Taming the scalar mass problem with a singlet Higgs boson

Anirban Kundu*
Theory Group, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta - 700064, India

Sreerup Raychaudhuri†
Theoretical Physics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay - 400005, India

(Received 27 October 1994; revised manuscript received 9 March 1995)

We investigate the fine-tuning problem in the standard model and show that Higgs boson and top quark masses consistent with current experimental bounds cannot be obtained unless one extends the particle spectrum. A minimal extension which achieves this involves the addition of a singlet real scalar but this model is not very predictive. With a discrete symmetry and the further addition of one generation of vectorlike fermions one can get a solution which leads to a phenomenologically viable prediction for the mass of the standard model Higgs boson.

PACS number(s): 14.80.Cp, 12.60.−i, 12.60.Fr, 14.80.Bn

I. INTRODUCTION

Electroweak precision tests at the CERN e^+e^- collider LEP and the recently reported discovery of the top quark at Fermilab [1] have established beyond reasonable doubt the fact that the standard model (SM) is an excellent description of fundamental interactions at least up to the electroweak symmetry-breaking scale. Nevertheless, there is a general belief that the SM does not tell us the whole story, but merely provides an effective Lagrangian of a deeper underlying theory which is yet to be established. One of the chief reasons for such a belief is the so-called *fine-tuning* problem.

In a nutshell, the fine-tuning problem is the following. The masses of scalars, specifically the Higgs boson, receive radiative corrections which are quadratically divergent. If the SM ceased to be applicable at a scale Λ, the mass of the Higgs boson would, therefore, be driven to the same order Λ. That this cannot be so is known from the fact that this would result in a strongly interacting scalar sector where perturbation theory would break down. One is, therefore, driven to argue that the tree-level mass of the Higgs boson must cancel with the radiatively induced self-energy function to yield acceptable values of the physical mass of the Higgs boson (60 GeV–1 TeV). Taking Λ to be the symmetry-breaking scale of grand unified theories (GUT's), i.e., $\Lambda \sim 10^{16}$ GeV, this implies an unnatural cancellation of about 26–28 orders of magnitude.

The fine-tuning problem described above affects the masses of scalars only, since the masses of fermions and vector bosons are protected by chiral and gauge symmetries, so that their radiative corrections can have only logarithmic divergences. This can be clearly seen on computation of radiative corrections to, say, the mass of the Z boson, where the quadratic divergences in individual diagrams will cancel in the final result. With this idea in mind, an elegant restatement of the fine-tuning solution to the problem of runaway corrections to scalar masses is the so-called *Veltman condition* [2]. Assuming that the underlying theory has some yet-to-be-discovered symmetry which protects the scalar mass, one simply sets to zero the sum of computed quadratic divergences in the radiative corrections to the scalar self-energy. Clearly this implies some relation between the physical Higgs boson mass and the masses of other particles such as the top quark and the gauge bosons. The *explanation* of such a relationship must lie, as already stated, in the underlying theory. For a phenomenological study, however, the Veltman condition is very useful, since it reduces, to some degree, the arbitrariness in the choice of top quark and Higgs boson masses in the SM.

Application of the Veltman condition to a model implies a little more than mere cancellation of the coefficients of the quadratic divergences in self-energy diagrams of the scalars. The condition must not change with renormalization group (RG) flow of the couplings. Thus, if $f(g_i,m_i)$ be the net coefficient of the quadratic divergence in question, then the Veltman condition is

$$f(g_i,m_i) \sim \frac{v^2}{\Lambda^2} \tag{1}$$

where $v = \langle 0|H^0|0\rangle$. Stability under RG flow requires

$$\frac{d}{dt} f(g_i,m_i) = 0 \tag{2}$$

where $t \equiv \ln(Q^2/\mu^2)$. These two equations would lead to a unique prediction for m_t and m_H in the SM. Unfortunately one does not obtain any real solution to these equations (see Sec. II). The literature is fairly rich in deeper discussions of this problem, including calculations involving two- and three-loop β functions for the running coupling constants and proper modes of regularization. Some of these are touched upon in the following section.

One solution to the fine-tuning problem lies in banishing fundamental scalars from the theory altogether. Attempts have been made in this direction, but without conspicuous success. The other solution to this dilemma seems to lie in extension of the SM beyond its minimal particle content.

*Electronic address: akundu@saha.ernet.in
†Electronic address: sreerup@theory.tifr.res.in

Typical of such solutions is supersymmetry, where pairing of bosons and fermions occurs in such a way that contributions to $f(g_i, m_i)$ cancel pairwise for every SM particle and its superpartner. Even the minimal supersymmetric extension of the SM, however, requires the addition of 66 particles to the SM. It is desirable, therefore, to consider minimal extensions of the SM particle spectrum to see if the Veltman condition can be satisfied more economically.

In this article, we first consider the addition of vectorlike singlet and doublet fermions to the SM and show that this extension also fails to yield a real solution. However, the addition of a singlet real scalar (which has no interactions with the gauge sector but interacts with the Higgs doublet) can satisfy the Veltman condition. This simple-minded scenario, however, involves a large number of free parameters and hence has no predictive power. An alternative scenario, in which one considers a singlet real scalar with a spontaneously broken discrete symmetry $h \to -h$ can reduce the number of free parameters but this scenario cannot accommodate a RG-stable Veltman condition for a top quark mass greater than 102 GeV. However, the further addition of one generation of vectorlike fermions leads, not only to acceptable solutions of the Veltman condition (for both doublet and singlet) for all values of the top quark mass allowed by experiment, but also to potentially interesting predictions such as a Higgs boson mass in the range 300 to 400 GeV. The vectorlike fermions can naturally appear in some superstring-inspired models. Thus one does not introduce any new scale below the GUT one, in spite of considering new particles. In short, unless one can satisfy the Veltman condition, the introduction of new physics will not reduce the severity of the fine-tuning problem.

The plan of this paper is as follows. In Sec. II, we discuss the Veltman condition in the SM and show that it has no real solution. We also show that inclusion of vectorlike singlet or doublet fermions does not improve the situation. Section III is devoted to two models, one with a singlet real scalar and the other with a singlet real scalar as well as vectorlike fermions. Finally, our conclusions are given in Sec. IV.

II. VELTMAN CONDITION IN THE SM

In this work, we consider the coefficients of quadratic divergences generated at the one-loop level, anticipating that contributions from higher orders will be suppressed by powers of the coupling constants. To this order, then, the Veltman condition in the SM has the form

$$|m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2| \leqslant \frac{16\pi^2}{3\Lambda^2} v^2 m_H^2. \tag{3}$$

In the limit $\Lambda \gg v$, this leads to a simple relation

$$m_H^2 \simeq 4m_t^2 - 2m_W^2 - m_Z^2 \tag{4}$$

which yields $m_H = 182 \pm 22$ GeV for $m_t = 174 \pm 17$ GeV. In determining the above, we set $m_W = 80.2$ GeV and $m_Z = 91.2$ GeV. If we allow new physics to appear at a lower scale, say 10 TeV, in which case the right side of Eq. (3) is of the order of m_H^2, the uncertainty in m_H is increased by about 5 GeV either way.

We can rewrite Eq. (4) using the tree-level relations between masses and coupling constants in the SM. This leads to the alternative form

$$8\lambda + g_1^2 + 3g_2^2 - 8g_t^2 \simeq 0 \tag{5}$$

where $m_H^2 = 2\lambda v^2$, $m_t = g_t v/\sqrt{2}$, and g_1 and g_2 are the coupling constants for $U(1)_Y$ and $SU(2)_L$, respectively. If we now impose RG stability on this equation, we demand

$$\frac{d}{dt}[8\lambda + g_1^2 + 3g_2^2 - 8g_t^2] = 0. \tag{6}$$

Using the well-known[1] β functions of the SM, viz.

$$16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 6g_t^2\lambda - \frac{3}{2}g_1^2\lambda - \frac{9}{2}g_2^2\lambda - 3g_t^4 + \frac{3}{16}g_1^4 + \frac{3}{8}g_1^2 g_2^2 + \frac{9}{16}g_2^4, \tag{7}$$

$$16\pi^2 \frac{dg_t}{dt} = \left(\frac{9}{4}g_t^2 - \frac{17}{24}g_1^2 - \frac{9}{8}g_2^2 - 4g_3^2\right) g_t, \tag{8}$$

$$16\pi^2 \frac{dg_1}{dt} = \frac{41}{12}g_1^3, \tag{9}$$

$$16\pi^2 \frac{dg_2}{dt} = -\frac{19}{12}g_2^3, \tag{10}$$

$$16\pi^2 \frac{dg_3}{dt} = -\frac{7}{2}g_3^3\theta(Q^2 - m_t^2) - \frac{23}{6}g_3^3\theta(m_t^2 - Q^2), \tag{11}$$

we obtain

$$72\lambda^2 + 36g_t^2\lambda - 45g_t^4 - 9g_1^2\lambda - 27g_2^2\lambda + \frac{25}{4}g_1^4 - \frac{15}{4}g_2^4 + \frac{9}{4}g_1^2 g_2^2 + 48g_3^2 g_t^2 + \frac{17}{2}g_1^2 g_t^2 + \frac{27}{2}g_2^2 g_t^2 = 0. \tag{12}$$

Numerical studies show that Eqs. (5) and (12) have no real solutions for m_t and m_H in the range 10 GeV$< m_t <$2 TeV. This tells us that, even if the Veltman condition is satisfied at a low energy scale, it is not valid when we go to high energies, where the problem of runaway scalar masses reappears.

Some authors [3] have argued that the strong coupling g_3 should not appear in the above analysis, since mass generation is essentially an electroweak phenomenon. We do not agree with this point of view, as g_3 appears only in the RG evolution of g_t, where its role is known to be important. In any case, exclusion of g_3 does not improve matters significantly. One does, indeed, get a real solution, but for $m_t = 117$ GeV, a value which is ruled out by the current Collider Detector at Fermilab (CDF) data [1]. We also note, in passing, that even if one considers a lower value of Λ one

[1]We use the convention $t \equiv \ln(Q^2/\mu^2)$ rather than $t \equiv \ln(\sqrt{Q^2}/\mu)$ as a result of which our β functions are half those in the latter convention.

does not obtain real solutions, though, in this case, the fine-tuning problem itself is not so severe.

It is appropriate, at this juncture, to briefly consider some further discussions on the subject which are available in the literature. In the first place, it may be noted that we have obtained all the above results using a simple cutoff regularization. It has been shown [4] that this is appropriate since dimensional regularization cannot isolate quadratic and logarithmic divergences. The Veltman condition can, however, be derived elegantly using point-splitting techniques for regularization [5,6]. Another set of papers [7,8] have discussed two-loop Veltman conditions and shown that the stability conditions at one loop can lead to the cancellation at two loops and similarly for the next order, though this appears to break down at the fourth order. However, as in Ref. [3], these considerations ignore the contribution of the strong coupling constant and thereby get lower values for the top quark mass, which are already ruled out by experiments [1]. Finally, it should be noted that, if one requires the Veltman condition to hold and imposes the additional constraint that the self-energy corrections to the top quark mass vanish, then one predicts [9] $m_t = 170$ GeV and $m_H = 300$ GeV. However, the second condition is rather *ad hoc*.

Let us now consider an extension of the SM particle spectrum by a single generation of exotic vectorlike singlet or doublet fermions. We have not discussed an extra sequential generation, or a generation of mirror fermions, since these are severely constrained by electroweak precision tests at LEP 1 [10]. A lower bound on the masses of vectorlike fermions from LEP 1 data is 45 GeV. Apart from this, vectorlike singlets are not at all constrained by these data, while doublets are merely constrained by the oblique parameter T to be nearly mass degenerate. However, these masses play little or no role in the subsequent discussion.

To study the Veltman condition taking these fermions into account, one notes that they can have gauge-invariant mass terms and can also couple to the SM gauge bosons according to their quantum number assignments. Taking these into account, one now obtains β functions

$$16\pi^2 \frac{dg_1}{dt} = \frac{187}{36} g_1^3, \tag{13}$$

$$16\pi^2 \frac{dg_2}{dt} = -\frac{19}{12} g_2^3, \tag{14}$$

$$16\pi^2 \frac{dg_3}{dt} = -\frac{17}{6} g_3^3 \theta(Q^2 - m_t^2) - \frac{19}{6} g_3^3 \theta(m_t^2 - Q^2) \tag{15}$$

for vectorlike singlets and

$$16\pi^2 \frac{dg_1}{dt} = \frac{139}{36} g_1^3, \tag{16}$$

$$16\pi^2 \frac{dg_2}{dt} = -\frac{1}{4} g_2^3, \tag{17}$$

$$16\pi^2 \frac{dg_3}{dt} = -\frac{17}{6} g_3^3 \theta(Q^2 - m_t^2) - \frac{19}{6} g_3^3 \theta(m_t^2 - Q^2) \tag{18}$$

for vectorlike doublets. Consequently, Eq. (12) gets modified to

$$72\lambda^2 + 36 g_t^2 \lambda - 45 g_t^4 - 9 g_1^2 \lambda - 27 g_2^2 \lambda + \frac{107}{12} g_1^4 - \frac{15}{4} g_2^4 + \frac{9}{4} g_1^2 g_2^2 + 48 g_3^2 g_t^2 + \frac{17}{2} g_1^2 g_t^2 + \frac{27}{2} g_2^2 g_t^2 = 0 \tag{19}$$

and

$$72\lambda^2 + 36 g_t^2 \lambda - 45 g_t^4 - 9 g_1^2 \lambda - 27 g_2^2 \lambda + \frac{83}{12} g_1^4 + \frac{9}{4} g_2^4 + \frac{9}{4} g_1^2 g_2^2 + 48 g_3^2 g_t^2 + \frac{17}{2} g_1^2 g_t^2 + \frac{27}{2} g_2^2 g_t^2 = 0, \tag{20}$$

for vectorlike singlets and doublets, respectively.

Numerical studies of the above equation again reveal that there is no real solution for m_t and m_H as in the SM. This is true even if we consider more than one extra generation since the system of equations changes little unless the number of extra generations is very large. We conclude, therefore, that mere inclusion of vectorlike fermions does not provide a solution to the fine-tuning problem.

III. THE SINGLET HIGGS BOSON OPTION

Let us now consider the minimal extension of the SM scalar sector by a singlet real scalar (h^0) which has all $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers equal to zero and hence does not couple with any of the gauge bosons of the SM. Thus the presence of h^0 does not change Eqs. (9)–(11).

We shall make two assumptions about the scalar potential. First, the potential is bounded from below, which is, strictly speaking, a necessary requirement and not an assumption. Second, h^0 and H^0, the SM Higgs boson, do not mix with each other. (If they do, some quantitative results may change but no qualitative change of what we will discuss takes place.) We shall consider two different scenarios which satisfy these criteria.

Scenario I. The most general scalar potential involving the h^0 and the SM doublet Φ has the form:

$$\mathcal{V}_{\rm scalar} = -m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 + eh + \tilde{m}^2 h^2 + dh^3 + \tilde{\lambda} h^4 + c(\Phi^\dagger \Phi) h + a (\Phi^\dagger \Phi) h^2 \tag{21}$$

with $\lambda, \tilde{\lambda} > 0$. Spontaneous breaking of the electroweak symmetry immediately leads to the relations

$$m_h^2 = 2\tilde{m}^2 + a v^2, \tag{22}$$

$$m_H^2 = 2\lambda v^2. \tag{23}$$

The Veltman conditions will now incorporate the couplings λ, $\tilde{\lambda}$, and a (which give rise to quadratic divergences) as well as the ones arising in the minimal SM. The RG equations for these couplings now become

$$16\pi^2\frac{d\lambda}{dt}=12\lambda^2+6g_t^2\lambda-\frac{3}{2}g_1^2\lambda-\frac{9}{2}g_2^2\lambda-3g_t^4+6a^2-\frac{1}{2}\lambda c^2+\frac{3}{16}g_1^4+\frac{3}{8}g_1^2g_2^2+\frac{9}{16}g_2^4, \tag{24}$$

$$16\pi^2\frac{d\tilde{\lambda}}{dt}=36\tilde{\lambda}^2+a^2-18\tilde{\lambda}d^2, \tag{25}$$

$$16\pi^2\frac{da}{dt}=\left(36\lambda+72\tilde{\lambda}+6g_t^2-\frac{1}{2}c^2-6d^2-\frac{3}{2}g_1^2-\frac{9}{2}g_2^2\right)a. \tag{26}$$

The Veltman condition for the singlet field is

$$3\tilde{\lambda}+a=0 \tag{27}$$

and its stability condition becomes

$$a\left[a-\left(4\lambda+\frac{2}{3}g_t^2-\frac{1}{6}g_1^2-\frac{1}{2}g_2^2-\frac{1}{18}c^2+\frac{4}{3}d^2\right)\right]=0. \tag{28}$$

The Veltman condition for the SM Higgs boson now becomes

$$8\lambda+\frac{4}{3}a+g_1^2+3g_2^2-8g_t^2=0, \tag{29}$$

and the corresponding stability condition is easily found using Eqs. (24) and (26). It is easy to check numerically that with a rather large c and correspondingly small d one can always satisfy all the constraints including the stability condition for h^0 with the top quark mass well within the CDF limit [1]. This hardly comes as a surprise, since the model has seven free parameters $(a,c,d,e,\tilde{\lambda},\tilde{m}^2,g_t)$ while there are just five constraints, viz., the Veltman conditions for the singlet scalar and the SM Higgs boson with the corresponding stability conditions and the requirement that 150 GeV $<m_t<$ 200 GeV.

It is rather interesting that the mere addition of a singlet scalar can help us to obtain phenomenologically allowed solutions to the fine-tuning problem. However, due to the presence of so many undetermined parameters this scenario has little or no predictive power. Only if the model can be embedded in a deeper underlying theory (one which incorporates the symmetry behind the Veltman condition, perhaps?) can we expect relations between some of these parameters, in which case the scenario would regain interest. At the present state of our knowledge, it appears pointless to pursue this approach any further.

Scenario II. A small modification in the above, however, leads to a much more attractive scenario. Let us introduce the singlet scalar together with a discrete symmetry $h\to -h$, which is spontaneously broken to yield a vacuum expectation value (VEV) v' of the singlet field. The unbroken potential is

$$\mathscr{V}_{\rm scalar}=-m^2\Phi^\dagger\Phi+\lambda(\Phi^\dagger\Phi)^2-\tilde{m}^2h^2+\tilde{\lambda}h^4+a(\Phi^\dagger\Phi)h^2 \tag{30}$$

with $\tilde{m}^2>0$, which is essentially like a $\lambda\phi^4$ theory coupled to the SM.[2] The symmetry-breaking pattern now yields the relations

$$m_h^2=4\tilde{m}^2-2av^2, \tag{31}$$

$$m_H^2=2m^2-2av'^2. \tag{32}$$

Notice that the number of undetermined parameters in this model has now reduced to five, since the discrete symmetry forces us to set $c=d=e=0$ and we have introduced the extra parameter v'. One can naively expect to find a solution, therefore, in which all the parameters are fixed. (In the absence of a vacuum expectation value for the singlet h^0, there are just four undetermined parameters which do not allow sufficient play to satisfy the five constraints detailed above.)

The RG equations for the couplings λ, $\tilde{\lambda}$, and a, for this scenario, are obtained simply by setting $c=d=0$ in Eqs. (24)–(26). Using these in the Veltman condition (27) for the singlet field we get the RG stability condition

$$a\left[a-\left(4\lambda+\frac{2}{3}g_t^2-\frac{1}{6}g_1^2-\frac{1}{2}g_2^2\right)\right]=0 \tag{33}$$

which is just Eq. (28) with $c=d=0$. It is now trivial to verify that Eqs. (27) and (33) have no nontrivial solution for $m_t>102$ GeV which is the threshold for which the quantity in parentheses (in the last equation) becomes positive. When coupled also with the Veltman condition and the RG stability condition of the SM Higgs boson, the set of equations have no real solution although the number of contraints is equal to the number of undetermined parameters.[3]

We conclude, therefore, that the inclusion of just a singlet scalar field, in this scenario, cannot make the fine-tuning problem vanish at all scales up to Λ.

Let us, therefore, enrich the particle spectrum in this scenario further by adding one generation of exotic vectorlike fermions (F). A purely phenomenological study [11] of the one-loop-induced process $Z^0\to h^0\gamma$ has shown that there are practically no contraints on this kind of scenario from LEP-1 data. We have seen, in the previous section, that these extra fermions will modify the β functions of the theory. Their Yukawa couplings with h^0 are given by

$$\mathscr{L}_{h\bar{F}F}=-\zeta_F h\bar{F}F. \tag{34}$$

Notice that the discrete symmetry $h\to -h$ should now include $F\to i\gamma_5 F$. It is now clear that unless the discrete symmetry is broken spontaneously the vectorlike fermions cannot develop mass terms. Observe that this modification introduces four new parameters in our analysis, viz., ζ_U,

[2]The discrete symmetry must be broken, however, to allow us to introduce Yukawa couplings of the singlet scalar with massive vectorlike fermions, which, we shall see, are required to solve the Veltman conditions.

[3]The apparent paradox created by this statement is resolved if we recall that there are other constraints arising from the positivity of the couplings λ and $\tilde{\lambda}$. In a strict sense, then, there are more constraints than free parameters.

ζ_D, ζ_N, and ζ_E (the suffixes are self-explanatory). The masses of these fermions are given by

$$m_F = v' \zeta_F . \tag{35}$$

The electroweak precision data force us to take $\zeta_U \simeq \zeta_D$ and $\zeta_N \simeq \zeta_E$ for vectorlike doublet fermions. A similar assumption about the Yukawa couplings of vectorlike singlet fermions may not be far from the truth. Thus we have added two more unknowns to the set of equations to be solved and should, therefore, expect a solution. For simplicity, we shall initially set $\zeta_U = \zeta_D = \zeta_N = \zeta_E$, so that there is only one extra parameter to play with and consider modifications of this scheme subsequently.

As the vectorlike fermions do not couple with the SM Higgs boson, the Veltman condition for H^0 is given by Eq. (29) while Eq. (27) will be modified to

$$3\tilde{\lambda} + a - Z^2 = 0, \tag{36}$$

where

$$Z^2 = \sum N_c \zeta^2 = 3(\zeta_U^2 + \zeta_D^2) + (\zeta_N^2 + \zeta_E^2). \tag{37}$$

Equations (28) and (29) will be modified to

$$16\pi^2 \frac{d\tilde{\lambda}}{dt} = 36\tilde{\lambda}^2 + a^2 + 4\tilde{\lambda} Z^2 - \sum N_c \zeta^4, \tag{38}$$

$$16\pi^2 \frac{da}{dt} = \left(36\lambda + 72\tilde{\lambda} + 6g_t^2 + 4Z^2 - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 \right) a. \tag{39}$$

The β functions for the ζ's are

$$16\pi^2 \frac{d\zeta_U}{dt} = \left(\frac{3}{2} \zeta_U^2 + Z^2 - \frac{4}{3} g_1^2 - 4g_3^2 \right) \zeta_U , \tag{40}$$

$$16\pi^2 \frac{d\zeta_D}{dt} = \left(\frac{3}{2} \zeta_D^2 + Z^2 - \frac{1}{3} g_1^2 - 4g_3^2 \right) \zeta_D , \tag{41}$$

$$16\pi^2 \frac{d\zeta_N}{dt} = \left(\frac{3}{2} \zeta_N^2 + Z^2 \right) \zeta_N , \tag{42}$$

$$16\pi^2 \frac{d\zeta_E}{dt} = \left(\frac{3}{2} \zeta_E^2 + Z^2 - 3g_1^2 \right) \zeta_E \tag{43}$$

for vectorlike singlet fermions, and

$$16\pi^2 \frac{d\zeta_U}{dt} = \left(\frac{3}{2} \zeta_U^2 + Z^2 - \frac{1}{12} g_1^2 - \frac{21}{8} g_2^2 - 4g_3^2 \right) \zeta_U , \tag{44}$$

$$16\pi^2 \frac{d\zeta_D}{dt} = \left(\frac{3}{2} \zeta_D^2 + Z^2 - \frac{1}{12} g_1^2 - \frac{21}{8} g_2^2 - 4g_3^2 \right) \zeta_D , \tag{45}$$

$$16\pi^2 \frac{d\zeta_N}{dt} = \left(\frac{3}{2} \zeta_N^2 + Z^2 - \frac{3}{4} g_1^2 - \frac{21}{8} g_2^2 \right) \zeta_N , \tag{46}$$

$$16\pi^2 \frac{d\zeta_E}{dt} = \left(\frac{3}{2} \zeta_E^2 + Z^2 - \frac{3}{4} g_1^2 - \frac{21}{8} g_2^2 \right) \zeta_E \tag{47}$$

for vectorlike doublet fermions. The RG stability equations are

$$72\lambda^2 + 36g_t^2\lambda - 45g_t^4 + 36a^2 + 36a\lambda + 72a\tilde{\lambda} + 6g_t^2 a - 9g_1^2\lambda - 27g_2^2\lambda + \frac{107}{12} g_1^4 - \frac{15}{4} g_2^4 + \frac{9}{4} g_1^2 g_2^2 + 48 g_3^2 g_t^2 + \frac{17}{2} g_1^2 g_t^2 + \frac{27}{2} g_2^2 g_t^2 - \frac{3}{2} a g_1^2 - \frac{9}{2} a g_2^2 + 4aZ^2 = 0, \tag{48}$$

$$108\tilde{\lambda}^2 + 3a^2 + 36a\lambda + 72a\tilde{\lambda} + 6g_t^2 a - \frac{3}{2} g_1^2 a - \frac{9}{2} g_2^2 a + (12\tilde{\lambda} + 4a)Z^2 - 2Z^4 - 6\sum N_c \zeta^4 + g_1^2(8\zeta_U^2 + 2\zeta_D^2 + 6\zeta_E^2) + 24 g_3^2(\zeta_U^2 + \zeta_D^2) = 0, \tag{49}$$

for vectorlike singlets, and

$$72\lambda^2 + 36g_t^2\lambda - 45g_t^4 + 36a^2 + 36a\lambda + 72a\tilde{\lambda} + 6g_t^2 a - 9g_1^2\lambda - 27g_2^2\lambda + \frac{83}{12} g_1^4 + \frac{9}{4} g_2^4 + \frac{9}{4} g_1^2 g_2^2 + 48 g_3^2 g_t^2 + \frac{17}{2} g_1^2 g_t^2 + \frac{27}{2} g_2^2 g_t^2 - \frac{3}{2} a g_1^2 - \frac{9}{2} a g_2^2 + 4aZ^2 = 0, \tag{50}$$

$$108\tilde{\lambda}^2 + 3a^2 + 36a\lambda + 72a\tilde{\lambda} + 6g_t^2 a - \frac{3}{2} g_1^2 a - \frac{9}{2} g_2^2 a + (12\tilde{\lambda} + 4a)Z^2 - 2Z^4 - 6\sum N_c \zeta^4 + \frac{1}{2} g_1^2(\zeta_U^2 + \zeta_D^2 + 3\zeta_E^2 + 3\zeta_N^2) + \frac{21}{4} g_2^2 Z^2 + 24 g_3^2(\zeta_U^2 + \zeta_D^2) = 0, \tag{51}$$

TABLE I. The predicted parameters for vectorlike singlet fermions with m_t as input. All masses are in GeV. We have set $\zeta_U = \zeta_D = \zeta_E = \zeta_N = \zeta$.

m_t	λ	$\tilde{\lambda}$	a	ζ	m_H	m_h/m_F
150	0.80	1.06	-1.39	0.47	311	6.15
155	0.86	1.11	-1.45	0.49	323	6.14
160	0.92	1.16	-1.49	0.50	334	6.11
165	0.98	1.22	-1.53	0.52	344	6.06
170	1.04	1.27	-1.56	0.53	355	6.01
175	1.11	1.33	-1.64	0.54	367	6.01
180	1.18	1.39	-1.70	0.56	378	6.01
185	1.25	1.45	-1.76	0.57	389	5.99
190	1.32	1.51	-1.81	0.58	400	5.96
195	1.39	1.58	-1.85	0.60	410	5.91
200	1.47	1.64	-1.94	0.61	422	5.93

for vectorlike doublets.

The set of equations is now solved numerically in each case and our results are shown in Table I for vectorlike singlet fermions and Table II for vectorlike doublet fermions. We are able, for the top quark mass m_t as input, to predict values for the couplings $\lambda, \tilde{\lambda}, a$, and ζ. Moreover, we can also use these as inputs to predict values of the SM Higgs boson mass m_H. Masses of the singlet Higgs boson and the vectorlike fermions cannot be predicted, but one can obtain a ratio m_h/m_F which roughly tells us that this scenario will be valid only if there are relatively light exotic fermions. Such fermions should definitely be seen at a 500 GeV e^+e^- collider, if not at LEP-2, and are an important prediction of the model. As explained above, we have taken $\zeta_U = \zeta_D = \zeta_N = \zeta_E$ to obtain the results given in these tables. Assuming that the ζ's are of the same order of magnitude, we have checked that the results hardly change if we take $\zeta_N = \zeta_E = 2\zeta_U = 2\zeta_D$ or $\zeta_U = \zeta_D = 2\zeta_N = 2\zeta_E$ for the vectorlike singlet fermions. For the vectorlike doublet fermions, λ does not change by more than 2–3%. This is illustrated in Table III. The change in the predicted ratio m_h/m_E from Table II is due to the fact that the corresponding Yukawa coupling carries a factor of one-half compared with that for

TABLE II. The predicted parameters for vectorlike doublet fermions with m_t as input. All masses are in GeV. We have set $\zeta_U = \zeta_D = \zeta_E = \zeta_N = \zeta$.

m_t	λ	$\tilde{\lambda}$	a	ζ	m_H	m_h/m_F
150	0.81	1.08	-1.45	0.47	313	6.21
155	0.86	1.13	-1.45	0.49	322	6.09
160	0.92	1.18	-1.49	0.50	333	6.07
165	0.99	1.24	-1.59	0.51	346	6.10
170	1.05	1.39	-1.61	0.56	356	5.90
175	1.12	1.35	-1.69	0.54	368	6.05
180	1.19	1.41	-1.76	0.55	379	6.05
185	1.26	1.47	-1.82	0.56	390	6.03
190	1.33	1.53	-1.87	0.58	401	6.00
195	1.40	1.59	-1.90	0.59	411	5.96
200	1.48	1.66	-1.99	0.61	423	5.96

TABLE III. The predicted parameters for vectorlike doublet fermions with m_t as input. All masses are in GeV. We have set $\zeta_U = \zeta_D = 2\zeta_E = 2\zeta_N = 2\zeta$. This results in a value for m_h/m_E roughly double that in Table II.

m_t	λ	$\tilde{\lambda}$	a	ζ	m_H	m_h/m_E
150	0.83	1.15	-1.56	0.26	316	11.27
160	0.95	1.25	-1.67	0.28	339	11.19
170	1.07	1.36	-1.73	0.30	359	10.99
180	1.21	1.48	-1.88	0.31	382	10.97
190	1.35	1.61	-1.99	0.33	404	10.86
200	1.50	1.75	-2.11	0.34	426	10.78

quarks. It seems, therefore, to be a safe assumption to take all ζ's equal. With such a relation among the ζ's, one can simultaneously solve four equations at some particular point of the $\lambda, \tilde{\lambda}$ space for a given m_t. It is noteworthy that the solution comes out in the perturbative domain of the couplings, i.e., $|\lambda|, |\tilde{\lambda}|, |a|, |\zeta^2| \leqslant 4\pi$. The prediction of the mass of the SM Higgs boson also comes out in the range 300–400 GeV, which appears to be in the right ballpark if one accepts current fits to LEP 1 data for the Higgs boson mass.

IV. CONCLUSIONS

We have shown that the Veltman condition in the SM together with its RG stability fails to produce any acceptable solution. This leads us to consider minimal extensions of the SM, first in the fermionic sector by introducing vectorlike exotic fermions, and then in the scalar sector by introducing a singlet real scalar. Extension of the standard model in this manner is, of course, *ad hoc*, and an improved theory (in which the standard model is embedded) should explain why such particles are present, apart from providing the underlying symmetry reflected in the Veltman condition. Nevertheless, even without such a theory, it is interesting to see that the first option fails to produce any real solution to the Veltman condition. The second provides acceptable solutions, but leads to the introduction of a great many undetermined parameters which makes the model completely unpredictive. A discrete symmetry removes some of these parameters, but also restricts solutions to the range $m_t < 102$ GeV, which is ruled out by experiment. However, when we consider vectorlike fermions as well as a singlet scalar (together with the discrete symmetry) in the particle spectrum, not only do we get solutions to the Veltman conditions for the two scalars, but we also get a prediction of m_H, which is in the experimentally favored range. The couplings also come out to be perturbative in nature, which is essential for the self-consistency of the entire scheme. This appears to be an encouraging result which should motivate searches for singlet Higgs bosons and exotic vectorlike fermions at the upcoming colliders.

ACKNOWLEDGMENTS

The authors are grateful to Debajyoti Choudhury and Ashoke Sen for a number of illuminating suggestions. A.K.

acknowledges the Theoretical Physics Seminar Circuit scheme of the Department of Science and Technology, Government of India, for funding his visit to the Theoretical Physics Group of the Tata Institute of Fundamental Research. He would also like to thank the Institute for warm hospitality. The work of S.R. was partially supported by a project (DO No. SR/SY/P-08/92) of the Department of Science and Technology, Government of India.

[1] CDF collaboration, F. Abe *et al.*, Phys. Rev. D **50**, 2966 (1994).
[2] M. Veltman, Acta Phys. Pol. B **12**, 437 (1981).
[3] E. Ma, Mod. Phys. Lett. A **7**, 2741 (1992).
[4] M.B. Einhorn and D.R.T. Jones, Phys. Rev. D **46**, 5206 (1992).
[5] P. Osland and T.T. Wu, Phys. Lett. B **291**, 315 (1992); Z. Phys. C **55**, 585 (1992).
[6] C. Newton, P. Osland, and T.T. Wu, Z. Phys. C **61**, 441 (1994).
[7] M.S. Al-sarhi, I. Jack, and D.R.T. Jones, Z. Phys. C **55**, 283 (1992); Nucl. Phys. **B345**, 431 (1990); I. Jack and D.R.T. Jones, *ibid.* **B342**, 127 (1990); Phys. Lett. B **234**, 321 (1990).
[8] J. Kubo, K. Sibold, and W. Zimmermann, Phys. Lett. B **220**, 191 (1989).
[9] G.L. Castro and J. Pestieau, Mod. Phys. Lett. A **10**, 1155 (1995).
[10] G. Bhattacharyya *et al.*, Phys. Rev. Lett. **64**, 2870 (1990).
[11] S. Raychaudhuri and A. Raychaudhuri, Phys. Rev. D **44**, 2663 (1991).