Long-distance contribution to $s \to d\gamma$ and implications for $\Omega^- \to \Xi^- \gamma$, $B_s \to B_d^* \gamma$, and $b \to s\gamma$

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We estimate the long-distance (LD) contribution to the magnetic part of the $s \to d\gamma$ transition using the vector meson dominance approximation ($V = \rho, \omega, \psi_i$). We find that this contribution may be significantly larger than the short-distance (SD) contribution to $s \to d\gamma$ and could possibly saturate the present experimental upper bound on the $\Omega^- \to \Xi^- \gamma$ decay rate, $\Gamma^{\max}_{\Omega^- \to \Xi^- \gamma} \simeq 3.7 \times 10^{-9}$ eV. For the decay $B_s \to B_d^* \gamma$, which is driven by $s \to d\gamma$ as well, we obtain an upper bound on the branching ratio $B(B_s \to B_d^* \gamma) < 3 \times 10^{-8}$ from $\Gamma^{\max}_{\Omega^- \to \Xi^- \gamma}$. Barring the possibility that the quantum chromodynamics coefficient $a_2(m_s^2)$ is much smaller than 1, $\Gamma^{\max}_{\Omega^- \to \Xi^- \gamma}$ also implies the approximate relation $\frac{2}{3}\Sigma_i g^2_{\psi_i}(0)/m^2_{\psi_i} \simeq \frac{1}{2} g^2_\rho(0)/m^2_\rho + \frac{1}{6} g^2_\omega(0)/m^2_\omega$. This relation agrees quantitatively with a recent independent estimate of the left-hand side by Deshpande, He, and Trampetic, confirming that the LD contributions to $b \to s\gamma$ are small. We find that these amount to an increase of $(4 \pm 2)\%$ in the magnitude of the $b \to s\gamma$ transition amplitude, relative to the SD contribution alone.

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I. INTRODUCTION AND OVERVIEW

The investigation of the quark radiative transition $b \to s\gamma$ has been an important focus of attention in recent years [1] both because of experimental measurements [2] and because long-distance (LD) corrections to the standard model (SM) predictions for the short-distance (SD) contributions are estimated to be small [3]. (For exclusive $B \to K^* \gamma$ decays see Ref. [4].) Thus, this transition constitutes an excellent laboratory to test the SM or possible high energy deviations thereof [5]. It has been pointed out recently [6] that for the $c \to u\gamma$ transition the situation is reversed, with the LD contributions dominating over the SD ones by many orders of magnitude.

In this paper we investigate the analogous quark transition $s \to d\gamma$ and two exclusive hadronic processes, $\Omega^- \to \Xi^- \gamma$ and $B_s \to B_d^* \gamma$, where it plays an important role. Throughout this paper we are concerned with the magnetic transition only, since the charge-radius one vanishes for real photons.

The SD contribution to $s \to d\gamma$ has been investigated before (see, e.g., [7–9]) and we simply repeat the calculations, using updated values for the relevant QCD coefficients. Applying the quark model formalism of Ref. [10] we find that the SD $s \to d\gamma$ contribution (by itself) to the $\Omega^- \to \Xi^- \gamma$ decay rate is far below (by a factor of order 600) the present experimental upper limit [11]:

$$\Gamma(\Omega^- \to \Xi^- \gamma) < 3.7 \times 10^{-9} \text{ eV (90\% C.L.)}. \tag{1}$$

Hadronic LD effects that involve light mesons in loops are estimated to be small [8,12], comparable to the SD contributions.

On the other hand, by using a vector meson dominance (VMD) approximation for the LD contribution to the $s \to d\gamma$ transition (along the lines discussed by Deshpande, He, and Trampetic [3] for $b \to s\gamma$), we find LD contributions that are likely to be significantly larger than the SD ones. In fact, the rate for $\Omega^- \to \Xi^- \gamma$ may not be far from the experimental bound (1), due to this VMD contribution. The resulting VMD amplitude is approximately proportional to

$$a_2(m_s^2)\left[\frac{2}{3}\sum_i \frac{g^2_{\psi_i}(0)}{m^2_{\psi_i}} - \frac{1}{2}\frac{g^2_\rho(0)}{m^2_\rho} - \frac{1}{6}\frac{g^2_\omega(0)}{m^2_\omega}\right], \tag{2}$$

where $a_2(m_s^2)$ is a quantum chromodynamics (QCD) coefficient [13] and the $g_V(0)$'s are the usual vector meson-photon couplings, evaluated at $q^2 = 0$. Although a direct estimate of $a_2(m_s^2)$ is not reliable because we are well into the low energy region where perturbation theory cannot be trusted, we may use the phenomenologically determined value $|a_2(m_c^2)| = 0.55 \pm 0.1$ [13] as a hint to assume $|a_2(m_s^2)| \gtrsim 0.5$. We then apply the formalism of Ref. [10] to get an expression for the $\Omega^- \to \Xi^- \gamma$ decay rate from our SD+VMD $s \to d\gamma$ amplitude. (Notice that there are no pole contributions to this decay.) It turns out that if the above rough estimate $|a_2(m_s^2)| \gtrsim 0.5$ is correct then the experimental limit (1) can be satisfied only if the contribution of the ψ_i resonances in the parentheses of Eq. (2) cancels the ρ and ω meson contributions (which can be reliably obtained from the ρ and ω leptonic widths [14]), at a level of 30% or better accuracy. The limit (1) then forces the approximate relation at $q^2 = 0$:

$$\frac{2}{3}\sum_i \frac{g^2_{\psi_i}(0)}{m^2_{\psi_i}} \simeq \frac{1}{2}\frac{g^2_\rho(0)}{m^2_\rho} + \frac{1}{6}\frac{g^2_\omega(0)}{m^2_\omega} \tag{3}$$

which is highly nontrivial, and may be interpreted as a remnant of the badly broken $SU(4)_F$ symmetry.

The relation (3) turns out to be very useful for the $b \to s\gamma$ decays. As noted in Ref. [3], in the VMD approximation the main LD contributions to this decay can be expressed in terms of the left-hand side (LHS) of Eq. (3). In

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Ref. [3], the sum on the left-hand side of Eq. (3) is estimated by using measured leptonic widths of the ψ_i states and ψ photoproduction data as well as an assumption about the higher ψ excitations. We estimate this sum with better accuracy by replacing experimental values for $g_\rho(0)$ and $g_\omega(0)$ on the RHS of Eq. (3) and find very good quantitative agreement with the central value obtained in Ref. [3]. We thus confirm the main result of Ref. [3] that LD contributions to $b \to s\gamma$ are of order of a few percent. According to our explicit estimate, these corrections amount to an increase of $(4 \pm 2)\%$ in the magnitude of the $b \to s\gamma$ transition amplitude, relative to the SD contribution alone.

Finally, we also apply the SD + VMD approximation for $s \to d\gamma$ to the unusual decay mode $B_s \to B_d^* \gamma$, where the b quark plays the ''spectator'' role. We point out that this decay (followed by $B_d^* \to B_d \gamma$) has a clear experimental signature of two monochromatic photons of energies $\simeq 50$ MeV each. We find, using the limit of Eq. (1), a small but hopefully measurable branching ratio $B(B_s \to B_d^* \gamma) < 3 \times 10^{-8}$.

II. SD CONTRIBUTION TO THE $s \to d\gamma$ AMPLITUDE

The SD amplitude relevant to the $s \to d\gamma$ transition can be expressed as

$$A_{\mathrm{SD}} = -\frac{e}{8\pi^2}\frac{G_F}{\sqrt{2}} F_2(\mu^2)\bar{d}\sigma^{\mu\nu}[m_s R + m_d L] s F_{\mu\nu}, \quad (4)$$

where m_s, m_d are current quark masses and $F_2(\mu^2)$ is a form factor evaluated at a low scale $\mu \geqslant 0(m_s)$ which includes (dominant) QCD corrections. Early estimates of $F_2(\mu^2)$ [7,8,12] were in the approximate range $0.15-0.36$ [15] while we obtain by explicit calculation, using $\alpha_s(m_c^2) \simeq 0.3$, $\alpha_s(\mu^2) = 0.9$ in the formulas given in Ref. [16], a somewhat smaller value $F_2(\mu^2) \simeq 0.1$, which will be used below (see also Ref. [9]).

III. LD CONTRIBUTION TO $s \to d\gamma$

To estimate the LD contribution to $s \to d\gamma$ we use the VMD approximation in analogy to the formalism used in Ref. [3] for $b \to s\gamma$. As an intermediate step one defines a transverse amplitude $A[s \to dV(q)]_T$ ($V = \psi_i, \rho, \omega$ in this case) and then introduces the V to γ conversion vertices, setting $q^2 = 0$. Using Gordon decomposition we find that the LD amplitude for the $s \to d\gamma$ transition is

$$A_{\mathrm{LD}} = -e\frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^* a_2(\mu^2)\left(\frac{2}{3}\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} - \frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2} - \frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2}\right)\frac{1}{M_s^2 - M_d^2}\bar{d}\sigma^{\mu\nu}[M_s R - M_d L] s F_{\mu\nu}, \quad (5)$$

where we have used $V_{cs}V_{cd}^* \simeq -V_{us}V_{ud}^*$, $a_2(\mu^2)$ is a QCD coefficient the value of which is taken from phenomenology in the context of the factorization approximation [13], and the $g_V(q^2)$ factors are defined in the usual way, e.g., $\langle \psi(q) | \bar{c}\gamma_\mu c | 0 \rangle = i g_\psi(q^2)\epsilon_\mu^+(q)$. We have not included possible contributions from the ρ and ω radial excitations ($\rho', \rho'', \ldots, \omega', \omega'', \ldots$) because we think that their contribution is much smaller and is already taken into account to a significant degree in the SD amplitude (4). The ψ excitations should be included however, because they are narrow resonances that are clearly distinguished from the $c\bar{c}$ continuum. Note that due to the hadronic nature of the VMD approximation, M_s and M_d should correspond to ''constituent'' mass parameters. [The use of ''constituent quark'' spinors in deriving (5) should take into account to some extent nonperturbative effects such as chiral symmetry breaking and confinement.] In any case, it turns out that only the combination $(M_s^2 + M_d^2)/(M_s^2 - M_d^2)^2$ which has a similar magnitude for ''constituent'' or ''current'' s,d quark masses, appears in our applications (see Secs. IV and V) when the interference between the (presumably) dominant LD contribution and the SD contribution is neglected.

It is difficult to estimate the coefficient $a_2(\mu^2)$ for $\mu \geqslant O(m_s)$ appearing in Eq. (5). However, relying on the phenomenologically obtained value $|a_2(m_c^2)| = 0.55 \pm 0.1$ [13] we take $|a_2(m_s^2)| \gtrsim 0.5$.

The couplings $g_{\psi_i}(m_{\psi_i}^2)$, $g_\rho(m_\rho^2)$, $g_\omega(m_\rho^2)$ are readily obtained from leptonic decays of these mesons, but their extrapolated values at $q^2 = 0$ are less trivial, especially for the ψ_i states. Photoproduction data seems to indicate that $g_\rho^2(0) \simeq g^2(m_\rho^2)$, $g_\omega^2(0) \simeq g^2(m_\omega^2)$ [17,18]. On the other hand, estimates in Ref. [3] using ψ photoproduction data [18–20] give $g_\psi^2(0) = (0.12 \pm 0.04) g_\psi^2(m_\psi^2)$. In Ref. [3] it is also assumed that the same ratio holds for the excitations ψ', ψ'', etc.

Making use of the above estimates as well as of the leptonic widths of the relevant vector mesons [14] we obtain the numerical values $g_\rho^2(0)/m_\rho^2 \simeq 0.047$ GeV^2, $g_\omega^2(0)/m_\omega^2 \simeq 0.038$ GeV^2, and $\Sigma_i[g_{\psi_i}^2(0)/m_{\psi_i}^2] \simeq 0.041$ GeV^2. The first two estimates should be accurate to about 10% while the latter must be considered only as a rough estimate, with an uncertainty of at least 40%. Once we derive the approximate relation (3) we will be able to give a far more reliable estimate of $\Sigma_i[g_{\psi_i}^2(0)/m_{\psi_i}^2]$, which is consistent with the above central value.

IV. APPLICATION TO THE DECAY $\Omega^- \to \Xi^- \gamma$ AND ITS CONSEQUENCES

The process $\Omega^- \to \Xi^- \gamma$ has a special place [8,15] among the hyperon radiative decays [21], since the quark composition of the participating hadrons precludes W exchange among pairs of valence quarks to induce this decay. A similar situation occurs in $\Xi^- \to \Sigma^- \gamma$ decay. Accordingly, these decays have been singled out as possible windows for the detection of the short-distance electroweak [10,12,15] or strong penguins [22]. Using the present knowledge on the QCD corrections to the effective nonleptonic Hamiltonian as discussed in Sec. II, the electroweak penguin contribution to the rate turns out to be lower by more than 2 orders of magnitude [see Eq. (8) below] than the present experimental upper limit on Ω^- radiative decay [Eq. (1)]. A similar result is given by the calculation of gluonic penguins [22].

The structure of the nonleptonic $\Delta s=1$ Hamiltonian does not allow for pole contributions in the $\Omega^-\to\Xi^-\gamma$ decays. Kogan and Shifman [8] have calculated the two-particle intermediate "s-channel" contributions to this decay of which $\Xi^0\pi^-$ is the largest. From the imaginary part they found the unitarity limit $B(\Omega^-\to\Xi^-\gamma)\geqslant 0.8\times 10^{-5}$ and the inclusion of the real part increases this figure by a factor of 1.5 only. Thus, the "s-channel" contributions are lower than Eq. (1) by a factor of about 40.

On the other hand, a VMD approach [23] to the hyperon radiative decays on the hadronic level, which uses $\mathrm{SU}(6)_W$ symmetry to determine the parity-violating couplings of vector mesons to baryons from the nonleptonic hyperon decays, finds a branching ratio for $\Omega^-\to\Xi^-\gamma$ which is already at the limit of Eq. (1) or even slightly higher. Actually, had we neglected the contributions from ψ_i our result due to light vector mesons only would lead to a rate which is also larger than the experimental limit.

In view of the above-mentioned results we turn now to the calculation of $\Omega^-\to\Xi^-\gamma$ by using as basic assumption the dominance of the "t-channel" $s\to dV$ transition for the long-distance radiative process. We shall use the quark model of Ref. [10] to estimate the rate for the decay $\Omega^-\to\Xi^-\gamma$, from the SD and LD contributions to the $s\to d\gamma$ quark decay amplitude obtained in previous sections.

For notational convenience, we define the constants $v\equiv|V_{cs}V_{cd}^*|\simeq 0.22$ and

$$C_{\mathrm{VMD}}\equiv\left(\frac{2}{3}\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2}-\frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2}-\frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2}\right).$$

The relative sign of the SD and LD contributions is determined by the theory [3] so that the full amplitude for the $s\to d\gamma$ transition can be written as

$$\begin{aligned}A_{\mathrm{tot}}(s\to d\gamma)&=A_{\mathrm{SD}}+A_{\mathrm{LD}}\\&=-\frac{eG_F}{\sqrt{2}}\bar{d}\sigma^{\mu\nu}\left[\left(\frac{m_sF_2}{8\pi^2}+\frac{va_2C_{\mathrm{VMD}}M_s}{M_s^2-M_d^2}\right)R\right.\\&\quad\left.+\left(\frac{m_dF_2}{8\pi^2}-\frac{va_2C_{\mathrm{VMD}}M_d}{M_s^2-M_d^2}\right)L\right]sF_{\mu\nu}.\end{aligned}\tag{6}$$

Following Ref. [10] we then obtain

$$\begin{aligned}\Gamma(\Omega^-\to\Xi^-\gamma)&=\frac{\alpha G_F^2}{12\pi^4}\left(\frac{m_{\Xi^-}}{m_{\Omega^-}}\right)|\vec{q}|^3\\&\quad\times\left[\left(m_sF_2+\frac{8\pi^2va_2C_{\mathrm{VMD}}M_s}{M_s^2-M_d^2}\right)^2\right.\\&\quad\left.+\left(m_dF_2-\frac{8\pi^2va_2C_{\mathrm{VMD}}M_d}{M_s^2-M_d^2}\right)^2\right],\end{aligned}\tag{7}$$

where $\vec{q}$ is the photon momentum in the Ω^- rest frame and the separate SD and LD contributions are exhibited explicitly.

In the absence of LD (VMD) contributions, we would obtain (for $m_s\simeq 175$ MeV, $m_d\simeq 10$ MeV, $F_2\simeq 0.1$, see Sec. II)

$$\Gamma_{\mathrm{SD}}(\Omega^-\to\Xi^-\gamma)\simeq 6.4\times 10^{-12}\ \ \mathrm{eV}\tag{8}$$

which is far below the present experimental bound of $\Gamma_{\mathrm{expt}}(\Omega^-\to\Xi^-\gamma)<3.7\times 10^{-9}$ eV. On the other hand, the large theoretical uncertainty of over 40% in the value of the sum $\Sigma_i[g_{\psi_i}^2(0)/m_{\psi_i}^2]$ (see Sec. III) which appears in C_{VMD}, would allow the LD contribution to saturate this experimental bound. In fact, the experimental limit can be used to constrain C_{VMD} and hence $\Sigma_i[g_{\psi_i}^2(0)/m_{\psi_i}^2]$. Using typical values $M_s\simeq 0.5$ GeV, $M_d\simeq 0.35$ GeV for the constituent quark masses and $|a_2|\gtrsim 0.5$ (see Sec. III), we find

$$\begin{aligned}|C_{\mathrm{VMD}}|&=\left|\frac{2}{3}\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2}-\frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2}-\frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2}\right|\\&<0.01\ \ \mathrm{GeV}^2.\end{aligned}\tag{9}$$

This constraint would be only slightly different, had we used current quark mass parameters instead of M_s and M_d.

The bound in Eq. (9) represents a remarkable cancellation at the 30% level, considering that

$$\frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2}+\frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2}\simeq 0.030\ \ \mathrm{GeV}^2$$

(see Sec. III). We presume that this effect may stem from the combination of the Glashow-Iliopoulos-Maiani (GIM) [24] mechanism and the underlying $\mathrm{SU}(4)_F$ symmetry, which if exact would give a full cancellation (after inclusion of $\rho',\rho'',\ldots,\ \omega',\omega''\ldots$ states). The $\mathrm{SU}(4)_F$ symmetry is known to be badly broken by the large mass of the c quark. However, here we are comparing the form factors $g_{\psi_i}^2(q^2)$, $g_\rho^2(q^2)$, $g_\omega^2(q^2)$ at a common scale $q^2=0$, which seems to "restore" this symmetry to some extent. We have noticed that if $|g_\phi(0)|\simeq|g_\phi(m_\phi^2)|$ [17,18], leading through ϕ leptonic width data [14] to $|g_\phi(0)|\simeq 0.24\ \mathrm{GeV}^2$, a completely analogous near cancellation occurs for the quantity

$$C'_{\mathrm{VMD}}\equiv-\frac{1}{3}\frac{g_\phi^2(0)}{m_\phi^2}+\frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2}-\frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2},$$

which is relevant to LD effects in $c\to u\gamma$ decay [6]. We obtain $C'_{\mathrm{VMD}}\simeq -1.4\times 10^{-3}\ \mathrm{GeV}^2$, which represents a cancellation at a level better than 10% for which presumably the $\mathrm{SU}(3)_F$ symmetry is responsible.

We note that the upper bound (9) on $|C_{\mathrm{VMD}}|$ tells us that although the LD effects are likely to dominate $s\to d\gamma$, they can be at most a factor of about 25 larger than the SD contribution in the amplitude. This represents an intermediate situation between the $b\to s\gamma$ decays where the SD contribution clearly dominates [3,4,25] and the $c\to u\gamma$ decays where the SD effects are completely negligible relative to the LD ones [6].

Since the process $\Xi^-\to\Sigma^-\gamma$ has underlying physics similar to $\Omega^-\to\Xi^-\gamma$, we must consider the effect of $s\to dV\to d\gamma$ on it as well. Using C_{VMD} of Eq. (9), we find that the contribution to the branching ratio is $B(\Xi^-\to\Sigma^-\gamma)_{\mathrm{LD}}<1.1\times 10^{-4}$, while the measured rate for it is $B(\Xi^-\to\Sigma^-\gamma)=(1.27\pm 0.23)\times 10^{-4}$ [14]. Hence, since we expect C_{VMD} to be somewhat lower that its upper limit, the contribution of $s \to dV$ to this decay can be accommodated with the two-particle ''s-channel'' contributions which were shown [8, 26] to account for a rate at the 10^{-4} level.

We also wish to comment on the well-known result of Gilman and Wise [10], that the $s \to d\gamma$ transition cannot be the dominant one for all hyperon decays. In fact, they proved [10] that if $\Sigma^+ \to p\gamma$ is driven by $s \to d\gamma$, other radiative decays are predicted to be much larger than the observed rates. Using again $C_{\rm VMD}$ of Eq.(9) we calculate the contribution of $s \to d\gamma$ to the well-measured $\Sigma^+ \to p\gamma$ decay and we find that it accounts for at most 1.1% of the observed branching ratio [14] of $(1.25 \pm 0.07) \times 10^{-3}$.

The above considerations on $\Xi^- \to \Sigma^- \gamma$ and on the typical pole decay $\Sigma^+ \to p\gamma$ demonstrate the consistency of our use of the long-distance $s \to d\gamma$ transition as the expected dominant contribution in $\Omega^- \to \Xi^- \gamma$.

V. IMPLICATIONS FOR THE LD CONTRIBUTION TO $b \to s\gamma$

Because

$$\left(\frac{1}{2}\frac{g_\rho^2(0)}{m_\rho^2} + \frac{1}{6}\frac{g_\omega^2(0)}{m_\omega^2}\right) \simeq 0.030 \ \ \mathrm{GeV}^2,$$

Eq. (9) implies that the approximate relation given in Eq. (3) must hold to an accuracy of order 30%. This then independently determines

$$\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} = 0.045 \pm 0.016 \ \ \mathrm{GeV}^2, \tag{10}$$

where our uncertainty in the values of $g_\rho(0)$ and $g_\omega(0)$ has been folded in. Notice that this result is in very good agreement with the central value ($\simeq 0.041$) estimated from ψ photoproduction data in Ref. [3], but the uncertainties there were larger (above 40%). Our results thus confirm previous assertions that the LD corrections are at the few percent level only [3,4] and further show that these contributions are well under control. The amplitude for $b \to s\gamma$ including SD and LD contributions can be expressed as [3]

$$A_{\rm tot}(b \to s\gamma) = -\frac{eG_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[\frac{1}{4\pi^2} m_b C_7^{\rm eff}(m_b) - a_2(m_b^2)\frac{2}{3m_b}\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2}\right] \bar{s}\sigma^{\mu\nu} RbF_{\mu\nu}, \tag{11}$$

where m_s, M_s have been neglected compared to m_b. Using $a_2(m_b^2) \simeq 0.24 \pm 0.04$ [3], $C_7^{\rm eff}(m_b) = -0.30 \pm 0.03$ [16], and $m_b = 4.8 \pm 0.2$ GeV, we find that the LD contribution increases the magnitude of the amplitude by $(4 \pm 2)\%$.

VI. APPLICATION TO $B_s \to B_d^* \gamma$

Another process where the $s \to d\gamma$ quark transition will dominate is $B_s \to B_d^* \gamma$. There are no pole contributions and we explicitly estimated the LD contribution from light meson loops to be smaller but comparable with the SD $s \to d\gamma$ contributions. This is an unusual B_s meson decay in the sense that it represents the decay of the *light* quark in a $\bar{Q}q$ system. Also, it has a clear signature: two photons with energies of about 50 MeV and 46 MeV (the second one coming from the decay $B_d^* \to B_d \gamma$), followed by a usual B_d decay.

We roughly estimate the $B_s \to B_d^* \gamma$ decay rate from our $s \to d\gamma$ amplitude (6) by assuming that the spatial wave functions of the s quark in the B_s meson and the d quark in the B_d^* meson are similar, and noting that the photon energy ($=50$ MeV) is small compared to the average momentum (~ 700 MeV) of the light quark in the bound state. A ''free quark'' approximation should then give a reasonable estimate of the transition amplitude. In terms of the amplitude $s \to d\gamma$ Eq. (6) we obtain, for the decay rate,

$$\Gamma(B_s \to B_d^* \gamma) = \frac{\alpha}{16\pi^4} G_F^2 |\vec{q}|^3 \left[\left(m_s F_2 + \frac{8\pi^2 v a_2 C_{\rm VMD} M_s}{M_s^2 - M_d^2}\right)^2 + \left(m_d F_2 - \frac{8\pi^2 v a_2 C_{\rm VMD} M_d}{M_s^2 - M_d^2}\right)^2\right] \tag{12}$$

where $\vec{q}$ is the photon momentum in the B_s rest frame. Comparing to Eq. (7) and using the upper bound (1) we obtain

$$\Gamma(B_s \to B_d^* \gamma) < 1.4 \times 10^{-20} \ \ \mathrm{GeV}. \tag{13}$$

Then, the present central value for the B_s lifetime $\tau_{B_s} \simeq 1.34 \times 10^{-12}$ s [14] gives a bound on the branching ratio, $B(B_s \to B_d^* \gamma) < 3 \times 10^{-8}$. Although this is a very rare decay mode, its unique signature and the large number of B_s mesons expected at B meson factories and at the CERN Large Hadron Collider (LHC)-$B \sim 2 \times 10^{11}$ [27] make it interesting.

VII. CONCLUSIONS

Using a VMD approximation, we found that the LD contribution to the $s \to d\gamma$ transition may be significantly larger than the SD one, and could even lead to a saturation of the present experimental upper limit on the decay rate for $\Omega^- \to \Xi^- \gamma$ [Eq. (1)]. This result throws new light on this decay mode. A further tightening of this upper limit or a measurement of the $\Omega^- \to \Xi^- \gamma$ rate would provide us with very useful information about the relative importance of the LD and SD contributions to $s \to d\gamma$. The present upper bound already implies a nontrivial cancellation at a level of 30% or better in the LD contribution. The resulting approximate relation [Eq. (3)] allowed us to estimate the relative importance of the LD contribution to the $b \to s\gamma$ transition amplitude. Our estimate of $(4 \pm 2)\%$ for this relative LD contribution, which is calculated by assuming vector meson dominance for it, agrees with earlier ones, which had larger uncertainties. Because the unusual process $B_s \to B_d^* \gamma$ is also dominated by an $s \to d\gamma$ transition, its decay rate is related to that of $\Omega^- \to \Xi^- \gamma$. We find that a present limit on the latter [Eq. (1)] implies an upper bound for the branching ratio $B(B_s \to B_d^* \gamma) < 3 \times 10^{-8}$, which is small but hopefully accessible in future experiments.

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