Bremsstrahlung correction for baryon β decays in the four-body region of the Dalitz plot: Charged baryons

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The analytical formula and the corresponding numerical results for the bremsstrahlung of semileptonic decays of charged baryons, in the region of the Dalitz plot that covers the four-body events which were not considered in previous calculations, are obtained. The formula is very accurate because it includes all the terms of the order α times the momentum transfer. The model dependence of the radiative correction is kept in a general form which is suitable for a model-independent experimental analysis. The bremsstrahlung contribution in terms of the energies of the produced fermions is obtained by analytical means. Our result is suitable for high statistics decays of ordinary baryons as well as for medium statistics decays of charm baryons.

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I. INTRODUCTION

The application of the radiative corrections in the analysis of the experimental data is indispensable, due to the recent experimental progress [1] of high-energy hyperon beams and the improving precision of the measurements in the hyperon β-decay processes.

In previous papers [2–5] we have obtained the formulas and the numerical values for the radiative corrections to the Dalitz plot of semileptonic decays of charged and neutral hyperons in the region where the three-body (TBR) process takes place. The high precision corrections include terms of the order of $\alpha q/\pi M_1$ (with q the four-momentum transfer and M_1 the mass of the decaying baryon). The numerical values of the radiative corrections to the lepton energy spectrum due to the TBR were also evaluated in Ref. [5].

To obtain the complete spectra of energy, the contributions of the events in which a real photon is always emitted are required. The Dalitz plot region which is not allowed kinematically for a three-body decay but where the four-body events take place is called the four-body region (FBR).

It is the purpose of this paper to obtain the model-independent high-precision analytical formulas which contain all the terms of order $\alpha q/\pi M_1$ for the bremsstrahlung contributions in the FBR of semileptonic decays of charged hyperons. With these results the previous ones are complemented in order to obtain the whole bremsstrahlung contributions to the energy spectra of the fermionic and baryonic emitted particles, and to the decay rate of the process.

As it is already known [3], the model dependence of the radiative corrections appears with the inclusion of the high precision terms. Fortunately the Low theorem [6,7], which states that there is no model dependence if one knows the experimental values of the electromagnetic static parameters of the baryons, solves this problem.

In Sec. II the kinematical relations used for the calculation of the bremsstrahlung in the FBR are presented. In Sec. III the bremsstrahlung amplitude with all the $\alpha q/\pi M_1$ terms included is reviewed. A closed expression for the several bremsstrahlung contributions is obtained by analytical means. In Sec. IV all the partial results are collected into a final completely integrated analytical expression. The numerical corresponding results are obtained and compared with the ones published in Refs. [8,9]. Finally, we include an Appendix which contains a summary with the definitions used along the different stages of the analytical integration.

II. KINEMATICS

In this section we present the notation we use for the four-body process we are interested in:

$$A(p_1)\to B(p_2)+e(\ell)+\bar{\nu}_e(p_\nu)+\gamma(k), \tag{1}$$

where the emission of a real photon takes place. The four-momenta and masses of the particles involved in baryon semileptonic decays will be denoted by $p_1=(E_1,\vec{p}_1)$, $p_2=(E_2,\vec{p}_2)$, $\ell=(E,\vec{\ell})$, $p_\nu=(E_\nu,\vec{p}_\nu)$, and $k=(k_0,\vec{k})$, and by M_1,M_2,m,m_ν, and m_k, respectively. We shall assume throughout this paper that $m_\nu=0$, and $m_k=0$ as corresponds to real photons. We specialize our calculations as in Ref. [2] to a coordinate frame in the rest system of the decaying particle A with the Z axis along the electron three-momentum and the X axis oriented so that the final baryon three-momentum is in the first or fourth quadrants of the X-Z plane. To be more explicit,

$$\vec{\ell}=|\vec{\ell}|(0,\,0,\,1),$$

$$\vec{k}=|\vec{k}|(\sqrt{(1-x^2)}\cos\phi_k,\ \ \sqrt{(1-x^2)}\sin\phi_k,x)$$

$$\vec{p}_2=|\vec{p}_2|(\sqrt{(1-y^2)},\,0,y),$$

and

$$\hat{k}\cdot\hat{p}_2=z. \tag{2}$$

We are interested in the process (1), when it takes place in the four-body region of the Dalitz plot defined by

$$M_2\leqslant E_2\leqslant E_2^{\min},\quad -1<y<+1,$$

and

$$m \leq E \leq E_B$$

with

$$E_2^{\substack{\max\\ \min}} = \frac{1}{2}(M_1 - E \pm \ell) + \frac{M_2^2}{2(M_1 - E \pm \ell)}$$

and

$$E_B = \frac{(M_1^2 - M_2)^2 + m^2}{2(M_1 - M_2)}. \tag{3}$$

This region corresponds to the region AOB in Fig. 1 of Ref. [2], where a more detailed explanation about the TBR and FBR can be found, see also Ref. [8].

For completeness, we explicitly show in this section the list of kinematical relations used in the expansion of the amplitudes of the nondivergent components of the radiative correction.

$$\vec{p}_2 \cdot \vec{p}_\nu = -|\vec{p}_2|(|\vec{p}_2| + |\vec{\ell}|y_0) + \frac{F}{2D}(E\beta x + E_\nu^0),$$

$$\vec{\ell} \cdot \vec{p}_\nu = -|\vec{\ell}|(|\vec{\ell}| + |\vec{p_2}|y_0) + \frac{F}{2} - \frac{|\vec{\ell}|x}{2D}F,$$

$$\hat{k} \cdot \vec{p}_\nu = E_\nu^0 - \frac{F}{2D} - D,$$

$$\vec{p}_2 \cdot \vec{\ell} = |\vec{\ell}||\vec{p}_2|y_0 - \frac{F}{2},$$

$$\vec{p}_2 \cdot \hat{k} = D - E_\nu^0 - E\beta x,$$

$$\vec{\ell} \cdot \hat{k} = E\beta x, \quad \text{and} \quad \hat{k} = \frac{\vec{k}}{k_0}. \tag{4}$$

Here y_0 is the cosine of the angle between the electron and the baryon B three-momenta when no radiation is emitted, i.e., $k_0 = 0$:

$$y_0 = \frac{E_\nu^{02} - \vec{p}_2^{\,2} - \vec{\ell}^{\,2}}{2|\vec{p}_2||\vec{\ell}_2|}, \tag{5}$$

where

$$E_\nu^0 = M_1 - E_2 - E, \tag{6}$$

$E_\nu = E_\nu^0 - k_0$ is the energy of the neutrino in Eq. (1),

$$k_0 = \frac{F}{2D} \tag{7}$$

is the energy of the photon, and

$$F = 2|\vec{p}_2||\vec{\ell}|(y_0 - y), \quad D = \frac{k \cdot p_\nu}{k_0}, \quad \text{and} \quad \beta = \frac{|\vec{\ell}|}{E}. \tag{8}$$

III. BREMSSTRAHLUNG DIFFERENTIAL DECAY RATE

In this section the emission of the real photon in Eq. (1) is described as a radiative correction to the Dalitz plot of the semileptonic decay of the charged hyperon. The transition amplitude for process (1) without the emission of a real photon is

$$M_0 = \frac{G_v}{\sqrt{2}} \bar{u}_B W_\mu u_A \bar{u}_\ell O_\mu v_\nu, \tag{9}$$

where

$$W_\mu = f_1(q^2)\gamma_\mu + \frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu + \frac{f_3(q^2)}{M_1}q_\mu + \left[g_1(q^2)\gamma_\mu + \frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu + \frac{g_3(q^2)}{M_1}q_\mu\right]\gamma_5, \tag{10}$$

$O_\mu = \gamma_\mu(1+\gamma_5)$ and $q = p_1 - p_2$ is the four-momentum transfer. Our metric and γ-matrix conventions are those of Ref. [2].

In order to obtain the bremsstrahlung correction to the Dalitz plot we can follow the discussions and adapt the results of Secs. III and IV of Ref. [3]. Thus, there is no need to enter into details here.

We shall first obtain the amplitude of this process and right afterward we shall consider separately the amplitude which generates the infrared divergence in the TBR. Next, we shall obtain a complete expression for the differential bremsstrahlung decay rate that gives the Dalitz plot with radiative corrections of process (1).

What we want is the amplitude of process (1) with all the $\alpha q/\pi M_1$ terms. It has been shown [10] that these terms can be obtained in a model independent fashion by virtue of the Low theorem [6,7].

We only have to reproduce the model-independent amplitude, in terms of the Dirac form factors, given in Eq. (17) of Ref. [3]:

$$M_B = M_1 + M_2 + M_3, \tag{11}$$

with

$$M_1 = eM_0\left(\frac{\epsilon \cdot \ell}{\ell \cdot k} - \frac{\epsilon \cdot p_1}{p_1 \cdot k}\right), \tag{12}$$

$$M_2 = \frac{eG_v}{\sqrt{2}} \epsilon_\mu \bar{u}_B W_\lambda u_A \bar{u}_\ell \frac{\gamma_\mu \not{k}}{2\ell \cdot k} O_\lambda v_\nu, \tag{13}$$

$$M_3=\frac{G_v}{\sqrt{2}}\bar{u}_{\ell}O_\lambda v_\nu \epsilon_\mu \bar{u}_B\bigg[\frac{eW_\lambda \not{k}\gamma_\mu}{2p_1\cdot k}-\kappa_1 W_\lambda \frac{\not{p}_1+M_1}{2p_1\cdot k}\sigma_{\mu\nu}k_\nu + \kappa_2\sigma_{\mu\nu}k_\nu\frac{\not{p}_2+M_2}{2p_2\cdot k}W_\lambda + e\left(\frac{p_{1\mu}k_\lambda}{p_1\cdot k}-g_{\mu\nu}\right)\left(\frac{f_3-f_2}{M_1} + \gamma_5\frac{g_3-g_2}{M_1}\right)+e\left(\frac{p_{1\mu}k_\lambda}{p_1\cdot k}-g_{\mu\nu}\right)(\sigma_{\lambda\nu}+g_{\lambda\nu}) \times\left(\frac{f_2+g_2\gamma_5}{M_1}\right)\bigg]u_A. \tag{14}$$

κ_1 and κ_2 are the anomalous magnetic moments of A and B given in Eqs. (21) and (22) in Ref. [3]. M_0 and W_λ are given in Eqs. (9) and (10). ϵ_μ is the photon polarization four vector.

In order to calculate the bremsstrahlung contribution to the radiative corrections in the required order $\alpha q/\pi M_1$, we shall trace a close parallelism with the calculation of Ref. [3].

The evaluation of the Dalitz plot is performed by standard trace calculations leaving as the relevant independent variables the energies E_2 and E of the emitted baryon and the electron, respectively. The square of M_B summed over spins can be split, after trace calculations, into the sum of three contributions:

$$\sum_{\text{spins}}|M_B|^2=\sum_{\text{spins}}M_1^2+\sum_{\text{spins}}(2M_1M_2+M_2^2) +\sum_{\text{spins}}2(M_1M_3+M_2M_3). \tag{15}$$

The term M_3^2 will contribute to order $\alpha q^2/\pi M_1^2$ and higher and thus it is not included in (15). The first summand in (15) is explicitly given by

$$\sum_{\text{spins}}M_1^2=\frac{e^2G_v^2}{2}\frac{4M_1}{M_2 m m_\nu}\frac{4\Sigma(\epsilon\cdot\ell)^2}{(2\ell\cdot k)^2}[A_0'+A_1-k_0B_0], \tag{16}$$

where

$$A_0'=Q_1EE_\nu^0-Q_2E(\vec{p}_2^{\,2}+|\vec{p}_2||\vec{\ell}|y_0)-Q_3(\vec{\ell}^{\,2}+|\vec{p}_2||\vec{\ell}|y_0) +Q_4E_\nu^0|\vec{p}_2||\vec{\ell}|y_0, \tag{17}$$

$$A_1=\tfrac{F}{2}(Q_2E+Q_3-Q_4E_\nu^0), \tag{18}$$

and

$$B_0=Q_1E+Q_2E(D-E_\nu^0-E\beta x) +Q_3E\beta x+Q_4\left(|\vec{p}_2||\vec{\ell}|y_0-\frac{F}{2}\right). \tag{19}$$

The coefficients Q_i, $i=1,\dots,4$, are given in Eqs. (16)–(19) in Ref. [2]. The A_0' in Eq. (17) is the same coefficient as the one that modulates the TBR infrared divergent term.

In order to proceed as in Ref. [3] let us write the differential decay rate of (1) as

$$d\Gamma_B=d\Gamma_B^{\mathrm{I}}+d\Gamma_B^{\mathrm{II}}+d\Gamma_B^{\mathrm{III}}, \tag{20}$$

with

$$d\Gamma_B^{\mathrm{I}}=d\Gamma_B^{A_0}+d\Gamma_B^{A_1}+d\Gamma_B^{B_0}. \tag{21}$$

$d\Gamma_B^{\mathrm{I}}, d\Gamma_B^{\mathrm{II}}$, and $d\Gamma_B^{\mathrm{III}}$ contain the contributions from the first, second and third summands in (15). Now, we adapt Eq. (38) in Ref. [2], and we obtain, for $d\Gamma_B^{\mathrm{I}}$ in first place,

$$d\Gamma_B^{A_0}=\frac{\alpha}{\pi}\frac{|\vec{p}_2||\vec{\ell}|}{2\pi}A_0'd\Omega\int_0^{2\pi}d\phi_k \times\int_{-1}^{+1}dy\frac{1}{F}\beta^2\int_{-1}^{1}dx\frac{1-x^2}{(1-\beta x)^2}, \tag{22}$$

where

$$d\Omega=\frac{G_v^2}{2}\frac{dE_2dE\; d\Omega_\ell d\phi_2}{(2\pi)^5}2M_1, \tag{23}$$

in the second place,

$$d\Gamma_B^{A_1}=\frac{\alpha}{\pi}\frac{|\vec{p}_2||\vec{\ell}|}{2\pi}d\Omega\cdot\int_0^{2\pi}d\phi_k\int_{-1}^{+1}dy\frac{1}{2}\beta^2 \times\int_{-1}^{1}dx\frac{1-x^2}{(1-\beta x)^2}(Q_2E+Q_3-Q_4E_\nu^0) \tag{24}$$

and, in the third place,

$$d\Gamma_B^{B_0}=-\frac{\alpha}{\pi}\frac{|\vec{p}_2||\vec{\ell}|}{2\pi}d\Omega \times\left(\int_0^{2\pi}\frac{d\phi_k}{D}\int_{-1}^{+1}dy\frac{\beta^2}{2}\int_{-1}^{1}dx\frac{1-x^2}{(1-\beta x)^2}B_0\right). \tag{25}$$

The integration in Eq. (22) is straightforwardly performed.

The overall integrated result does not depend on the infrared divergence cut-off λ which appears in Ref. [2]. The integrated expression for $d\Gamma_B^{A_0}$ becomes

$$d\Gamma_B^{A_0}=\frac{\alpha}{\pi}A_0' 2\ln\left|\frac{(1+y_0)}{(y_0-1)}\right|\left(\frac{\operatorname{arctanh}\beta}{\beta}-1\right)d\Omega. \tag{26}$$

The A_0' and $d\Omega$ are given in Eqs. (17) and (23), respectively.

As a guide to perform the lengthy integrations and in order to reproduce our results, the definitions used all along the different stages of the analytical integration are given in the Appendix.

The integrated expression for the second component of $d\Gamma_B^{\mathrm{I}}$ in Eq. (24) is

$$d\Gamma_B^{A_1}=\frac{\alpha}{\pi}d\Omega|\vec{p}_2||\vec{\ell}|(Q_2E+Q_3-Q_4E_\nu^0)\theta_0^T, \tag{27}$$

θ_0^T is given in Eqs. (45) in the Appendix.

The third part of $d\Gamma^{\mathrm{I}}$ given in Eq. (25) becomes

$$d\Gamma_B^{B_0}=\frac{\alpha}{\pi}d\Omega\sum_{i=0}^{13}H_i^{B_0}\theta_i^T, \tag{28}$$

where

$$H_0^{B_0}=-|\vec{p}_2||\vec{l}|EQ_2, \quad H_1^{B_0}=0,$$

$$H_2^{B_0}=\frac{|\vec{p}_2|\beta m^2}{2}[Q_1+Q_3-(Q_2+Q_4)(E_\nu^0+E)],$$

$$H_3^{B_0}=\frac{|\vec{p}_2|\beta m^2}{2}[Q_2E-Q_3+Q_4(E_\nu^0+2E)]$$
$$+|\vec{p}_2||\vec{l}|E[(Q_2+Q_4)(E_\nu^0+E)-(Q_1+Q_3)],$$

$$H_4^{B_0}=\frac{-|\vec{p}_2||\vec{l}|m^2Q_4}{2}+\frac{|\vec{p}_2||\vec{l}|E}{2}[Q_1+2Q_3$$
$$-2(Q_2+Q_4)E-(Q_2+2Q_4)E_\nu^0],$$

$$H_5^{B_0}=\frac{|\vec{p}_2|\vec{l}^2}{2}[Q_3-Q_2E-Q_4(E_\nu^0+2E)],$$

$$H_6^{B_0}=H_7^{B_0}=H_8^{B_0}=H_9^{B_0}=0\ ,$$

$$H_{10}^{B_0}=-\frac{|\vec{l}|}{|\vec{p}_2|}H_{13}^{B_0}, \quad H_{11}^{B_0}=\frac{m^2}{E^2}H_{13}^{B_0}, \quad H_{12}^{B_0}=-2H_{13}^{B_0},$$

$$H_{13}^{B_0}=Q_4\frac{\vec{l}^2}{2}\vec{p}_2^{\,2},$$

and the θ_i^T are also defined in Eqs. (45) in the Appendix.

The partial decay rate $d\Gamma_B^{\mathrm{I}}$ is now compactly given by

$$d\Gamma_B^{\mathrm{I}}=\frac{\alpha}{\pi}d\Omega\sum_{i=0}^{13}H_i\theta_i^T, \tag{29}$$

where the H_i's with $i=1,\ldots,13$ are given in Eqs. (43) in Ref. [3].

Now, we turn to the other terms in $d\Gamma_B$. The obtained results for $d\Gamma_B^{\mathrm{II}}$, and $d\Gamma_B^{\mathrm{III}}$ have the same form of the ones displayed in Eqs. (44) and (45) in Ref. [3], but instead of the θ_i's given in Ref. [3] the new θ_i^T's, which are displayed in the Appendix [Eqs. (45)], appear.

The integrated expressions in terms of the relevant independent variables, the energies E_2 and E of the emitted baryon and the electron are, respectively,

$$d\Gamma_B^{\mathrm{II}}=\frac{\alpha}{\pi}d\Omega\sum_{i=0}^{15}H_{i+14}\theta_i^T \tag{30}$$

and

$$d\Gamma_B^{\mathrm{III}}=\frac{\alpha}{\pi}d\Omega\sum_{i=0}^{16}H_{i+30}\theta_i^T, \tag{31}$$

with the H_j $j=14,\ldots,46$ are given in Eqs. (44) and (45) in Ref. [3]. The preceding partial results are still to be collected in Eq. (20): namely,

$$d\Gamma_B=d\Gamma_B^{\mathrm{I}}+d\Gamma_B^{\mathrm{II}}+d\Gamma_B^{\mathrm{III}}.$$

IV. FINAL RESULT AND CONCLUSIONS

Now we are in a position to obtain the bremsstrahlung correction in the FBR, up to order $\alpha q/\pi M_1$. Our complete result is given by

$$d\Gamma_B(A\to Be\nu)=\frac{\alpha}{\pi}d\Omega\sum_{i=0}^{16}H_i'\theta_i^T. \tag{32}$$

The analytical results obtained for the θ_i^T's are

$$\theta_0^T=2\,(I_1-2), \quad \theta_1^T=(I_1-2)\ln\left|\frac{y_0+1}{y_0-1}\right|, \tag{33}$$

$$\theta_2^T=\frac{1}{\beta|\vec{p}_2|}\left[\frac{I_2^-}{1+\beta a^-}-\frac{I_2^+}{1+\beta a^+}+\frac{E^2}{m^2}\left(I_2^+-I_2^-+\beta\ln\left|\frac{I_3^-}{I_3^+}\right|\right)\right]$$
$$+\frac{2I_1}{E(1+\beta a^-)(1+\beta a^+)},$$

$$\theta_3^T=\frac{I_1}{|\vec{p}_2|}\ln\left|\frac{1+\beta a^+}{1+\beta a^-}\right|+\frac{1}{\beta|\vec{p}_2|}\left(L\left[\frac{1-\beta}{1+\beta a^-}\right]-L\left[\frac{1-\beta}{1+\beta a^+}\right]\right.$$
$$\left.+L\left[\frac{1+\beta}{1+\beta a^+}\right]-L\left[\frac{1+\beta}{1+\beta a^-}\right]\right),$$

$$\theta_4^T=\frac{1}{|\vec{p}_2|}\left[a^+I_2^+-a^-I_2^-+\ln\left|\frac{I_3^-}{I_3^+}\right|\right],$$

$$\theta_5^T=\frac{1}{2|\vec{p}_2|}\left[(1-a^{+2})I_2^+-(1-a^{-2})I_2^-+4\frac{|\vec{p}_2|}{E\beta}\right],$$

$$\theta_6^T=2\frac{(y_0-a^-)}{(1+\beta a^-)^2}(I_2^-+\beta I_1)-2\frac{(y_0+a^+)}{(1+\beta a^+)^2}(I_2^++\beta I_1)$$
$$+2\left[2+\beta\left(\frac{y_0-a^-}{1+\beta a^-}-\frac{y_0+a^+}{1+\beta a^+}\right)\right]I_4,$$

$$\theta_7^T=2\left[2I_1+\frac{y_0-a^-}{1+\beta a^-}(\beta I_1+I_2^-)-\frac{y_0+a^+}{1+\beta a^+}(\beta I_1+I_2^+)\right],$$

$$\theta_8^T=2\,[4+(y_0-a^-)I_2^--(y_0+a^+)I_2^+],$$

$$\theta_9^T=24\,E+2[6(E_\nu^0-E)+\beta(G^{T-}+G^{T+})]I_1+2(G^{T-}I_2^-$$
$$+G^{T+}I_2^+)+2|\vec{p}_2|\left[\frac{(y_0-a^-)^2I_3^-}{1+\beta a^-}-\frac{(y_0+a^+)^2I_3^+}{1+\beta a^+}\right],$$

$$\theta_{10}^T=\frac{1}{3|\vec{p}_2|}\left[2(a^{-2}-a^{+2})-a^{-3}I_2^-+a^{+3}I_2^++\ln\left|\frac{I_3^-}{I_3^+}\right|\right],$$

$$\theta_{11}^T=\frac{2(I_4-I_1)}{\beta|\vec{p}_2|},\quad \theta_{12}^T=\frac{\theta_0^T}{\beta|\vec{p}_2|},\quad \theta_{13}^T=0,$$

$$\theta_{14}^T=2[(2-a^-I_2^-)(y_0-a^-)-(2-a^+I_2^+)(y_0+a^+)],$$

$$\theta_{15}^T=24E_\nu^0+4\beta E[a^-(y_0-a^-)I_2^- - a^+(y_0+a^+)I_2^+] +2|\vec{p}_2|[(y_0-a^-)^2I_3^- -(y_0+a^+)^2I_3^+],$$

$$\theta_{16}^T=24E^2(I_1-2)+8(E_\nu^{02}-2E^2\beta^2)I_1+4E\beta|\vec{p}_2| \times\left[\frac{(y_0-a^-)^2}{1+\beta a^-}(\beta I_1+I_2^-)-\frac{(y_0+a^+)^2}{1+\beta a^+}(\beta I_1+I_2^+)\right],$$

where

$$a^{\pm}=\frac{E_\nu^0\pm|\vec{p}_2|}{E\beta}, \tag{34}$$

$I_1, I_2^{\pm}, I_3^{\pm}, I_4$, and $G^{T\pm}$ are given by

$$I_1=\frac{2\ \arctan h\beta}{\beta},\quad I_2^{\pm}=\ \ln\left|\frac{1+a^{\pm}}{a^{\pm}-1}\right|, \quad I_3^{\pm}=\frac{2}{(a^{\pm})^2-1},\quad I_4=\frac{2}{1-\beta^2}, \tag{35}$$

and

$$G^{T\pm}=\mp\beta\left[\frac{2Ea^{\pm}(y_0\pm a^{\pm})}{(1+\beta a^{\pm})}+\frac{|\vec{p}_2|(y_0\pm a^{\pm})^2}{(1+\beta a^{\pm})^2}\right]. \tag{36}$$

For completeness, we explicitly show new simplified expressions for the form-factor-dependent coefficients H_i''s (given in Appendix D Ref. [4]):

$$H_0'=E\beta|\vec{p}_2|\left[\frac{1}{2}(Q_3-Q_4E_\nu^0)-\frac{E}{2M_1}[(f_1+g_1)^2+4f_2g_1 +2(f_1f_3-g_1g_2)]\right], \tag{37}$$

$$H_1'=E\{E_\nu^0Q_1-Q_2|\vec{p}_2|^2-Q_3\beta^2E+\beta|\vec{p}_2|y_0(E_\nu^0Q_4-Q_3 -EQ_2)\},$$

$$H_2'=\frac{|\vec{p}_2|}{2E}m^2\beta[-(Q_1+Q_3)E_\nu^0+(Q_2+Q_4)E|\vec{p}_2|\beta y_0 +Q_4(E+E_\nu^0)^2+Q_2|\vec{p}_2|^2],$$

$$H_3'=\beta|\vec{p}_2|\left\{\frac{E}{4}[[2E_\nu^0-E(1+\beta^2)](Q_1+Q_3) +2\beta|\vec{p}_2|y_0(E_\nu^0Q_4-Q_3-EQ_2)-2|\vec{p}_2|^2Q_2+(E_\nu^0+E) \times[E(1+\beta^2)(Q_2+3Q_4)-2(Q_3+2EQ_4)]] +m^2\left[\frac{h^+}{e}(E_\nu^0+2E)+[f_1^2+g_1^2+2(f_1f_3-g_1g_2)]\frac{E_\nu^0}{2M_1} -g_1(f_1+f_2+g_2)\frac{(2E+E_\nu^0)}{M_1}\right]\right\},$$

$$H_4'=\frac{E\beta|\vec{p}_2|}{2}\left\{\tfrac{1}{2}E[Q_1-E_\nu^0Q_2-E(Q_2+Q_4)+3(Q_3 -E_\nu^0Q_4)]+\tfrac{1}{2}[-E^2\beta^2Q_2+|\vec{p}_2|^2Q_4+E_\nu^0(2Q_3 -3E_\nu^0Q_4)]-\frac{EE_\nu^0}{M_1}[(f_1-g_1)^2+2f_1f_3]+\frac{1}{M_1}[f_1^2-g_1^2 +2(g_1g_2+f_1f_3)]|\vec{p}_2|\beta Ey_0+\frac{2E}{M_1}g_1g_2[2E_\nu^0+E(4 -3\beta^2)]+\frac{2E}{M_1}g_1[2E(f_1+f_2-g_2)+f_2(E_\nu^0+\beta^2E)] +\frac{2h^-}{e}\beta^2E^2-2E[E_\nu^0+2E(1-\beta^2)]\frac{h^+}{e}\right\},$$

$$H_5'=\frac{|\vec{p}_2|E^2\beta^2}{4}\left\{Q_1-(E+E_\nu^0)Q_2+3Q_3-(3E+7E_\nu^0)Q_4 -4(2E+E_\nu^0)\frac{h^+}{e}-8E_\nu^0\frac{h^-}{e}+\frac{4}{M_1}[E_\nu^0(f_1^2+2f_1f_3) +(2E-E_\nu^0)(f_1+f_2)g_1+(2E+3E_\nu^0)g_1g_2]\right\},$$

$$H_6'=\frac{|\vec{p}_2|(1-\beta^2)E\beta}{4}[Q_1+Q_3-(Q_2+Q_4)(E_\nu^0+E)],$$

$$H_7'=-\frac{E\beta|\vec{p}_2|}{4}\left\{\frac{(2E-E_\nu^0)}{E}(Q_1+Q_3)+\frac{|\vec{p}_2|^2}{E}(Q_2+Q_4) +2(\beta^2-1)EQ_4+|\vec{p}_2|\beta y_0\left(Q_2+3Q_4-2\frac{h^+}{e}\right)-2(E_\nu^0 +E\beta^2)\frac{h^+}{e}+[(\beta^2-3)E-2E_\nu^0]Q_2+\frac{E}{M_1}(1-\beta^2)[f_1^2 +g_1^2-g_1(2f_1+3f_2+g_2)]+g_1(f_2-g_2)\left(\frac{M_2}{E}\right) \times\left(\frac{2E_2-M_1}{M_2}-\frac{M_2}{M_1}\right)+\frac{2m^2}{EM_1}(f_1f_3-g_1g_2)\right\},$$

TABLE I. Comparison of the results of Ref. [9] with our results for the relative correction to the $(E,E2)$ distribution in the four-body region.

					Process $\Sigma\rightarrow ne\bar{\nu}$							
y/x		0.05			0.15			0.25			0.35	
0.8021	2.5	2.52	2.30									
0.7998	1.9	1.94	1.76									
0.7975	1.5	1.50	1.36	3.5	3.54	3.24						
0.7951	1.2	1.12	1.02	2.2	2.22	2.01						
0.7928	0.9	0.83	0.75	1.5	1.52	1.36						
0.7905	0.6	0.58	0.52	1.0	1.01	0.88	1.9	1.95	1.78			
0.7882	0.4	0.36	0.33	0.6	0.62	0.51	1.1	1.07	0.94	2.9	3.04	2.81
0.7859	0.2	0.17	0.16	0.3	0.28	0.19	0.4	0.45	0.35	0.8	0.86	0.77

$$H_8'=\frac{E\beta|\vec{p}_2|}{4}\Bigg\{Q_1+Q_3-(2E+E_\nu^0)Q_2+(E_\nu^0-E)Q_4 +2E_\nu^0\left(\frac{2h^- - h^+}{e}\right)+[(f_1-g_1)^2+2f_1f_3]\left(\frac{E-2E_\nu^0}{M_1}\right) +g_1\frac{[(2f_2+g_2)(E_\nu^0-2E)+g_2E_\nu^0]}{M_1}\Bigg\},$$

$$H_9'=\frac{|\vec{p}_2|\beta}{8}[-(Q_1+Q_3)+(Q_2+Q_4)(E+E_\nu^0)],$$

$$H_{10}'=\frac{(E\beta)^3|\vec{p}_2|}{4}\Bigg\{-(Q_2+5Q_4)-4\left[\frac{3h^- +2h^+}{e}\right] +\frac{2}{M_1}[3f_1(f_1+2f_3)+g_1(g_1-4f_1-6f_2+8g_2)]\Bigg\},$$

$$H_{11}'=0,$$

$$H_{12}'=(E\beta)^2|\vec{p}_2|^2\Bigg\{\frac{h^+}{e}-\frac{Q_4}{2}+\frac{1}{2M_1}[(f_1+g_1)^2 +2g_1(3f_2-2g_2)+2f_1f_3]\Bigg\},$$

$$H_{13}'=\frac{(E\beta)^2|\vec{p}_2|^2}{2}\Bigg\{Q_4-2\frac{h^+}{e}+\frac{1}{M_1}[2f_2(f_1-g_1)+g_1^2-f_1^2 -2f_2^2-2f_1f_3]\Bigg\},$$

$$H_{14}'=\frac{(E\beta)^2|\vec{p}_2|}{4M_1}[(f_1-g_1)^2-4f_2g_1 +2(f_1f_3-g_1g_2)-M_1Q_2],$$

$$H_{15}'=\frac{E\beta|\vec{p}_2|}{8}\Bigg\{-(Q_2+Q_4)-4\frac{h^-}{e}+\frac{2}{M_1}[(f_1-g_1)^2-2f_2g_1 +2f_1f_3]\Bigg\},$$

$$H_{16}'=\frac{|\vec{p}_2|}{4M_1}\beta\left[-M_1\frac{h^+}{e}+g_1(g_2-f_2)\right].$$

We have used the definition

$$h^\pm=-g_1^2(\kappa_1+\kappa_2)\pm f_1g_1(\kappa_2-\kappa_1). \tag{38}$$

With Eq. (32) we have the analytic result for the bremsstrahlung part of the Dalitz plot of the semileptonic decay of charged hyperons, Eq. (1) at the FBR. With Eq. (32), and Eq. (42) given in Ref. [3] we have a full analytic result for the bremsstrahlung part of the complete Dalitz plot of semileptonic decay of unpolarized charged hyperons.

Those results are of high precision, are model independent, and are useful for processes where the momentum transfer is not small and therefore cannot be neglected.

The expected error by the omission of higher order terms is of the order of $\alpha q/\pi M_1^2\approx 0.0006$ in charm decay. If the accompanying factors amount to one order of magnitude increase, then we can estimate an upper bound to the theoretical uncertainty of 0.6%.

There are no other analytical results (for the FBR contributions) available in the literature to which compare our expressions (32). There are some numerical analysis available [8,9], except that there the anomalous magnetic moments of particles A and B and the q^2 dependence of the form factors are ignored while in this paper they have been included. On the other hand, it is worth mentioning that there are some efforts to develop a Monte Carlo method for photon bremsstrahlung calculations in semileptonic decays [11]. The analytical precise result and the Monte Carlo results might be complementary each to the other.

In order to compare with the numerical results in Table I(a) of Refs. [9] we present in Table I the results of the evaluation of the relative corrections (RC's) in %, caused by bremsstrahlung events, which fall outside the TBR Dalitz plot.

Our calculations use the same values of the form factors employed in Ref. [9], they are displayed in Table II, along with the anomalous and total magnetic moments of the baryons involved.

In Table I we repeat in the first columns for each value of x, the numerical values for the RC's to the Dalitz plot from Ref. [9]. The numbers shown in the second and third columns are obtained from Eq. (32), taking into account the

TABLE II. Numerical values of the form factors and of the anomalous and the total magnetic moments used in our calculations. $\kappa_1, \kappa_2, \mu_1$, and μ_2 are given in nuclear magnetons.

Process	f_1	f_2	g_1	κ_1	κ_2	μ_1	μ_2
$\Sigma \rightarrow n e \nu$	1.0	−1.139	−0.310	−0.373	−1.9130	−1.157	−1.9130

anomalous magnetic moments and following the definitions given in Eq. (4.2′) in Ref. [9]. The numbers shown in the third column of this table do contain the model contribution and the q^2/M_1^2 dependence of the form factors which are explained in detail in Refs. [5].

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APPENDIX

The order of integration which is followed in our procedure, is first to integrate over the azimuthal angle of the photon ϕ_k, second, over the cosine of the polar angle of the final baryon y, and, third, over the cosine of the polar angle of the photon x. There are four integrals over ϕ_k they were evaluated previously and are given by Eqs. (64)–(67) of Ref. [2]. These are

$$K_0 = \int_0^{2\pi} d\phi_k = 2\pi,$$

$$K_1(y,x) = \int_0^{2\pi} d\phi_k \frac{1}{D(x,y,\phi_k)} = 2\pi \frac{1}{\sqrt{R}},$$

$$K_2(y,x) = \int_0^{2\pi} d\phi_k \frac{1}{D^2(x,y,\phi_k)} = 2\pi \frac{a'}{\sqrt{R^3}},$$

$$K_3(y,x) = \int_0^{2\pi} d\phi_k \frac{1}{D^3(x,y,\phi_k)} = 2\pi \left[\frac{1}{\sqrt{R^3}} + \frac{3b'^2}{2\sqrt{R^5}} \right]. \tag{A1}$$

Here

$$R = a'^2 - b'^2, \quad a' = E_\nu^0 + |\vec{l}|x + |\vec{p}_2|xy,$$
$$b' = |\vec{p}_2| \sqrt{(1-x^2)(1-y^2)}. \tag{A2}$$

Otherwise, R can be written as

$$R = ry^2 + sy + t, \tag{A3}$$

where

$$r = \vec{p}_2^{\,2}, \quad s = 2|\vec{p}_2|x(E_\nu^0 + |\vec{l}|x),$$
$$t = (E_\nu^0 + |\vec{l}|x)^2 - |\vec{p}_2|^2(1-x^2). \tag{A4}$$

The analytical integration over the polar angle y of the final baryon presents no in-principle difficulty. All the integrals that appear are standard and can be explicitly performed. However, the major difficulty comes from the length of the expressions involved. The integrals over y are written in the same way as those in Refs. [2–4], they are

$$\int_{-1}^{1} dy K_1(y,x) \equiv 2\pi \xi_1^T(x),$$

$$\int_{-1}^{1} dy F(y) K_2(y,x) \equiv 2\pi [\xi_2^T(x) - 2|\vec{l}|x \xi_1^T(x)],$$

$$\int_{-1}^{1} dy F^2(y) K_3(y,x) \equiv 2\pi [\xi_3^T(x) - 2|\vec{l}|^2 \times [2 - 3(1-x^2)] \xi_1^T(x)],$$

$$\int_{-1}^{1} dy\, y K_1(y,x) \equiv 2\pi \left[\xi_4^T(x) - x \frac{(E_\nu^0 + |\vec{l}|x)}{p_2} \xi_1^T(x) \right],$$

$$\int_{-1}^{1} dy\ F^2(y) K_2(y,x) \equiv 2\pi [\xi_5^T(x) + 4|\vec{l}|^2 [(E_\nu^0 + |\vec{l}|x) \times (1-3x^2) - 2p_2 y_0 x] \xi_1^T(x)]. \tag{A5}$$

The new $\xi_i^T(x)$ come out to be

$$\xi_1^T(x) = |\vec{p}_2|^{-1} \ln \left| \frac{x+a^+}{x+a^-} \right|,$$

$$\xi_2^T(x) = 2\left(2 + \frac{y_0 - a^-}{x+a^-} - \frac{y_0 + a^+}{x+a^+} \right),$$

$$\xi_3^T(x) = 12(E_\nu^0 - E\beta x) + 4E\beta \left(a^- \frac{(y_0 - a^-)}{(x+a^-)} - a^+ \frac{(y_0 + a^+)}{(x+a^+)} \right) + 2|\vec{p}_2| \left(\frac{(y_0 - a^-)^2}{(x+a^-)^2} - \frac{(y_0 + a^+)^2}{(x+a^+)^2} \right), \quad \xi_4^T(x) = \frac{2x}{|\vec{p}_2|},$$

$$\xi_5^T(x) = 8E_\nu^{0\,2} + 4E\beta \left[(6x^2 - 4)E\beta + |\vec{p}_2| \left(\frac{(y_0 - a^-)^2}{(x+a^-)} - \frac{(y_0 + a^+)^2}{(x+a^+)} \right) \right], \tag{A6}$$

where $a^\pm$ are given in Eq. (34). The integrals over x generate the θ_i^T's in the following way:

$$\theta_0^T=\beta^2\int_{-1}^{1}dx\,\frac{(1-x^2)}{(1-\beta x)^2},\quad \theta_{4-n}^T=\int_{-1}^{1}dx\,\frac{\xi_1^T(x)}{(1-\beta x)^n},$$

$$\theta_5^T=\int_{-1}^{1}dx\;x\xi_1^T(x),\quad \theta_{8-n}^T=\int_{-1}^{1}dx\,\frac{\xi_2^T(x)}{(1-\beta x)^n},$$

$$\theta_9^T=\int_{-1}^{1}dx\,\frac{\xi_3^T(x)}{1-\beta x},\quad \theta_{10}^T=\int_{-1}^{1}dx\;x^2\xi_1^T(x),$$

$$\theta_{13-n}^T=\int_{-1}^{1}dx\,\frac{\xi_4^T(x)}{(1-\beta x)^n},\quad \theta_{14}^T=\int_{-1}^{1}dx\;x\xi_2^T(x),$$

$$\theta_{15}^T=\int_{-1}^{1}dx\;\xi_3^T(x),\quad \theta_{16}^T=\int_{-1}^{1}dx\,\frac{\xi_5^T(x)}{1-\beta x},\quad n=0,1,2. \tag{A7}$$

θ_1^T is given in Eq. (33).

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