Neutrino mass and oscillation angle phenomena within the asymmetric left-right models

O. Boyarkin*
International Centre for Theoretical Physics, Trieste 34100, Italy

D. Rein
III Physikalisches Institut, Rheinisch-Westfälische Technische Hochschule, Aachen, Germany
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The light and heavy Majorana neutrinos which appear naturally in the $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ model are investigated. The analysis of the electron neutrino flux propagating through magnetic and matter fields is presented. The cross section of the reaction $e^-e^- \longrightarrow W_k^- W_n^-$ is calculated and its dependence on the mass of the right-handed neutrino and the oscillation angle is investigated. The process $e^+e^- \longrightarrow W_k^+ W_n^-$ is also included in our analysis.

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I. INTRODUCTION

A precise knowledge of the intrinsic properties of the neutrino would have a large impact on our understanding of both astrophysics and particle physics. At present the experimental values of neutrino masses, lifetimes, decay modes, the oscillation angles, and multipole moments (MM) are still inconclusive. In addition the question remains open as to whether the neutrino is a Majorana or Dirac particle. The standard model (SM) predicts three massless neutrinos while in almost every extension of the SM there are more than three massive neutrinos. In this paper we will draw attention to the possible manifestations of the left-right (LR) symmetry from the point of view of the experiments with cosmic neutrinos and the collider experiments going through neutrino exchange. We shall compare the results of the standard model (SM) with those obtained within the asymmetric LR model (ALRM), which was first discussed in Ref. [1]. Our results will be represented for the ALRM proposed in Ref. [2]. This model has one more additional parameter φ called the LR symmetry violating angle. Varying φ one may obtain all the possible versions of the LR models. The SM is reproduced in the SM particles sector at $\varphi=0$ and when the following conditions are fulfilled:

$$\Phi=\xi=0, \tag{1}$$

where Φ and ξ are the mixing angles of the neutral and charged gauge bosons, respectively, and

$$g_L=es_W,\ g'^{-1}=\sqrt{c_W^2e^{-2}-g_R^{-2}},\ \mid g_R\mid > g_Ls_Wc_W^{-1}, \tag{2}$$

where g_L, g_R, and g' are the couplings constants (CC's) of $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ gauge groups, respectively, and $c_W=\cos\theta_W$, $s_W=\sin\theta_W$. In any LR model there are light left-handed and heavy right-handed neutrinos in each lepton generation. Depending on the choice of the Higgs boson sector the neutrinos could be either Dirac or Majorana particles. The heavy and the light neutrinos of the same flavor are obtained from mixing the mass eigenstates ν_l and N_l according to

$$\nu_{lL}=\nu_l\cos\theta_{\nu_lN_l}+N_l\sin\theta_{\nu_lN_l}\ , \tag{3}$$
$$\nu_{lR}=-\nu_l\sin\theta_{\nu_lN_l}+N_l\sin\theta_{\nu_lN_l}\ , \tag{4}$$

where $\theta_{\nu_lN_l}$ is the oscillation angle in the vacuum. Mixing between different generations is also allowed. The present limit on the mass of the electron neutrino is [3]

$$m_{\nu_e}<8\ \text{eV}, \tag{5}$$

whereas for its heavy counterpart N_l the following mass bounds have been derived [4,5]:

$$(377\text{–}413)\ \text{GeV}\left(\frac{1\ \text{TeV}}{m_{W_2}}\right)^4 < m_{N_e} < 2e^{-1}\sqrt{\sin\theta_W(m_{W_1}^2+m_{W_2}^2)}, \tag{6}$$

where m_{W_i} $(i=1,2)$ denote the masses of the W_1 and W_2 gauge bosons, respectively, and the numbers in parentheses reflect the uncertainty from the nuclear matrix element of the neutrinoless double-β decay

$$^{76}\text{Ge}\longrightarrow^{76}\text{Se}\ e^+e^- \tag{7}$$

on which the lower bound estimate is based. Information about the oscillation parameters could be obtained from accelerator and nuclear reactor experiments and from available data on atmospheric and solar neutrinos (for Review see [6]). The aim of this paper is to consider the influence of neutrino parameters on neutrino

*Permanent address: Grodno State University, Grodno 230023, Belarus, Russia.

oscillations and $e^-e^-(e^+)$ collider and very high energy cosmic-ray experiments. The paper is organized as follows. In Sec. II we investigate the Majorana neutrinos propagating through magnetic and dense matter fields. Section III is devoted to the investigation of the indirect signatures of the neutrino parameters in the total cross section of the reactions

$$e^-e^- \longrightarrow W_k^- W_n^- \qquad (k,n=1,2), \tag{8}$$

$$e^+e^- \longrightarrow W_k^+ W_n^-. \tag{9}$$

Section IV summarizes our conclusions.

II. NEUTRINO OSCILLATIONS

In this section we consider the propagation of massive Majorana neutrinos through matter and a magnetic field taking into account their electromagnetic properties. We have in mind neutrinos emanating from the inner core of a star which on their way out have to traverse a sizable magnetic field. The electromagnetic quantities we are interested in are caused by radiative corrections which at the one loop level have their origin in two kinds of diagrams shown on Fig. 1, where $V=\gamma, Z_{1,2}$. The subject has already been addressed by Voloshin, Vysotsky, and Okun in Ref. [7], however, only in the framework of the symmetric left-right model assuming Dirac neutrinos and disregarding right-handed charged current neutrino-electron interaction. Within the SM Majorana neutrinos have been discussed in Refs. [8,9].

We base our investigation on the most general form of the left-right gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [2] and take into account right-handed charged and neutral currents. The basic interaction including the processes defined in Fig. 1(a,b) can be formulated in terms of the following effective Lagrangians:

$$\mathcal{L}_a = \frac{i}{2} a_{L,R} \overline{\nu}_{L,R} \partial_\mu V_{\mu\nu} \gamma_\nu \gamma_5 \nu_{L,R} + \text{conj}, \tag{10}$$

$$\mathcal{L}_b = \tfrac{1}{2} \mu_{L,R} \overline{\nu}_{R,L} \sigma_{\mu\nu} V_{\mu\nu} \nu_{L,R} + \text{conj}, \tag{11}$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. These give rise to the appearance of so-called anapole electromagnetic (weak) moments [10] described by the coefficients $a_{L,R}$ (which might be different for every lepton family, e.g., $a^e_{L,R} \neq a^\mu_{L,R}$) and electromagnetic (weak) dipole moments described by $\mu_{L,R}$. We should stress that in the one-flavor approximation for the Majorana neutrino the anapole moment is the only electromagnetic form factor allowed by the CTP invariance. For massless neutrinos it is connected with the neutrino electric charge radius (ECR). In the framework of the SM the ECR has been calculated in [11]. The results display the dependence on the top quark and the Higgs boson masses and the neutrino flavor. For example, at $m_t = 90$ GeV and $m_H = 100$ GeV one obtains

$$\langle r^2 \rangle^e_L \approx (48.7 \pm 3.6) \times 10^{-34}\ \text{cm}^2, \tag{12}$$

$$\langle r^2 \rangle^\mu_L \approx (79.6 \pm 3.6) \times 10^{-34}\ \text{cm}^2\ . \tag{13}$$

The existing experimental constraints on ECR are [12]

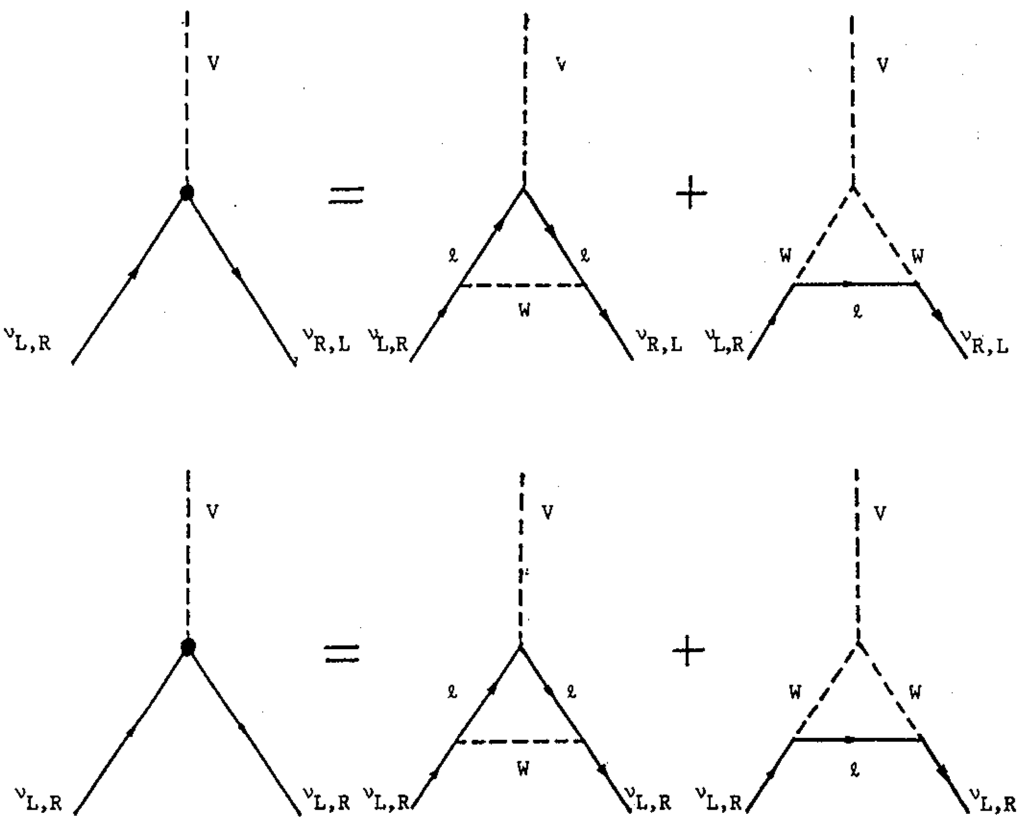

FIG. 1. The Feynman diagrams giving the contributions to the electroweak multipole moments.

$$\langle r^2\rangle_L^{e,\mu} \le (1\text{–}3)^2 \times 10^{-32}\ \mathrm{cm}^2,$$
$$\langle r^2\rangle_R^{e} \le 2 \times 10^{-33}\ \mathrm{cm}^2.$$

Within the SM the influence of the ECR and anapole moments of neutrinos has been considered for the disintegration of deuteron by neutrinos [13]. It has been shown that the ECR contribution can be as large as $\approx 1.2\%$. Hence, these quantities should be taken into account when investigating the neutrino phenomena.

The appearance of a neutrino dipole moment is also possible in the SM but its value is very small

$$(\mu_{\nu_l})_{\mathrm{SM}} \approx 2.7 \times 10^{-10}\mu_B\frac{m_{\nu_l}}{m_p}, \qquad (14)$$

where m_{ν_l} and m_p are the masses of the neutrino and the proton, respectively, and μ_B is a Bohr magneton. Such a dipole moment could be much larger in ALRM where it is given by [14]

$$\mu_{\nu_N} \approx \frac{g_R}{2g_L}\sin 2\xi[1.3 \times 10^{-9}(V_{i\tau}^\dagger U_{\tau i}) + 1.0 \times 10^{-10} \times(V_{i\mu}^\dagger U_{\mu i}) + 0.2 \times 10^{-14}(V_{ie}^\dagger U_{ei})]\mu_B \qquad (15)$$

where $m_{\nu_\alpha} = U_{\alpha i}m_{\nu_i}$, $m_{N_\alpha} = V_{\alpha i}m_{N_i}$, $\alpha = e, \mu, \tau$.

The current limits on magnetic dipole moment are coming from reactor experiments and cosmological observations. There is one bound on the electron neutrino moment inferred from laboratory experiments of neutrino-electron scattering and $e^+e^- \longrightarrow \gamma\nu\bar{\nu}$ process [15] giving $\mu_{\nu_e} < 10.8 \times 10^{-10}\mu_B$. Astrophysical considerations have led to a somewhat more stringent upper bound [16] $\mu_{\nu_e} < 7.9 \times 10^{-10}\mu_B$. In the literature the possibility of measuring μ_{ν_e} by use of the transition radiation of neutrinos crossing the interface between two media [17] is also discussed. For the Dirac particle the values of the anapole moment could be estimated by comparison with the diagonal elements of the magnetic moment μ_{ll}. In the case of Majorana neutrinos μ_{ll} are equal to zero and only the off diagonal elements (transition magnetic moments) can exist.

First we consider the case when the mixing between different generations is absent and the electron left- and right-handed neutrinos are connected with the mass eigenstates by (3) and (4). Then, the contributions to the multipole moments (MM's) are caused by the virtual electrons only. This leads to the small values of MM's. For example, at $\xi = 3.1 \times 10^{-2}$ the transition magnetic moment is as small as $1.7\times10^{-18}\mu_B$. It is well known that the quantity coming into play in the solar neutrino problem is $\mu B_\perp$, instead of μ and $B_\perp$ individually. For our further discussion it will be useful to recall information about the solar interior magnetic profiles. Little is known about the configuration and the strength of the solar interior magnetic field. We can only make observations of the magnetic activity at the Sun's surface and infer the field inside. The maximal value of the field in the central core of the Sun should not exceed 0.5×10^8 G. In the radiative zone the field could be as large as $10^4 - 10^5$ G. In the convective zone the magnetic field displays the 11.2-year cycle. This cycle is characterized by the creation of the so-called active regions, the areas of the developing magnetic field at the Sun's surface. When in these regions the field strength rises above about 500 G sunspots begin to form. The largest of them could reach a size about $R_\odot$ in diameter. Sunspot fields are the strongest, ranging beyond 5×10^3 G. They expand to the corona to levels of 10^5 km. For example, at the height of 10^3 km the strength could be as strong as 10^3 G (we recall that in an atmosphere beyond the sunspot the mean magnetic field is about 1 G). Therefore, $\mu B_\perp$ decreases from 3×10^{-18} eV at the center up to 10^{-23} eV at the active regions of the Sun's atmosphere. It is clear that for an explanation of the anticorrelation between sunspot activity and solar neutrino flux behavior we should change the oscillation scheme (3) and (4). To increase the value of the transition magnetic moment one has to introduce the mixing between different neutrino generations. Let us investigate the following oscillation scheme:

$$\nu_{eL} = \nu_1\cos\theta_L + \nu_2\sin\theta_L,$$
$$\nu_{XL} = -\nu_1\sin\theta_L + \nu_2\cos\theta_L,$$
$$N_{eR} = \nu_3\cos\theta_R + \nu_4\sin\theta_R,$$
$$N_{XR} = -\nu_3\sin\theta_R + \nu_4\sin\theta_R,$$

where $X = \mu, \tau$ and for the sake of simplicity we shall set $\theta_L = \theta_R = \theta$. Of course, the full description in two flavor bases demands the consideration of the 8-component wave function including the antiparticles too. However, in order to display the typical features of the ALRM it is enough to limit oneself to the investigation of the evolution of the state vector

$$\Psi^T = (\nu_{eL}, \nu_{XL}, N_{eR}, N_{XR}).$$

Before writing the evolution equation for Ψ we note some magnetic field characteristics which we are also going to take into account. The magnetic field both in the convective zone and in the atmosphere above spot is characterized by the geometrical (or topological) phase $\Phi(z)$ and its derivative $\dot{\Phi}(z)$

$$\tan\Phi(z) = \frac{B_y}{B_x}.$$

At least in the regions below and above sunspots this field has nonpotential character [18], namely,

$$(\mathrm{rot} \times \vec{B})_z = j_z \ne 0\ .$$

The longitudinal electric currents could change both the values and the directions after the solar flare [19]. Further we shall assume the matter to be electric neutral and disregard the velocity of the Sun's matter. Then by the standard way from the Lagrangian averaged over the matter distribution we obtain the neutrino transmission equation in a Schrödinger-like form

$$i\frac{d}{dz}\Psi = \begin{pmatrix} \mathcal{H}_{\nu_e\nu_X} & Me^{i\Phi} \\ M^\dagger e^{-i\Phi} & \mathcal{H}_{N_eN_X} \end{pmatrix}\Psi, \qquad (16)$$

where

$$\mathcal{H}_{\nu_e\nu_X} = \begin{pmatrix} \delta_c^{12} + V_{eL} + a_{eL}j_z + \Sigma & -\delta_s^{12} \\ -\delta_s^{12} & -\delta_c^{12} + V_{XL} + a_{XL}j_z + \Sigma \end{pmatrix},$$

$$\delta_{c(s)}^{12} = \frac{m_1^2 - m_2^2}{4E}\cos 2\theta(\sin 2\theta), \quad V_{eL,R} = V_{L,R}^C + V_{L,R}^N,$$

$$V_{XL,R} = V_{L,R}^N, \quad V_L^C = \sqrt{2}G_F N_e, \quad V_R^C = \frac{g_R^2}{4}\left(\frac{\sin^2\xi}{m_{W_1}^2} + \frac{\cos^2\xi}{m_{W_2}^2}\right)N_e,$$

$$V_{L,R}^N = \frac{N_n}{4}\sum_{j=1}^{2}\frac{g_{V_j}^{\nu_{L,R}} g_{V_j}^n}{m_{Z_j}^2}, \quad \Sigma = \frac{m_1^2 + m_2^2 - m_3^2 - m_4^2}{8E},$$

$$g_{V_1}^f = ec_W^{-1}[s_W^{-1}c_\phi(T_{3L}^f - 2Q_f s_W^2) + s_\phi\omega^{-1}(T_{3L}^f + T_{3R}^f c_W^2 e^{-2}g_R^2 - 2Q_f)], \quad g_{V2}^f = g_{V1}^f\left(\phi \to \phi + \frac{\pi}{2}\right)$$

$$c_W = \cos\theta_W, \quad s_W = \sin\theta_W, \quad s_\phi = \cos\phi, \quad c_\phi = \cos\phi, \quad \omega = \sqrt{c_W^2 e^{-2} g_R^2 - 1},$$

$$M = \begin{pmatrix} \mu_{ee} & \mu_{eX} \\ -\mu_{eX} & \mu_{XX} \end{pmatrix}, \quad \mathcal{H}_{N_e N_X} = \mathcal{H}_{\nu_e\nu_X}(\{L, m_1, m_2, j_z\} \longrightarrow \{R, m_3, m_4, -j_z\}).$$

N_e and N_n are the densities of electrons and neutrons, respectively. We have chosen the propagation direction of the neutrino as the axis of the spin quantization. Hereafter we assume that $\varphi = 0$ and use the notations of Ref. [2].

Having performed a phase rotation

$$\Psi' = S\Psi, \tag{17}$$

where $S = \mathrm{diag}\{e^{\frac{i\Phi}{2}}, e^{\frac{i\Phi}{2}}, e^{\frac{-i\Phi}{2}}, e^{\frac{-i\Phi}{2}}\}$ and $\Psi'^T = (\nu'_{eL}, \nu'_{XR}, N'_{eR}, N'_{XR})$ is the vector state of neutrino system in the reference frame rotating with the same angular velocity as the transverse magnetic field, we exclude the phase factors from the nondiagonal elements of the Hamiltonian. Since $|\Psi'|^2 = |\Psi|^2$ we will drop the prime sign for the sake of simplicity. The expression for the transformed Hamiltonian follows from the old one [see (16)] by the substitution

$$e^{\pm i\Phi} \longrightarrow 1, \; V_{L,R} \longrightarrow V_{L,R} \pm \frac{\dot{\Phi}}{2}\,. \tag{18}$$

Now we shall concentrate on the ν_L resonant conversions only. There are three possible crossing resonances: (a) the MSW resonance ($\nu_{eL} \longrightarrow \nu_{\mu L}$); (b) the spin-flipping (SF) resonance ($\nu_{eL} \longrightarrow N_{eR}$); (c) the spin-flavor flipping (SFF) resonance ($\nu_{eL} \longrightarrow N_{XR}$). The conditions for them can be written as

$$\Delta_{\mathrm{MSW}} = 2\delta_c^{12} + V_{eL} - V_{XL} + (a_{eL} - a_{XL})j_z = 0,$$

$$\Delta_{\mathrm{SF}} = \delta_c^- + V_{eL} - V_{eR} + (a_{eL} + a_{eR})j_z + \dot{\Phi} + 2\Sigma = 0,$$

$$\Delta_{\mathrm{SFF}} = \delta_c^+ + V_{eL} - V_{XR} + (a_{eL} + a_{XR})j_z + \dot{\Phi} + 2\Sigma = 0,$$

where $\delta^\pm = \delta_c^{12} \pm \delta_c^{34}$.

Their widths are defined according to

$$\delta N_e(\mathrm{MSW}) \sim [N_e(\mathrm{MSW}) - (a_{eL} - a_{XL})j_z(\sqrt{2}G_F)^{-1}] \times \tan 2\theta$$

$$\delta N_e(\mathrm{SF}) \sim \frac{2\mu_{ee}B_\perp N_e(\mathrm{SF})}{\delta_c^- + (a_{eL} + a_{eR})j_z + \dot{\Phi} + 2\Sigma},$$

$$\delta N_e(\mathrm{SFF}) \sim \frac{2\mu_{eX}B_\perp N_e(\mathrm{SFF})}{\delta_c^+ + (a_{eL} + a_{XR})j_z + \dot{\Phi} + 2\Sigma},$$

where $N_e(i)$ (i=MSW, SF, SFF) is the density at which the i resonance occurs.

If these resonance regions are well separated, namely, the following conditions are satisfied:

$$\delta N_e(i) + \delta N_e(k) < N_e(i) - N_e(k) \tag{19}$$

then we can interpret them independently from each other. For such decoupled resonances the transition probability is given by

$$D^i = \exp[-\gamma^i(z_i)F_i], \tag{20}$$

where the adiabaticity parameters $\gamma^i(z)$ are

$$\gamma^{\mathrm{MSW}}(z) = \frac{16(\delta_s^{12})^2}{\sin^3 2\theta_{\mathrm{MSW}} \mid \frac{d}{dz}[V_{eL} - V_{XL} + (a_{eL} - a_{XL})j_z] \mid},$$

$$\gamma^{\mathrm{SF}}(z) = \frac{8(\mu_{ee}B_\perp)^2}{\sin^3\theta_{\mathrm{SF}} \mid \frac{d}{dz}[V_{eR} - V_{eL} - \dot{\Phi} - (a_{eL} + a_{eR})j_z]},$$

$$\gamma^{\rm SFF}(z) = \frac{8(\mu_{eX}B_\perp)^2}{\sin^3\theta_{\rm SFF} \mid \frac{d}{dz}[V_{XR} - V_{eL} - \dot{\Phi} - (a_{eL} + a_{XR})j_z] \mid},$$

$\sin^2\theta_i = \frac{C_i^2}{\Delta_i^2 + C_i^2}$, $C_{\rm MSW} = 2\delta_s^{12}$, $C_{\rm SF} = 2\mu_{ee}B_\perp$, $C_{\rm SFF} = 2\mu_{eX}B_\perp$, z_i is the z coordinate of the i resonance point, and the quantity F depends on the kind of resonance and how N_e, $\dot{\Phi}$, and j_z vary with z near the resonance. For example, when j_z is constant, the $F_{\rm MSW}$ equals either $\frac{\pi}{4}$ or $\frac{\pi}{4}(1-\tan^2\theta)$ for the linear and exponential variations of N_e, respectively.

Here we should emphasize the characteristic feature of the ALRM. The N_{kR} ($k = e, \mu, \tau$) are not the sterile particles, because the ALRM predicts their interactions with matter caused by the existence of the right-handed currents. However, the probability of the N_{kR} absorption by the neutrino detectors is weakened by the factor

$$\frac{g_R^4 m_{W_1}^4}{g_L^4 m_{W_2}^4}$$

unlike the corresponding probability of their left-handed counterpart. Analogously the creation probabilities of the left- and right-handed solar electron neutrinos are connected in the same way.

If the adiabaticity condition (AC) is fulfilled for these resonances the ν_{eL} will be completely converted to ν_{XL}, N_{eR}, and N_{XR}. In the case of violating AC the amount of the transformed neutrinos depends on the degree of the adiabaticity. Further it will be useful to have the analytical expression for the survival probability of a ν_{eL} in the more general case, namely, in a nonadiabatic one. Let us assume for the sake of simplicity that $\theta = 0$. As a result μ_{ee} becomes vanishing small because it receives the contributions coming from virtual electrons only. Then in the first order of the perturbation theory the $\nu_{eL} \longrightarrow N_{XR}$ resonant conversion is allowed only. For the case of the steady rotation of $B_\perp(\dot{\Phi} = \text{const})$, the constant value of j_z and an exponential falloff in the solar density near the resonance the survival probability is determined by expression

$$P(\nu_{eL} \longrightarrow \nu_{eL}) = \tfrac{1}{2}[1 + (1 - 2\mathcal{D}^{\rm SFF})\cos 2\theta_0^{\rm SFF}], \quad (21)$$

where $F_{\rm SFF} = \frac{\pi}{4}$ and subscript 0 at the effective mixing angle $\theta^{\rm SFF}$ means that it is defined at the point of creation of a ν_{eL}. We recall that the adiabaticity conditions are most difficult to satisfy in the level crossing points. So, if the adiabaticity parameter $\gamma(z_i) \gg 1$, then the neutrino propagation is adiabatic everywhere. In the case when $\gamma(z_i)$ is close to or smaller than unity, the contribution of $\mathcal{D}$ becomes sizable. We see that depending on the choice of the structural parameters (SP's) of the ALRM the expressions for the probabilities, the location, and the distances between resonances are changed. Investigating all the possible energy crossings levels we could also face the case of resonance permutations. Here the main question we are interested in is the value of possible deviations from the SM. Let us consider expression (21). Here we have the following sources of the deviations from the SM. The first source is connected with the values of the magnetic moments in both models [compare Eqs. (14) and (15)]. The second one comes from the quantity δ_c^{34} being equal to zero in the SM. Finally the third source is caused by the extra $SU(2)_R$ sector. The contributions from the right-handed charged and neutral currents modify the matter potential. In expression (21) they enter the definition of $\cos 2\theta_0^{\rm SFF}$. If we consider the neutrino system consisting from ν_{eL}, ν_{XL} and their right-handed counterpart within the SM then the matter potential entering the $\cos 2\theta_0^{\rm SFF}$ will be larger. At the center of the Sun this potential could exceed that for the ALRM case on the value of order 45%. In our estimation we have used the following values of the SP's of the ALRM:

$$\begin{aligned} m_{W_1} &= 80.13 \text{ GeV}, \quad m_{W_2} = 477 \text{ GeV}, \\ m_{Z_1} &= 91.77 \text{ GeV}, \quad m_{Z_2} = 800 \text{ GeV}, \\ \Phi &= 9.6 \times 10^{-3}, \quad \xi = 3.1 \times 10^{-2}, \\ \theta_{\nu_e N_e} &= 5 \times 10^{-2}, \quad g_R = 1.6 g_L , \end{aligned} \quad (22)$$

which are in accordance with the existing experimental bounds (see, e.g., Ref. [20]).

There is also one interesting possibility connected with the appearance of $a_{L,R}[\nabla \times \vec{B}]_z$ and $\dot{\Phi}$ terms in the Hamiltonian. We briefly discuss it considering the $\nu_{eL} \longrightarrow N_{eR}$ resonance conversion. Let us consider the propagation of the ν_{eL} flux through the sunspot before and after the flare. In order to make our arguments as definite as possible we assume that in the convective zone the following inequalities are satisfied:

$$\delta_c^- + 2\Sigma < 0,$$

$$\delta_c^- + 2\Sigma + \kappa_{b,a} < 0,$$

$$|\kappa_b| > |\kappa_a|,$$

where $\kappa = \dot{\Phi} + (a_{eL} + a_{eR})j_z$ and the subscripts b and a mean that κ is defined in the pre- and postflare periods. As for the $\kappa_{b,a}$ dependence on z we shall take $\kappa_{b,a}$ being constant in the convective zone and decreasing quicker than the electron density in the atmosphere. Next let us assume that in the preflare period the resonance condition is satisfied in the convective zone. Therefore, we have the intersection of the two curves $V_{eR} - V_{eL}$ and $\delta_c^- + 2\Sigma + \kappa_a$ at the point z_l. However, if the intersection point does not lie on the upper border of the convective zone z_u the resonance will also occur in the atmosphere. Now let us introduce the quantities

$$\epsilon = |\kappa_b - \kappa_a|,$$

$$\Delta V = V_{eL}(z_l) - V_{eR}(z_l) - V_{eL}(z_u) + V_{eR}(z_u).$$

Then, in the postflare period the following situations are possible. (a) $\epsilon < \Delta V$. The resonance conversion occurs twice. (b) $\epsilon = \Delta V$. The resonance conversion occurs once only. (c) $\epsilon > \Delta V$. The resonance disappears.

III. EXPERIMENTS WITH ELECTRON AND POSITRON BEAMS

In this section we discuss the impact of the Majorana neutrino scenario both for collider experiments and for experiments with cosmic rays. We shall be limited by the case when only the mixing angle between the left- and right-handed neutrino of the same flavor is not equal to zero [see, Eqs. (3) and (4)]. In our calculations we shall use the Feynman rules for the case of Majorana neutrinos which has been formulated in Ref. [21]. As it follows from them the cross sections of lepton flavor conserving processes have the same form both for the Dirac and Majorana neutrinos. A reliable test of the nature of the neutrino involved is the process which is connected (by time reversal symmetry) to neutrinoless double-β decay, i.e.,

$$e^-e^- \longrightarrow W^-_k W^-_n \quad (k,n=1,2). \tag{23}$$

We recall that it is forbidden if the neutrino is a Dirac particle. The reaction (23) will be accessible to cosmic rays and to the e^-e^- option of LEP II and NLC (TLC). The experimental program is motivated not only by looking for lepton flavor violation but also by searching for doubly charged leptons and Higgs boson particles.

Within the symmetric LRM the process (23) has been studied in Refs. [22–25]. In Refs. [22,24] it was assumed that the reaction (23) was caused by the neutrino exchange only. In Refs. [23,25] the model was used in which this process proceeds not only via a neutrino exchange in t and u channels but via a doubly charged Higgs Δ_{--} exchange in the s channel. However the authors restricted their discussion to the cases of the LL and RR polarized e^-e^- beams with $k=n$. It should be noted that the presence of the diagram with Δ_{--} leads to the cancellation in the cross section of the terms which violate the unitarity. The conclusions of Refs. [22,24] do not agree between each other. In Ref. [22] in the high-energy limit the total cross section grows as a linear function of s, while in Ref. [24] it increases as the square of s.

In the left-right model we can introduce the Higgs boson triplets $\Delta_L(1,0,1)$ and $\Delta_R(0,1,1)$ (in brackets their numbers I_L, I_R, and $\frac{Y}{2}$ are given) in order to obtain the spontaneous breaking the underlying gauge symmetry to the symmetry $\mathrm{U}(1)_Q$ as observed in the real world. Their electrically neutral component acquires VEV v_L and v_R, respectively. As a result we have two physical doubly charged Higgs boson scalars Δ^L_{--} and Δ^R_{--}. Their interactions with charged fermions and gauge bosons are described by the Lagrangian

$$\mathcal{L}_{\Delta^{L,R}_{--}} = -[g^2_{L,R} v_{L,R} W_{\mu L,R} W_{\mu L,R} + \tfrac{1}{2} h \bar{l}^c (1 \pm \gamma_5) l] \Delta^{L,R}_{--} \tag{24}$$

where the superscript c means the charge conjugation and h together with v_R and v_L defines the left-handed and the right-handed neutrino masses m_ν and m_N according to

$$h v_R = m_N \cos^2\alpha + m_\nu \sin^2\alpha,$$

$$h v_L = m_N \sin^2\alpha + m_\nu \cos^2\alpha,$$

where $\alpha = \theta_{\nu_e N_e}$. Let us calculate the cross section of process (23) within the ALRM. This reaction is represented by the Feynman diagrams pictured in Fig. 2. The main difference of this model from the SM is that if in the latter the reaction (23) occurs for left-polarized electrons ($e^-_L e^-_L$) only, whereas in the ALRM the contribution to the cross section arise from both $e^-_{L,R} e^-_{L,R}$ and $e^-_{L,R} e^-_{R,L}$ contributions. The identity

$$\bar{e}\gamma_\mu (1 \pm \gamma_5) \nu = -\bar{\nu}\gamma_\mu (1 \mp \gamma_5) e^c \tag{25}$$

will be useful in our calculations. For example, with its help the matrix element corresponding to the process

$$e^-_L(k_1) e^-_R(k_2) \longrightarrow W^-_k(p_1) W^-_n(p_2) \tag{26}$$

takes the form

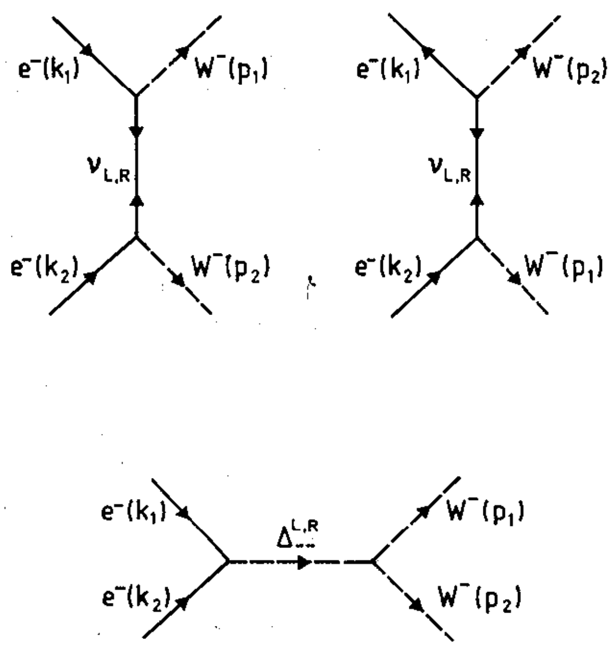

FIG. 2. The Feynman diagrams corresponding to the process $e^-e^- \longrightarrow W^-_k W^-_n$.

$$M_L R = \frac{1}{2}(\sin 2\alpha) g_L g_R b_L^{(k)} b_R^{(n)} \bar{e}(k_1)\gamma_\mu (1+\gamma_5)\left(\frac{(\hat{k}_1-\hat{p}_1)+m_\nu}{(k_1-p_1)^2-m_\nu^2} - \frac{(\hat{k}_1-\hat{p}_1)+m_N}{(k_1-p_1)^2-m_N^2}\right)$$
$$\times \gamma_\lambda (1+\gamma_5) e^c(k_2) W_\mu^{*(k)}(p_1) W_\lambda^{*(n)}(p_2) + (k \leftrightarrow n), \tag{27}$$

where $b_{L,R}^{(k)} = \frac{1}{2}\{[\pm 1 + (-1)^k]\sin\xi + [1 \mp (-1)^k]\cos\xi\}$. To constant this expression coincides with the amplitude of t channel of the reaction

$$f_i \overline{f_i} \longrightarrow W_k^- Z_n \quad (k,n = 1,2), \tag{28}$$

where i is the flavor of a fermion f. This process was studied within the SM ($k = n = 1$) in Ref. [26] whereas within the LRM it was done in Ref. [27]. Therefore we can use these results and write down the differential cross section in the form

$$\frac{d\sigma_{L,R}^{(k,n)}}{dt} = \frac{(g_L g_R m_N^2 \sin 2\alpha)^2}{512\pi s^2}\left[\left(\frac{b_R^{(n)} b_L^{(k)}}{t-m_N^2}\right)^2 B_3^{(kn)}(s,t,u) + \left(\frac{b_R^{(k)} b_L^{(n)}}{u-m_N^2}\right)^2 B_3^{(kn)}(s,u,t) + \frac{b_L^{(k)} b_L^{(n)} b_R^{(k)} b_R^{(n)}}{(t-m_N^2)(u-m_N^2)} B_4^{(kn)}(s,t,u)\right], \tag{29}$$

where the functions $B_{3,4}^{(kn)}(s,u,t)$ are defined in Ref. [27]. Hereafter, for the sake of simplicity, we take the mass of the light neutrino to be equal to zero. It should be noted that in the SM without a scalar triplet, the ν_{eL} gets a seesaw mass equal to $\frac{-m^2}{M}$ with mixing angle $\alpha = \frac{m}{M}$. Setting its mass to zero also requires α to be zero. However, in the ALRM the light neutrino also receives a mass from Δ_L, so it is possible to make its mass zero from the cancellation of this against the seesaw contribution, and have a nonzero α.

The corresponding calculations for the differential cross section of the reaction

$$e_L^- e_L^- \longrightarrow W_n^- W_k^- \tag{30}$$

yield the expression

$$\frac{d\sigma_L^{(kn)}}{dt} = \frac{(g_L^2 m_N b_L^{(k)} b_L^{(n)} \sin^2\alpha)^2}{512\pi s^2}\Bigg\{\left(3s - \frac{Fs^2(s-2m_{W_n}^2-2m_{W_n}^2)}{4m_{W_n}^2 m_{W_k}^2} - \frac{st(s-m_{W_n}^2-m_{W_k}^2)}{m_{W_n}^2 m_{W_k}^2}\right)\frac{1}{(t-m_N^2)^2}$$
$$+\frac{4s}{(s-m_{\Delta_L}^2)^2+\Gamma_{\Delta_L}^2 m_{\Delta_L}^2}\left[\left(8+\frac{(s-m_{W_n}^2-m_{W_k}^2)}{m_{W_n}^2 m_{W_k}^2}\right)+\frac{s-m_{\Delta_L}^2}{t-m_N^2}\left(8+\frac{2t(m_{W_n}^2+m_{W_k}^2-s)}{m_{W_n}^2 m_{W_k}^2}\right)\right]+(u\leftrightarrow t)$$
$$+\left(2s+\frac{Fs^2(s-2m_{W_n}^2-2m_{W_k}^2)}{2m_{W_n}^2 m_{W_k}^2}\right)\frac{1}{(t-m_N^2)(u-m_N^2)}\Bigg\} \tag{31}$$

where $F = 4(ut - m_{W_n}^2 m_{W_k}^2)/s^2$ and Γ_{Δ_L} is the decay width of Δ_{--}^L Higgs boson scalar. Considering only the prominent decay mode of $\Delta_{--}^{L,R}$ [28] into $\tau^-\tau^-$ we obtain for the Higgs boson width

$$\Gamma_{\Delta_{L,R}} = \frac{m_N^2}{16\pi v_{L,R}^2} m_{\Delta_{L,R}}\left(\frac{m_\tau}{m_{\Delta_{L,R}}}\right)^2\sqrt{1-\frac{4m_\tau^2}{m_{\Delta_{L,R}}^2}},$$

where m_τ is the τ-lepton mass. The differential cross section of the reaction

$$e_R^- e_R^- \longrightarrow W_k^- W_n^-, \tag{32}$$

follows from (28) by the substitution

$$g_L \to g_R,\ b_L^{(k)} \to b_R^{(k)},\ \alpha \to \alpha + \frac{\pi}{2},\ m_{\Delta_L} \to m_{\Delta_R}\,. \tag{33}$$

The total cross section of reaction (26) is obtained from (29) after multiplication by $\frac{\beta s}{2}$ and substitution

$$\left.\begin{matrix}\frac{B_3^{(kn)}(s,t,u)}{(t-m_N^2)^2} \\ \frac{B_3^{(kn)}(s,u,t)}{(u-m_N^2)^2}\end{matrix}\right\} \to D_3^{(kn)}(s)\,,$$

$$\frac{B_4^{(kn)}(s,t,u)}{(t-m_N^2)(u-m_N^2)} \to D_4^{(kn)}(s)\,, \tag{34}$$

where

$$D_3^{(kn)}(s) = \frac{1}{m_{W_n}^2 m_{W_k}^2}\left(1 + \frac{(m_{W_n}^2 + m_{W_k}^2 - s - 2m_N^2)\ln L_\nu}{2\beta s}\right)$$
$$+\frac{1}{m_N^4}\left(\frac{[4m_{W_n}^2 m_{W_k}^2 - 2m_N^2(m_{W_n}^2 + m_{W_k}^2 - s)](\ln L_\nu - \ln L)}{\beta s m_N^2} - 4\right) - \frac{s(m_{W_n}^2 + m_{W_k}^2)}{C_\nu m_{W_n}^2 m_{W_k}^2},$$
$$D_4^{(kn)}(s) = -\frac{2[2s(m_{W_n}^2 + m_{W_k}^2) + C_\nu]\ln L_\nu}{\beta s m_{W_n}^2 m_{W_k}^2(m_{W_n}^2 + m_{W_k}^2 - s - 2m_N^2)}$$
$$-\frac{1}{m_{W_n}^2 m_{W_k}^2} - \frac{8(m_{W_n}^2 + m_{W_k}^2)}{\beta m_N^2(m_N^2 - m_{W_n}^2 - m_{W_k}^2 + s)}\left(\frac{\ln L_\nu}{m_{W_n}^2 + m_{W_k}^2 - s - 2m_N^2} - \frac{\ln L}{m_{W_n}^2 + m_{W_k}^2 - s}\right),$$
$$C_\nu = m_N^2(m_{W_n}^2 + m_{W_k}^2 - s - m_N^2) - m_{W_n}^2 m_{W_k}^2, \quad L_\nu = \left|\frac{m_{W_n}^2 + m_{W_k}^2 - s - 2m_N^2 + \beta s}{m_{W_n}^2 + m_{W_k}^2 - s - 2m_N^2 - \beta s}\right|,$$
$$L = L_\nu\,|_{m_N=0}, \quad \beta = \sqrt{\left(1 - \frac{m_{W_n}^2 + m_{W_K}^2}{s}\right)^2 - \left(\frac{2m_{W_n}m_{W_k}}{s}\right)^2}.$$

For the total cross section of reaction (30) we find the expression

$$\sigma_L^{(kn)} = \beta\frac{(g_L^2 m_N b_L^{(k)} b_L^{(n)} \sin^2\alpha)^2}{256\pi s}\left\{\frac{2s - 4(m_{W_n}^2 + m_{W_k}^2)}{m_{W_n}^2 m_{W_k}^2} - s\left(3 + \frac{m_N^4 + m_{W_n}^2 m_{W_k}^2}{m_{W_n}^2 m_{W_k}^2}\right)C_\nu^{-1}\right.$$
$$+\frac{\ln L_\nu}{\beta s}\left[\frac{2sm_N^2 + 2(m_{W_n}^2 + m_{W_k}^2 - s - 2m_N^2)(m_{W_n}^2 + m_{W_k}^2)}{m_{W_n}^2 m_{W_k}^2}\right.$$
$$\left.+\frac{2}{m_{W_n}^2 + m_{W_k}^2 - s - 2m_N^2}\left(s + \frac{s - 2m_{W_n}^2 - 2m_{W_k}^2}{m_{W_n}^2 m_{W_k}^2}C_\nu\right)\right]$$
$$+\frac{1}{(s - m_{\Delta_L}^2)^2 + \Gamma_{\Delta_L}^2 m_{\Delta_L}^2}\left[16s + \frac{2s(s - m_{W_n}^2 - m_{W_k}^2)^2}{m_{W_n}^2 m_{W_k}^2} + 16(s - m_{\Delta_L}^2)\right.$$
$$\left.\left.\times\left(\frac{\ln L_\nu}{\beta} + \frac{s(m_{W_n}^2 + m_{W_k}^2 - s)}{4m_{W_n}^2 m_{W_k}^2}\left(1 + \frac{m_N^2 \ln L_\nu}{\beta s}\right)\right)\right]\right\}. \quad (35)$$

Having done the replacement (33) in the expression (35) we obtain the total cross section of the reaction (32). Now we can compare our results with those of Refs. [22–25]. The analytical expressions for the cross sections have been obtained in Refs. [22,25] only. At $k = n$ and the initial $e^-_{L(R)}e^-_{L(R)}$ beam our cross sections coincide with those calculated in Ref. [25]. If we disregard the contribution of Δ^L_{--} to the σ_L^{11} ($m_{\Delta_L} \longrightarrow \infty$) then the high energy limit of (35) leads to the formulas of Ref. [22].

Now we discuss the asymptotic behavior of the cross sections obtained above. From expressions (34) and (35) we can see that their partial contributions violate unitarity. To be definite, in the case of the LR electron beam they increase as a linear function of s, while for LL or RR electron beams those tend to be constant when $s \longrightarrow \infty$. However, in all cases the total cross sections resulting from the sum of those contributions have a good asymptotic behavior

$$\sigma \sim \frac{\ln s}{s}. \quad (36)$$

It is caused by the cancellation among the partial contributions. We should stress that the reasons leading to (36) are quite different for LR and LL (or RR) polarized electron beams. In the former case the Δ contribution to the cross section is absent and the cancellation is connected with the spin behavior of the virtual neutrino. If the neutrino flips the helicity between the acts of emission and absorption then the amplitudes coming from ν_e and N_e exchanges are proportional to the neutrino masses. Since we neglected m_{ν_e} only the N_e exchange term will contribute. On the contrary, when the neutrino does not flip its helicity (this case is realized for LR electron beams) we have two nonvanishing terms. They are caused by ν_e and N_e exchanges and have opposite signs. As the calculations show, disregarding the contribution of the light neutrino would then lead to a cross section proportional to s. As it follows from the expression for the cross section of the reaction (23) this reaction could be a good tool for the definition of such parameters of the ALRM as m_N, g_R, α, and ξ. In our numerical calculation we shall, in what follows, use the values of the SP's of the ALRM defined by (22) and set m_{Δ_L} and m_{Δ_R} equal to 100 and 110 GeV, respectively. We remind the reader that the masses values of the doubly charged Higgs bosons are free parameters of the theory in analogy to the SM Higgs boson mass. The values of $\sigma^{(kn)}$ strongly depend on m_N. For example, at $m_N = 100$ GeV and

$g_L = g_R$, in the energy region up to 200 GeV we have $\sigma_L^{(11)} \approx 8\sigma_R^{(11)} \approx 8 \times 10^{-2}$ fb whereas $\sigma_{LR}^{(11)}$ is about a few$\times 10^{-3}$ fb. At increasing m_N the contribution from $\sigma_{LR}^{(11)}$ becomes dominant. For example, at $m_N = 1$ TeV and $\sqrt{s} = 200$ GeV we have

$$\sigma_R^{(11)} \approx 3.3 \times 10^{-3}\ \text{pb}, \quad \sigma_L^{(11)} \approx 2.6 \times 10^{-2}\ \text{pb},$$

$$\sigma_{LR}^{(11)} \approx 5 \times 10^{-2}\ \text{pb}\ .$$

It should be noted here that at the sufficiently great m_N ($m_N > 5$ TeV) the cross section of reaction (23) could reach the values which are compatible and even greater than the cross section of the reaction

$$e^- e^+ \longrightarrow W_1^- W_1^+ .$$

In Fig. 3 we represent $\sigma_{LR}^{(11)}$ versus $\sqrt{s}$ for $m_N = 1$ and 1.4 TeV ($g_L = g_R$). At $k = 1$ and $n = 2$ the situation is as follows. When the values of m_N are of the order of a few hundred GeV the main contribution comes from the RR-polarized e^- beams. For example, at $m_N = 100$ GeV, the $\sigma_R^{(12)}$ exceeds the $\sigma_{LR}^{(12)}$ on one order of magnitude and its value is about 7×10^{-1} pb ($\sigma_L^{(12)} \approx 10^{-5} \times \sigma_R^{(12)}$). Again with the increase of m_N the contribution from $\sigma_{LR}^{(12)}$ gains dominance. For example, at $m_N = 1$ TeV, $\sigma_{LR}^{(12)}$ is twice as much as $\sigma_R^{(12)}$. In Fig. 4 we give $\sigma_{LR}^{(12)}$ as a function of $\sqrt{s}$ for $m_N = 1$ and 1.4 TeV ($g_L = g_R$). In the case $k = n = 2$ the reaction (23) mainly goes for the $e_R^- e_R^-$ beams. Only at the enormous values of m_N (~ 100 TeV), $\sigma_{LR}^{(22)}$ may be compatible with $\sigma_R^{(22)}$. In Fig. 5 we plot $\sigma_R^{(22)}$ as a function of $\sqrt{s}$ for $m_N = 1$ and 1.4 TeV ($g_L = g_R$). It is also important to recall that the dependence of $\sigma_R^{(kn)}$ and $\sigma_L^{(kn)}$ on m_N is much weaker than the one of $\sigma_{LR}^{(22)}$. For example, at $k = n = 2$ varying m_N from 10^3 GeV up to 10^2 GeV leads to the decrease of $\sigma_R^{(22)}$ and $\sigma_{LR}^{(22)}$ on 1 and 3 orders of magnitude, respectively. We should stress that in the case

$$m_{\Delta_{L,R}} < (m_{W_n} + m_{W_k})$$

results obtained above weakly depend on the assumed value of $m_{\Delta_{L,R}}$. In the contrary case there should be the s-channel resonance giving the increase of the cross sections for $e_L^- e_L^-$ and $e_R^- e_R^-$ beams. Note, in particular, that the signature of doubly charged Higgs bosons would strongly support the idea of a left-right approach.

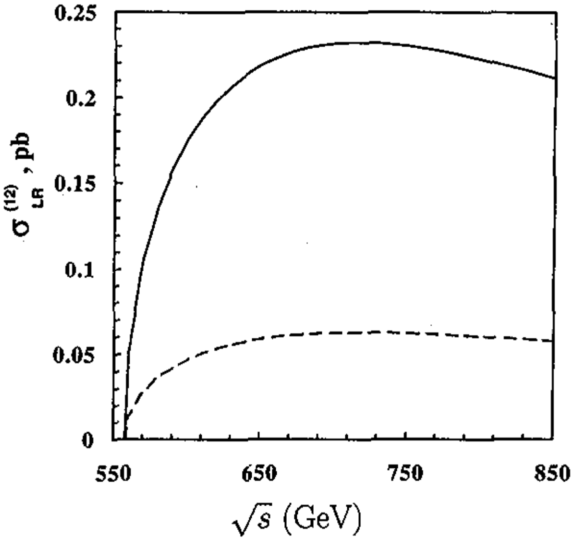

FIG. 4. $\sigma_{LR}^{(12)}$ as a function of $\sqrt{s}$ at $m_N = 1$ (dashed line) and $m_N = 1.4$ TeV (solid line).

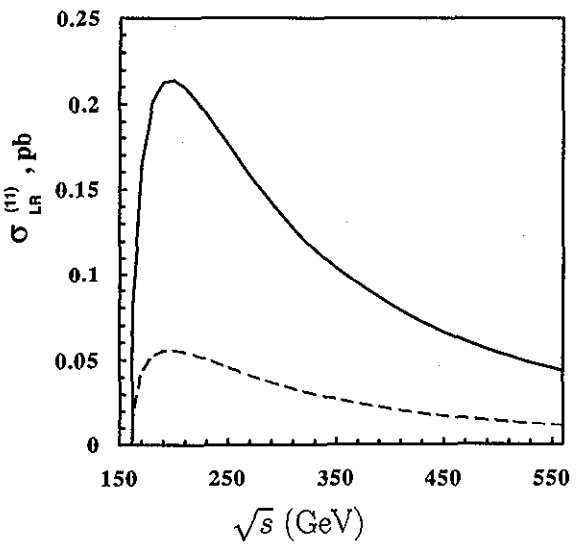

FIG. 3. The total cross section of the reaction $e_L^- e_R^- \longrightarrow W_1^- W_1^-$ versus $\sqrt{s}$ at $m_N = 1$ (dashed line) and $m_N = 1.4$ TeV (solid line).

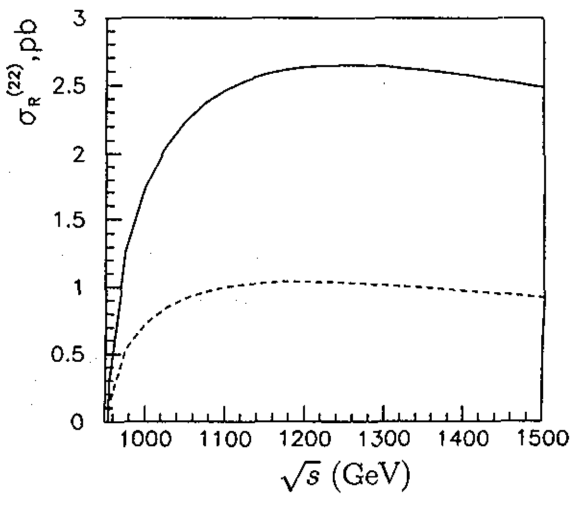

FIG. 5. $\sigma_R^{(22)}$ versus $\sqrt{s}$ at $m_N = 1$ (dashed line) and $m_N = 1.4$ TeV (solid line).

Now we should pay some attention to the question of detectability of the processes

$$e^-e^- \longrightarrow W_k^- W_n^- \longrightarrow l_i^- l_j^- + \text{neutrinos}, \tag{37}$$

$$e^-e^- \longrightarrow W_k^- W_n^- \longrightarrow l_i^- + jet + \text{neutrinos}, \tag{38}$$

where i and j are the fermion flavors. The most clean signature is the case $i, j = e, \mu$ plus missing momentum carried away by neutrinos. In order to eliminate the major background coming from RC to elastic e^-e^- scattering we should demand $i \neq j$ or the sizable $p_\perp$ of outgoing leptons. The cut on $p_\perp$ helps to reduce the QED background caused by radiative pair production

$$e^-e^- \longrightarrow e^-e^-\mu^-\mu^+\tau^-\tau^+. \tag{39}$$

Other relevant background processes are

$$e^-e^- \longrightarrow e^-e^- Z_k \longrightarrow e^-e^- l_i^- l_i^+, \tag{40}$$

$$e^-e^- \longrightarrow e^-e^- Z_k \longrightarrow e^-e^- \nu_l \overline{\nu_l}, \tag{41}$$

and

$$e^-e^- \longrightarrow e^-e^- W_k^- W_n^+ \longrightarrow e^-e^- l_i^- l_j^+ \nu_i \overline{\nu_j}. \tag{42}$$

The main difference between the final states of the reactions (37)and (38) and (40)–(42) is that in the former case they consist of like-sign leptons pair only. Therefore, it is easy to distinguish the W^-W^- signal from these backgrounds by means of the charged final-state leptons identification. We also have the background caused by multihadronic events

$$e^-e^- \longrightarrow e^-e^- W_k^- W_n^+ q_i \overline{q_i}. \tag{43}$$

For this reaction there is no possibility to observe the high $p_\perp$ of the final like-sign nonelectron leptons. Hence, we conclude that the cut on $p_\perp$ and the charged lepton identification of the outgoing leptons will be very useful in order to reduce backgrounds for the $W_k^- W_n^-$ signal.

Now let us consider the process

$$e^+e^- \longrightarrow W_k^- W_n^+ \quad (k, n = 1, 2). \tag{44}$$

The cross section of reaction (44) does not depend on the assumed (Dirac or Majorana) nature of the neutrino. This reaction proceeds through the s and t channels. All the couplings of the electroweak interaction will enter here. As a result the violating unitarity terms which enter the partial cross sections are canceled themselves and the asymptotic behavior of the total cross sections defined by expression (36). The advantage of reaction (44) compared with (23) is the possibility to measure the trilinear gauge boson couplings (electromagnetic and weak). The investigation of the weak trilinear gauge boson couplings can be studied for $k \neq n$ because in this case the channel connected with the γ exchange is closed. At $\alpha = 0$ the reaction (44) has been considered in [2b] where one was shown that the total cross section $\sigma_{\lambda\overline{\lambda}}^{(kn)}$ where λ and $\overline{\lambda}$ are the helicities of the electron and positron, respectively, is sensitive to the m_N variations. So we shall also concentrate on $\sigma_{\lambda\overline{\lambda}}^{(kn)}$. The expression for the total cross section of the reaction (44) follows from the corresponding one of Ref. [2b] by the substitutions

$$\mathcal{D}_2^{(kn)} \longrightarrow \mathcal{D}_2^{(kn)} \cos^2\alpha + \mathcal{D}_{2\nu}^{(kn)} \sin^2\alpha\,, \tag{45}$$

$$\mathcal{D}_{2\nu}^{(kn)} \longrightarrow \mathcal{D}_{2\nu}^{(kn)} \cos^2\alpha + \mathcal{D}_2^{(kn)} \sin^2\alpha,$$

$$\mathcal{D}_{3\nu}^{(kn)} \longrightarrow \mathcal{D}_{3R}^{(kn)}(\alpha),$$

$$\mathcal{D}_3^{(kn)} \longrightarrow \mathcal{D}_{3L}^{(kn)}(\alpha),$$

where

$$\begin{aligned}
\mathcal{D}_{3R}^{(kn)}(\alpha) = {} & \frac{s}{m_{W_n}^2 m_{W_k}^2}\left(m_{W_n}^2 + m_{W_k}^2 + \frac{\beta^2 s}{12}\right) - 4 \\
& + \frac{2\ln L}{\beta s}\left((m_{W_n}^2 + m_{W_k}^2 - s)[1 - \cos^2\alpha(2 - \cos^2\alpha)] + \frac{m_{W_n}^2 m_{W_k}^2 \sin^2 2\alpha}{2m_N^2}\right) \\
& + \sin^2 2\alpha + \frac{m_N^2\cos^2\alpha}{m_{W_n}^2 m_{W_k}^2}\left(\frac{m_{W_n}^2 + m_{W_k}^2 - s}{2} - \frac{s m_N^2 (m_{W_n}^2 + m_{W_k}^2)\cos^2\alpha}{C_\nu} - m_N^2(1 + \cos^2\alpha)\right) \\
& + \frac{m_N^2 \cos^2\alpha \ln L_\nu}{m_{W_n}^2 m_{W_k}^2 \beta s}\left[C_\nu\left(1 + \frac{4m_{W_n}^2 m_{W_k}^2 \sin^2\alpha}{m_N^4}\right)\right. \\
& \left. + \frac{1}{2}\cos^2\alpha\left(m_N^2 + \frac{4m_{W_n}^2 m_{W_k}^2}{m_N^2}\right)(m_{W_n}^2 + m_{W_k}^2 - s - 2m_N^2) + 2s(m_{W_n}^2 + m_{W_k}^2)\right],
\end{aligned}$$

$$\mathcal{D}_{3L}^{(kn)}(\alpha) = \mathcal{D}_{3R}^{(kn)}\left(\alpha + \frac{\pi}{2}\right).$$

We note that at $\alpha = 0$ the dependence on m_N could only be studied by the investigation of $\sigma^{(kn)}_{-1,1}$ [2], because for the $e^-_R e^+_L$ initial $(-\lambda = \overline{\lambda} = 1)$ the dominant channel with the ν_L exchange is closed and the contributions coming from the diagram with the ν_R exchange becomes observable. However, when we take into account the neutrino oscillation angle the situation is totally different. In this case both $\sigma^{(kn)}_{-1,1}$ and $\sigma^{(kn)}_{1,1}$ depend on m_N. The cross sections for initially LL and RR polarized e^-e^+ beams identically vanish and the unpolarized cross section $\sigma^{(kn)}$ is defined by

$$\sigma^{(kn)} = \tfrac{1}{4}(\sigma^{(kn)}_{1,-1} + \sigma^{(kn)}_{-1,1}). \tag{46}$$

Therefore, we can get information about m_N investigating the case of the initially unpolarized e^+e^- beams too.

In order to pursue the dependence of the total cross section on α and m_N it is useful to introduce the quantities

$$\delta^{(kn)}_{\lambda\overline{\lambda}}(\alpha) = \frac{\sigma^{(kn)}_{\lambda\overline{\lambda}}(\alpha) - \sigma^{(kn)}_{\lambda\overline{\lambda}}(0)}{\sqrt{\sigma^{(kn)}_{\lambda\overline{\lambda}}(0)}}\sqrt{LT}, \tag{47}$$

$$\delta_{\lambda\overline{\lambda}}(m_N) = \frac{(\sigma_{\lambda\overline{\lambda}})_{\rm SM} - \sigma^{(11)}_{\lambda\overline{\lambda}}}{\sqrt{(\sigma_{\lambda\overline{\lambda}})_{\rm SM}}}\sqrt{LT}, \tag{48}$$

where $\sigma^{(kn)}_{\lambda\overline{\lambda}}(\alpha)$ is the ALRM total cross section at the oscillation angle α, $(\sigma_{\lambda\overline{\lambda}})_{\rm SM}$ is the total cross section of the SM, and LT is the integrated luminosity. It is obvious that these quantities define the sensitivity of the experiment to the deviations caused by α and m_N and give the deviations expressed in the standard error units.

We start the analysis of the reaction (44) from the case of the $W^+_1W^-_1$ production. The value of $\delta^{(11)}_{\rm LR}(\alpha)$ is very small over the whole energy region at any m_N. In contrast, for the initial $e^-_R e^+_L$, $\delta^{(11)}_{RL}(\alpha)$ could be large enough to be detectable. In Fig. 6 we display $\delta^{(11)}_{RL}(\alpha)$ versus $\sqrt{s}$ for different values of m_N $(g_L = g_R)$. We see that $\delta^{(11)}_{RL}(\alpha)$ increases with the growth of m_N and at $m_N = 1$ TeV can reach 1.1σ. The degree of the longitudinal polarization of e^-e^+ beams at present is about 0.4. Changing λ $(\overline{\lambda})$ from -1 (1) till -0.4 (0.4) leads to the near vanishingly small values of $\delta^{(11)}_{\lambda\overline{\lambda}}(\alpha)$. So, the deviations connected with the oscillation angle will be statistically significant at very big m_N $(m_N > 1$ TeV), only.

In Fig. 7 we display $\delta_{\lambda\overline{\lambda}}(m_N)$ versus $\sqrt{s}$ at $m_N = 100$ and 1000 GeV $(g_L = g_R)$. From Fig. 7 it follows that in the case of the completely RL-polarized e^-e^+ beams we have a good chance to observe the clean signal of the LR symmetry at 2σ level (95% C.L.) in the LEP II energy region. The increase of the right-handed neutrino mass leads to the growth of $\delta_{\lambda\overline{\lambda}}(m_N)$. It is worth stressing that $\delta_{LR}(m_N)$ reaches its peak of energy at about 190 GeV and then starts to decrease till the Z_2 resonance.

The investigation $W^+_1W^-_2$ pair production does not give any information about α. As the analysis shows $\delta^{(12)}_{\lambda\overline{\lambda}}(\alpha)$ is very small at any values of m_N. In Fig. 8 we display $\sigma^{(12)}_{1,-1}$ and $\sigma^{(12)}_{-1,1}$ versus $\sqrt{s}$ for $m_N = 100$ and 1000 GeV $(g_L = g_R)$. The decrease of $\sigma^{(12)}_{-1,1}$ for a very massive right-handed neutrino is due to the fact that at $s \ll m_N^2$ the functions $\mathcal{D}^{(kn)}_{2\nu}$ and $\mathcal{D}^{(kn)}_{3R}$ decreases with the growth of m_N.

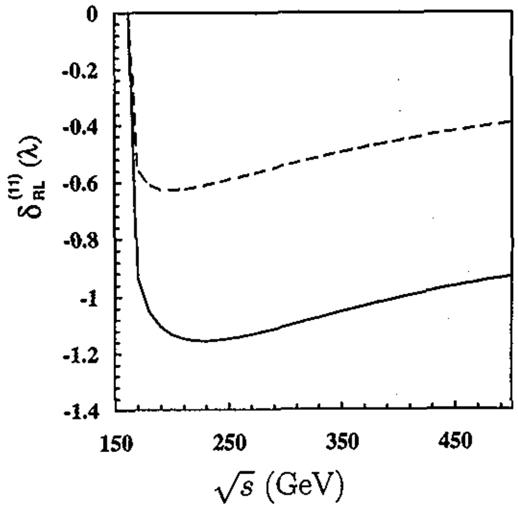

FIG. 6. $\delta^{(11)}_{RL}(\alpha)$ as function of $\sqrt{s}$ at $m_N = 1$ (dashed line) and $m_N = 1.4$ TeV (solid line).

In Fig. 9 we depict the total cross section of the reaction $e^+e^- \longrightarrow W^+_2W^-_2$ as a function of $\sqrt{s}$ for the different values of the right-handed neutrino masses. Again we are limited by consideration of the symmetric case. For the LR polarized e^+e^- beams the total cross section

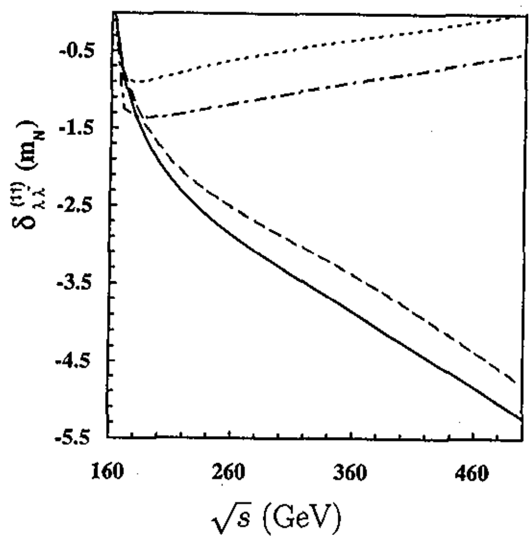

FIG. 7. $\delta^{(11)}_{\lambda\overline{\lambda}}(m_N)$ versus $\sqrt{s}$. The dotted (dash-dotted) line corresponds to the initial $e^-_L e^+_R$ at $m_N = 100$ (1000) GeV. The dashed (solid) one goes to the initial $e^-_R e^+_L$ at $m_N = 100$ (1000) GeV.

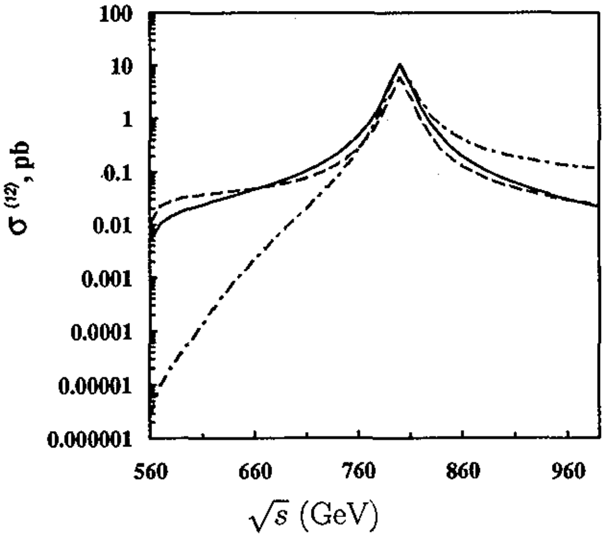

FIG. 8. The total cross section of the reaction $e^+e^- \longrightarrow W_1^+W_2^-$ versus $\sqrt{s}$: (a) at $m_N = 100$ GeV for $\lambda = -\bar{\lambda} = 1$—dashed line; (b) at $m_N = 100$ (1000) GeV for $-\lambda = \bar{\lambda} = 1$—solid (dash-dotted) line.

is smaller on the 2 orders of magnitude. We see that the decrease of the total cross section for the great values of m_N also takes place. In the case $m_N = 1$ TeV, $\sigma_{LR}^{(22)}$ reaches its maximum in an energy range of about 1.6 TeV and only after that starts to decrease accordingly to (36).

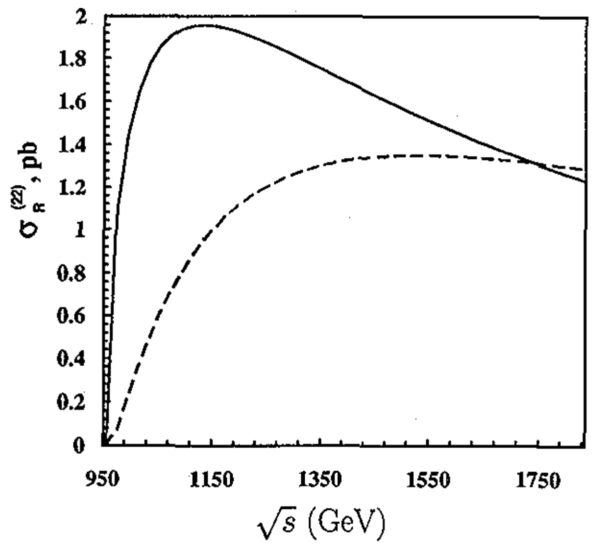

FIG. 9. The total cross section of the reaction $e^-e^+ \longrightarrow W_2^-W_2^+$ versus $\sqrt{s}$. The solid and dashed lines correspond to $-\lambda = \bar{\lambda} = 1$ at $m_N = 100$ and 1000 GeV, respectively.

IV. CONCLUSIONS

Within the asymmetric left-right model the electron *neutrino oscillations have been considered*. We have investigated the neutrino flux propagating through matter and magnetic fields. The neutrino behavior within the ALRM and the standard model of electroweak interactions is qualitatively similar but differs if we compare numbers (for the oscillation parameters). In particular the contributions coming from the $SU(2)_R$ sector of the extended model could be sizable. For example, for the convective zone of the Sun they could reach 10% of its $SU(2)_L$ counterpart. This fact is very important because the neutrino retain directional information on their sources of origin and can thus provide the detailed description of the stars interior.

It has been shown that it could be possible to observe the correlation between the neutrino flux and the solar flare events. This effect is determined by the values of the neutrino anapole moments and such characteristics of the electromagnetic field of the Sun as the longitudinal electric current and the twisting rate.

Analytical expressions of the cross section of the inverse neutrinoless double-β decay ($e^-e^- \longrightarrow W_k^-W_n^-$) in the case of the polarized e^- beams have been obtained within the framework of the ALRM. The cross section asymptotic behavior is in accordance with unitarity bounds. In the case $k = n = 1$ this reaction mainly goes for LR polarized electrons. As the SM predicts σ_{LR} being equal to zero the observation of the nonvanishing cross section will be a clean signal of the LR symmetry of the electroweak interaction. The obtained $\sigma_{LR}^{(11)}$ has maximum in the energy region of about 200 GeV. With the increase of m_N, $\sigma_{LR}^{(11)}$ grows and could reach values on the order of 0.3 pb at $m_N = 1.4$ TeV.

Concerning $e^-e^- \longrightarrow W_1^-W_2^-$, the leading contribution comes from LR-polarized electron beams. Again at the increasing m_N the enhancement mechanism of the total cross section takes place. At the chosen values of SP's the maximum of $\sigma^{(12)}$ is in an energy region ~ 700 GeV. Having reached its maximum the total cross section starts to decrease as $s^{-1}\ln s$.

The $e^-e^- \longrightarrow W_2^-W_2^-$ production is mainly due to RR-polarized electron beams. Its dependence on m_N is very similar to that for $\sigma^{(11)}$ and $\sigma^{(12)}$.

There is substantial uncertainty associated with the masses of $\Delta_{--}^{L,R}$ [29]. Its principle source is connected with the choice of the Higgs boson potential. Our analysis has shown that the influence of the values of $m_{\Delta_{L,R}}$ is negligible when

$$\sqrt{s} \neq m_{\Delta_{L,R}} \pm \frac{\Gamma_{\Delta_{L,R}}}{2}.$$

We should stress that the resonance enhancement of the inverse neutrinoless double-β decay takes place for the initially $e_L^-e_L^-$ and $e_R^-e_R^-$ and is absent in the case LR polarized e^- beam. As a final remark we remind the reader that the general feature of the reactions discussed above is their extreme sensitivity to such parameters of the ALRM as ξ, α, and g_R.

Turning to $e^+e^- \longrightarrow W_k^+W_n^-$ reaction, analogous investigation has shown that the dependence on m_N is also the general feature of the total cross section for all its final states while the dependence on α could not be measurable.

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