Form factor for the Dalitz decay of K_L

K. Terasaki
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-01, Japan

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The Dalitz decay of the K_L meson is investigated in consistency with the approximate $|\Delta\mathbf{I}|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays, the K_L-K_S mass difference, and the $K_L\to\gamma\gamma$ decay. The form factor for the Dalitz decay is compared with the existing data from $K_L\to\gamma e^+e^-$ and $\gamma\mu^+\mu^-$. It is demonstrated that vector meson poles can play an important role.

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A recent result on the $K_L\to\gamma\gamma^*$ form factor measured through the Dalitz decay $K_L\to\gamma\mu^+\mu^-$ [1] seems to be not compatible, in the lower photon-mass region, with the previous ones through the decay $K_L\to\gamma e^+e^-$ [2,3]. The existing theoretical analyses [4] in which a contribution of the K^*-meson pole is canceled on the photon mass shell seems to favor the latter. In this article, we investigate the form factor from another perspective, in which the K^* pole survives even on the photon mass shell, in consistency with the other weak interactions of K mesons, for example, the approximate $|\Delta\mathbf{I}|=\frac{1}{2}$ rule in $K\to\pi\pi$ decays, the K_L-K_S mass difference (Δm_K), and the $K_L\to\gamma\gamma$ decay.

We have studied the approximate $|\Delta\mathbf{I}|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays and the K_L-K_S mass difference (Δm_K) as reviewed briefly below. The amplitude for the $K(\mathbf{p})\to\pi_1(\mathbf{k_1})\pi_2(\mathbf{k_2})$ decay can be approximated in the form

$$M(K\to\pi_1\pi_2)\simeq M_{\mathrm{ETC}}(K\to\pi_1\pi_2)+M_S(K\to\pi_1\pi_2), \tag{1}$$

which can be obtained [5,6] by extrapolating $\mathbf{k_2}\to 0$ in the infinite momentum frame (IMF) of the parent particle (i.e., $\mathbf{p}\to\infty$). M_{ETC} and M_S are given by

$$M_{\mathrm{ETC}}(K\to\pi_1\pi_2)=-\frac{i}{\sqrt{2}f_\pi}\langle\pi_1|[V_{\bar{\pi}_2},H_w]|K\rangle+(\pi_1\leftrightarrow\pi_2) \tag{2}$$

and

$$M_S(K\to\pi_1\pi_2)=-\frac{i}{\sqrt{2}f_\pi}\left\{\sum_n\left(\frac{m_\pi^2-m_K^2}{m_n^2-m_K^2}\right)\langle\pi_1|A_{\bar{\pi}_2}|n\rangle\langle n|H_w|K\rangle+\sum_{\ell}\left(\frac{m_\pi^2-m_K^2}{m_\ell^2-m_\pi^2}\right)\langle\pi_1|H_w|\ell\rangle\langle\ell|A_{\bar{\pi}_2}|K\rangle\right\}+(\pi_1\leftrightarrow\pi_2), \tag{3}$$

where V_π and A_π denote the isospin and the corresponding axial charge, respectively. The isospin $\mathrm{SU}_I(2)$ symmetry is always assumed. Thus the amplitude is governed by *asymptotic matrix elements* (matrix elements taken between single hadron states with infinite momentum) of the effective weak Hamiltonian H_w. M_{ETC} and ground-state-meson contribution to M_S are given by asymptotic *ground-state-meson* matrix elements of H_w (taken between the ground-state-meson states with infinite momentum). It has been shown, from two different but complemental approaches, i.e., one is based on commutation relations among charges and currents [6,7] and the other based on more intuitive quark-line arguments [8] which will be examined again, that the asymptotic ground-state-meson matrix elements satisfy the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule (*the asymptotic* $|\Delta\mathbf{I}|=\frac{1}{2}$ rule). Contributions of excited mesons to M_S are expected to be small since their masses are much higher than m_K and overlappings of their wave functions with those of the ground-state $\{q\bar{q}\}_0$ mesons will be small. For the $K^+\to\pi^+\pi^0$ decay, both of M_{ETC} and $M_S^{\{q\bar{q}\}_0}$ (ground-state meson contribution to M_S) vanish because of the asymptotic $|\Delta\mathbf{I}|=\frac{1}{2}$ rule mentioned above. The small violation of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays arises from exotic $(qq)(\bar{q}\bar{q})$ meson contributions to M_S in the present perspective [8]. The amplitude for the $K_S\to\pi^+\pi^-$ is dominated by $M_{\mathrm{ETC}}+M_S^{\{q\bar{q}\}_0}$ and satisfies the approximate $|\Delta\mathbf{I}|=\frac{1}{2}$ rule.

The K_L-K_S mass difference (Δm_K) can be written [9] by a sum of short distance contribution $(\Delta m_K)_{\mathrm{short}}$ and long distance contribution $(\Delta m_K)_{\mathrm{long}}$. However, in the present perspective, $(\Delta m_K)_{\mathrm{short}}$ vanishes [10] as will be reviewed later. The vanishing $(\Delta m_K)_{\mathrm{short}}$ and the asymptotic $|\Delta\mathbf{I}|=\frac{1}{2}$ rule for the ground-state-meson matrix elements of H_w mentioned before are compatible with each other [10] as seen below. Inserting the commutation relation, $[V_{K^0},O_4^\dagger]=O_{\Delta S=2}$, between $\langle K^0|$ and $|\bar{K}^0\rangle$, we obtain [11]

$$\langle K^0|O_{\Delta S=2}|\bar{K}^0\rangle=\sqrt{2}\langle\pi^0|O_4|K^0\rangle, \tag{4}$$

where $O_{\Delta S=2}$ is the operator arising from the so-called box diagram [12] and O_4 is the $|\Delta\mathbf{I}|=\frac{3}{2}$ component of H_w. Therefore the asymptotic $|\Delta\mathbf{I}|=\frac{1}{2}$ rule for the ground-state-meson matrix elements of H_w leads to the vanishing $(\Delta m_K)_{\text{short}}$ and vice versa. The long distance contribution, $(\Delta m_K)_{\text{long}}$, can be decomposed into a sum of pole and continuum contributions. The latter will be dominated by contributions of $(\pi\pi)$ intermediate states, $(\Delta m_K)_{\pi\pi}$. We here take [13] $(\Delta m_K)_{\pi\pi}/\Gamma_{K_S}=0.22\pm0.03$, which seems to be well constrained. Therefore we concentrate on the pole contribution which is approximated by [14]

$$(\Delta m_K)_{\text{pole}}\simeq\left\{\sum_{P_i}\frac{|\langle K_L^0|H_w|P_i\rangle|^2}{2m_K(m_K^2-m_{P_i}^2)}-\sum_{V_i}\frac{|\langle K_S^0|H_w|V_i\rangle|^2}{2m_K(m_K^2-m_{V_i}^2)}\right\} \tag{5}$$

in the IMF, where $P_i=\pi^0,\ \eta,\ \eta',\ \iota$ [a glueball or a glue-rich meson with $J^{PC}=0^{-+}$] and $V_i=\rho^0,\ \omega,\ \phi$. In this way, the K_L-K_S mass difference also can be given by the asymptotic ground-state-meson matrix elements of H_w.

Now we study the $K_L\to\gamma\gamma^{(*)}$ decay. It is known [12] that the short distance contribution to the $K_L\to\gamma\gamma$ decay is too small. Therefore we need to consider long distance contributions to reproduce the observed rate for this decay. Among the possible long distance effects, pole contributions will be most important since contributions of two and more pion intermediate states are suppressed due to the approximate CP invariance and the small phase space volume, respectively. However, it has been shown [15], in a chiral Lagrangian approach, that a sum of π^0, η, and η' meson pole amplitudes with the usual η-η' mixing (the mixing angle $\theta_P\simeq-20°$ [16]) is hard to reproduce the observation [16] $\Gamma(K_L\to\gamma\gamma)_{\text{expt}}=(7.26\pm0.35)\times10^{-12}$ eV. Therefore, one has to take into account some other contributions. For example, a possible role of the glueball (ι) through the quark-loop (or the so-called penguin) effects has been considered in Ref. [17]. (However, it will be discussed later that the ι contribution will be small in the present perspective.) The pseudoscalar (PS) meson pole amplitude for the $K_L\to\gamma\gamma$ decay given in Ref. [17] will be extrapolated to the off-shell region $(k^2\neq0)$ in the form

$$A_P(K_L\to\gamma\gamma^*(k^2))=\sum_{P_i}\frac{\langle K_L|H_w|P_i\rangle A(P_i\to\gamma\gamma)}{(m_K^2-m_{P_i}^2)(1-k^2/\Lambda_P^2)}, \tag{6}$$

where $P_i=\pi^0,\eta,\eta'$, and ι, since the observed form factors for the π^0, η, $\eta'\to\gamma\gamma^*$ decays are well described in the form [18] $\sim(1-k^2/\Lambda_P^2)^{-1}$ with $\Lambda_P\simeq m_\rho$.

Another possible contribution to the $K_L\to\gamma\gamma$ decay will be the K^* meson pole. However, there have been arguments [19] against the K^* pole contribution. The basic ideas of these arguments were, on general grounds, that *only* the $\sim F_{\mu\nu}V_{\mu\nu}$ type of photon-vector meson coupling should be allowed because of the gauge invariance and, on the field algebra hypothesis, that a sum of the K^*-V-γ $(V=\rho^0,\ \omega,\ \phi)$ transition amplitudes should be canceled by the direct K^*-γ coupling. It is true that the $\sim A_\mu V_\mu$ type of photon-vector meson coupling by itself violates the gauge invariance but, if the V interactions are generated by the minimal principle, or equivalently, if a negative photon mass term is combined with the $\sim A_\mu V_\mu$ coupling, the gauge invariance can be all right [20]. Therefore we can get rid of the first problem. Next, the long distance K^*-V-γ coupling cannot be canceled by the direct K^*-γ since the short distance K^*-γ coupling should vanish in the standard model [21]. Therefore, in the theory of weak interactions based on the field algebra, the long distance K^*-V-γ also have to vanish. However, nonleptonic weak Hamiltonian in the theoretical frame work of the field algebra [19] is given by *symmetric* products of *left-handed* currents and its dominant part transforms like $\mathbf{8_s}$ of $\mathrm{SU}_f(3)$ [no $|\Delta\mathbf{I}|=\frac{3}{2}$ part is included] in contrast with the standard model in which the effective weak Hamiltonian is approximately written in the form [22]

$$H_w\simeq c_-O_-+c_+O_++(c_pO_p)+\text{ H.c.}, \tag{7}$$

where $O_\pm$ (and O_p) are normal ordered operators. The main term c_-O_- transforms like $\mathbf{8}_a$ (not $\mathbf{8}_s$) of $\mathrm{SU}_f(3)$ and the coefficient c_+ of O_+ which is the origin of the $|\Delta\mathbf{I}|=\frac{3}{2}$ interactions is not very small (and the penguin term O_p includes right-handed components). The above structure of H_w in the standard model is very much different from the weak Hamiltonian embedded within the theoretical framework of the field algebra. Thus the theory of weak interactions based on the field algebra seems to be far from the standard one. This trouble seems to be caused because the weak interactions were embedded unreasonably into the theoretical framework of the field algebra which is one of low energy effective theories to derive the vector meson dominance. Therefore the weak interactions and the vector meson dominance will have to be treated separately for the present. Since it is known that another theory [23] to derive the vector meson dominance is possible, we now need not be constrained by the field algebra even though we use the vector meson dominance. We can actually write down, in a gauge invariant form, the $K_L\to\gamma\gamma$ amplitude involving the K^*-V-γ coupling (but not the direct K^*-γ). Its off-shell amplitude is given by

$$\begin{aligned}A_{K^*}(K_L\to\gamma\gamma^*(k^2))\\ &=\sum_{V_i}\sum_{V_j}\sqrt{2}X_{V_i}X_{V_j}A(K^0\to K^{*0}V_i)\langle K^{*0}|H_w|V_j\rangle_{\lambda=\pm1}\\ &\quad\times\left\{\frac{1}{m_{V_i}^2(m_{K^*}^2-k^2)(m_{V_j}^2-k^2)}+\frac{1}{(m_{V_i}^2-k^2)m_{K^*}^2m_{V_j}^2}\right\}\end{aligned} \tag{8}$$

with $V_i=\rho^0$, ω, and ϕ, where $X_{V_i}=em_{V_i}^2/f_{V_i}$ (f_{V_i} is the usual photon-vector meson transition moment) is the photon-vector meson coupling strength. The subscript λ of the matrix element $\langle K^{*0}|H_w|V_j\rangle_{\lambda=\pm1}$ denotes the helicity of the vector meson states which sandwich H_w. Then the amplitude for the $K_L\to\gamma\gamma^*$ decay, which is dominated by the pole contribution as discussed before, can be approximated by

$$\begin{aligned}A(K_L\to\gamma\gamma^*(k^2))\simeq{}&A_P(K_L\to\gamma\gamma^*(k^2))\\ &+A_{K^*}(K_L\to\gamma\gamma^*(k^2))\end{aligned} \tag{9}$$

and then the form factor for the $K_L\to\gamma\gamma^*$ is given by

$$f(k^2)=\frac{A(K_L\to\gamma\gamma^*(k^2))}{A(K_L\to\gamma\gamma)}, \tag{10}$$

where $A(K_L\to\gamma\gamma)=A(K_L\to\gamma\gamma^*(k^2=0))$.

Now we evaluate Eq. (9) with Eqs. (6) and (8) in the IMF because of consistency with the prior investigations of the $K\to\pi\pi$ decays and Δm_K. Then the $K_L\to\gamma\gamma^*$ amplitude also is governed by *asymptotic* matrix elements of H_w. Constraints on the asymptotic matrix elements of H_w taken between $\lambda=0$ states have already been obtained from two different approaches [6–8]. We here review the intuitive quark-line argument [8]. The normal ordered operators $O_\pm$ can be expanded into a sum of products of (a) two annihilation and two creation operators, (b) one annihilation and three creation operators, (c) one creation and three annihilation operators, and (d) four annihilation or four creation operators of quarks and antiquarks. We associate these products of annihilation and creation operators with different types of weak vertices by requiring the usual connectedness of the quark lines. In this procedure, we have to be careful with the order of the quark(s) and antiquark(s). For (a), we utilize the two annihilation and the two creation operators to annihilate and create, respectively, the quarks and the antiquarks belonging to the meson states $|\{q\bar{q}\}\rangle$ and $\langle\{q\bar{q}\}|$ in the asymptotic matrix elements of $O_\pm$. However, in cases (b) and (c) we now have to add a spectator quark or antiquark to reach $\langle\{qq\bar{q}\bar{q}\}|O_\pm|\{q\bar{q}\}\rangle$ and $\langle\{q\bar{q}\}|O_\pm|\{qq\bar{q}\bar{q}\}\rangle$, where the four quark mesons $\{qq\bar{q}\bar{q}\}$ are classified into the following four types [24]: $\{qq\bar{q}\bar{q}\}=[qq][\bar{q}\bar{q}]\oplus(qq)(\bar{q}\bar{q})\oplus\{[qq](\bar{q}\bar{q})\pm(qq)[\bar{q}\bar{q}]\}$. ( ) and [ ] denote symmetry and antisymmetry, respectively, under the exchange of flavors between them. The first two components, $[qq][\bar{q}\bar{q}]$ and $(qq)(\bar{q}\bar{q})$, can have $J^{P(C)}=0^{+(+)}$ but the last $\{[qq](\bar{q}\bar{q})\pm(qq)[\bar{q}\bar{q}]\}$ have only $J^P=1^+$.

Noting the antisymmetry property of K meson wave function under exchange of quark and antiquark composing it [25], we obtain, in the IMF,

$$\langle\{q\bar{q}\}_0|O_+|\{q\bar{q}\}_0\rangle=0. \tag{11}$$

In the same way,

$$\langle[qq][\bar{q}\bar{q}]|O_+|\{q\bar{q}\}_0\rangle=0 \tag{12}$$

and

$$\langle(qq)(\bar{q}\bar{q})|O_-|\{q\bar{q}\}_0\rangle=0 \tag{13}$$

can also be obtained [8]. These are quite reasonable from the symmetry property of the wave functions of the $[qq][\bar{q}\bar{q}]$ and $(qq)(\bar{q}\bar{q})$ mesons under the exchange of the flavors of quarks and antiquarks. The penguin term always satisfies the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule. Therefore Eq. (11) implies that the asymptotic ground-state-meson matrix elements of H_w satisfy the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule. Nonvanishing $\langle(qq)(\bar{q}\bar{q})|O_+|\{q\bar{q}\}_0\rangle$ can give the natural origin of the small violation of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule in the $K\to\pi\pi$ decays. The same procedure as the above to obtain Eq. (11) leads us to $\langle\pi^0|O_4|K^0\rangle=0$, where O_4 is a component of O_+ and provides the $|\Delta\mathbf{I}|=\frac{3}{2}$ part of H_w, and also $\langle K^0|O_{\Delta S=2}|\bar{K}^0\rangle=0$. Therefore the asymptotic $|\Delta\mathbf{I}|=\frac{1}{2}$ rule for the ground-state-meson matrix elements of H_w and the vanishing short distance contribution in the K_L-K_S mass difference are compatible with each other in the present perspective as discussed earlier. We extended the same quark-line argument to asymptotic matrix elements of charm changing Hamiltonians and reproduced well a large violation of charm counterparts of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule, a long-standing puzzle, $\Gamma(D^0\to K^+K^-)/\Gamma(D^0\to\pi^+\pi^-)\sim 3$, etc., in consistency with the other charm meson decays [26]. In these arguments, only the $\lambda=0$ matrix elements of H_w take part. We here present the results on the asymptotic matrix elements of H_w between two pseudoscalar (PS) meson states [10],

$$\langle\pi^+|H_w|K^+\rangle=-\sqrt{2}\langle\pi^0|H_w|K^0\rangle=(1+r_p^{(0)})H^{(\mathrm{PS})}, \tag{14}$$

$$\sqrt{2}\langle\eta_0|H_w|K^0\rangle=-(1-r_p^{(0)})H^{(\mathrm{PS})},$$
$$\langle\eta_s|H_w|K^0\rangle=r_P^{(0)}H^{(\mathrm{PS})}, \tag{15}$$

$$\langle g_0|H_w|K^0\rangle=r_p^{(0)}H_g^{(\mathrm{PS})}, \tag{16}$$

where η_0, η_s, and g_0 are $(u\bar{u}+d\bar{d})/\sqrt{2}$, $(s\bar{s})$ and glueball components with $J^{PC}=0^{-+}$, respectively. They mix to realize the physical η, η', and ι. In this paper, we consider the usual η-η' mixing with the mixing angle $\theta_P\simeq-20°$ [16] (ι is treated approximately as a glueball). $r_p^{(0)}$ denotes the unknown fractional contribution of the penguin (or the quark loop) diagrams relative to those of the c_-O_- in these asymptotic matrix elements taken between the helicity $\lambda=0$ states. $H^{(\mathrm{PS})}$ and $H_g^{(\mathrm{PS})}$ provide the normalizations of these *asymptotic* matrix elements taken between the PS meson states and between g_0 and K^0 meson states, respectively. The matrix elements of H_w including vector meson state(s), $\langle V|H_w|K\rangle$ and $\langle V|H_w|K^*\rangle_{\lambda=0,\pm1}$, also can be parametrized in the same way. The asymptotic matrix elements taken between helicity $\lambda=0$ states, $\langle P|H_w|K\rangle$ and $\langle V|H_w|K\rangle$, can be related to each other, for example, as

$$\langle\rho^0|H_w|K^0\rangle=\pm\langle\pi^0|H_w|K^0\rangle,\ \text{etc.}, \tag{17}$$

by using algebraic calculations [6,7]. We take the positive sign as in our previous papers [6–8,10,26].

The size of PS meson matrix elements of H_w will be estimated later by using our hard pion amplitude for the $K_S\to\pi^+\pi^-$ decay and the observed value of $\Gamma(K_S\to\pi^+\pi^-)$. However, we do not know how to estimate reliably the size of matrix elements of H_w taken between $\lambda=\pm1$ vector meson states. (We could use the factorization prescription to estimate it as Bergström *et al.* [4] did. However, we wonder if the prescription is reliable since H_w is not a simple product of currents but normal ordered operator and its matrix elements cannot easily be factorized.) Therefore, we here parametrize the relative size between these matrix elements with $\lambda=\pm1$ and $\lambda=0$ by

$$\langle\rho^0|H_w|K^{*0}\rangle_{\lambda=\pm1}=\alpha\langle\pi^0|H_w|K^0\rangle, \tag{18}$$

where α is a parameter introduced. The following helicity argument suggests $\alpha>1$ (contrary to the factorization by Bergström *et al.* [4]) and $r_p^{(0)}>|c_p/c_-|$. The operators $O_\pm$ in

Eq. (7) are of normal ordered products of left-handed (LH) currents and the penguin operator, O_p, includes right-handed (RH) components. Now we consider Lorentz invariant matrix elements of H_w. In the center of mass system of K or K^*, the helicity of a pair of light quark and antiquark produced through LH currents will be dominated by $\lambda = \pm 1$ at short distance. Therefore $\langle P|O_-|K\rangle$ might be reduced (the so-called helicity suppression) relatively to $\langle V|O_-|K^*\rangle_{\lambda=\pm 1}$ and $\langle P|O_p|K\rangle$ if short distance physics survives in these ground-state-meson matrix elements of H_w. Because of its small coefficient (and the possible helicity suppression), we here neglect the penguin contribution to $\langle V|H_w|K^*\rangle_{\lambda=\pm 1}$. Even though we take this approximation, $\langle \rho|H_w|K^*\rangle_{\lambda=\pm 1}$ still satisfy the same selection rule as $\langle \pi|H_w|K\rangle$.

To compare our result on the $K_L \to \gamma\gamma^*$ with experiments, we estimate values of parameters involved in the amplitude. We, first, estimate the size of $\langle \pi|H_w|K\rangle$. Neglecting small contributions of excited-state-meson poles and substituting the constraints on the asymptotic ground-state-meson matrix elements of H_w, Eqs. (14) and (17), into Eq. (1) with Eqs. (2) and (3), we obtain $A(K_S \to \pi^+\pi^-) \simeq (f_\pi)^{-1}\langle \pi^+|H_w|K^+\rangle \times 1.2e^{i\delta_0}$, where δ_0 ($\simeq 60°$) is the $I=0$ $\pi\pi$ phase shift at m_K, and therefore [10] $|\langle K_L|H_w|\pi^0\rangle| \simeq 1.9\times 10^{-7} m_K^2$ from the observed decay rate [16], $\Gamma(K_S \to \pi^+\pi^-)_{\rm expt} \simeq 0.77\times 10^{10}$ sec^{-1}.

The values of photon-vector meson couplings, X_ρ, X_ω, and X_ϕ, which are involved in the amplitude for the $K_L \to \gamma\gamma$ decay can be estimated from the observed cross sections for the photoproductions of ρ^0, ω, and ϕ mesons; $X_\rho = 0.033 \pm 0.003$ GeV^2, $X_\omega = 0.011 \pm 0.001$ GeV^2, and $X_\phi = -0.018 \pm 0.004$ GeV^2 on the photon mass shell [27]. The sizes of the amplitudes, $A(\pi^0 \to \gamma\gamma)$, $A(\eta \to \gamma\gamma)$, $A(\eta' \to \gamma\gamma)$, and $A(\iota \to \gamma\gamma)$, are estimated from the observed decay rates [16], $\Gamma(\pi^0 \to \gamma\gamma)_{\rm expt} = 7.7 \pm 0.6$ eV, $\Gamma(\eta \to \gamma\gamma)_{\rm expt} = 0.46 \pm 0.04$ keV, $\Gamma(\eta' \to \gamma\gamma)_{\rm expt} = 4.26 \pm 0.19$ keV, and $\Gamma(\iota \to \gamma\gamma)_{\rm expt} < 1.2$ keV. We determine the relative signs among these amplitudes by using the quark model. Then we obtain $A(\pi^0 \to \gamma\gamma) \simeq 3.5\times 10^{-5}$ (MeV)$^{-1}$,

$$A(\eta \to \gamma\gamma)/A(\pi^0 \to \gamma\gamma) \simeq 0.94,$$

$$A(\eta' \to \gamma\gamma)/A(\pi^0 \to \gamma\gamma) \simeq 1.24,$$

and

$$A(\iota \to \gamma\gamma)/A(\pi^0 \to \gamma\gamma) < 0.36.$$

Even though we use the upper limit as the value of the last ratio, our result on the $K_L \to \gamma\gamma$ is not very sensitive to it as will be seen later. The coupling $A(K^0 \to K^{*0}\rho^0)$ is estimated to be $A(K^0 \to K^{*0}\rho^0) \simeq -0.86$ (GeV)$^{-1}$ from the above value of $A(\pi^0 \to \gamma\gamma)$ by assuming the vector meson dominance and the asymptotic SU$_f$(3) symmetry [28] (or the nonet symmetry). The estimated value of $A(K^0 \to K^{*0}\rho^0)$ reproduces well $\Gamma(K^{*0} \to K^0\gamma)_{\rm expt}$. Because of the negative sign of $A(K^0 \to K^{*0}\rho^0)$ opposite to $A(\pi^0 \to \omega\rho^0)$, the K^* pole amplitude, $A_{K^*}(K_L \to \gamma\gamma^*)$, interferes destructively with the PS meson pole amplitude, $A_P(K_L \to \gamma\gamma^*)$.

Substituting the parametrization of asymptotic ground-state-meson matrix elements of H_w, Eqs. (14)–(16), and the similar ones for the asymptotic matrix elements, $\langle \rho^0(\omega,\phi)|H_w|K^0\rangle$, into Eq. (5), we can express $(\Delta m_K)_{\rm pole}/\Gamma_{K_S}$ as a function of $r_p^{(0)}$ and $r_g = H_g^{(\rm PS)}/H^{(\rm PS)}$, where Γ_{K_S} is the total decay rate of K_S. We restrict the value of r_g to be $|r_g| < 1$ since the overlapping of the wave functions between the glueball and K meson is expected to be smaller than that between two ordinary PS mesons. Then the contribution of the glueball (ι) will be much smaller than those of π^0, η, and η' because of the high mass of ι.

By using the constraints on the asymptotic matrix elements of H_w, Eqs. (14)–(16), and similar ones including vector meson states, the parametrization, Eq. (18), and the values of $|\langle \pi^0|H_w|K^0\rangle|$, X_V's and $A(P \to \gamma\gamma)$'s estimated above, the amplitude, $A(K_L \to \gamma\gamma)$, and the form factor, $f(x)$, for the $K_L \to \gamma\gamma^*$ can be given as functions of $r_p^{(0)}$, r_g, and α. Before our result on $\Gamma(K_L \to \gamma\gamma)$ is compared with the observation, the K_L-K_S mass difference, Δm_K, is fitted to the measured value. In the present perspective, $\Delta m_K/\Gamma_{K_S}$ is approximately given by a sum of $(\Delta m_K)_{\rm pole}/\Gamma_{K_S}$ and $(\Delta m_K)_{\pi\pi}/\Gamma_{K_S}$ as discussed earlier, where the former has been given as a function of $r_p^{(0)}$ and r_g before and the latter has been constrained as $(\Delta m_K)_{\pi\pi}/\Gamma_{K_S} = 0.22 \pm 0.03$ in Ref. [13]. Our result is not very sensitive to r_g as expected. Therefore we neglect the ι contribution to Δm_K. Then, for $r_p^{(0)} \simeq 0.31$, our $\Delta m_K/\Gamma_{K_S}$ reproduces well the observed value [16], $(\Delta m_K/\Gamma_{K_S})_{\rm expt} = 0.476 \pm 0.002$. Next, we compare our result on $\Gamma(K_L \to \gamma\gamma)$ with the experimental data cited before. Our result is again not very sensitive to r_g as long as $|r_g| < 1$ as discussed before. It implies that the glueball contribution to this decay is not very important in contrast with Ref. [17]. Therefore we neglect the glueball contribution. Then, for two sets of values of $r_p^{(0)}$ and α, i.e., (i) for $0.30 < r_p^{(0)} < 0.32$ and $3.4 < \alpha < 3.5$, and (ii) for $0.29 < r_p^{(0)} < 0.32$ and $1.3 < \alpha < 1.4$, we can reproduce $(\Delta m_K/\Gamma_{K_S})_{\rm expt}$ and $\Gamma(K_L \to \gamma\gamma)_{\rm expt}$, simultaneously. The allowed values of $r_p^{(0)}$ around 0.3 are much larger than that of $|c_p/c_-|$ expected in the perturbation theory [22] and imply that the penguin contribution is important, although still not dominant, in the asymptotic ground-state-meson matrix elements of H_w. The allowed values of α larger than unity seems to be a remnant of the short distance physics as suggested before. The above (i) and (ii) lead to $|A_{K^*}(K_L \to \gamma\gamma)| > |A_P(K_L \to \gamma\gamma)|$ and $|A_{K^*}(K_L \to \gamma\gamma)| < |A_P(K_L \to \gamma\gamma)|$, respectively. If the K^* pole contribution to $K_L \to \gamma\gamma$ were neglected, its observed rate could not be reproduced for reasonable values of $r_p^{(0)}$ and r_g. The calculated values of the form factor (i) for $r_p^{(0)} = 0.31$ and $\alpha = 3.42$ and (ii) for $r_p^{(0)} = 0.31$ and $\alpha = 1.34$ are compared with the experimental data in Fig. 1. The data from the $K_L \to \gamma e^+e^-$ at BNL [2] and at CERN [3] are consistent with each other. However, they are not compatible with the new data from the $K_L \to \gamma\mu^+\mu^-$ at Fermilab [1] in lower x region. The existing theoretical analyses [4] in which the K^*-meson pole contribution is canceled on the photon mass shell favor the former data. Therefore their predictions on the ratio, $R_{\gamma e^+e^-} = \Gamma(K_L \to \gamma e^+e^-)/\Gamma(K_L \to \gamma\gamma)$, are compatible with the experiments [2,3], $(R_{e^+e^-\gamma})_{\rm expt} \simeq 1.6\times 10^{-2}$, while their values of the ratio, $R_{\gamma\mu^+\mu^-}$

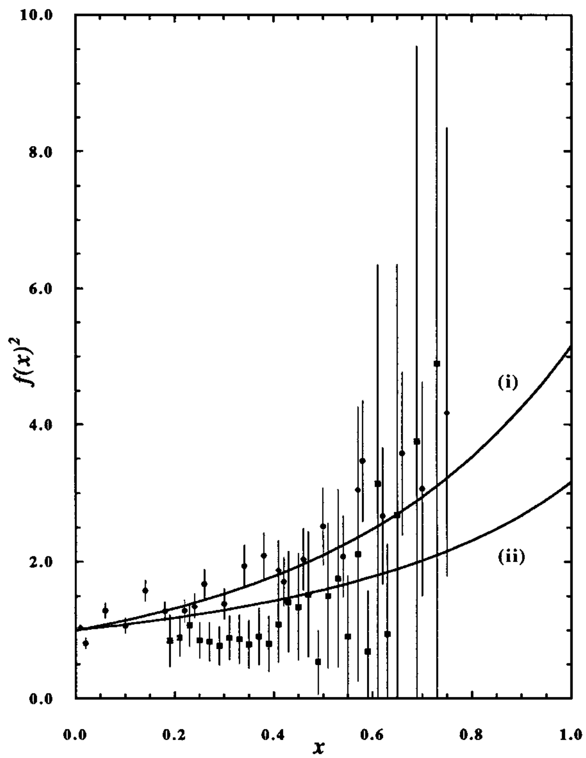

FIG. 1. The square of the form factor $f(x)^2$ for $K_L \to \gamma\gamma^*$ with $x = k^2/m_K^2$. The solid curves are the calculated $f(x)^2$ with $\Lambda_P = 0.72$ GeV, (i) for $r_p^{(0)} = 0.31$, $r_g = 0$, and $\alpha = 3.42$ and (ii) for $r_p^{(0)} = 0.31$, $r_g = 0$, and $\alpha = 1.42$, respectively. The data points (the solid circles, the diamonds, and the squares) are taken from Refs. [2,3,1], respectively.

$= \Gamma(K_L \to \gamma\mu^+\mu^-)/\Gamma(K_L \to \gamma\gamma)$, are much larger than the recent measurement [1], $(R_{\mu^+\mu^-\gamma})_{\rm expt} = (5.66 \pm 0.59) \times 10^{-4}$. (For example, the prediction by Ko [4] is higher by about 3σ than the observation [1].) However, our results on the form factor are between the data from the $K_L \to \gamma e^+e^-$ and the $K_L \to \gamma\mu^+\mu^-$ in the $0.2 < x < 0.4$ region and consistent with the data at higher x within the large errors. Our calculated value of the ratio, $R_{\mu^+\mu^-\gamma} \simeq 5.5 \times 10^{-4}$, in case (ii) reproduces well the measured one while the same ratio in case (i) ($\simeq 6.7 \times 10^{-4}$) may be a little too large. Our values, $R_{\gamma e^+e^-} \simeq 5.1 \times 10^{-3}$ in case (i) and $\simeq 4.9 \times 10^{-3}$ in case (ii), are about $\frac{1}{3}$ of the measured one since our results on the form factor are less than the measurements at lower x.

In summary, we have investigated the Dalitz decays of K_L from an entirely new perspective in which the main term, $c_- O_-$, of H_w does produce the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule for the *asymptotic* ground-state-meson matrix elements of H_w and consequently the amplitudes for the $K \to \pi\pi$ decays satisfy the approximate $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. From the same perspective, the K_L-K_S mass difference and the rate for the $K_L \to \gamma\gamma$ decay in which the K^*-meson pole is included in contrast with the other theoretical analyses [4] have been calculated. Their observed values have been reproduced simultaneously by two sets of values of included parameters which imply that the penguin contribution is important, although still not dominant, in asymptotic ground-state-meson matrix elements of H_w and that the helicity ± 1 matrix elements are larger than the corresponding ones with the helicity zero. Therefore, in nonleptonic weak interactions of K mesons, long distance hadron physics is important although short distance physics still survives in the asymptotic matrix elements of H_w taken between ground-state-meson states. The same values of the parameters provide two different results on the form factor for the Dalitz decay. The experimental data from the $K_L \to \gamma e^+e^-$ [2,3] and from the $K_L \to \gamma\mu^+\mu^-$ [1] are still not compatible with each other in the lower x region although they are consistent with each other within the large errors at higher x. The existing theoretical analyses [4] in which the K^* meson pole contribution is canceled on the photon mass shell favor the former data. However, our results on the form factor are between two different data in the lower x region while they are consistent with the data within large errors at higher x. Consequently, our results on the ratio, $\Gamma(K_L \to \gamma e^+e^-)/\Gamma(K_L \to \gamma\gamma)$, are $\sim \frac{1}{3}$ of the observed one while one of our solutions reproduces well the observed value of $\Gamma(K_L \to \gamma\mu^+\mu^-)/\Gamma(K_L \to \gamma\gamma)$. To determine the form factor for the Dalitz decay of K_L, more measurements and theoretical investigations will be needed.

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