m_{\tilde{b}_L}\geqslant 60$ GeV. As will be discussed below, this imposes an additional constraint on the c parameter as a function of m_0.

It is convenient to define the ratio

$$R_{s\tilde{g}\tilde{g}}=\frac{\Gamma(b\to s\tilde{g}\tilde{g})}{\Gamma(b\to c\bar{u}d)+\Gamma(b\to c\bar{u}s)},\tag{19}$$

where the denominator is given by

$$\Gamma(b\to c\bar{u}d)+\Gamma(b\to c\bar{u}s)=\frac{3G_F^2m_b^5|K_{cb}|^2}{192\pi^3}I_0\left(\frac{m_c^2}{m_b^2},0,0\right)\eta J.\tag{20}$$

The expression for the phase space factor I_0 and the values of leading-log anomalous dimension enhancement η and next-to-leading corrections enhancement J can be found in the literature (e.g., [7,8]) and their product is of order 1. Combining Eqs. (18), (19), and (20), we get

$$R_{s\tilde{g}\tilde{g}}=\frac{128}{27}\left(\frac{\alpha_S}{\alpha}\right)^2\frac{\sin^4(\theta_W)c^2}{(m_0^2+cm_t^2)^2}\left(\frac{m_Wm_t}{m_0}\right)^4\left|\frac{K_{ts}^*}{K_{cb}}\right|^2.\tag{21}$$

If the ratio $R_{s\tilde{g}\tilde{g}}$ gets as high as 20%, then the branching ratio of the nonstandard model decay $b\to s\tilde{g}\tilde{g}$ is more than 14%. This could completely account for the discrepancy between $\mathscr{B}_{\rm SL}(B)|_{\rm expt}$ and $\mathscr{B}_{\rm SL}(B)|_{\rm QCD}$. The current experimental data can be fit if $R_{s\tilde{g}\tilde{g}}=(22.68\pm0.52)\%$.

In general, by requiring $R_{s\tilde{g}\tilde{g}}$ to have a certain value, one gets c as a function of the universal scalar mass m_0. This function is plotted in Fig. 1 for several different values of $R_{s\tilde{g}\tilde{g}}$. We have used $\alpha_S(m_b)=0.18$, $\alpha(m_b)=1/133$, $\sin^2\theta_W=0.232$, and $m_t=170$ GeV in all numerical calculations. Also, we have taken advantage of the equality $|K_{ts}|\simeq|K_{cb}|$. From Fig. 1 it can be seen that the needed nonleptonic enhancement in b decays can be obtained using the contribution from the process $b\to s\tilde{g}\tilde{g}$ for reasonable values of m_0 and c. The necessary values of c as a function of m_0 are intermediate between those considered by Refs. [10,11]. Our calculation is made at the parton level. If there is a hadronization suppression of the right-hand side of Eq. (21) because of the masses of glueballinos then the required value of c in Fig. 1 will be greater. If the suppression is by more than an order of magnitude, our proposal might no longer be relevant.

The lower bound on the b squark mass of 60 GeV mentioned above does not interfere with any of the curves plotted in Fig. 1. In fact, the mass of the b squark is certainly above 75 GeV for $m_0\geqslant 80$ GeV. The $m_{\tilde{b}_L}$ as a function of m_0 obtained using Eqs. (10) and (21) is plotted in Fig. 2. This has an interesting implication for the problem of the b excess in Z decays. A possible explanation of the b anomaly could have been a Z decay into the b squark and the b antisquark. For that to be possible, the b squarks would have to have a mass less than $M_Z/2$. In order to explain the b anomaly using this process, one would need the ratio $\Gamma(Z\to\tilde{b}\bar{\tilde{b}})/\Gamma(Z\to b\bar{b})$ to be about 2%. This is possible, however, only if $m_{\tilde{b}_L}\approx 0.47M_Z$. This imposes the following constraint on the c parameter through Eq. (10):

$$c=\frac{(0.47M_Z)^2-m_0^2}{m_t^2}.\tag{22}$$

Unfortunately, this constraint is incompatible with the c dependence on m_0 that we got from the analysis of the B semileptonic branching ratio. In addition, such a light b squark would lead to unacceptably large contributions from $Z\to b\bar{\tilde{b}}\tilde{g}$. Therefore, in the light gluino scenario one has to rule out the possibility of b squarks being lighter than $M_Z/2$. Nevertheless, there are other mechanisms, that can explain the b excess [3] without contradiction with our current analysis. The current calculation is not sensitive to the gluino mass varying in the range of the low mass window (0 GeV–0.7 GeV).

It is interesting to compare the decay rate (18) to the decay rate of $b\to sg$, because processes such as this could also account for the missing 14–20% in the hadronic branching ratio of the B [8]. We use the formula for $\Gamma_{\rm SUSY}(b\to sg)$ given in [12], that corresponds to the processes with a squark and a gluino exchange within a loop and an external gluon attached either to the gluino line or the squark line. The decay rate is given by

$$\Gamma_{\rm SUSY}(b\to sg)=\frac{\alpha_S^3}{16\pi^2}m_b^5\left(1-\frac{m_s^2}{m_b^2}\right)^3\left(1+\frac{m_s^2}{m_b^2}\right)\left(\frac{cm_t^2}{\tilde{m}^4}\right)^2 \times|K_{tb}K_{ts}^*|^2\left\{A\sqrt{x}\left[\frac{1}{3}g_d(x)-3g_c(x)\right] -\left[\frac{1}{6}f_b(x)-\frac{3}{2}f_a(x)\right]\right\}^2, \tag{23}$$

with $x=m_{\tilde{g}}^2/\tilde{m}^2$ and $\tilde{m}^2=\frac{1}{2}(m_{\tilde{b}}^2+m_{\tilde{s}}^2)$. The functions g and f are given in [12] but for the purpose of our comparison we need only their limits as $x\to 0$. These are

$$\lim_{x\to 0}f_a(x)=\frac{1}{3};\quad \lim_{x\to 0}f_b(x)=\frac{1}{6}. \tag{24}$$

The decay rate in the case of the gluino with a negligible mass is then equal to

$$\tilde{\Gamma}_{\rm SUSY}(b\to sg)=\frac{289}{20\,736}\frac{\alpha_S^3}{\pi^2}m_b^5\frac{c^2m_t^4}{\tilde{m}^8}|K_{tb}K_{ts}^*|^2, \tag{25}$$

where the terms proportional to m_s^2/m_b^2 and its higher powers were neglected. Dividing Eq. (25) by Eq. (18), one gets

$$\frac{\tilde{\Gamma}_{\rm SUSY}(b\to sg)}{\Gamma(b\to s\tilde{g}\tilde{g})}=\frac{289}{768}\frac{\alpha_S}{\pi}, \tag{26}$$

indicating that the contribution to the total b width from $b\to s\tilde{g}\tilde{g}$ decay is dominant over the one from $b\to sg$ in the light gluino scenario. The mechanism of [12] for the nonleptonic enhancement is only consistent with a b squark above $M_Z/2$ if the gluino is heavier than 2 GeV and m_0 is less than 150 GeV.

In conclusion, we can say that, assuming that the gluino is light, the decay $b\to s\tilde{g}\tilde{g}$ provides a plausible explanation of the gap between $\mathcal{B}_{\rm SL}(B)|_{\rm expt}$ and $\mathcal{B}_{\rm SL}(B)|_{\rm QCD}$. The final state gluinos in b decay could hadronize into the gluino-gluon or gluino-gluino bound states discussed in [5] or merely into intrinsic gluino components of normal hadrons. Since the values of the c parameter and the universal scalar mass are not yet well determined, the branching ratio of $b\to s\tilde{g}\tilde{g}$ may provide a useful constraint as experiments improve.

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