Reconciling supersymmetric grand unification with $lpha_s(m_Z)pprox 0.11$

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We argue that supersymmetric grand unification of gauge couplings is not incompatible with small α_s , even without large GUT-scale corrections, if one relaxes a usual universal gaugino mass assumption. A commonly assumed relation $M_2 \simeq m_{\tilde{g}}/3$ is in gross contradiction with $\alpha_s \approx 0.11$. Instead, a small α_s favors $M_2 \gg m_{\tilde{g}}$. If this is indeed the case our observation casts doubt on another commonly used relation $M_1 \simeq 0.5M_2$ which originates from the same constraint of a common gaugino mass at the GUT scale. One firm prediction emerging within the small α_s scenario with the unconstrained gaugino masses is the existence of a relatively light gluino below ~ 200 GeV.

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I. INTRODUCTION

One of the testing grounds for various models of grand unification is calculating the strong coupling constant $\alpha_s(m_Z)$ using, as input, the experimental values of the electromagnetic coupling constant α and $\sin^2 \theta_W$, where θ_W is the Weinberg angle. These calculations have been repeatedly carried out in different models and under different assumptions (for recent reviews see, e.g., Ref. [1]). It has been shown, in particular, that the simplest grand unification based on the standard model (SM) and SU(5) gauge group leads to too small a value of the strong coupling constant $\alpha_s(m_Z) = 0.073 \pm 0.002$ [2] and is, thus, ruled out [3]. In contrast, supersymmetric models generally predicted $\alpha_s(m_Z)$ in agreement [3] with experimental data available at that time.

A straightforward supersymmetrization of SM gives rise to the minimal supersymmetric standard model (MSSM) [4]. Actually, to fully specify the model one has to make an additional assumption about the pattern of supersymmetry (SUSY) breaking. The most popular mechanism is that of soft breaking in which one adds to the Lagrangian all possible soft SUSY-breaking terms and treats them as independent parameters. Such terms arise, e.g., when the MSSM is coupled to supergravity [5]. This mechanism of generating soft terms is so deeply rooted that quite often in the current literature no distinction is made between the MSSM per se and the MSSM plus the assumptions of the minimal supergravitybased SUSY-breaking. In fact, an overwhelming majority of papers devoted to even purely phenomenological studies of the MSSM assume some (but typically not all) relations stemming from minimal supergravity, e.g., the relation between the mass parameters of the gauginos of SU(2) and U(1).

Encouraged by early studies [3], many authors (see, e.g., Refs. [6-9]) then studied unification in the context

strictive. In particular, in the context of minimal N = 1supergravity the masses of all gauginos, gluinos of SU(3), W-inos of SU(2), and the B-ino of U(1), turn out to be the same at the Planck scale. Similarly, the soft mass parameters of all squarks and sleptons are equal at that scale. In this restrictive model, which was called constrained MSSM (CMSSM) [8], one assumes universal masses for all the gauginos $(m_{1/2})$ and all the scalars (m_0) at the grand unified theory (GUT) scale, and often additionally imposes a mechanism of radiative electroweak symmetry breaking (EWSB) [10]. Accepting these assumptions one arrives at quite definite predictions for the spectra of masses of the model at the weak scale and for $\alpha_s(m_Z)$. For example, the gluino turns out to be roughly three times heavier than W-ino [4]. Furthermore, $\alpha_s(m_Z)$ generally decreases with increasing $m_{1/2}$ and m_0 . Restricting $m_{1/2}$ and m_0 (or alternatively all the masses) below roughly 1 TeV leads to $\alpha_s(m_Z) \gtrsim 0.12$ [8, 2]. For example, an updated analysis of Ref. [11] quotes $\alpha_s(m_Z) = 0.129 \pm 0.008$. The theoretical error here is mostly due to uncertainty associated with the so-called threshold corrections at the GUT and low (SUSY-breaking) scales and higher-dimensional nonrenormalizable operators (NRO's) in the GUT scale Lagrangian. The above prediction for $\alpha_s(m_Z)$ was considered as a great success and the strongest evidence in favor of the MSSM in light of the fact that, as was believed, the direct measurement of the strong coupling constant at the CERN e^+e^- collider LEP and SLAC Large Detector (SLD) yields $\alpha_s(m_Z) = 0.125 \pm 0.05$ [12].

of the MSSM coupled to minimal supergravity. The set of SUSY-breaking terms generated this way is quite re-

Recently it has been pointed out, however, that QCD cannot tolerate such a large value of the coupling constant [13]. A wealth of low-energy data indicates that $\alpha_s(m_Z)$ must be very close to 0.11 [14], three standard deviations below the alleged LEP or SLD value. A theoretically clean method of determining α_s by extracting α_s from deep inelastic scattering (DIS) [15] gives $\alpha_s(m_Z) = 0.112 \pm 0.005$. Another reliable approach is using [20, 21] (Euclidean) QCD sum rules. The observation of Ref. [13] motivated a new analysis of the Υ sum

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rules [21] claiming the record accuracy achieved so far:

$$\alpha_s(m_Z) = 0.109 \pm 0.001. \tag{1}$$

Similar values are obtained in lattice QCD: $\alpha_s(m_Z) =$ 0.110 ± 0.006 (*cc* spectrum) [16] and 0.115 ± 0.002 (*bb* spectrum) [17]. The apparent clash between the low-energy determinations of the strong coupling constant and those at the Z peak may be explained (see, e.g., Refs. [18, 19, 34, 13) by contributions going beyond SM which were not taken into account in the global fits. It should be stressed that the two scenarios, large α_s versus small α_s , cannot coexist peacefully, as it is sometimes implied in the current literature. Our starting point is the assumption that the large α_s option [22], inconsistent with crucial features of QCD, will eventually evaporate and the value of the strong coupling constant at m_Z will stabilize close to 0.11. (In fact, in Ref. [23] it has been argued that the systematic error usually quoted in the LEP number is grossly underestimated, and that at present LEP experiments can only claim $0.10 \lesssim \alpha_s(m_Z) \lesssim 0.15$.)

The question arises whether grand unification within the framework of the MSSM can accommodate small $\alpha_s \approx 0.11$. This study addresses this question. Our task is to sort out assumptions (sometimes implicit) which are inevitable in analyses of this type and to find out which assumptions of the CMSSM absolutely preclude one from descending to small $\alpha_s(m_Z)$ and, therefore, have to be relaxed.

There are several possible ways to reconcile the prediction for $\alpha_s(m_Z)$ in supersymmetric grand unification with $\alpha_s(m_Z) \approx 0.11$. One is to remain in the context of the CMSSM with strict gauge coupling unification but to adopt a heavy SUSY scenario with the SUSY mass spectra significantly exceeding 1 TeV. This scenario would not only put SUSY into both theoretical and experimental oblivion but is also, for the most part, inconsistent with our expectations that the lightest supersymmetric particle (LSP) should be neutral and/or with the lower bound on the age of the Universe of at least some 10 billion years [8]. Another possibility is to invoke large enough negative corrections due to GUT-scale physics. The issue has been reanalyzed very recently [11,24-26]. Under a socalled no-conspiracy assumption it was found in Ref. [11] that $\alpha_s(m_Z) > 0.12$. Relaxing this assumption one can, in principle, construct models of the CMSSM with large negative contributions coming from GUT scale threshold corrections and the NRO's which could decrease the value of $\alpha_s(m_Z)$ by ~ 10% [11, 24]. [Alternatively, one can entertain the possibility of an intermediate scale [27]around 10^{11} GeV whose existence is motivated by other reasons. In this case, however, many more unknowns affect the running of the gauge couplings and one cannot really talk about predicting $\alpha_s(m_Z)$.] Although it may well happen that the GUT-scale and NRO corrections are negative and large, the guiding idea of grand unification becomes less predictive. Indeed, by appropriately complicating GUT-scale physics one could, perhaps, achieve gauge coupling unification even in the standard model.

Below we want to point out an alternative mechanism of lowering $\alpha_s(m_Z)$, which does not require GUT scale corrections but does not exclude them either. We will adopt a down-to-earth, purely phenomenological attitude, with no assumptions about mechanisms of SUSY breaking. We do not assume N = 1 supergravity, nor any mass relations associated with this scheme, for instance, the equality of the gaugino masses at the GUT scale. In the traditional approach, one does assume not only the existence of an underlying GUT gauge group but also a rather specific mechanism of generating soft masses by coupling the MSSM, or its parent GUT model, to N = 1supergravity where local SUSY breaks via a gaugino condensate in the hidden sector. In this scenario gaugino masses at the GUT scale are equal. This mechanism may be considered by some as most attractive but it is certainly not the only one possible. In particular, one can imagine breaking SUSY in the visible sector below the GUT scale, in which case the gaugino masses at the GUT scale are zero and at the electroweak scale they are a priori unrelated [28]. If no theoretical scheme for the mass generation of SUSY partners is specified one is free to consider any values of these masses. Our task is to try to find out what pattern of masses may be preferred by phenomenology. We consider the MSSM and limit ourselves to a "minimal set" of assumptions: (i) all gauge coupling constants are exactly equal to each other at the GUT scale; (ii) the SUSY-breaking terms do not significantly exceed 1 TeV.

We will show that by relaxing the CMSSM to the MSSM one can easily descend to $\alpha_s(m_Z) \approx 0.11$. The only effect which is actually important in dramatically reducing the minimal value of $\alpha_s(m_Z)$ is untying the gluino and W-ino masses. One firm conclusion is a relatively light gluino (in the ballpark of 100 GeV and typically below 200 GeV) and a relatively heavy W-ino (at least a few hundred GeV), i.e., a relation opposite to the one emerging in the CMSSM and, more generally, any scenario with universal gaugino masses. This summarizes our main results.

II. CALCULATING $\alpha_s(M_Z)$ FROM GRAND UNIFICATION

A. Procedure

The procedure for predicting $\alpha_s(m_Z)$ assuming gauge coupling unification has been adequately described in the literature (see, e.g., Ref. [8] and references therein), and we will only summarize it briefly here. The strategy is simple: the coupling constants α_1 and α_2 (which are known more accurately than α_s) are evolved from their experimental values at m_Z up to the point where they intersect (which thus defines the unification scale M_X and the gauge strength α_X). At that point one identifies α_s with α_X and runs it down to m_Z , thus predicting the value of $\alpha_s(m_Z)$ as a function of input parameters. Oneand two-loop corrections are taken into account.

The renormalization group equations (RGE's) for the gauge couplings are given by

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2 + \text{two loops},\tag{2}$$

where $i = 1, 2, 3, t \equiv \log(Q/m_Z)$ and $\alpha_1 \equiv \frac{5}{3}\alpha_Y$. The one-loop coefficients b_i of the β functions for the gauge couplings change across each new running mass threshold. In the MSSM they can be parametrized as [6, 29, 8]

$$b_{1} = \frac{41}{10} + \frac{2}{5}\theta_{\widetilde{H}} + \frac{1}{10}\theta_{H_{2}} + \frac{1}{5}\sum_{i=1}^{5} \{\frac{1}{12}(\theta_{\tilde{u}_{L_{i}}} + \theta_{\tilde{d}_{L_{i}}}) + \frac{4}{3}\theta_{\tilde{u}_{R_{i}}} + \frac{1}{3}\theta_{\tilde{d}_{R_{i}}} + \frac{1}{4}(\theta_{\tilde{e}_{L_{i}}} + \theta_{\tilde{\nu}_{L_{i}}}) + \theta_{\tilde{e}_{R_{i}}}\},$$
(3)

$$b_{2} = -\frac{19}{6} + \frac{4}{3}\theta_{\widetilde{W}} + \frac{2}{3}\theta_{\widetilde{H}} + \frac{1}{6}\theta_{H_{2}} + \frac{1}{2}\sum_{i=1}^{3} \{\theta_{\tilde{u}_{L_{i}}}\theta_{\tilde{d}_{L_{i}}} + \frac{1}{3}\theta_{\tilde{e}_{L_{i}}}\theta_{\tilde{\nu}_{L_{i}}}\},\tag{4}$$

$$b_{3} = -7 + 2\theta_{\tilde{g}} + \frac{1}{6} \sum_{i=1}^{3} \{\theta_{\tilde{u}_{L_{i}}} + \theta_{\tilde{d}_{L_{i}}} + \theta_{\tilde{u}_{R_{i}}} + \theta_{\tilde{d}_{R_{i}}}\},\tag{5}$$

where $\theta_x \equiv \theta(Q^2 - m_x^2)$.

In Eqs. (3)-(5) \tilde{H} stands for the (mass degenerate) Higgsino fields, \overline{W} for the W-inos, the partners of the SU(2) gauge bosons $(m_{\widetilde{W}} \equiv M_2)$, and \widetilde{g} stands for the gluino, all taken to be mass eigenstates in this approximation. Also, in this approximation H_2 stands for a heavy Higgs boson doublet, as explained in Ref. [8]. (The full two-loop gauge coupling β functions for the SM and the MSSM which we use in actual calculations can be found, e.g., in Ref. [30].)

Equations (3)-(5) represent so-called leading logarithmic approximations and involve some simplifications. However, as we will argue later, it will be sufficient to present the basic points of our analysis and answer the question how low one could descend in the values of $\alpha_s(m_Z)$ assuming only strict unification of the gauge couplings in the MSSM.

The prediction for $\alpha_s(m_Z)$ depends on the adopted values of the input parameters: α , $\sin^2 \theta_W(m_Z)$, and m_t . It also receives corrections from the two-loop gauge and Yukawa contributions, scheme dependence [modified minimal subtraction (MS) versus dimensional reduction with modified minimal subtraction (\overline{DR}) , mass thresholds at the electroweak scale and, finally, the GUT-scale mass thresholds and NRO contributions. We will discuss these effects in turn now.

The input values of α_1 and α_2 at $Q = m_Z$ can be extracted from the experimental values of $\alpha(m_Z)$ and $\sin^2 \theta_W(m_Z)$. For the electromagnetic coupling we take [31]

$$\alpha(m_Z) = \frac{1}{127.9 \pm 0.1}.\tag{6}$$

Recently, three groups have reanalyzed $\alpha(m_Z)$ [32] and obtained basically similar results: $\alpha(m_Z)^{-1} = 127.96 \pm$ 0.06 (Martin and Zeppenfeld), 127.87 ± 0.10 (Eidelman and Jegerlehner), and 128.05 ± 0.10 (Swartz). Adopting even the largest (central) value of Swartz would shift $\alpha_s(m_Z)$ up by only 0.001 [11].

The range of input values of $\sin^2 \theta_W(m_Z)$ is rather critical. This sensitivity is due to the fact that $\alpha_2(Q)$ does not change between $Q = m_Z$ and the GUT scale $Q = M_X$ as much as the other two couplings. Thus, a small increase in $\sin^2 \theta_W(m_Z)$ has an enhanced (and negative) effect on the resulting value of $\alpha_s(m_Z)$. Following Ref. [11] we assume [33]

$$\sin^2 \theta_W(m_Z) = 0.2316 \pm 0.0003 - 0.88 \times 10^{-7} \,\text{GeV}^2 \left[m_t^2 - (160 \,\text{GeV})^2 \right]. \tag{7}$$

Moreover, the global analysis of Ref. [34] implies that in the MSSM $m_t = 160 \pm 13 \,\text{GeV}$. Recently, both the Collider Detector at Fermilab (CDF) and D0 Collaborations have reported discovery of the top quark and quoted somewhat higher mass ranges: $m_t = 176 \pm 8 \pm 10 \,\text{GeV}$ (CDF) [35] and $m_t = 199 \pm 20 \pm 22 \text{ GeV}$ (D0) [36]. Such high (central) values of m_t would lower $\sin^2 \theta_W(m_Z)$ and increase $\alpha_s(m_Z)$ by 0.002 and 0.005, respectively.

Including the two-loop terms in the RGE's increases $\alpha_s(m_Z)$ by about 10%. There are two types of contributions to $\alpha_s(m_Z)$ at the two-loop level. Pure gauge terms yield $\Delta \alpha_s(m_Z) = 0.012$ if one assumes SUSY in both one- and two-loop coefficients of the β function all the way down to $Q = m_Z$. This is the most important correction to the one-loop value of $\alpha_s(m_Z)$. If, instead, the two-loop coefficients of the pure gauge part are changed to their SM values at Q = 1 TeV, one finds an additional shift $\Delta \alpha_s(m_Z) \approx 0.0007$. Since this shift is negligibly small we keep the two-loop coefficients supersymmetric all the way down to m_Z . Corrections due to the Yukawa-coupling contribution to the RGE's are also small, although negative [11]. In the limit of large top Yukawa coupling $(h_t \simeq 1, h_b \simeq 0 \simeq h_{\tau})$, as in the small tan $\beta \simeq 1$ scenario) one finds $\Delta \alpha_s(m_Z) = -0.0015$ while even in the extreme case of the large $\tan\beta$ scenario $(h_t \simeq h_b \simeq h_\tau \simeq 1) \ \Delta \alpha_s(m_Z) = -0.004$, in agreement with Ref. [11].

Above Q = 1 TeV we also change from the conventional MS scheme, that we use throughout this paper, to the fully supersymmetric $\overline{\mathrm{DR}}$ scheme. The corresponding shift in $\alpha_s(m_Z)$ is about 0.0002 and is negligible numerically [2, 8, 29].

Before proceeding to discussing in more detail the contribution from one-loop threshold effects, a remark is in order on possible corrections from the GUT-scale mass thresholds and NRO's. Since in this paper we look for an alternative way of lowering $\alpha_s(m_Z)$, we switch off all corrections from the GUT-scale physics whatsoever. As was noted previously [29, 2, 24, 11] they are GUTmodel dependent and, in principle, can be sizable. For

instance, according to Refs. [2, 11] the corresponding effect in $\alpha_s(m_Z)$ can be as large as ~ 0.008; a factor of 2.5 larger effect is needed, however, to ensure $\alpha_s(m_Z) \approx 0.11$. Building a fully elaborated and phenomenologically acceptable model of this type seems to be a task for the future.

What remains to be done is to explain our treatment of the mass thresholds at the electroweak and SUSY scales. We use the usual steplike approximation in the coefficients of the β function, Eqs. (3)–(5). In the one-loop coefficients the jumps occur at the positions of the masses of the individual particles while in the two-loop coefficients it is sufficient, to our accuracy, to consider one jump at a common SUSY scale, as explained above. As a matter of fact, with no loss of accuracy, we take this scale in the two-loop coefficients to be lower than m_Z so that in our evolution from M_X down to m_Z we treat the two-loop coefficients as fully supersymmetric. Also, the t quark is not frozen at m_t in the two-loop coefficients. It is well known that the steplike approximation is not absolutely accurate in the problem of the coupling constant evolution (see, e.g., Ref. [37] for a recent discussion), especially if the mass thresholds are rather close to m_Z , as is the case with t quark. We find that the other thresholds are far less important, since, as we vary their positions, the effect of the variation mimics the nonlogarithmic corrections omitted in the step approximation. The error in $\alpha_s(m_Z)$ due to the inaccuracy of our approximation of the α_s evolution at m_t is less than 1% and is. thus, unimportant.

B. MSSM with gauge unification only

The question we want to address is whether supersymmetric grand unification necessarily predicts large values of $\alpha_s(m_Z) \gtrsim 0.12$ as long as all SUSY masses are restricted to lie below 1 TeV. This is indeed the case in the CMSSM with additional assumptions of common gaugino mass and common scalar mass, as described in the Introduction.

In order to track the role of these mass relations we begin by treating the masses of the different types of states as completely independent parameters. We choose to remain open-minded and not biased by any additional (even well-motivated) assumptions about the parameters involved, other than the basic idea of gauge coupling unification. Thus, we assume no relation between squarks and sleptons, or between the gauginos. (Actually, the structure of supersymmetry alone forces certain relations



FIG. 1. Dependence of $\alpha_s(m_Z)$ on the mass of individual states entering the one-loop thresholds, as in Eqs. (3)-(5). The masses of all other states are set to either 100 GeV (dash) or 1 TeV (dots) and $m_t = 160$ GeV. Also plotted (thick solid) is $\alpha_s^{\min}(m_Z)$ - the lowest range of $\alpha_s(m_Z)$ obtained by choosing other mass parameters in such a way as to minimize it (as in the last row of Table I).

between sfermion masses and gaugino masses, thus disallowing, for example, very light squarks and very heavy gauginos [38]. We will see below that this will not have any substantial effect on our results.) We also do not impose a mechanism of radiative electroweak symmetry breaking. We will see *a posteriori* that requiring EWSB will not change our conclusions significantly.

In Fig. 1 we show $\alpha_s(m_Z)$ as a function of the mass of each relevant type of state. We assume all other masses to be degenerate and equal to either 100 GeV or 1 TeV. Generally, we will treat all squarks and all sleptons as mass degenerate. The only exception to this rule will be the scalar top states, \tilde{t}_L and \tilde{t}_R . This is because their masses are typically expected to be significantly different from the other squarks and from each other.

It is obvious from the form of the β functions, Eqs. (3)– (5), that the resulting value of $\alpha_s(m_Z)$ will most sensitively depend on two parameters only: the gluino mass $m_{\tilde{g}}$ and the soft mass parameter M_2 of the W-ino. The reasons are twofold: not only are their β -function coefficients among the largest but also they change only one out of the three b_i 's. Figure 1 clearly confirms our expectation. Also, Table I shows $\alpha_s(m_Z)$ for several choices

TABLE I. $\alpha_s(m_Z)$ for several choices of mass parameters (assumed between 100 GeV and 1 TeV) and $m_t = 160$ GeV. Last row displays the case for which the smallest $\alpha_s(m_Z)$ was found.

M_2	$m_{\widetilde{q}}$	$m_{\tilde{l}}$	$m_{ ilde{q}}$	$m_{\tilde{t}_{I}}$	$m_{\tilde{t}_B}$	$m_{\widetilde{H}}$	m_{H_2}	$\alpha_s(m_Z)$
$100{ m GeV}$	$100{ m GeV}$	$100{ m GeV}$	$100\mathrm{GeV}$	100 GeV	$100{ m GeV}$	100 GeV	100 GeV	0.127
$500{ m GeV}$	$100{ m GeV}$	$100{ m GeV}$	$100{ m GeV}$	$100{ m GeV}$	$100{ m GeV}$	$100{ m GeV}$	$100{ m GeV}$	0.118
$1{ m TeV}$	$1 \mathrm{TeV}$	$1{ m TeV}$	$1 \mathrm{TeV}$	$1{ m TeV}$	$1 { m TeV}$	$1 \mathrm{TeV}$	$1{ m TeV}$	0.118
$1\mathrm{TeV}$	$500{ m GeV}$	$1 \mathrm{TeV}$	$100{ m GeV}$	$1\mathrm{TeV}$	$100{ m GeV}$	$1 \mathrm{TeV}$	$1 { m TeV}$	0.112
$1\mathrm{TeV}$	$100{ m GeV}$	$1\mathrm{TeV}$	$100{ m GeV}$	$1{ m TeV}$	$100{ m GeV}$	$1\mathrm{TeV}$	$1\mathrm{TeV}$	0.106

of relevant parameters. The first four rows are meant to demonstrate the dependence of $\alpha_s(m_Z)$ on M_2 and $m_{\tilde{q}}$.

We are interested in the lowest possible values of $\alpha_s(m_Z)$ allowed by (strict) grand unification. As it is obvious from Fig. 1, minimization of $\alpha_s(m_Z)$ requires minimizing $m_{\tilde{q}}$ and $m_{\tilde{t}_{P}}$ while simultaneously maximizing the masses of the W-ino, the sleptons, the Higgsino, and of the heavy Higgs bosons. We have also verified that, in order to minimize $\alpha_s(m_Z)$, one should also set $m_{\tilde{q}}$ $(m_{\tilde{t}_L})$ at its lowest (largest) possible value. Since the "standard" prediction for $\alpha_s(m_Z)$ emerging in the CMSSM is quoted above under the assumption that all sparticles are lighter than 1 TeV we accordingly restrict all the masses to that range. At the lower end we allow the masses to lie as low as 100 GeV. (Lowering this limit down to m_Z would not noticeably change $\alpha_s(m_Z)$ [40].) In the last row of Table I we show the lowest value of $\alpha_s(m_Z)$ obtained by varying all the mass parameters between 100 GeV and 1 TeV. Experimental bounds on most of those states are still less than m_Z . Even for $m_{\tilde{a}}$ and the masses of the squarks there are no inescapable lower bounds, other than roughly $m_Z/2$ from LEP [39]. (Very recently, the D0 Collaboration [39] has published new improved limits: $m_{\tilde{g}} > 144 \,\text{GeV}$ for any $m_{\tilde{g}}$ and $m_{\tilde{g}} > 212 \,\text{GeV}$ for $m_{\tilde{g}} = m_{\tilde{q}}$. Adopting these limits in the last row of Table I would increase $\alpha_s^{\min}(m_Z)$ by only 0.002 and 0.003, respectively.)

We also display in Fig. 1 $\alpha_s^{\min}(m_Z)$ (thick solid line) as a function of the mass of each individual state, while setting all the other masses as in the last row of Table I. It is clear that in general one can easily obtain values of $\alpha_s(m_Z)$ small enough to accommodate the range $\alpha_s(m_Z) \approx 0.11$ which we favor. Furthermore, $\alpha_s(m_Z)$ shows little dependence on the masses of the states other than the SU(2) and SU(3) gauginos. Therefore one actually has considerable freedom in choosing the other masses as desired. This justifies our approach of assuming all sleptons to be mass degenerate, and similarly with squarks. Furthermore, relatively weak dependence of $\alpha_s(m_Z)$ on the mass of the Higgsino (which we approximate by the Higgs/Higgsino mass parameter μ) shows that imposing EWSB would probably not lead to any strong increase in the lower bound on $\alpha_s(m_Z)$. This is because the conditions of EWSB determine μ in terms of (soft) Higgs mass parameters which influence $\alpha_s(m_Z)$ even less.

It is also evident from the gluino window of Fig. 1 that the mass of the gluino is strongly confined to rather small values in the range of a few hundred GeV only. This is a distinctive feature and a strong prediction of our approach. The exact value of the upper bound on $m_{\tilde{g}}$ that one allows clearly depends on how large GUT-related corrections one assumes and also how large values of $\alpha_s(m_Z)$ one is willing to accept.

On the other hand, the W-ino mass parameter M_2 should preferably be larger than $m_{\tilde{g}}$, contrary to what is commonly expected. This is clearly shown in Fig. 2 where, in the plane $(m_{\tilde{g}}, M_2)$, we plot the lowest allowed values of $\alpha_s(m_Z)$ found by assuming all other mass parameters as in the last row of Table I. It is clear that $\alpha_s(m_Z) \approx 0.11$ favors relatively small $m_{\tilde{g}}$ and large M_2 .

FIG. 2. Contours of constant $\alpha_s^{\min}(m_Z)$ in the $(m_{\widetilde{g}}, M_2)$ plane. All other mass parameters are chosen so as to minimize $\alpha_s(m_Z)$ (as in the last row of Table I) and $m_t = 160$ GeV.

C. Relating gaugino masses

Among perhaps the most commonly assumed, and least questioned, relations are the ones between the mass parameters of the gauginos:

$$M_1 = \frac{5}{3} \tan^2 \theta_{\rm W} M_2 \simeq 0.5 M_2, \tag{8}$$

$$M_2 = \frac{\alpha_2}{\alpha_s} m_{\widetilde{g}} \simeq 0.3 m_{\widetilde{g}}, \tag{9}$$

where the SUSY-breaking parameters M_1 , M_2 and $m_{\tilde{g}}$ of the *B*-ino, the *W*-ino, and the gluino states are evaluated at the electroweak scale. Virtually all phenomenological and experimental studies adopt at least the relation (8). Strictly speaking, however, both relations are not necessary in the context of the MSSM. They both originate from the assumption that, in minimal SU(5) N = 1supergravity, the kinetic term of the gauge bosons and gauginos is equal to a Kronecker δ . Clearly, a priori this assumption is not an indispensable part of the MSSM.

From our previous analysis it is evident that any additional assumption relating the masses of the W-ino and the gluino will have a significant impact on the prediction of $\alpha_s(m_Z)$. In Fig. 3 we plot $\alpha_s^{\min}(m_Z)$ versus $m_{\widetilde{g}}$ for $M_2 = x m_{\widetilde{g}}$. We set all the other masses in such a way as to minimize $\alpha_s(m_Z)$, as in the last row of Table I. We also show the lowest allowed $\alpha_s(m_Z)$ (thick solid curve) as a function of $m_{\widetilde{g}}$ only by setting also $M_2 = 1$ TeV. It is clear that the usually assumed ratio $x \approx 0.3$ forces $\alpha_s(m_Z)$ above ~ 0.120. To be consistent with $\alpha_s(m_Z) \approx 0.11$ the ratio $x \gtrsim 3$ is required. This corresponds to $M_2 \gtrsim 9 m_{\widetilde{g}}$ at the GUT scale.

Furthermore, Fig. 3 shows that the mass of the gluino must again be rather small, $m_{\tilde{g}} \leq 300 \text{ GeV}$, in the absence of large GUT-scale corrections, unless one allows for the *W*-ino mass parameter M_2 significantly above 1 TeV.





FIG. 3. $\alpha_s^{\min}(m_Z)$ versus $m_{\widetilde{g}}$ for several choices of x assuming $M_2 = x m_{\widetilde{g}}$. All other mass parameters are set in such a way as to minimize $\alpha_s(m_Z)$ (as in the last row of Table I), except $m_t = 160 \text{ GeV}$. For x = 0.3, the range $m_{\widetilde{g}} \lesssim 157 \text{ GeV}$ corresponds to (W-ino-like) chargino lighter than about 47 GeV excluded by LEP. For x = 3, $m_{\widetilde{g}} \lesssim 333 \text{ GeV}$ from requiring $M_2 < 1 \text{ TeV}$. As in Fig. 1 the thick solid curve represents $\alpha_s^{\min}(m_Z)$ - the lowest range of $\alpha_s(m_Z)$ obtained by choosing mass parameters, other than $m_{\widetilde{g}}$, in such a way as to minimize it (as in the last row of Table I). The value $x \approx 0.3$ represents the choice commonly made in the literature.

The above considerations put into doubt also the relation (8), which has its root in the same assumption of the equality of all the gaugino masses at the GUT scale. It is true that the mass parameter of the *B*-ino M_1 does not enter Eqs. (3)-(5) and cannot be directly related to M_2 and $m_{\tilde{g}}$. However, in the CMSSM the lightest neutralino almost invariably comes out to be an almost pure *B*-ino [7, 8] and $m_{\chi} \simeq M_1$. It is also an excellent dark matter candidate. There are also stringent limits on the cosmic abundance of exotic particles with color and electric charges. Requiring that the lightest (*B*ino-like) neutralino be lighter than the gluino, and thus a likely candidate for the lightest supersymmetric particle (LSP) leads to $M_1 \lesssim \frac{1}{3}M_2$ (or $M_1 \lesssim \frac{2}{3}M_2$ at M_X), thus violating the relation (8) [41].

Many phenomenological and dark matter properties of the neutralinos depend on the relation (8). Relaxing it may bear important consequences for neutralino detection in accelerators [42, 43] and in dark matter searches [42], as well as in placing bounds on other sparticles. Basically, the mass of the (lightest) *B*-ino-like neutralino is $m_{\chi} \simeq M_1$. Reducing the ratio M_1/M_2 leads to lighter neutralinos. The region of the plane (μ, M_2) (as it is usually presented) where χ remains mostly *B*-ino-like actually increases somewhat [42]. Also, even rather light neutralinos with mass in the range 3 GeV to a few tens of GeV are in principle not excluded and possess excellent dark matter properties $(\Omega_{\chi} h_0^2 \sim 1)$ [42].

Finally, it is worth commenting that, even in the con-

text of N = 1 supergravity one can relax the assumptions (8) and (9) [44,45]. This can be done by considering a general form of the kinetic term of the gauge and gaugino fields, rather than assuming it to be equal to unity. In this case one finds that the gauge couplings at M_X need not be equal (thus making the GUT energy scale M_X somewhat ill defined) and, in general, relations among gaugino masses become arbitrary. If, however, one assumes $M_X \ll m_{\text{Planck}}$ then one finds, at M_X , $m_{\widetilde{g}}/\alpha_s =$ $-\frac{3}{2}M_2/\alpha_2+\frac{5}{2}M_1/\alpha_1$ [44]. In the limit in which the gauge couplings are only slightly displaced from each other at M_X we find $(m_{\widetilde{g}}/M_2)_{|_{M_X}} \simeq -\frac{3}{2} + \frac{5}{2}(M_1/M_2)_{|_{M_X}}$. One solution is the usual $m_{\widetilde{g}} = M_2 = M_1$. But there exist also solutions to this relation which are consistent with small $\alpha_s(m_Z)$, for example $(m_{\widetilde{g}}/M_2)_{|_{M_X}} \simeq 0.1$ and $(M_1/M_2)|_{M_X} \simeq 0.64$, in agreement with what we have found above. Thus in principle it may be possible to reconcile $\alpha_s(m_Z) \approx 0.11$ with some nonminimal versions on N = 1 supergravity. However, in specific GUT models coupled to N = 1 supergravity, it may be very difficult, if not impossible, to accommodate such a large nonuniversality of gaugino masses while preserving almost exact gauge coupling unification [25, 46]. In this type of scenarios (even with general kinetic terms) the mechanism of lowering $\alpha_s(m_Z)$ may play at best only a subdominant role.

III. PHENOMENOLOGICAL CONSEQUENCES

The version of supersymmetric grand unification considered here leads to several distinct implications. One is the necessary existence of a relatively light gluino below ~ 200 GeV and preferably large W-ino mass parameter M_2 . The likely violation of the commonly assumed relations (8) and (9) may lead to many important consequences for placing bounds on various sparticles and to more promising prospects for neutralino dark matter searches.

Below we discuss how the existence of a light gluino affects possible solutions to the long-lasting anomaly of the $Z \rightarrow b\bar{b}$ width. Furthermore, $\alpha_s \approx 0.11$ may lead to a significant relaxation of the constraints on $\tan \beta$ from requiring $b-\tau$ mass unification. We discuss these points below.

A. Consequences of light gluino

If $\alpha_s(m_Z) \approx 0.11$ does indeed require the gluino mass to lie in the ballpark of 100 GeV, as was argued above, the question which immediately comes to one's mind is "what are other phenomenological implications of such a light gluino?"

First and foremost, with this mass, the gluino must be accessible to direct searches at the Tevatron. Currently, a gluino mass range up to about 200 GeV is probed [39] but no firm assumption-independent bounds can be drawn. On the other hand, with the Main Injector upgrade, the Tevatron experiments will be able to probe $m_{\tilde{g}}$ in the range up to 300 GeV. If the gluino is indeed found below some 240 GeV and no (W-ino-like) chargino is found at LEP II up to some 80 GeV, we will know that the relation (9) does not hold.

Second, light gluinos propagating in loops make the corresponding radiative corrections more pronounced. They can then become important in understanding several facts where hints on disagreement between observations and SM expectations were detected. The most well-known example of this type is the problem of α_s itself. As was noted in Refs. [47, 48] the gluino exchange correction to the $Zq\bar{q}$ vertices is positive so that the gluino correction enhances the hadronic width of Z, imitating in this way a larger value of α_s . Figure 2 of Ref. [47] shows that the correction can reach ~ 0.4% in each quark channel provided that $m_{\tilde{g}} \sim 100 \text{ GeV}$ and $m_{\tilde{q}} \sim 70 \text{ GeV}$. With such a correction the value of α_s measured at the Z peak slides down by ~ 10% solving the problem in full.

On the other hand, it seems extremely unlikely that the very same mechanism may be responsible for the alleged enhancement in the $b\bar{b}$ channel. Indeed, if we take the central value for the experimental $Z \to b\bar{b}$ width, the excess over the theoretical expectation amounts to ~ 7 MeV [1], a factor of 5 larger than the excess produced by the gluino correction above. One would have to descend to unacceptably low squark and gluino masses to get this factor of 5. Recently, another possible solution of the R_b problem was suggested in Ref. [49]. In this work the mass parameters of the MSSM were also considered as apriori unrelated. It was shown that, in order to induce large enough SUSY correction to reconcile the measured value of R_b with the SM prediction, a relatively light (below roughly 80 GeV) Higgsino-like chargino is required. The authors also need at least one top squark with a significant \tilde{t}_R component in the same mass range. In order to examine what predictions for $\alpha_s(m_Z)$ this scenario leads to we have set the Higgsino mass parameter μ and $m_{\tilde{t}_R}$ at m_Z , and chosen all other mass parameters in such a way as to minimize $\alpha_s(m_Z)$, as before. We find $\alpha_s(m_Z)\gtrsim 0.11.$

Another problem where the relatively light gluino can help is the deficit of the semileptonic branching ratio in B mesons and the charm multiplicity [50]. Theoretical calculations of these quantities are at a rather advanced stage now. Both perturbative and nonperturbative effects have been considered. The most detailed analysis of the nonperturbative effects is carried out in Ref. [50], with the conclusion that they can be essentially neglected in the problem at hand. As for perturbative calculations, they have been repeatedly discussed in the literature. (See, e.g., recent papers [51, 52] and references therein.) The theoretical prediction turns out to be rather sensitive to the choice of the value of α_s and the normalization scale μ relevant to the process. Smaller values of μ and larger values α_s tend to enhance the nonleptonic width and, thus, lower the prediction for the semileptonic branching ratio. On the contrary, larger values of μ and smaller α_s suppress the nonleptonic width and enhance the branching ratio. The theoretical prediction can be made marginally compatible [52] with the data on the semileptonic branching ratio [53] provided that α_s is chosen on the high side and μ on the low side. At the same time, if $\alpha_s(m_Z) \approx 0.11$ the prediction for $B_{\rm sl}(B)$ does not fall lower than 11.5% [54], while the corresponding experimental number is $(10.43 \pm 0.24)\%$ [53]. Moreover, no reasonable choice of the parameters above allows one to eliminate a very substantial deficit in the charm multiplicity.

Both discrepancies evaporate if the B nonleptonic decays receive a contribution from the $b \rightarrow s +$ gluon transition, at the level of $\sim 15\%$ of the total width. Then the theoretical prediction for $B_{\rm sl}(B)$ shifts down to 10.4%; simultaneously, the charm multiplicity turns out to be within error bars. As was observed in Ref. [55], in supersymmetric models such a transition can naturally arise, with the right strength, if the gluino and squark masses lie in the 100 GeV ballpark. What is important is that the additional graphs giving rise to $b \rightarrow s +$ gluon transition do not spoil the $b \rightarrow s +$ photon transition. Indeed, the ratio of the photon to gluon probabilities is $(Q_d^2 \alpha)/(\alpha_s \eta^2)$ where $Q_d = 1/3$ is the down quark electric charge, and η is a numerical factor including, among other effects, an enhancement of the $b \rightarrow s +$ gluon transition due to the gluon radiative corrections. According to Ref. [55] $\eta \sim 2.5$ to 3. With $\alpha_s \approx 0.11$ the ratio is close to 10^{-3} . This means that the $b \rightarrow s +$ gluon transition can well contribute at the level of 15%; the corresponding contribution to the $b \rightarrow s\gamma$ is at the level of 10^{-4} , which is quite acceptable phenomenologically [56].

B. $b-\tau$ unification

It has been argued that, in the MSSM alone, with no additional mass relations, the requirement of strict $b-\tau$ mass unification can only be achieved in a relatively very narrow region of the $(m_t, \tan\beta)$ plane for a wide range of $\alpha_s(m_Z)$ [57, 58]. However, if $\alpha_s(m_Z)$ is small ~ 0.11, the above strong relation between $\tan\beta$ and m_t can be significantly relaxed provided that strict unification condition $h_b/h_{\tau} = 1$ at the GUT scale is somewhat reduced (~ 10%) [58,59,8]. GUT-scale uncertainties of this size are actually typically present in GUT's [58].

We want to stress, however, that the running of h_b/h_τ is most sensitive to α_s at large Q close to the GUT scale. In order to relax the relation $(\tan\beta - m_t)$ one therefore needs smaller values of α_s close to the GUT scale. This can be achieved by invoking large enough GUT scale corrections but not by the mechanism considered here.

IV. CONCLUSIONS

The observation that the gauge coupling constants, which look so different at the electroweak scale, evolve and converge at a scale somewhat smaller than the Planck mass was crucial in the original idea of grand unification [60]. Later on, with more accurate data and more precise calculations available, it turned out that the gauge couplings do not intersect at one point. The fact that we are off by only a relatively very small amount is very encouraging and shows that the original idea is viable, and only details must be adjusted. This first led people from the SM to the MSSM. This work concludes that, if $\alpha_s(m_Z)$ is indeed close to 0.11, the gluino must be rather light, $m_{\tilde{g}} \sim 100 \text{ GeV}$, and thus accessible to present direct searches. It is also gratifying to note that, with the mass of the gluino lying in this ballpark, other problems (such as the R_b excess at LEP, a deficit of the semileptonic branching ratio of B mesons, etc.) might find their solutions as well. Finally, many studies of SUSY, including mass bounds on sparticles and dark

 P. Langacker, Precision Tests of the Standard Electroweak Model, edited by P. Langacker (World Scientific, Singapore, 1994); presented at the International Symposium on Radiative Corrections, Gaitlinburg, Tennessee, 1994 (unpublished); Report No. hep-ph/9411247 (unpublished).

- [2] P. Langacker and N. Polonsky, Phys. Rev. D 47, 4028 (1993). [For an update see N. Polonsky, Ph.D. thesis, University of Pennsylvania, 1994.]
- [3] Earlier calculations were phrased in different terms, but the conclusions were the same; see, e.g., U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. B 260, 443 (1991); J. Ellis, S. Kelley, and D.V. Nanopoulos, *ibid.* 260, 131 (1991); P. Langacker and M.-X. Luo, Phys. Rev. D 44, 817 (1991); F. Anselmo, L. Cifarelli, A. Peterman, and A. Zichichi, Nuovo Cimento A 104, 1817 (1991); 105, 581 (1992).
- [4] For reviews, see, e.g., H.-P. Nilles, Phys. Rep. 110, 1 (1984); H.E. Haber and G.L. Kane, *ibid.* 117, 75 (1985);
 L.E. Ibáñez and G.G. Ross, in *Perspectives in Higgs*, edited by G.L. Kane (World Scientific, Singapore, 1993);
 R. Mohapatra, *Unification and Supersymmetry*, 2nd ed. (Springer-Verlag, Berlin, 1992).
- [5] A. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. 49, 970 (1982); L.E. Ibáñez, Phys. Lett. 118B, 73 (1982); J. Ellis, D.V. Nanopoulos, and K. Tamvakis, *ibid.* 121B, 123 (1983); H.P. Nilles, M. Srednicki, and D. Wyler, *ibid.* 120B, 346 (1982); R. Barbieri, S. Ferrara, and C. Savoy, *ibid.* 119B, 343 (1982).
- [6] R.G. Roberts and G.G. Ross, Nucl. Phys. B377, 571 (1992).
- [7] R.G. Roberts and L. Roszkowski, Phys. Lett. B 309, 329 (1993).
- [8] G. Kane, C. Kolda, L. Roszkowski, and J. Wells, Phys. Rev. D 49, 6173 (1994).
- [9] R. Arnowitt and P. Nath, Phys. Lett. B 287, 89 (1992);
 S. Kelley, J.L. Lopez, D.V. Nanopoulos, H. Pois, and K. Yuan, *ibid.* 273, 423 (1991); D.J. Castaño, E.J. Piard, and P. Ramond, Phys. Rev. D 49, 4882 (1994); B. de Carlos and A. Casas, Phys. Lett. B 309, 320 (1993);
 V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. D 49, 4908 (1994).
- [10] L.E. Ibáñez and G.G. Ross, Phys. Lett. 110B, 215 (1982); K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Progr. Theor. Phys. 68, 927 (1982); L. Alvarez-Gaumé, J. Polchinski, and M. Wise, Nucl. Phys. B221, 495 (1983); J. Ellis, D.V. Nanopoulos, and K. Tamvakis [5].
- [11] P. Langacker and N. Polonsky, Phys. Rev. D 52, 3081 (1995).
- [12] Reference [1] quotes even a higher value, $\alpha_s(m_Z) = 0.127 \pm 0.005$. The errors are believed to be dominated by

matter searches, rely on the mass relations (8) and (9). This analysis provides arguments for relaxing them.

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theoretical uncertainties. The most exhaustive theoretical analysis is done for the total hadronic width $\Gamma(Z \rightarrow$ hadrons). The error quoted above, $\Delta \alpha_s(m_Z) = \pm 0.005$, is essentially determined by analyzing $\Gamma(Z \rightarrow$ hadrons).

- [13] M. Shifman, Mod. Phys. Lett. A 10, 605 (1995).
- [14] G. Altarelli, in QCD 20 Years Later, Proceedings of the Workshop, Aachen, Germany, 1992, edited by P. Zerwas and H. Kastrup (World Scientific, Singapore, 1993), Vol. 1, p. 172; S. Bethke, in QCD '94, Proceedings of the International Conference, Montpellier, France, 1994, edited by S. Narison [Nucl. Phys. B (Proc. Suppl.) 39B (1995).
- [15] An example of the DIS data analysis can be found, e.g., in M. Virchaux and A. Milsztajn, Phys. Lett. B 274, 221 (1992); A. Martin, W. Stirling, and R. Roberts, *ibid.* 354, 155 (1995). A nice compilation is given in [14].
- [16] A.X. El-Khadra, G. Hockney, A. Kronfeld, and P. Mackenzie, Phys. Rev. Lett. 69, 729 (1992).
- [17] C. Davies, K. Hornbostel, G.P. Lepage, A. Lidsey, J. Shigemitsu, and J. Sloan, Phys. Lett. B 345, 42 (1995).
- [18] G. Altarelli, R. Barbieri, and F. Caravaglios, in *Results and Perspectives* in *Particle Physics*, Proceedings of the 1993 Rencontres de Physique de la Vallée d'Aoste, edited by M. Greco (Editions Frontières, Gif-sur-Yvette, 1993), p. 351.
- [19] A. Blondel and C. Verzegnassi, Phys. Lett. B 311, 346 (1993).
- [20] S. Eidelman, L. Kurdadze and A. Vainshtein, Phys. Lett. 82B, 278 (1979).
- [21] M. Voloshin, Int. J. Mod. Phys. A 10, 2865 (1995).
- [22] The following terminology is used throughout the paper: if $\alpha_s(m_Z)$ is 0.12 or larger it will be referred to as "large α_s "; if $\alpha_s(m_Z)$ is close to 0.11 it will be referred to as "small α_s ."
- [23] M. Consoli and F. Ferroni, Phys. Lett. B 349, 375 (1995).
- [24] D. Ring, S. Urano, and R. Arnowitt, Phys. Rev. D 52, 6623 (1995).
- [25] T. Dasgupta, P. Mamales, and P. Nath, Phys. Rev. D 52, 5366 (1995).
- [26] L. Clavelli and P.W. Coulter, Phys. Rev. D 51, 3913 (1995).
- [27] D.-G. Lee and R.N. Mohapatra, Phys. Rev. D 52, 4125 (1995).
- [28] M. Dine, A.E. Nelson, Y. Nir, and Y. Shirman, Phys. Rev. D (to be published).
- [29] J. Ellis, S. Kelley, and D.V. Nanopoulos, Nucl. Phys. B373, 55 (1992); Phys. Lett. B 287, 95 (1992); J. Hisano, H. Murayama, and T. Yanagida, Phys. Rev. Lett. 69, 1014 (1992).
- [30] V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. D 47, 1093 (1993).
- [31] Particle Data Group (PDG), L. Montanet et al., Phys.

Rev. D 50, 1173 (1994).

- [32] A. Martin and D. Zeppenfeld, Phys. Lett. B **345**, 558 (1995); M.L. Swartz, "Reevaluation of the Hadronic Contribution to $\alpha(M_Z)^2$," Report No. SLAC-PUB-6710, hep-ph/9411353 (unpublished); S. Eidelman and F. Jegerlehner, Z. Phys. C **67**, 585 (1995).
- [33] Let us parenthetically note that some authors investigate a wider range of variations of $\sin^2 \theta_W(m_Z)$. For instance, in the first paper of [24] the highest value considered was $\sin^2 \theta_W(m_Z) = 0.2327$. As was explained above inching $\sin^2 \theta_W(m_Z)$ up one lowers the minimal value of $\alpha_s(m_Z)$. Thus, these authors find that in the CMSSM with the Planck mass correction switched off is $\alpha_s(m_Z)_{\min} \approx 0.114$. The question whether the value $\sin^2 \theta_W(m_Z) = 0.2327$ is admissible left open; we stick to Eq. (7).
- [34] J. Erler and P. Langacker, Phys. Rev. D 52, 441 (1995).
- [35] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 74, 2626 (1995).
- [36] D0 Collaboration, S. Abachi et al., Phys. Rev. Lett. 74, 2632 (1995).
- [37] J. Chyla and A. Kataev, in *Reports*, Proceedings of the Workshop Group on Precision calculations for the Z. Resonance, Geneva, Switzerland, 1994, edited by D. Badin *et al.* (CERN Report No. 95-03, Geneva, 1995), Report No. hep-ph/9502383 (unpublished).
- [38] L.E. Ibáñez and C. López, Nucl. Phys. B233, 511 (1984);
 L.E. Ibáñez, C. López, and C. Muñoz, *ibid.* B256, 218 (1985).
- [39] See, e.g., L. Galtieri, in Beyond the Standard Model IV, Proceeding of the Conference, Lake Tahoe, 1994, edited by J. Gunion, T. Han, and J. Ohnemus (World Scientific, Singapore, 1995); D0 Collaboration, A. Jonckheere, *ibid.*; D0 Collaboration, S. Eno, in Electroweak Interactions and Unified Theories, Proceedings of the XXX Rencontre de Moriond, Les Arcs, France, 1995, edited by J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, in press); D0 Collaboration, M. Paterno, in Proceedings of Les Rencontres de Physique de la Vallee d'Aoste, La Thuile, Italy, 1995, edited by G. Belletini (unpublished); CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 69 3439 (1992); D0 Collaboration, S. Abachi et al., *ibid.* 75, 618 (1995).
- [40] In a recent work of J. Bagger *et al.*, Phys. Lett. B **348**, 443 (1995), the nonleading mass-threshold effects were found to be substantial in the CMSSM for $m_{1/2} <$ 150 GeV – a corrected evolution curve significantly deviates from the leading log (LL) result. The reason is quite obvious: a jump in the slope of the LL evolution curve is due to the fact that at $m_{1/2} \approx 120$ GeV the W-ino becomes lighter than Z and freezes out. Nothing of the kind

happens in our analysis and we generally do not expect the nonleading mass-threshold effects to be large.

- [41] L.R. thanks Dennis Silverman for this remark.
- [42] K. Griest and L. Roszkowski, Phys. Rev. D 46, 3309 (1992).
- [43] A. Bartl, H. Fraas, W. Majerotto, and N. Oshima, Phys. Rev. D 40, 1594 (1989); M. Drees and X. Tata, *ibid.* 43, 1971 (1991).
- [44] J. Ellis, K. Enqvist, D. Nanopoulos, and K. Tamvakis, Phys. Lett. 155B, 381 (1985).
- [45] M. Drees, Phys. Lett. 158B, 409 (1985).
- [46] R. Arnowitt (private communication); K. Kounnas (private communication).
- [47] K. Hagiwara and H. Murayama, Phys. Lett. B 246, 533 (1990).
- [48] G. Bhattacharyya and A. Raychaudhuri, Phys. Rev. D 47, 2014 (1993); A. Djouadi, M. Drees, and H. Konig, *ibid.* 48, 308 (1993); N. Krasnikov, Phys. Lett. B 318, 174 (1993).
- [49] J. Wells, G. Kane, and C. Kolda, Phys. Lett. B 338, 219 (1994).
- [50] I. Bigi, B. Blok, M. Shifman, and A. Vainshtein, Phys. Lett. B 323, 408 (1994).
- [51] E. Bagan, P. Ball, and V. Braun, Nucl. Phys. B432, 3 (1994); E. Bagan, P. Ball, V. Braun, and P. Gosdzinsky, Phys. Lett. B 342, 362 (1995).
- [52] E. Bagan, P. Ball, B. Fiol, and P. Gosdzinsky, Phys. Lett. B 351, 546 (1995).
- [53] R. Patterson, in Proceedings of the International Conference on High Energy Physics, Glasgow, Scotland, 1994, edited by P. J. Bussey and I. G. Knowles (IOP, London, 1995); P. Roudeau, *ibid.*
- [54] A. Kagan (private communication); and (unpublished).
- [55] A. Kagan, Phys. Rev. D 51, 6196 (1995).
- [56] CLEO Collaboration, M.S. Alam et al., Phys. Rev. Lett. 74, 2885 (1995).
- [57] V. Barger, M.S. Berger, P. Ohmann, and R.J.N. Phillips, Phys. Lett. B **314**, 351 (1993); M. Carena, S. Pokorski, and C. Wagner, Nucl. Phys. **B406**, 59 (1993).
- [58] P. Langacker and N. Polonsky, Phys. Rev. D 49, 1454 (1994).
- [59] V. Barger, M.S. Berger, and P. Ohmann, in Supersymmetry and Unification of Fundamental Interaction (SUSY 93), Proceedings of the International Workshop, Boston, Massachusetts, 1993, edited by P. Nath (World Scientific, Singapore, 1993), Report No. hep/ph-9305236 (unpublished).
- [60] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Georgi, H.R. Quinn, and S. Weinberg, *ibid.* 33, 451 (1974).