Relation of constituent quark models to QCD: Why several simple models work "so well"

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We discuss the relationship between exact QCD and constituent quark models (nonrelativistic, bag, or others) to clarify why different models work reasonably in many cases. For this we use the general parametrization method [G. Morpurgo, Phys. Rev. D **40**, 2997 (1989)] now expressed in terms of the standard current quark fields (m_u and m_d a few MeV; $m_s \approx 150$ MeV, at the usual mass renormalization point $q=1$ GeV). The method provides for several quantities the most general exact form of the spin-flavor structure derivable from the QCD Lagrangian. We can thus determine for many important quantities (masses of lowest baryons and mesons, baryon magnetic moments, semileptonic decays, etc.), from a fit to the data, the coefficients of the parametrization, the same ones that a direct QCD calculation, if feasible, would give. It turns out that only a few coefficients are relatively important. Because different models, each with its few free parameters, can produce these terms by some choice of parameters, one can see why models so different as the nonrelativistic or quasichiral models work "well." Finally, expressing the coefficients in the general parametrization dimensionally in terms of current quark masses and Λ, we find that the m_s expansion of broken SU(3)$\times$SU(3) is just an expansion in $\Delta m/(\xi\Lambda) \approx m_s/(\xi\Lambda) \approx 0.3$. The ξ's determined from different data are rather close (from 2.3 to 3.7). The resulting effective light quark masses in constituent models are of order $(\xi\Lambda)$. None of the above conclusions depend on whether or not the chiral limit $m_u, m_d, m_s \to 0$ is mathematically sound.

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I. INTRODUCTION

The connection between QCD and the nonrelativistic quark model [1] (NRQM) of light hadrons (called also the naive model) has been rather mysterious for many years. The NRQM not only works qualitatively in the classification of light hadrons, but also leads to fairly good *quantitative* predictions. How can this happen and be compatible with a description so dissimilar as the chiral one? Can both descriptions be derived from QCD? If so, what is the relationship between the two, and what is the relationship between current (quasichiral) quarks and constituent ones, and, first, what are constituent quarks? Incidentally, the last two questions apply not only to the NRQM but also to any constituent or potential model, such as the MIT bag model [3,2(b)], of "relativistic"-type with Dirac four-spinors.

To exemplify, consider the De Rújula, Georgi, and Glashow (DGG) treatment [4], which was the first attempt to connect the NRQM to the QCD Hamiltonian, starting the "QCD-inspired" treatments. DGG write the QCD Hamiltonian and calculate, in the semirelativistic Fermi-Breit approximation, the one-gluon exchange QCD potential between a quark and an antiquark or between two quarks. By examining the effect of the hyperfine interaction $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k$ on the masses of the lowest hadron states, DGG derive a value for ΔM. From their Eqs. (5) and (11) they obtain for $\Delta M/M_\lambda$ values around 0.35 and conclude "The value of $M_{\mathscr{P}}/M_\lambda$ given by (5) or (11) does not coincide with the value obtained from the pseudoscalar meson masses via current algebra. Ours are effective masses of quarks bound in hadrons, not the masses appearing in the phenomenological Lagrangians describing the breaking of SU(3)$\times$SU(3)." This statement is correct, but cryptic. It leaves obscure the reason why, having set the task of calculating the hadron masses in terms of parameters of the QCD Hamiltonian (including masses), DGG must conclude that the masses in their final formulas are something different; moreover, this "effective quark mass" appears abruptly in their treatment, without having been defined. Of course, in QCD the quark masses are running; yet it is not clear from DGG how the quark masses M_i and M_k in their term $(M_i M_k)^{-1}(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)$ of the hadron masses are related to the masses m in the Lagrangian of QCD.

Recently we have shown [5–7] that a description of constituent-type can be derived exactly from a relativistic field theory of quarks and gluons, such as QCD. It appeared that any constituent model is nothing but a convenient parametrization of certain physical quantities (e.g., hadron masses, magnetic moments, etc.) in the spin-flavor space. It was shown [5] that the general structure of this parametrization can be derived exactly (and thus relativistically, though noncovariantly) from QCD, using general properties of QCD, namely, the flavor structure of the Lagrangian and the fact that gluons are flavorless and neutral. However, we left open the relationship between current and constituent quarks, in particular their masses; we shall fill this gap here.

We divide the presentation into two parts, starting with pure QCD and concluding with models. In the first part (Secs. I–VII) we express the parameters of the general parametrization in terms of the QCD masses of current quarks, and show that (a) expanding the parameters in powers of $\Delta m = m_s - m$, the scale of this expansion ($\approx 3\Lambda$) extracted from the baryon and meson masses is found to coincide with the standard scale of the m_s expansion in broken SU(3) $\times$SU(3) and (b) one is led to a natural definition of the effective mass of a light "constituent quark," the scale of $\approx 3\Lambda$ (more precisely, from 2.3Λ to 3.7Λ) just mentioned.

In the second part (Secs. VIII–XI) the general parametrization is shown to allow us to check any proposed constituent quark model in a way more useful than a direct comparison of the model predictions to masses, magnetic moments, etc.; applying the analysis to several models, we clarify why many are "so good."

Two points on notation and language are in order. (a) Above, we spoke of "current" quarks and "quasichiral" quarks. The two are synonymous and refer to the light (u,d,s) quark fields in the Lagrangian of QCD renormalized at $q \geqslant 1$ GeV. (b) Except when noted, M will be used to denote the effective mass of constituent quarks. Instead, $m(q)$ will be the running quark masses of QCD at the renormalization point q; for the standard $q=1$ GeV, we omit writing q and call such masses simply m_u, m_d, or m_s (m_u and m_d of a few MeV and $m_s \approx 120$–180 MeV).

II. THE GENERAL PARAMETRIZATION OF BARYON MASSES IN TERMS OF CURRENT QUARKS

Call H_{QCD} the exact Hamiltonian of QCD. Its strong part is

$$\begin{aligned} H_{\mathrm{QCD}} &= H_c + \int d^3\mathbf{x}[m(\bar{u}u+\bar{d}d+\bar{s}s)+\Delta m \bar{s}s] \\ &\equiv H_c + \int d^3\mathbf{x}(m\bar{\Psi}\Psi + \Delta m \bar{\Psi} P^s \Psi), \end{aligned} \tag{1}$$

where, for simplicity, we neglected intrinsic isospin breaking, setting

$$m_u = m_d \equiv m, \quad \Delta m \equiv m_s - m, \tag{2}$$

and H_c is the chiral-invariant part of the Hamiltonian. On the right in (1) Ψ is the quark field,

$$\Psi(x) = \begin{vmatrix} u(x) \\ d(x) \\ s(x) \end{vmatrix}, \tag{3}$$

and P^s is the projector on the strange quark field

$$\begin{aligned} &\begin{vmatrix} 0 & & \\ & 0 & \\ & & 1 \end{vmatrix} \equiv (1-\lambda_8)/3, \\ &P^s s = s, \quad P^s u = 0, \quad P^s d = 0. \end{aligned} \tag{4}$$

As to flavor, it is broken only by the Δm term. Recall that M_K^2/M_π^2 calculated using H_{QCD} (1) gives [8] $m_s/m \approx 25$ [or, depending on the corrections [9], $(m_s/m)=8$–25]. This same QCD Hamiltonian leads to the equally time-honored value (from 45 to $\approx$60 MeV) of the πN σ term [10,11].

Now consider the general parametrization. In Refs. [5–7] we selected the renormalization point of the running quark masses in the region of low q's, so as to have H_{QCD} expressed in terms of renormalized quark fields with mass values in a range typical of those usually assigned to constituent quarks. But, in fact, the parametrization in [5–7] is independent of the choice of the renormalization point of the quark masses. Because here we intend to relate constituent and current quarks, we now think of the QCD Hamiltonian expressed in terms of the standard quark fields with standard masses [8,10], a few MeV for m_u and m_d, corresponding to the conventional renormalization point at $q \approx 1$ GeV. The coefficients of the parametrization are also thought of as expressed in terms of the standard masses. Accordingly, we adopt the standard u, d, and s for quark fields instead of $\mathscr{P}$, $\mathscr{N}$, and λ of Ref. [5]. We stress that adopting this standard choice of renormalization point does not alter the deduction [5] of the general parametrization. We recall briefly below such a treatment.

We start with the masses of the lowest **8** and **10** baryons:

$$M_i = \langle \psi_i | H_{\mathrm{QCD}} | \psi_i \rangle. \tag{5}$$

Here H_{QCD} is the QCD Hamiltonian (1) and $|\psi_i\rangle$ is the exact eigenstate of the ith **8** and **10** baryon *at rest*:

$$H_{\mathrm{QCD}} |\psi_i\rangle = M_i |\psi_i\rangle. \tag{6}$$

To parametrize a property of the lowest baryons, one imagines constructing the exact baryon eigenstates $|\psi_i\rangle$ by applying a unitary transformation V to a set of simple three-quark states $|\phi_i\rangle$ (important: in [5–7] we called these states $|\phi_i\rangle$ "model states"; now, to avoid any possibility of confusion with constituent quark models, we will refer to $|\phi_i\rangle$'s as "*auxiliary* states"):

$$|\psi_i\rangle = V|\phi_i\rangle. \tag{7}$$

The auxiliary states $|\phi_i\rangle$ and V are defined in Ref. [5], where it was shown how, in principle, V can be constructed (see also Sec. IV).

As shown in [5] it is convenient to select the wave function ϕ_i of the auxiliary states as products of a space (or momentum) factor $X_{L=0}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ with orbital angular momentum $L=0$ and a symmetrical spin-flavor factor W_i constructed in terms of the spin-flavor variables of three quarks:

$$\phi_i = X_{L=0}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\cdot W_i. \tag{8}$$

W_i accounts for all the angular momentum of the state i, and therefore has necessarily the SU(6) spin-flavor structure. For instance, for the proton p (S means symmetrization over 1,2,3) and for Δ^{++} it is

$$W_p(\uparrow) = (18)^{-1/2} S[\alpha_1(\alpha_2\beta_3-\alpha_3\beta_2)u_1u_2d_3], \tag{9}$$

$$W_{\Delta^{++}}(\uparrow) = \alpha_1\alpha_2\alpha_3 u_1u_2u_3. \tag{10}$$

We underline that all *physical results* (e.g., the baryon masses to be considered now) are obviously independent of the choice of the auxiliary states $|\phi_i\rangle$. They only depend on H_{QCD}.

Let us recall the general parametrization [5,6(d),6(f)] of the masses M_i of the **8** and **10** baryons. As shown in Ref. [5] it is (i specifies the baryon)

$$\begin{aligned} M_i &= \langle \psi_i | H_{\mathrm{QCD}} | \psi_i \rangle = \langle \phi_i | V^\dagger H_{\mathrm{QCD}} V | \phi_i \rangle \\ &= \langle W_i | \text{“parametrized mass”} | W_i \rangle, \end{aligned} \tag{11}$$

where the last form is what we call the "general parametrization." [W_i are the spin-flavor functions defined in (8); the fact that the space variables have disappeared from the last form of (11) is due to the factorizability (8) of ϕ_i.] From Refs. [5,6](f) it is (compare also Appendix A)

$$\text{“parametrized mass”} = M_0 + B\sum_i P_i^s + C\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)$$
$$+D\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(P_i^s+P_k^s)$$
$$+E\sum_{\substack{i\neq k\neq j\\(i>k)}}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)P_j^s + a\sum_{i>k}P_i^sP_k^s$$
$$+b\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)P_i^sP_k^s$$
$$+c\sum_{\substack{i\neq k\neq j\\(i>k)}}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(P_i^s+P_k^s)P_j^s$$
$$+dP_1^sP_2^sP_3^s. \quad (12)$$

In (12) the $\boldsymbol{\sigma}_i$'s are the Pauli matrices; the projectors P_i^s on the strange quark were defined above in (4).

A few comments on (12) follow. Because the different masses of the lowest octet and decuplet baryons are 8 (barring e.m. and isospin corrections), Eq. (12), containing nine parameters (M_0,B,C,D,E,a,b,c,d), is certainly true, no matter what is the underlying theory. Nevertheless, Eq. (12) can be regarded as an exact deduction from QCD in the following sense: We could not write the parametrization (12) if the exact states $|\psi_i\rangle$ were not related, as in (7), to a set $|\phi_i\rangle$ of *three-quark*, *no-gluon* states. This is the feature of QCD that enters. This being clear, the exact parametrization (12) is not trivial; the values of the eight parameters obtained fitting the masses [in the analysis only $(a+b)$ intervenes] decrease strongly, moving to terms with increasing number of indices (that is [6(f)], with increasing number of gluons exchanged and/or flavor-breaking P_k^s factors).

Note that, in deriving (12) from QCD, $\Delta m\bar{\Psi}P^s\Psi$ in the Lagrangian is treated exactly; Eq. (12) is always true, in particular no matter how large is Δm in the QCD Lagrangian.

In [5,6(f)] we gave M_0, B, C, D, E, $(a+b)$, c, and d. Here, we reanalyze the data (Appendix B) to determine $M_0,B,\ldots,d$ after subtraction of electromagnetic and intrinsic isospin effects, using (for wide resonances) both the pole [12] and the conventional masses [13]. The pole values of parameters in (12) are given below in (13), omitting errors, if unimportant. The parameters from conventional baryon masses are similar (Appendix B), but the small ones are not identical. The pole parameters (in MeV) are

$$M_0=1076,\quad B=192,$$
$$C=45.6,\quad D=-13.8\pm0.3,\quad (a+b)=-16\pm1.4, \quad (13)$$
$$E=5.1\pm0.3,\quad c=-1.1\pm0.7,\quad d=4\pm3.$$

The hierarchy of these numbers is evident. The values (13) decrease so strongly that, omitting c and d, the following mass formula results [6(d)]:

$$\tfrac{1}{2}(P+\Xi^0)+T=\tfrac{1}{4}(3\Lambda+2\Sigma^+-\Sigma^0). \quad (14)$$

The symbols stand for masses and T is the following combination of decuplet masses:

$$T=\Xi^{*-}-\tfrac{1}{2}(\Omega+\Sigma^{*-}). \quad (15)$$

The combinations of masses in (14) are independent of electromagnetic effects to zero order in flavor breaking. This is the reason for the charge combinations in (14) and (15). The data satisfy Eq. (14) as follows:

$$\text{left-hand side}=1132.6\pm1.2,$$
$$\text{right-hand side}=1132.6\pm0.1, \quad (16)$$

an impressive agreement confirming the smallness of the terms neglected in (12). One more remark: A QCD calculation, if feasible, would express each $(M_0,B,\ldots,c,d)$ in (12) in terms of the quantities in the QCD Lagrangian, the running quark masses—normalized at any q that we like to select—and the dimensional (mass) parameter Λ of QCD [$\alpha_s(q^2)=4\pi(\beta_0\ln q^2/\Lambda^2)^{-1}$ for $q\gg1$ GeV]; for instance (recall that we set $m_u=m_d\equiv m$):

$$M_0\equiv\Lambda\hat{M}_0(m(q)/\Lambda,m_s(q)/\Lambda), \quad (17)$$

where $\hat{A}$ is some function, and similarly for B, C, D, E, a, b, c, and d. To simplify the notation, in what follows we set $\Lambda=1$, reinstalling Λ when appropriate, and suppress the caret on the RHS of M_0, etc.

Note finally that, while the derivation of the general parametrization needs only general properties of the Lagrangian of QCD, the asymptotic freedom typical of QCD enters when we introduce $\Lambda\approx150$–200 MeV in (17), as will be essential in what follows.

III. PARAMETRIZATION OF OCTET BARYON MAGNETIC MOMENTS AND MESON MASSES

For later use, we display also the parametrizations of magnetic moments of octet baryons [5] and the masses of lowest meson nonets [6(b)].

Baryon magnetic moments

Introduce the magnetic moment operator in the rest frame,

$$\mathcal{M}=\frac{1}{2}\int d^3\mathbf{r}[\mathbf{r}\times\mathbf{j}(\mathbf{r})], \quad (18)$$

where $\mathbf{j}(\mathbf{r})$ is the space part of the electromagnetic current at $t=0$:

$$j_\mu(x)=ie[\tfrac{2}{3}\bar{u}(x)\gamma_\mu u(x)-\tfrac{1}{3}\bar{d}(x)\gamma_\mu d(x)-\tfrac{1}{3}\bar{s}(x)\gamma_\mu s(x)]$$
$$\equiv\frac{ie}{2}[\bar{\Psi}(x)(\lambda_3+\tfrac{1}{3}\lambda_8)\gamma_\mu\Psi(x)]$$
$$\equiv ie[\bar{\Psi}(x)Q\gamma_\mu\Psi(x)]. \quad (19)$$

The charge Q in (19) is, in terms of the λ's or of the projectors P^u, P^d, and P^s,

$$Q=\tfrac{1}{2}(\lambda_3+\tfrac{1}{3}\lambda_8)=\tfrac{2}{3}P^u-\tfrac{1}{3}P^d-\tfrac{1}{3}P^s. \tag{20}$$

The formula that now replaces (11) for the masses is

$$\begin{aligned} M_i&=\langle\psi_i|\mathscr{M}_z|\psi_i\rangle=\langle\phi_i|V^\dagger\mathscr{M}_zV|\phi_i\rangle\\ &=\langle W_i|\text{``parametrized magn. moment''}|W_i\rangle. \end{aligned} \tag{21}$$

Here we give the general parametrization of magnetic moments, keeping only terms linear in P^s. For baryon masses Eq. (12) was exact to all orders in P^s; the same is true for Eq. (28) below for meson masses. For the magnetic moments we might easily write the parametrization to all orders in P^s, but then we would have too many parameters to make it useful. The terms neglected, bilinear or cubic in P^s, are expected to be at most 5% of the dominant term [6(f)]. Keeping only terms linear in P^s the parametrization of the magnetic moments of the baryon octet has eight terms [5,14]:

$$\text{``parametrized magn. moment''}=\sum_{\nu=0}^{7} g_\nu(\mathbf{G}_\nu)_z \tag{22}$$

with

$$\mathbf{G}_0=\mathrm{Tr}[QP^s]\sum_i \boldsymbol{\sigma}_i,\quad \mathbf{G}_1=\sum_i Q_i\boldsymbol{\sigma}_i,\quad \mathbf{G}_2=\sum_i Q_iP_i^s\boldsymbol{\sigma}_i,$$

$$\mathbf{G}_3=\sum_{i\neq k} Q_i\boldsymbol{\sigma}_k,\quad \mathbf{G}_4=\sum_{i\neq k} Q_iP_i^s\boldsymbol{\sigma}_k,\quad \mathbf{G}_5=\sum_{i\neq k} Q_iP_k^s\boldsymbol{\sigma}_k, \tag{23}$$

$$\mathbf{G}_6=\sum_{i\neq k} Q_i\boldsymbol{\sigma}_iP_k^s,\quad \mathbf{G}_7=\sum_{i\neq k\neq j} Q_iP_k^s\boldsymbol{\sigma}_j.$$

As remarked in [7(a)] the coefficient g_0 of $\mathbf{G}_0$ is expected, due to general arguments, to be $\approx 10^2$ times smaller than g_1 and therefore negligible. Omitting $\mathbf{G}_0$, the data determine the other seven coefficients $g_1, g_2, \ldots, g_7$. Fitting the observed moments gives (in proton magnetons)

$$\begin{aligned} &g_1=2.79,\quad g_2=-0.94,\quad g_3=-0.076,\quad g_4=0.41,\\ &g_5=0.097,\quad g_6=-0.134,\quad g_7=0.155, \end{aligned} \tag{24}$$

showing that the first two terms are appreciably larger than the remaining ones; thus one understands why the "naive" NRQM (in which only g_1 and g_2 are kept) gives a fair description of the magnetic moments. Indeed, neglecting, besides g_0, all coefficients from g_3 to g_7 we remain with the "naive" [1(a)] additive form of the "parametrized magnetic moment" operators: namely,

$$g_1\sum_i [1+(g_2/g_1)P_1^s]Q_i\boldsymbol{\sigma}_i. \tag{25}$$

With the above values of g_1 and g_2,

$$g_1=2.79,\quad g_2/g_1=-0.34, \tag{26}$$

(25) gives a fit (Ref. [5], Fig. 1) correct to about 15% of all octet magnetic moments. Note that keeping g_0 and neglecting g_7, which is also expected [6(f)] to be small because it is a two-gluon exchange, flavor-breaking term, the values of g_1 and g_2 in (22) would remain essentially the same as listed in (24).

Meson masses

The general parametrization of meson masses proceeds as for baryons. Here we give the parametrization only for the lowest pseudoscalar and vector mesons with isospin $I\neq 0$ (that is, π, K, ρ, and K^*). The $I=0$ mesons (η, η', ω, and ϕ) are treated in [6(b)]. Equation (11) now becomes

$$\begin{aligned} M_i&=\langle\psi_i|H_{\mathrm{QCD}}|\psi_i\rangle=\langle\phi_i|V^\dagger H_{\mathrm{QCD}}V|\phi_i\rangle\\ &=\langle w_i(1,2)|\text{``parametrized mass''}|w_i(1,2)\rangle, \end{aligned} \tag{27}$$

where

$$\begin{aligned} \text{``parametrized mass''}&=A+B(P_1^s+P_2^s)+C\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\\ &\quad+D\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2(P_1^s+P_2^s). \end{aligned} \tag{28}$$

Similar formulas ([5,6(b)]) could be written for any power of the masses.

Once more the parametrization (28) is exact, to all orders in P^s. Again this formula looks trivial: four masses and four parameters. But two aspects of (28) are not trivial, as for baryons: (1) Its structure is typical of a NRQM description, yet (28) follows exactly from QCD, and (2) the coefficients decrease in magnitude from A to D [see Eq. (31)].

In the last form of (27) the w_i's are the spin-flavor functions

$$w_i(1,2)=\chi_i(1,2)f_i(1,2) \tag{29}$$

for the auxiliary states $|\phi_i\rangle$ of a quark (1)–antiquark (2) corresponding to each meson π, K, ρ, and K^*; in (29) the χ_i's are obviously a singlet spin function for π and K, and a triplet for ρ and K^*; the f_i's are the flavor functions, e.g., $u_1\bar{s}_2$ for a K^+ or K^{*+}. In (28) A, B, C, and D are four real parameters. The other symbols are obvious. Of course [see (17)] A, B, C, and D are Λ times functions of $m(q)/\Lambda$, $m_s(q)/\Lambda$, that could be determined if we were able to calculate with QCD.

Recalling that $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2=-3$ for $J=0$ and $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2=+1$ for $J=1$, the meson masses (indicated with the meson symbols) are

$$\begin{aligned} &\pi=A-3C(=138),\quad K=A-3C+B-3D(=495),\\ &\rho=A+C(=770),\quad K^*=A+C+B+D(=894). \end{aligned} \tag{30}$$

Therefore (in MeV),

$$A=612,\quad B=182,\quad C=158,\quad D=-58. \tag{31}$$

We conclude with the following remark. The pion mass is

$$\pi=A-3C=138 \tag{32}$$

with $A=612$ and $C=158$. The idea that in the perfect chiral limit (mass zero of u and d quarks) the pion would be massless, that is

$$A-3C=0, \tag{33}$$

is almost universally held. Then the pion is looked on as a quasi Goldstone boson, getting its mass from explicit breaking of chiral symmetry due to the small u and d quark masses. This description, extended to all mesons of the lowest pseudoscalar octet, makes them all quasi Goldstone bosons. We will not discuss this standard chiral picture [which accounts for, but is not strictly required by, the great classical successes of current algebra plus PCAC (partial conservation of axial vector current)]. We must add, nevertheless, that the pion mass on the RHS of $A-3C=138$ is not so small on the scale of the parameters A, B, C, and D. Because of the 3 in front of C, a percentually minor change of C (possibly produced in QCD by a comparatively small change of α_s) might equally well lead to $A-3C\cong 0$ or to $A-3C$, say 350, making the pion mass comparable to the others in the octet.

IV. THE DEPENDENCE OF THE TRANSFORMATION V ON THE RENORMALIZATION POINT OF THE RUNNING QUARK MASSES IN THE LAGRANGIAN

We digress briefly to examine more closely the question of the choice of the mass renormalization point in constructing the parametrization. We stated that the parametrization [thus V in Eqs. (7), (11), (21), or (27)] can be introduced, in principle, for any choice of the renormalization point q for the quark masses in the QCD Hamiltonian; also for $q\approx 1$ GeV, and, therefore, m_u and m_d a few MeV. As already stated, it is important to have this clear because in Refs. [5–7] we were thinking of the QCD Hamiltonian expressed in terms of quark masses renormalized at a low value of q, so as to have u and d masses in the range—a few hundred MeV—usually assigned to constituent quarks. That choice is possible but unnecessary. Here we have adopted the conventional choice.

To see where the renormalization point enters in V, we express, therefore, from now on, the QCD Hamiltonian in terms of quark fields with masses $m(q)$ defined at some definite freely chosen renormalization four-momentum q. Decompose the QCD Hamiltonian as

$$H_{\mathrm{QCD}}=H_a+H_b\,, \tag{34}$$

where H_b is the quark-gluon interaction plus the flavor-breaking mass term and H_a is all the rest; thus H_a is flavor invariant [all quarks in it with mass $m(q)$]. Introduce a complete set of states $|\nu(q)\rangle$ of H_a:

$$H_a|\nu(q)\rangle=E_a(q)|\nu(q)\rangle, \tag{35}$$

where q indicates the selected renormalization point. To be definite we refer below to baryons; for mesons everything goes similarly.

Write the auxiliary states $|\phi_i\rangle$, introduced in (11), as

$$|\phi_i\rangle=\sum_{\mathbf{p},r} C^i_{r_1r_2r_3}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)a^\dagger_{\mathbf{p}_1r_1}a^\dagger_{\mathbf{p}_2r_2}a^\dagger_{\mathbf{p}_3r_3}|0\rangle, \tag{36}$$

where $\Sigma_{\mathbf{p},r}$ stands for $\Sigma_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3,r_1r_2r_3}$. In (36) the $a^\dagger_{\mathbf{p}r}$ are creation operators of quarks of momentum $\mathbf{p}$ and spin-flavor-color index r (we omit color except when necessary); $|0\rangle$ is the vacuum state, in a Fock space of quarks and gluons. In constructing the auxiliary states the masses of quarks u, d, and also s are taken equal. For simplicity—this is not necessary—we identify this value with the (common) mass $m(q)$ of u and d at the renormalization point q. In (36) $C^i_{r_1r_2r_3}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$ is the momentum space function of the auxiliary state of the three quarks in the rest frame; $C^i_{r_1r_2r_3}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$ contains a factor $\delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)$. The quark spin states are taken as four-spinors with the upper components

$$\begin{vmatrix}1\\0\end{vmatrix} \quad \text{or} \quad \begin{vmatrix}0\\1\end{vmatrix}$$

and two zeros in the lower components. A transformation of Foldy-Wouthuysen type is part of V. The auxiliary states $|\phi_i\rangle$ (36) can be seen [5] as the lowest (degenerate) eigenstates of some auxiliary Hamiltonian $\mathscr{H}$, defined in the Fock sector of three quarks, no antiquark, no gluon; $\mathscr{H}$ is useful only to show how V can be constructed by the adiabatic procedure (Appendix to Ref. [5]). As already stated, no physical result, in particular baryon or meson masses or magnetic moments, depends on the choice of the auxiliary states, that is, of $\mathscr{H}$.

We now characterize V. The transformation V is simply a correspondence between a certain set of auxiliary states $|\phi_i\rangle$ and the exact states $|\psi_i\rangle$ of interest. To characterize V, expand the exact state $|\psi_i\rangle$ in the complete set of states $|\nu(q)\rangle$ of Eq. (35):

$$|\psi_i\rangle=\sum_{\nu(q)}|\nu(q)\rangle\langle\nu(q)|V|\phi_i\rangle. \tag{37}$$

In Eq. (37) the sum (that is, the expansion of the exact state in terms of Fock quark-gluon states) extends to all possible eigenstates $\nu(q)$ of H_a. Clearly Eq. (37) defines V through

$$\langle\nu(q)|V|\phi_i\rangle\equiv\langle\nu(q)|\psi_i\rangle \tag{38}$$

for any $\nu(q)$; thus V can be defined by selecting freely the renormalization point q of the running quark mass, as long as, for any q, the states $\nu(q)$ are a complete set. Less formally, Eq. (37) means that the exact state $|\psi_i\rangle$ has an extremely complicated structure in Fock space. Schematically,

$$|\psi_i\rangle=|qqq\rangle+|qqq\bar{q}q\rangle+|qqq,\mathrm{Gluons}\rangle+\cdots\ , \tag{39}$$

where the ellipsis indicates states (in the $\mathbf{P}=\mathbf{0}$ frame) with any number n of quarks, $n-3$ antiquarks, and any number of gluons, provided only that the conserved quantum numbers of the Fock states on the RHS of (39) (color, charge, baryonic number, strangeness, parity, angular momentum) are the same as those of $|\psi_i\rangle$ ($|\phi_i\rangle$ has, of course, the same quantum numbers as $|\psi_i\rangle$).

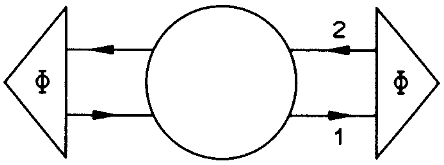

FIG. 1. Schematic diagram representing the ''external'' lines 1 and 2 and the ''internal'' box in the general parametrization of a meson property; ϕ is the auxiliary state. The ''box'' contains all sorts of gluon lines and quark closed loops.

V. THE EXPANSION OF THE MESON MASS PARAMETERS IN TERMS OF $\Delta m \equiv m_s - m \cong m_s$

We now analyze the mass parametrization of the lowest hadrons. For simplicity consider first the $I \neq 0$ mesons. The result of a full QCD calculation of the ''parametrized mass'' (28), showing the most general dependence of the ''parametrized mass'' on the quark masses in the QCD Lagrangian, can be written as

''parametrized mass''

$$= \phi(m, \Delta m | m + \Delta m P_1^s, m + \Delta m P_2^s) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) F(m, \Delta m | m + \Delta m P_1^s, m + \Delta m P_2^s). \tag{40}$$

Here the two functions ϕ and F of m and Δm multiplying the spin-independent and spin-dependent parts are assumed to result from a QCD calculation of $V^\dagger H_{\rm QCD} V$ after contraction of all creation and destruction operators and integration on the space (or momentum) variables. That is, we think of ϕ and F as calculated, from first principles, in QCD.

The functions ϕ and F depend on the masses in the QCD Lagrangian, m and Δm, in two different ways, as illustrated in Fig. 1. (a) A first dependence comes from the external lines and carries the indices of the quarks in the auxiliary state; the QCD Lagrangian shows that this dependence entails Δm multiplied by the projectors P_i^s. If, doing the QCD calculation, we keep all P_i^s [without exploiting $(P_i^s)^n = P_i^s$], the dependence of ϕ and F on the P_i^s's is uniquely determined. In Eq. (40) this dependence on $\Delta m P_i^s$ appears in the arguments of ϕ and F on the right of the vertical bar. In fact, it is slightly more convenient, as we did, to insert as arguments on the right of the vertical bar in (40) $(m + \Delta m P_i^s)$ instead of $\Delta m P_i^s$. (b) The second dependence of ϕ and F on m and Δm comes from internal quark loops in the ''blob'' of Fig. 1. This dependence is noted in the arguments on the left of the vertical bar in ϕ and F. It goes without saying that, though the numerical values of the quark running masses at a given q are definite, we imply, in speaking of the m dependence of ϕ and F, that QCD makes sense also in a range of values of these masses (as QED can be expressed in terms of the electron mass, though the latter is 0.51 MeV).

Equation (40) can be written slightly more compactly as

$$A + B(P_1^s + P_2^s) + [C + D(P_1^s + P_2^s)](\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \equiv \phi(m, \Delta m | m_1, m_2) + F(m, \Delta m | m_1, m_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \tag{41}$$

where we set

$$m_1 = m + \Delta m P_1^s, \quad m_2 = m + \Delta m P_2^s, \tag{42}$$

and $\phi(m, \Delta m | m_1, m_2)$ and $F(m, \Delta m | m_1, m_2)$ are symmetric in 1,2. Exploiting now

$$(P_i^s)^n = P_i^s \quad (i = 1,2) \tag{43}$$

and recalling

$$m_s = m + \Delta m, \tag{44}$$

we obtain ($P_1^s P_2^s$ terms do not contribute)

$$\begin{aligned} \phi(m, \Delta m | m_1, m_2) &= \phi(m, \Delta m | m, m) + [\phi(m, \Delta m | m_s, m) - \phi(m, \Delta m | m, m)](P_1^s + P_2^s), \\ F(m, \Delta m | m_1, m_2) &= F(m, \Delta m | m, m) + [F(m, \Delta m | m_s, m) - F(m, \Delta m | m, m)](P_1^s + P_2^s), \end{aligned} \tag{45}$$

so that

$$\begin{aligned} A &= \phi(m, \Delta m | m, m), \\ B &= \phi(m, \Delta m | m_s, m) - \phi(m, \Delta m | m, m), \\ C &= F(m, \Delta m | m, m), \\ D &= F(m, \Delta m | m_s, m) - F(m, \Delta m | m, m). \end{aligned} \tag{46}$$

Reinstalling Λ (see the end of Sec. II), we have, more explicitly,

$$\begin{aligned} \phi(m, \Delta m | m, m) &\equiv \Lambda \phi(m/\Lambda, \Delta m/\Lambda | m/\Lambda, m/\Lambda), \\ F(m, \Delta m | m, m) &\equiv \Lambda F(m/\Lambda, \Delta m/\Lambda | m/\Lambda, m/\Lambda), \end{aligned} \tag{47}$$

with similar expressions for all other quantities. Again, with some exceptions, we set $\Lambda = 1$ in what follows.

The ratios B/A and D/C are

$$\begin{aligned} \frac{B}{A} &= \frac{\phi(m, \Delta m | m_s, m) - \phi(m, \Delta m | m, m)}{\phi(m, \Delta m | m, m)}, \\ \frac{D}{C} &= \frac{F(m, \Delta m | m_s, m) - F(m, \Delta m | m, m)}{F(m, \Delta m | m, m)}. \end{aligned} \tag{48}$$

From now on, to simplify the notation, we omit the arguments on the left of the vertical bar in all functions; we keep the memory of them by the notation

$$\phi(m, \Delta m | m, m) \equiv \phi(\ | m, m), \tag{49}$$

using a similar (| symbol for all the intervening functions. It is important to recognize that if a ratio such as those in Eq. (48) above is expanded in Δm, no contributions to first order in Δm arise from the Δm dependence of the functions ϕ and F in the arguments on the left of the vertical bar. In other words, for the first order terms in the above mentioned expansion, one can forget the Δm dependence of ϕ and F on the left of the bar and consider only the Δm dependence from $m_s = m + \Delta m$ in ϕ and F.

We now expand $F(m,\Delta m|m_s,m)$ and $F(m,\Delta m|m,m)$ as well as $\phi(m,\Delta m|m_s,m)$ and $\phi(m,\Delta m|m,m)$ in series of Δm, assuming the expansion possible at m. Reinstalling here Λ, we have

$$\begin{aligned}
&\phi(|(m+\Delta m)/\Lambda,m/\Lambda)/\phi(|m/\Lambda,m/\Lambda)\\
&\qquad=1+(\Delta m/\beta_0\Lambda)+\gamma_0(\Delta m/\beta_0\Lambda)^2+\cdots,\\
&F(|(m+\Delta m)/\Lambda,m/\Lambda)/F(|m/\Lambda,m/\Lambda)\\
&\qquad=1+(\Delta m/\beta_h\Lambda)+\gamma_h(\Delta m/\beta_h\Lambda)^2+\cdots,
\end{aligned} \tag{50}$$

where β and γ are some coefficients (the index h refers to the hyperfine terms, the index 0 to the spin-independent ones).

Recalling $\Delta m \gg m$ and thus $\Delta m \cong m_s$ we have

$$\begin{aligned}
B/A&=(m_s/\beta_0\Lambda)+\gamma_0(m_s/\beta_0\Lambda)^2+\cdots,\\
D/C&=(m_s/\beta_h\Lambda)+\gamma_h(m_s/\beta_h\Lambda)^2+\cdots.
\end{aligned} \tag{51}$$

If the series on the RHS of (51) converges fast enough, we have [compare the experimental values of A, B, C, and D in (31)]

$$|m_s/\beta_0\Lambda|\approx|B/A|=0.30, \quad |m_s/\beta_h\Lambda|\approx|D/C|=0.37. \tag{52}$$

Note that B/A and D/C (and therefore β_0 and β_h) have opposite signs.

In fact, assuming that the second term in the expansions (51) is of order (first term)^2 with an unknown sign, one should write, instead of (52),

$$|m_s/\beta_0\Lambda|=0.30\pm0.09, \quad |m_s/\beta_h\Lambda|=0.37\pm0.13. \tag{53}$$

This scale in m_s is compatible with that assumed in chiral perturbation theory [11], where the expansion parameter governing kaon physics is taken to be $[M(K)/S]^2$ ($M(K)$ $=$kaon mass and S a mass between that of the ρ and of a scalar meson $\approx$1 GeV, thus, $[M(K)/S]^2$ between 0.25 and 0.40.) Note that the actual values of the coefficients β depend, like Δm or m_s, on the chosen renormalization point q and we refer here to the standard point $q=1$ GeV.

However, we stress that, in determining these expansion parameters $|m_s/\beta\Lambda|$, no assumption is made about the existence of the chiral limit of SU(3)$\times$SU(3), that is, about the behavior of $\phi(|m,m)$ or $F(|m,m)$ near $m=0$. Nonanalyticity at $m\to0$ might imply that expanding from m up to m_s is not possible. But even then we can proceed exactly as above, only expanding in Δm near m_s and moving down to m. With trivial changes the same expansion holds. (More generally, none of the above or the following conclusions depends on whether the exact chiral limit $m_u,m_d,m_s\to0$ is mathematically sound or not.)

The scale of the expansion in m_s is derived here simply from the B/A ratio $+0.30$ or D/C ratio -0.37, typical flavor-breaking effects, e.g., $D/C=[F(|m_s/\Lambda,m/\Lambda)/F(|m/\Lambda,m/\Lambda)]-1=-0.37$. Indeed, long ago (before [8]), instead of the m_s expansion scale, one used to speak of a flavor-breaking expansion. Below we may use occasionally this language; because $m_s\cong\Delta m$, the two are equivalent; the difference, of course, with respect to old times is that now the expansion is not in $\Delta m/m$ (as it was originally) but in $\Delta m/(\beta\Lambda)$.

As to the convergence of the expansion (51), we now will see that it is supported by the data in the analogous case of baryon masses.

VI. THE EXPANSION OF BARYON MASS PARAMETERS IN TERMS OF $\Delta m \equiv m_s - m \cong m_s$

In the baryon masses [Eq. (12)] consider first the hyperfine terms, with coefficients C, D, E, b, and c. To $d=3d_h+d_0$ contribute the hyperfine term ($3d_h$) and the spin-independent one (d_0). Experimentally it is impossible to determine the magnitude of each. As to b, the data determine only $a+b$.

As for mesons (Sec. V) we write the coefficients of hyperfine terms as

$$\begin{aligned}
&\sum_{i>k}\Bigg[C+D(P_i^s+P_k^s)+E\sum_{j\neq k,i}P_j^s+bP_i^sP_k^s\\
&\qquad+c\sum_{j\neq k,i}(P_i^s+P_k^s)P_j^s+d_h\sum_{j\neq k,i}P_k^sP_i^sP_j^s\Bigg](\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)\\
&=\sum_{\substack{i\neq k\neq j\\(i>k)}}F(m,\Delta m|m_i,m_k;m_j)(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k),
\end{aligned} \tag{54}$$

where the notation $x,y;z$ in $F(m,\Delta m|x,y;z)$ recalls that such a function (derivable in principle from QCD) is symmetric in x,y, but not necessarily in z. As for mesons, we omit the arguments on the left of the bar, setting

$$F(m,\Delta m|m_i,m_k;m_j)\equiv F(|m_i,m_k;m_j). \tag{55}$$

Again set, as in (42),

$$m_i=m+\Delta m P_i^s \tag{56}$$

and use the property (43). The function $F(|x,y;z)$ on the RHS of (55) is determined for values of x, y, and z that can be either m or $m_s=m+\Delta m$. To simplify the formulas we write s for m_s. Thus

$$\begin{aligned}
C&=F(|m,m;m),\quad D=F(|s,m;m)-F(|m,m;m),\quad E=F(|m,m;s)-F(|m,m;m),\\
b&=F(|m,m;m)-2F(|s,m;m)+F(|s,s;m),\\
c&=F(|m,m;m)-F(|s,m;m)+F(|s,m;s)-F(|m,m;s),\\
d_h&=F(|s,s;s)-F(|m,m;m)+F(|m,m;s)-F(|s,s;m)+2\{F(|s,m;m)-F(|s,m;s)\}.
\end{aligned} \tag{57}$$

We also consider the spin-independent part of the parametrization. It is

$$M_0+B\sum_i P_i^s+a\sum_{i>k} P_i^s P_k^s+d_0 P_1^s P_2^s P_3^s=\phi(|m_1,m_2,m_3). \tag{58}$$

In (58) ϕ is, like F, a function of m and Δm and of the three indexed masses (or better of m_1/Λ, etc.) now symmetric in 1,2,3 (thus the notation $x,y;z$ in F is replaced by x,y,z). It is

$$\begin{gathered}M_0=\phi(|m,m,m), \quad B=\phi(|s,m,m)-\phi(|m,m,m),\\ a=\phi(|m,m,m)-2\phi(|s,m,m)+\phi(|s,s,m),\\ d_0=\phi(|s,s,s)-3\phi(|s,s,m)+3\phi(|s,m,m)-\phi(|m,m,m).\end{gathered} \tag{59}$$

Proceeding as for mesons, consider first the hyperfine terms and the ratio $D/C\cong-0.3$. Because it is

$$\frac{D}{C}=\frac{F(|m_s,m;m)-F(|m,m;m)}{F(|m,m;m)}, \tag{60}$$

we can again expand F in powers of $\Delta m/\Lambda$ at m. The expansion is similar to (50), but of course not identical, since the function F of three variables in (54) differs from the F in (41). We thus have

$$D/C=(m_s/\beta_h'\Lambda)+\gamma_h'(m_s/\beta_h'\Lambda)^2+\cdots \tag{61}$$

with β_h' and γ_h' replacing β_h and γ_h in (50). Because D/C is now -0.3, instead of -0.37 for mesons, (53) is replaced by

$$|m_s/\beta_h'\Lambda|\approx 0.30\pm 0.09 \tag{62}$$

(having again, arbitrarily, estimated the uncertainty as the square of the first term in the expansion) so that the flavor-breaking scale, or, if one prefers, the SU(3)×SU(3)-breaking scale, $(\beta_h'\Lambda)$ for baryons, is near to that, $(\beta_h\Lambda)$, for mesons. Note that the signs of β_h for mesons and baryons are the same. While for mesons the convergence of the expansion (50) was assumed, here the availability of more coefficients [and their strong decrease—see (13)] allows a check. We show first in general that the experimental hierarchy of the coefficients, together with Eqs. (57) and (59) expressing the coefficients in terms of ϕ and F, strongly indicates convergence in $(\Delta m/\Lambda)$; next we analyze the situation in more detail.

The general argument is as follows. Expanding in series of Δm the expressions of the coefficients given in (57) and (59), it appears immediately that the expansion starts with a term of order Δm for D and E of order $(\Delta m)^2$ for a, b, and c and of order $(\Delta m)^3$ for d_0 and d_h. Of course to see this we do not need (57) and (59). Just look at the number of P^s that multiply each coefficient, since in the Lagrangian only the product $\Delta m P^s$ intervenes. Thus,

$$D,E=O(\Delta m), \quad a,b,c=O(\Delta m^2), \quad d_0,d_h=O(\Delta m^3). \tag{63}$$

Here by $O(\Delta m^3)$ we mean, for instance, that the first non-vanishing term in an expansion in Δm (or better $\Delta m/\Lambda$) is of third order. This general result alone, together with the experimental values of the coefficients, suggests that terms associated with higher powers of $(\Delta m/\Lambda)$ are indeed smaller.

The power in $\Delta m/\Lambda$ is, however, not the only reason for the striking decrease in the coefficients. For a more detailed analysis, consider first $E/C\cong 0.11$. As noted in Ref. [6(f)], where the hierarchy of the coefficients was discussed in detail, the term associated with E has just one P^s and therefore is of the same order in $\Delta m/\Lambda$ as D. However, E multiplies a three-index term whereas D multiplies a two-index term. Three-index terms should arise, in a QCD calculation, from diagrams exchanging at least one more gluon than diagrams giving the main contribution to terms with two indices [15]. Because hyperfine terms represent chromomagnetic interactions of two dipoles, they should be, intrinsically, short range. We take $|E/D|\cong 0.37$ [16] as an estimate of the reduction due to this additional gluon (hard, on the average) and refer to [6(f)] and Appendix B for some additional detail.

With the reduction factor for "one gluon more" 0.37 and the flavor scale $\Delta m/(\beta_h'\Lambda)=0.3$, the order of magnitude of $|c|$ is expected to be $(0.37\times 0.3)|D|=0.11|D|\cong 1.5$. It is (Appendix B) $c=-1.1\pm 0.7$. We get an estimate for $|d_h|$ by multiplying $|c|$ by 0.3; it is $|d_h|=+0.3\pm 0.2$.

Consider now the parameters of the spin-independent terms M_0, B, a, and d_0. From B and M_0 we have

$$\begin{aligned}\frac{B}{M_0}&=\frac{\phi(|m_s,m,m)-\phi(|m,m,m)}{\phi(|m,m,m)}\\ &=(m_s/\beta_0'\Lambda)+\gamma_h'(m_s/\beta_0'\Lambda)^2+\cdots\cong 0.18,\end{aligned} \tag{64}$$

from which we estimate

$$(m_s/\beta_0'\Lambda)=0.18\pm 0.03. \tag{65}$$

The order of magnitude is comparable to that of $|m_s/\beta_h'\Lambda|$ (though it can differ by as much as 2). The sign of (65) is opposite to that from the hyperfine terms, as for mesons.

Coming to a, the data determine only $(a+b)=-16\pm 1.4$. Taking, as order of magnitude, $|b/D|\approx|D/C|$, we have $|b|\approx 4$. If $b>0$ we have $a\approx-20$; for $b<0$, we have $a\approx-12$. Therefore $|a/B|\cong 0.06\div 0.1$, which implies again a large reduction factor of a with respect to the additive term B. The different physical meaning of a and B does not, however, allow us to relate this reduction factor to that of E/D. With a similar depression factor, $|d_0|$ is expected to be $\approx 0.1|a|\approx 2$, leading to $|d|=|d_0+3d_h|\approx 2\pm 1$ (experiment: $d=4\pm 3$). In conclusion, the m_s/Λ expansion scale for spin-independent terms is near to that of hyperfine terms, though we do not have an equivalent for the "hard" gluon chromomagnetic argument (except the usual hand-waving one that soft gluons produce confinement and, after this is taken into account, the remaining ones are hard on the average).

For later use (Sec. VII) we add a remark. Consider the hyperfine terms for baryons. Assume that in (54) $F(|m_i,m_k;m_j)$ is approximately factorizable:

$F(|m_i, m_k; m_j) \approx f(m_i) f(m_k) \varphi(m_j)$. Put $\rho = [f(m_s) - f(m)]/f(m)$ and $\omega = [\varphi(m_s) - \varphi(m)]/\varphi(m)$. Equations (57) lead to

$$D/C = \rho \approx -0.30, \quad E/C = \omega \approx 0.1,$$

$$b/C = +\rho^2 \approx +0.09, \quad c/C = \rho\omega \approx -0.03,$$

$$d_h/C = +\rho^2 \omega \approx 0.01. \tag{66}$$

These agree with the above orders of magnitude. Incidentally, factorizability fixes the signs of b, c, and d_h.

VII. THE MAGNETIC MOMENTS OF OCTET BARYONS

The general parametrization of the baryon octet moments was given in Sec. III. Differently from baryon and meson masses [where formulas (12) and (28) of the parametrization are exact], we are limited for magnetic moments to terms linear in P^s. Recall that g_1 and g_2 in the general parametrization alone provide a fit to 15%. With only g_1 and g_2 the parametrization reduces to Eq. (25) and the physical meaning of

$$-g_2/g_1 = 0.34 \pm 0.01 \tag{67}$$

is obvious: $[1 + (g_2/g_1)] = 0.66$ is the ratio of strange to nonstrange quark magnetic moments.

The ''parametrized magnetic moment'' of the baryon octet is written compactly similarly to the ''parametrized baryon masses'' (Sec. V) with the help of three functions χ, η, and ξ of (m/Λ) and $(\Delta m/\Lambda)$. Again, these functions, in principle, are derivable from QCD. We have

$$\text{``parametrized magnetic moment''} = (\mathrm{Tr} Q P^s) \sum_{\substack{i \neq k \neq j \\ (i>k)}} \chi(m, \Delta m | m_i; m_k, m_j) \boldsymbol{\sigma}_i + \sum_{\substack{i \neq k \neq j \\ (i>k)}} \eta(m, \Delta m | m_i; m_k, m_j) Q_i \boldsymbol{\sigma}_i$$
$$+ \sum_{\substack{i \neq k \neq j \\ (i>k)}} \xi(m, \Delta m | m_i, m_k, m_j) Q_k \boldsymbol{\sigma}_i \tag{68}$$

with $m_i = m + \Delta m P_i^s$ and the functions χ and η symmetric in k and j. The $\mathrm{Tr} Q P^s$ term in (68) is of order $\Delta m/\Lambda$, and is related to g_0 in the general parametrization of Sec. III [$g_0 = \chi(m, \Delta m | m; m, m)$]. The other terms produce the spin-flavor structures $Q_i \boldsymbol{\sigma}_i$ and $Q_k \boldsymbol{\sigma}_i$ multiplied by unity or by products of P_j^s with up to three factors. Extracting from (68) all terms either with no P^s or linear in P^s, we reobtain, of course, the terms listed in the general parametrization of Sec. III. (Some terms bilinear in P^s are incorrectly absent in the list of Ref. [5], Eqs. (37)–(39). This is of no consequence because we never used for magnetic moments terms bilinear or cubic in P^s.)

Here we discuss the terms extracted from η in (68), the only ones of interest for Eq. (25) (in this comparison χ and ξ do not intervene). Again we shorten $\eta(m, \Delta m | m_1; m_2, m_3)$ to $\eta(|m_1; m_2, m_3)$. We get, identically,

$$\begin{aligned} \eta(|m_i; m_k, m_j) = {} & \eta(|m; m, m) + [\eta(|s; m, m) \\ & - \eta(m; m, m)] P_i^s + [\eta(|m; s, m) \\ & - \eta(|m; m, m)](P_k^s + P_j^s) \\ & + (\text{terms bilinear and cubic in } P^s). \end{aligned} \tag{69}$$

Thus, comparing with (25),

$$\begin{aligned} g_2/g_1 &= \frac{\eta(|s; m, m) - \eta(|m; m, m)}{\eta(|m; m, m)} \\ &= (\Delta m/\beta''\Lambda) + \gamma''(\Delta m/\beta''\Lambda)^2 + \cdots . \end{aligned} \tag{70}$$

Identifying (as in Secs. V and VI) Δm with m_s:

$$g_2/g_1 \cong (m_s/\beta''\Lambda) + \gamma''(m_s/\beta''\Lambda)^2 + \cdots , \tag{71}$$

where β'' and γ'' are coefficients in the expansion. Assuming, as previously, that the second term in the expansion has the order of magnitude of the square of the first, we have

$$(\Delta m/\beta''\Lambda) = -0.34 \pm 0.11. \tag{72}$$

A classical remark (to be inserted more properly in the ensuing sections, where we will consider constituent quarks) is this: Approximating magnetic moments by (25), and *defining* effective masses of quarks as inversely proportional to magnetic moments, the ratio between the effective masses of the strange and nonstrange quarks, *so defined*, is $(1 + g_2/g_1)^{-1} \cong 1.5 \pm 0.25$. The expansion (71) also implies an order of magnitude for the effective mass of a quark $\beta''\Lambda$. With the conventional choice $\Delta m = 150$ MeV, this is ≈ 450 MeV.

Coming back to QCD it is remarkable that β, β', and β'' in the expansions for the hyperfine parts of the meson and baryon masses (53) and (62) and baryon magnetic moments (72) are so close. Why it is so? Only a full QCD calculation can explain this, but a guess may help to relate the β' of the hyperfine mass term in baryons to the β'' in the Δm expansion of the magnetic moments (71). Assume that $F(|m_i, m_k; m_j)$ governing the hyperfine part of the baryon masses is approximately factorizable as mentioned at the end of Sec. VI (it is so in some models—see Sec. IX—and is anyway true to first order in $\Delta m/\Lambda$); factorizability means that $F(|m_i, m_k; m_j) = f(m_i) f(m_k) \varphi(m_j)$. Then D/C (60) is $[f(m_s) - f(m)]/f(m)$. If $\eta(|m_i; m_k, m_j)$ is also factorizable, that is, $\eta(|m_i;m_k,m_j)=t(m_i)r(m_k)r(m_j)$, we have $g_2/g_1=[t(m_s)-t(m)]/t(m)$. Then $(1/\beta')$ is the first order coefficient in the expansion of the quark-gluon chromomagnetic vertex in $\Delta m/\Lambda$, normalized to the vertex at $m_s=m$. As to $(1/\beta'')$, this is the same for the electromagnetic vertex. The similarity in structure of electromagnetic and chromomagnetic interactions suggests that to first order in m_s we have $[t(m_s)-t(m)]/t(m)=[f(m_s)-f(m)]/f(m)$, that is, $\beta'=\beta''$; note the equality in sign.

So far we have used only QCD (no assumption on models). The quarks in play were the standard current (quasichiral) quarks. From now on we shall deal, instead, with models, discussing how the parametrization provides a convenient way to test models. Before this we comment briefly on the notion of constituent quarks, which sometimes is the source of some confusion.

VIII. THE TWO MEANINGS OF "CONSTITUENT QUARK"

At present "constituent quark" has two meanings, both familiar, but rather different. We recall them only to avoid ambiguities in what follows.

In the first (less common) meaning, a constituent light quark is the QCD field after choosing a low q (near Λ) as the renormalization point for the mass [17,18]:

$$m_{\text{constituent}}=m(\text{at } q \text{ near } \Lambda). \tag{73}$$

This definition was implied in Refs. [5,6], when deriving the general parametrization. But, as noted, the derivation of the parametrization is independent of the renormalization point and can proceed using as quark fields the standard current fields; thus the q in $\nu(q)$ in (37) can be as high as we like.

Constituent quarks defined by something such as (73) would be related to the QCD Lagrangian. But it is hard to turn this definition, for light quarks, into something useful. At low q's perturbative QCD fails. With (73) the fact that $(\Delta m/m)$ differs for current and constituents, in spite of scale invariance, might be due to this failure and/or to Politzer's [17] q^{-2} term.

The second meaning of "constituent quark" is the more common. It dates back to the NRQM [1]; its continuing use is due to the above difficulty of reaching a really useful operational definition of the first type. In this second usage, constituent quarks are defined with reference to specific models. Their (effective) masses are just some among the many parameters in a calculation with the selected model. From now on constituent quarks will have this second meaning; we will use, as mentioned, M for the effective mass of a constituent quark.

IX. MODELS AND QCD

The proliferation of models of hadron structure in the past 20 years has brought a lack of predictive power: too many models, all "so good." Thus it is interesting to see why models work and record certain properties that a model should have to agree with some general consequences of QCD. The properties to be considered below are minimal properties: A model should satisfy them, but the model is not necessarily perfect if they are satisfied (a "perfect" model coincides with the true theory, say QCD).

Another question with models, raised in the Introduction, is the relation of the effective masses of constituent quarks and the masses of current quarks in the QCD Lagrangian. We will examine this also.

Consider, to exemplify, a model of baryon structure (the same applies to mesons). It should at least reproduce the masses of the lowest baryons and their magnetic moments. Of course, it should reproduce much more, as already stated. But below we concentrate on these, because these alone are sufficient to substantiate our point.

We will examine four classes of models: any nonrelativistic quark model, the semirelativistic QCD-inspired one-gluon-exchange DGG model, the MIT bag model, and the cloudy bag model. They are all characterized, at least at some stage in the calculation, by Hamiltonians with three quarks (for baryons). The cloudy bag model, which couples these quarks to pions, will be treated in the next section.

A. Nonrelativistic quark model

Consider a nonrelativistic quark model. Call H_{NR} a typical Hamiltonian for it, expressed in terms of the space, spin, and flavor coordinates of the three quarks (any quark variable has an index $j=1,2,3$). H_{NR} may be quite general; flavor has to be broken only by λ_8^j matrices, or, if electromagnetism is included, also by λ_3^j matrices. The eigenstates of H_{NR} in general will be mixtures of various orbital angular momenta; in other words, its lowest exact eigenfunctions $\psi_i(\text{NR})$ (i refers to a baryon in the octet or decuplet) may have configuration mixing. Yet, for the lowest baryons (octet plus decuplet) we may write

$$\psi_i(\text{NR})=V_{\text{NR}}\phi_i, \tag{74}$$

where ϕ_i is an auxiliary wave function having the product form of Eq. (8). In (74), of course, V_{NR} is a transformation producing, from the $L=0$ function ϕ, the exact configurationally mixed function ψ. This is certainly a much simpler transformation than the V introduced for QCD in Sec. II to construct the exact state $|\psi_i\rangle$ from the auxiliary state $|\phi_i\rangle$. There the V transformation had the gigantic task of dressing the $3q$ state with all sorts of $q\bar{q}$ pairs and gluons, plus producing configuration mixing, plus transforming two-component spinors into four-component ones. In the present case the V_{NR} transformation has just the task of producing configuration mixing. But formally Eq. (11) can be rewritten also in this case [in writing it we suppress NR in ψ(NR)]. Thus,

$$\begin{aligned} M_i &= \langle\psi_i|H_{\text{NR}}|\psi_i\rangle=\langle\phi_i|V_{\text{NR}}^{\dagger}H_{\text{NR}}V_{\text{NR}}|\phi_i\rangle \\ &= \langle W_i|(\text{“parametrized mass”})_{\text{NR}}|W_i\rangle. \end{aligned} \tag{75}$$

Because Eq. (12) for the "parametrized mass" in (11) follows only from the flavor dependence and invariance properties of the QCD Hamiltonian, with the factorizable choice of ϕ_i, the same expression (12) is true here. *Therefore a NRQM Hamiltonian gives a description of the masses of the lowest baryons identical to that of QCD, provided only that it has the number of parameters necessary to produce all terms in (12)*. Of course since, as we saw, many terms in (12) are small (in particular, those with three quark indices), even simple NR Hamiltonians may lead to good results. A similar argument holds for magnetic moments, where, in writing a NRQM Hamiltonian, one has to pay attention to gauge invariance. Also, in this case, the fact that the additive terms of the general QCD expression (25) already reproduce to 15% the magnetic moments, makes it not so miraculous that simple NR Hamiltonians with few parameters give a good account of the magnetic moments. What is of interest in this case, as already remarked and so far unexplained [6(f)], is that in the general QCD parametrization (23) the coefficient g_3 is so small (-0.076). It is this smallness that produces the classical ratio $\cong -\frac{3}{2}$ of the magnetic moments of proton and neutron (which fact [19] greatly contributed, in 1965, to the birth of the quark model).

Finally, though in this paper we did not treat this problem, the analysis [6(a)] of semileptonic decays of the lowest baryons leads to similar conclusions. Of course, many more properties should be considered (just think of excited hadronic states). Still, the conclusion is that simple NR models work because the number of important terms in the QCD general parametrization is relatively small.

B. The QCD-inspired model of De Rujula, Georgi, and Glashow

In their treatment DGG first calculate the one-gluon-exchange QCD potential $\mathscr{V}$ between two quarks in the Fermi-Breit approximation. Their three-body Hamiltonian is $H_{\mathrm{DGG}}=H_0+\mathscr{V}$, with H_0 flavor and spin independent. Treating $\mathscr{V}$ as a first order perturbation, the DGG baryon masses are

$$M_i=\langle \psi_i|H_{\mathrm{DGG}}|\psi_i\rangle \cong \langle \phi_i|H_{\mathrm{DGG}}|\phi_i\rangle$$
$$=\langle W_i|(\text{``parametrized mass''})_{\mathrm{DGG}}|W_i\rangle. \qquad (76)$$

In (76) H_{DGG} is the full DGG Hamiltonian, ψ_i the exact and ϕ_i the zero order eigenfunctions (with the effect of $\mathscr{V}$ in H_{DGG} neglected); the last expression in (76) arises from the third after integration on the space variables. Because H_0 is flavor and spin independent, the unperturbed zero order wave functions ϕ_i are flavor independent for all lowest octet and decuplet states and they are factorizable $\phi_i=X_{L=0}(r_1,r_2,r_3)W_i$ as in (8). In this treatment the baryon masses automatically have the form (12) predicted by the QCD general parametrization, except for the absence of terms with three different quark indices; these would be there if DGG had included the exchange of two or more gluons. Thus, for the lowest baryons, the ‘‘parametrized mass’’ again agrees with the general QCD parametrization (12), although it does not contain all the parameters in (12). This being clear, we pass to the question raised in the Introduction, namely, what is the meaning of the ‘‘effective masses’’ of quarks in the hyperfine term of DGG?

In DGG [4] the hyperfine contribution to the baryon masses from quarks 1 and 2 (one must then sum over all pairs of quarks) is

$$K_{\mathrm{DGG}}=\frac{4\pi}{9}\,\alpha_s\,\frac{1}{M_1M_2}\,\langle X_{L=0}|\,\delta^3(\mathbf{r}_{12})|X_{L=0}\rangle(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$
$$\equiv\frac{\tau}{M_1M_2}\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2), \qquad (77)$$

where the last expression is just a definition of τ:

$$\tau\equiv(4\pi/9)\,\alpha_s\langle X_{L=0}|\,\delta^3(\mathbf{r}_{12})|X_{L=0}\rangle. \qquad (78)$$

In (77) M_1 and M_2 are what DGG call the effective masses of quarks 1 and 2; they are not defined except by (77). In the Fermi-Breit treatment of a He-like atom containing, say, an electron and a muon, M_1 and M_2 would be the masses of the electron and muon. Here, on the other hand, they are not the masses of the current quarks that appear in the original Lagrangian of QCD from which DGG move. They have the dimensions of a mass and differ for a strange and nonstrange quark, but this is all. The question is: How are M_1 and M_2 related to quantities in the QCD Lagrangian? To simplify the answer, assume, following DGG, that $\langle X_{L=0}|\,\delta^3(\mathbf{r}_{12})|X_{L=0}\rangle$ is independent of M_1 and M_2 (this, essentially, corresponds to $X_{L=0}$ being uniform inside a sphere). Thus the only dependence of K_{DGG} on M_1 and M_2 is in $(M_1M_2)^{-1}$; note that this is factorizable.

Now go back to the exact general parametrization of baryon masses in QCD [Eq. (54)] and compare it with the DGG formula. If we wish the DGG result (77) to approximate QCD, $F(|m_i,m_k;m_j)$ in (54) must be factorizable, that is, $F(|m_i,m_k;m_j)=f(m_i)f(m_k)\varphi(m_j)$. We introduced factorizability at the end of Sec. VII, having in mind also the present application. To compare with the DGG one-gluon-exchange treatment one must put $\varphi(m_j)=1$ in $f(m_i)f(m_k)\varphi(m_j)$. Then comparing with (77) we have

$$\frac{M_1M_2}{\tau}=\frac{1}{f(m_1)f(m_2)}\equiv\frac{\Lambda^2}{\Lambda^3 f(m_1/\Lambda)f(m_2/\Lambda)}. \qquad (79)$$

In the last form we reinstalled the Λ’s to make the dimensions explicit. Clearly the DGG ‘‘effective masses’’ M_i in terms of QCD masses m_i and Λ are

$$(\tau/\Lambda^3)^{-1/2}M_i=\Lambda f^{-1}(m_i/\Lambda). \qquad (80)$$

This shows, as expected, that Λ is the QCD scale giving the effective mass scale of the constituent quarks.

Equation (80) shows clearly how the relationship of current and constituent quark masses depends on the model used to introduce the latter. We obtained (80) assuming that the integral in τ is independent of the M’s. Otherwise, the relation of the M’s to the QCD m’s is affected.

Similarly, consider $(\Delta M/M_\lambda)_{\mathrm{DGG}}$ and its relationship with the QCD masses. Comparing (77) and the general parametrization (12), $(\Delta M/M_\lambda)_{\mathrm{DGG}}=-D/C=0.3$. From (60)

$$D/C=[F(|m_s,m;m)-F(|m,m;m)]/F(|m,m;m)$$

and, in the factorized approximation, $D/C=[f(m_s)-f(m)]/f(m)$. Thus, reinstalling Λ,

$$0.3=-(D/C)=(\Delta M/M_\lambda)_{\mathrm{DGG}}=-[f(m_s/\Lambda)$$
$$-f(m/\Lambda)]/f(m/\Lambda)\approx-\Delta m/(\beta'\Lambda), \qquad (81)$$

having used in the last step the expansion (61). Equation (81) shows that there is no contradiction between $M_\lambda/M \approx 1.4$ and m_s/m equal to 25 or 10 (or ∞); of course, m_s/m is fundamental and M_λ/M model dependent; Eq. (81) displays the conceptual relationship between them. Again the model dependence enters in (81) because to get $(M_1 M_2)^{-1}$ in Eq. (77) [and therefore $-(D/C)=(\Delta M/M_\lambda)_{\mathrm{DGG}}$] one must assume, as noted in [20], that the integral in τ is mass independent. This assumption, nearly true in a potential well, is not so for a harmonic oscillator, or, worse, for a Coulomb potential. In such cases we would not have approximate factorizability, as can be seen easily in the analogous simpler case of mesons; $\Delta M/M_\lambda$ might be quite different.

C. The MIT bag model

We now turn briefly to the MIT bag model, where the quarks are relativistic (Dirac equation in a bag with four-component spinors; the quark masses in the model are taken very small or zero for u and d and, say, 100 MeV for s). We limit ourselves to considering here the case of the baryon masses, and specifically the hyperfine contribution. Note that in spite of the fully relativistic nature of the four-component spinors from which one starts, the hyperfine term appears, of course, at the end [21] in the Pauli form (82), in agreement with the general parametrization. Indeed, the hyperfine contribution to the baryon mass from quarks 1 and 2 (one must sum over the three pairs of quarks) is the expectation value of

$$K_{\mathrm{bag}}=8\alpha_s(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,\frac{\mu_1\mu_2}{R^3}\,I_{1,2} \tag{82}$$

on the W_i spin-flavor states. In (82) R is the radius of the bag, μ_1 and μ_2 are the chromomagnetic moments, and $I_{1,2}$ is an expression depending, as do the chromomagnetic moments, on the radius R. [In principle, R also in (82) might depend on the flavors of 1 and 2; in that case, it should be R_{12}.] The dependence on the quark masses m_i inserted in the model remains in μ_i and $I_{1,2}$. To a good approximation in (82) (and also in the baryon magnetic moments) intervene the effective masses of the quarks $M_i=(m_i^2+x^2/R^2)^{1/2}$ where x/R is the quark momentum in the bag ($x=2.04$ in the limit $mR\to 0$).

The main point of interest is the following. The hyperfine term (82) is contained in the general parametrization. However, in the simple version of the model treated so far, all terms with three indices, which in general also appear in the parametrization, are absent. Since these terms are relatively small, the situation is, in this respect, the same as in the DGG treatment with one-gluon exchange, in spite of the fact that the two models differ considerably.

Essentially the same conclusion is true for a variety of relativistic or semirelativistic quark models. Any of them can be successful (but not superior to others, in spite, often, of complicated calculations) provided that it reproduces the spin-flavor structure of the general parametrization and provided that it contains a number of parameters producing the dominant coefficients of it. Of course, one might object that different models will reveal differences in the calculation of hadron properties other than those considered here (think, e.g., of the spectrum of excited states). This is certainly true, but is of interest only if the models do not add too many additional parameters for this purpose.

X. EXCHANGE CURRENTS AND THE CLOUDY BAG MODEL

In the cloudy bag model we discuss only one point, the exchange pion current contribution to the baryon magnetic moments. The question is: Does the general QCD parametrization contain terms that can be interpreted as due to pion exchange currents? At first sight the question looks intriguing for the following reason: The Hamiltonian of the cloudy bag model contains, due to the coupling of pions to quarks, the Gell-Mann flavor matrices λ_1, λ_2, and λ_3. Thus one expects that the result of any calculation, say that of baryon magnetic moments, is expressed through λ_1 and λ_2 or [in SU(2)] τ_x and τ_y; indeed [22], the magnetic moments of proton and neutron receive, due to pion exchange, a contribution with the spin-flavor structure

$$\sum_{i\neq k}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_k)_z(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k). \tag{83}$$

On the other hand, the QCD Lagrangian (including electromagnetism) contains only the flavor matrices λ_8 and λ_3. They commute and form a closed algebra. Performing a pure QCD calculation, where virtual pions are $q\bar{q}$ aggregates, one expects that λ_1 and λ_2 (that is, τ_x and τ_y) cannot enter in the final result, in contrast with (83). Indeed, to derive the flavor structure of the general parametrization, we used the fact that operating with λ_3 and λ_8 (a closed algebra) one cannot produce other flavor matrices, which thus cannot appear in the final expression. How can this problem be solved?

Below we will show that, *in apparent contradiction with the argument given above*, a term such as (83) can arise from a QCD calculation; thus the cloudy bag model (and its pion exchange current) is compatible with QCD and we were *incorrect* in questioning this compatibility in Ref. [7(a)]. However, we will also show that, *in agreement with the previous argument*, the term (83) can be identically rewritten as a sum of the spin-flavor structures $\mathbf{G}_1$ and $\mathbf{G}_3$ in (22), not containing τ_x and τ_y at all (recall that $\mathbf{G}_1=\Sigma_i Q_i\boldsymbol{\sigma}_i, \mathbf{G}_3=\Sigma_{i\neq k}Q_i\boldsymbol{\sigma}_k$). Thus nothing changes in the general parametrization and the term (83) is not an unequivocal signature of pion exchange or of the cloudy bag model.

The proof is simple. Consider the Majorana space exchange operator P_x^{ik} exchanging the *space* coordinates of quarks i and k. In a QCD calculation of $V^\dagger\mathcal{M}V$ such operators may intervene; that is, $V^\dagger\mathcal{M}V$ for the baryon magnetic moments may contain space exchange terms. Then one can proceed in two fully equivalent ways. First, because the auxiliary function ϕ [Eq. (8)] is factorized as the product of a space factor $X_{L=0}$ times a spin-flavor factor W_i, we let P_x act on $X_{L=0}$ and integrate on $\mathbf{x}$. This is just the procedure adopted in deriving the general parametrization (22); the presence of P_x does not alter the result (22) (this will emerge clearly from Appendix A).

Second, operating, for simplicity, in SU(2), use now the symmetry of the whole wave function, and write $P_x^{ik}=(1+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)/4$ [a similar argument holds in

SU(3)]. Consider then a term of the form $\Sigma_{i\neq k}Q_i\boldsymbol{\sigma}_i P_x^{ik}$, the existence of which is possible in QCD. Rewrite it as

$$\sum_{i\neq k} Q_i\boldsymbol{\sigma}_i P_x^{ik}=\sum_{i\neq k} Q_i\boldsymbol{\sigma}_i(1+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(1+\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)/4. \quad (84)$$

Setting [in SU(2)] $Q_i=\frac{1}{2}\tau_{zi}+\frac{1}{6}$, using the identities

$$\tau_{zi}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)=\tau_{zk}-i(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_k)_z,$$

$$\sigma_{zi}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)=\sigma_{zk}-i(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k)_z, \quad (85)$$

and limiting oneself to terms that can contribute to the expectation value of the real functions W_i of the baryon octet and decuplet, one obtains, from (84),

$$4\sum_{i\neq k} Q_i\sigma_{iz}P_x^{ik}=\sum_{i\neq k}[(Q_i+Q_k)+\tfrac{1}{6}[(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)-1]](\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_k)_z - \tfrac{1}{2}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_k)_z(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k)_z. \quad (86)$$

Because $\Sigma_{i\neq k}[(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)-1](\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_k)_z$ gives zero when operating on the P, N, or Δ states, and because $P_x^{ik}X_{L=0}=X_{L=0}$, we remain with the identity, *valid only for the nonstrange baryons of octet and decuplet*:

$$\sum_{i\neq k}(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_k)_z(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_k)_z=-8\sum_i Q_i\sigma_{iz}+4\sum_{i\neq k}Q_i\sigma_{kz}. \quad (87)$$

Thus the exchange term of the cloudy bag model is already contained in the terms with coefficients g_1 and g_3 in the general parametrization (22). It may contribute (more or less) to g_1 and g_3 but it seems impossible to disentangle it from all other contributions to them. In other words, there is no specificity of the cloudy bag model in this respect, if one does not add other assumptions that may be interesting, but hard to prove.

XI. CONCLUSIONS

Several points were already listed in the Abstract, to which we refer. Here we wish to add or underline the following.

(1) Using the general parametrization starting from the standard QCD Lagrangian with quasichiral (or current) light quarks, we have shown that for each model the effective "constituent quark" masses M are related to the quark masses m of the QCD Lagrangian and to Λ_{QCD}; the existence of a Λ around 200 MeV intervenes in an essential way. In all sensible models the effective masses of light quarks are $\xi\Lambda$ with ξ a number between 2.3 and 3.7. There is, in principle, no contradiction in having $m_s/m\approx(8-25)$ and $M_\lambda/M\approx1.4$.

(2) The circumstance that different models may give, with a convenient choice of a few parameters in each of them, results in agreement with the data is due to the fact that the general parametrization derived from QCD usually contains [5,6(f)] only a few important terms. An appropriate selection of parameters in each model considered can produce these few important terms. This can be regarded, if one so wishes, as an extension of the so called "Cheshire cat" principle, introduced originally [23] to assess the equivalence of descriptions of hadronic phenomenology in terms of bag models with different radii.

(3) The important issue of predictivity of models remains the same as ever. Indeed, assuming that QCD is "the theory," that is, a full QCD calculation would reproduce all details of hadron physics, the extrapolation of any given model to new phenomena, beyond those where it has been tested and which were used to fix the parameters in the model, cannot be expected to be 100% exact. This applies to any model whatsoever. In this situation the "best" model is to some extent a matter of taste and, to a large extent, a matter of simplicity. To exemplify with a recent important achievement [24], consider the measurement of the magnetic moment of the Ω^-, an experiment of extraordinary precision: $\mu(\Omega^-)=(-2.024\pm0.056)\mu_N$. At the end of their paper the authors state that this measurement disagrees with the static quark model value of $-1.84\mu_N$ and express the hope that the result will provide a stringent test for future models of baryon structure. On the reality of this hope our point of view differs from that of the authors of [24]. It seems already remarkable that the NRQM *in its simplest form* predicts $\mu(\Omega^-)=-1.84\mu_N$, confirming the dominance of a few terms stated above. We saw, in fact, that the general parametrization for the magnetic moments of the octet baryons has a large variety of terms [Eqs. (22)–(24)] and the same is true for the decuplet baryons [7(a)]. Exploiting this variety, it would be trivial to add some terms to the "naive" ones and obtain the measured $\mu(\Omega^-)$ value, even if the latter were known with a precision still higher than the extraordinary one given in [24]. But this would not too be fruitful. Yes, the model so constructed would produce the measured $\mu(\Omega^-)$, but it would necessarily still be approximate for some other quantity; unless the "model" and the true theory were the same thing.

APPENDIX A: OUTLINE OF THE DERIVATION OF THE GENERAL PARAMETRIZATION

We outline the calculation of the expectation value of the field operator $V^\dagger H_{\mathrm{QCD}}V$ in the baryon three-quark auxiliary state $|\phi_i\rangle$; i.e., we outline the derivation of the fourth form of (11), expressing the masses as expectation values of a spin-flavor three-body operator [Eq. (12)] on the spin-flavor functions W_i. In the third form of Eq. (11) the only part of $V^\dagger H_{\mathrm{QCD}}V$ that contributes is its projection in the $|3q$,no gluon$\rangle$ Fock sector:

$$\tilde{H}=\sum_{3q,3q'}\sum |3q\rangle\langle 3q|V^\dagger H_{\mathrm{QCD}}V|3q'\rangle\langle 3q'|, \quad (A1)$$

where the sums in (A1) are on all possible three-quark, no-gluon Fock states. After normal ordering of all creation and destruction operators in $\tilde{H}$ and their contraction with those arising from $\langle\phi_i|$ and $|\phi_i\rangle$ [see Eq. (36)], the operator $\tilde{H}$ becomes a function only of the spin-flavor space variables of the three quarks in $|\phi_i\rangle$; thus parametrizing $\tilde{H}$ amounts to constructing the most general scalar operator of the $\boldsymbol{\sigma}_i$'s, λ_i's, and $\mathbf{r}_i$'s of the three quarks ($i=1,2,3$), including in it, of course, only those terms, that have a nonvanishing expectation value in ϕ_i. Note that (Sec. IV) we took the quarks in the auxiliary states $|\phi_i\rangle$ to be identical to those in the QCD

Lagrangian at the chosen renormalization point in the no-flavor-breaking limit. Due to this, the contraction of the creation and destruction operators in $V^{\dagger}H_{\mathrm{QCD}}V$ with those in the auxiliary states $|\phi_i\rangle$ [Eq. (36)] is straightforward. After this contraction, the projection $\tilde{H}$ of the field operator $V^{\dagger}H_{\mathrm{QCD}}V$ in the three-body sector becomes, as stated above, a scalar (i.e., rotation-invariant) function of the space $\mathbf{r}_i$, spin $\boldsymbol{\sigma}_i$, flavor f_i, and color operators of the three quarks (we suppress the color variables when possible). One has to write the most general expression of such an operator. We call it $\tilde{H}'$ (we use a different symbol because $\tilde{H}$ [Eq. (A1)] operates in Fock space, whereas $\tilde{H}'$, obtained after contraction of the field operators, is just a three-body quantum mechanical operator). The number of independent scalar operators in the spin-flavor space of three quarks is finite; we use for them, in general, the symbol $Y_{\mu}(\boldsymbol{\sigma},f)$ where μ specifies the operator to which we refer. Thus *the most general operator of the space and spin-flavor variables is necessarily*

$$\tilde{H}'=\sum_{\mu} R_{\mu}(\mathbf{r},\mathbf{r}')Y_{\mu}(\boldsymbol{\sigma},f), \tag{A2}$$

where $R_{\mu}(\mathbf{r},\mathbf{r}')$ are operators (not necessarily local) acting in the coordinate space of the three quarks. In (A2) $\mathbf{r}$ means

$$\mathbf{r}\equiv(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3).$$

To calculate a physical quantity such as a mass (as we are doing) one must form the expectation of (A2) on ϕ_i. Now a most important point: The auxiliary ϕ_i is arbitrary, provided that it has the correct quantum numbers of the state $|\psi_i\rangle$ under consideration (in this case an octet or decuplet baryon). With this proviso one can choose ϕ_i freely. For instance, the three-quark part of the correct $|\psi_i\rangle$ [first addend of the RHS of (39)] certainly has configuration mixing; still, one can select an auxiliary wave function ϕ_i without configuration mixing. It is the task of the transformation V to produce configuration mixing, and, of course, the whole complexity and variety of Fock states present on the right-hand side of (39). Thus we select the auxiliary wave function ϕ_i to be as simple as possible. An important feature in this choice is factorizability. That is, we select ϕ_i as in (8), the product of a space part $X_{L=0}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ with orbital angular momentum $L=0$ and a spin-flavor (color) factor $W_i(1,2,3)$ carrying the whole J [see (9) and (10)]. The factorization of ϕ_i implies for the expectation value (11) (that is, the mass M_i) the structure

$$M_i=\sum_{\mu} \langle X_{L=0}(\mathbf{r})|R_{\mu}(\mathbf{r},\mathbf{r}')|\phi_{L=0}(\mathbf{r}')\rangle\langle W_i|Y_{\mu}(\boldsymbol{\sigma},f)|W_i\rangle, \tag{A3}$$

that we also rewrite as

$$\sum_{\mu} g_{\mu}\langle W_i|Y_{\mu}(\boldsymbol{\sigma},f)|W_i\rangle \tag{A4}$$

with

$$g_{\mu}=\langle X_{L=0}(\mathbf{r})|R_{\mu}(\mathbf{r},\mathbf{r}')|X_{L=0}(\mathbf{r}')\rangle. \tag{A5}$$

Because the space part of the model wave function has, by construction, $L=0$, the operators $R_{\mu}(\mathbf{r},\mathbf{r}')$ in (A2) must be rotation invariant. Because $\tilde{H}'$ is a scalar, the $Y_{\mu}(\boldsymbol{\sigma},f)$ that enter in the parametrization of the masses M_i must be scalar operators, to be constructed only in terms of the spins $\boldsymbol{\sigma}_i$ of the three quarks; hence the parametrized masses are written as

$$\text{“parametrized mass”}=\sum_{\mu} g_{\mu}Y_{\mu}(\boldsymbol{\sigma},f) \tag{A6}$$

and the masses M_i are

$$\begin{aligned}M_i&=\langle\phi_i|\tilde{H}|\phi_i\rangle=\sum_{\mu} g_{\mu}\langle W_i|Y_{\mu}(\boldsymbol{\sigma},f)|W_i\rangle\\&\equiv\langle W_i|\text{“parametrized mass”}|W_i\rangle,\end{aligned} \tag{A7}$$

the result already given in Eq. (11).

To deduce the general parametrization (12) of the masses it is now sufficient to list all scalars $Y_{\mu}(\boldsymbol{\sigma},f)$ formed with the spins and flavors of the three quarks. Because the W_i's are symmetric in spin flavor, the only intervening spin-flavor structures are precisely those in Eq. (12). This is due to the following points.

(1) The only possible scalars constructed with the three spin Pauli $\boldsymbol{\sigma}_i$'s are 1 and $(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)$. The scalar $(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\boldsymbol{\sigma}_3$ (times any Hermitian real flavor operator) has vanishing expectation value on any real spin-flavor state of three particles, as the W_i's are.

(2) The only flavor operator in the strong Lagrangian is P^s. Thus only P_i^s, $P_i^sP_k^s$, and $P_1^sP_2^sP_3^s$ are possible flavor operators for three quarks; $(P_i^s)^n$ with any (integer) n reproduce P_i^s. Structures such as $\mathrm{Tr}(P_i^s)$ are numbers.

A similar procedure leads to Eq. (28) for the parametrized masses of the mesons (with $I\neq 0$). For the baryon magnetic moments (22) and (23), the $Y_{\nu}(\boldsymbol{\sigma},f)$'s in (A2) must then be axial vectors under rotations; keeping only terms linear in P^s one then obtains (22) and (23) [5].

APPENDIX B: THE COEFFICIENTS IN THE BARYON PARAMETRIZATION

In [5,6(d),6(f)] we determined the coefficients of the parametrization (12) from the baryon masses. For Δ, $\Sigma(1385)\equiv\Sigma^*$, and $\Xi(1530)\equiv\Xi^*$ we used for this in [6] the conventional masses (resonance peaks) as given in [13]. One of us [12] noted that it might be preferable to use the “pole” masses. For the “large” coefficients A, B, and C the differences between the values of the coefficients A, B, C, D, E, $(a+b)$, c, and d in (12) derived using the conventional or the pole masses are irrelevant or of little interest. For the smaller coefficients D, E, $(a+b)$, c, and d, the two determinations may differ significantly. Below we will list the coefficients obtained from the conventional and pole fits. Because the general parametrization (and therefore its coefficients) refers to the strong interaction only [the masses in (12) are the eigenvalues of H_{QCD}, without the electromagnetic interaction] it is necessary, especially for the smaller coefficients, to extract from the experimental masses the strong part. That is, to determine the coefficients of the parametrization (12), one must construct and use combinations of baryon masses independent of the e.m. and isospin breaking ($m_u \neq m_d$), at least to first order. We did this already [6(d)] when writing Eq. (14). Below we write these combinations of baryon masses. To determine the "large" coefficients M_0, B, and C, the precision stated above is unnecessary and we simply averaged the Coulomb and isospin effects:

$$M_0=(\overline{N}+\overline{\Delta})/2, \quad B=\Lambda-\overline{N}+3E, \quad C=(\overline{\Delta}-\overline{N})/6, \tag{B1}$$

where $\overline{N}=(n+p)/2$, $\overline{\Delta}=(\Delta^{++}+\Delta^{+}+\Delta^{0}+\Delta^{-})/4$, and E is the coefficient in (12) given below in Eq. (B3). As to the coefficients D, E, $(a+b)$, c, d, they are determined from the following Eqs. (B2) which are Coulomb and isospin independent to first order

$$\begin{aligned}
D&=(1/6)[(\Sigma^{*-}-\Delta^{-})+(\Sigma^{+}-p)],\\
E&=(1/6)(\Sigma^{*-}-\Delta^{-})+(1/12)(\Sigma^{0}-3\Lambda+2n),\\
c&=(1/3)[(\Xi^{*-}-\Xi^{-})-(\Sigma^{*-}-\Sigma^{-})]-2E,\\
a+b&=\Xi^{-}-\Sigma^{-}+(1/2)(\Sigma^{0}-3\Lambda+2n)+2c,\\
d&=\Omega-\Delta^{++}-3(\Xi^{*0}-\Sigma^{*+}),
\end{aligned} \tag{B2}$$

where in the above formulas Δ^- stays for

$$\Delta^{-}=\Delta^{++}+3(n-p).$$

From Eqs. (B3), using the conventional and pole values of the masses-also the latter are found in Ref. [13] (we recall, e.g., Δ^{++}(conv)=1231±1, Δ^{++}(pole)=1210.5±1) we get for the coefficients (in MeV):

M_0	B	C	D	E	$(a+b)$	c	d
pole: 1076	192	45.6±0.3	−13.8±0.3	5.1±0.3	−16±1.4	−1.1±0.7	4±3
conv: 1086	184	49.2±0.3	−16.4±0.2	2.5±0.2	−7.5±0.8	+3.1±0.4	−5.7±2

The pole values (first line) were already listed in (13). Note the appreciable difference in the smaller coefficients (E to d) according to the two determinations; although, for reasons on which we do not come back here [12], we tend to prefer the pole fit, in both cases the new [6d] octet-decuplet mass formula (14) is satisfied practically with the same remarkable precision (16) stated in the text.

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[15] Compare Fig. 1 of Ref. [6(f)] and its figure caption.

[16] The analysis of Ref. [6(f)], based on the conventional masses (not the pole ones) gave 0.2 instead of 0.37 for $|E/D|$ (see Appendix B in the present paper). The same analysis produced 0.33 for the flavor-breaking factor instead of the 0.3 given here.

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