Bose-Einstein source of intermittency in hadronic interactions

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The multiparticle Bose-Einstein correlations are the source of "intermittency" in high energy hadronic collisions. The power-law-like increase of factorial moments with decreasing bin size is obtained by a complete event weighing technique with a Gaussian approximation of space-time particle emitting source shape. The value of the source size parameter is found to be higher than the common one fitted with the help of the standard Hanbury Brown–Twiss procedure.

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The use of an intensity interferometry to determine space-time sizes of the particle emitting source is a well-established technique of high energy physics. The standard method based on the Hanbury Brown–Twiss [1] (HBT) effect is to fit the Fourier transform of the source space-time density to the two-particle correlation function. The probability of finding one of the two emitted particles with momentum p_1 and the second with p_2 is given by

$$P_{\{12\}}=\int |\Psi(x_1,x_2;p_1,p_2)|^2\rho(x_1)\rho(x_2)d^4x_1d^4x_2, \quad (1)$$

where x_1 and x_2 are the four-positions of the emission points, each of them distributed in the "source" according to ρ. If the particles are bosons a symmetrization in the amplitude Ψ evaluation leads to the well-known formula

$$P_{\{12\}}\sim 1+|\mathscr{F}_{12}|^2, \quad (2)$$

where $\mathscr{F}_{12}$ can be related to the source distribution by

$$\mathscr{F}_{12}=\int e^{(iq_{ij}x)}\rho(x)d^4x, \quad q_{ij}=p_i-p_j. \quad (3)$$

There could also be other interpretations of $\mathscr{F}_{12}$, such as, for example, the one given in Ref. [2] derived on the basis of the relativistic string fragmentation picture.

The particular choice of the source space-time distribution (or, more general, the form of $\mathscr{F}_{12}$) leaves some degrees of freedom here, but the results do not depend very much on that choice. The most popular is the Gaussian in space and exponential in time emission source shape. However for the present work we choose the form of $\mathscr{F}_{12}$ which is known as a Gaussian parametrization for its simplicity and because it is Lorentz invariant,

$$\mathscr{F}_{12}=e^{-(Q_{ij}R_0)^2/2}, \quad Q_{ij}^2=-(p_i-p_j)^2, \quad (4)$$

which leads to the well-known formula for the two-particle correlation function:

$$C_2(Q^2)=1+e^{-(QR_0)^2}. \quad (5)$$

The R_0 can still be interpreted as a measure of the space-time extension of the emitting source.

The idea presented briefly above has been used extensively to analyze different high energy physics data since the first work [3] of Goldhaber *et al.* Since that time many experimental and theoretical efforts have been made. Different source shapes were examined; some fine effects were predicted. Some difficulties were also found in the interpretation of the source shape while the source is moving very fast with respect to the laboratory system. However the main idea of the HBT effect remains unchanged.

In the mid 1980's due to the work of Białas and Peschanski [4], new interest in the particle correlation arose. The phenomenon called "intermittency" was found in the very small phase-space bin size analysis. Since the first measurements the experimentally available smallest bin size has been reduced more than an order of magnitude but, what is even more important, new techniques to study fine structures have been developed. The "intermittency" of the particle creation process seen by Białas and Peschanski, which is in fact the fractal (self-scaled) behavior of the multiparticle correlation measured at very small phase-space scales, contradicts the standard Bose-Einstein statistics driven description given by Eqs. (2) and (3). The intermittent picture of hadronic creation was also inconsistent with existing models of particle production (such as, e.g., the Lund hadronization model). The intermittent models such as the α model [4,5], geometrical branching model [6], one-dimensional model of intermittency by Dias de Deus [7] were invented but none of them achieved such a completeness and predictivity as high energy physics standards [Lund- or dual-parton- (DP-)type models]. On the other hand, the treatment of the "intermittency" as a real new phenomenon was still not so obvious. In Ref. [8] different data sets were examined and as the last conclusion it is stated that the intermittency is caused by Bose-Einstein correlations *in addition* to a mechanism responsible for the power-law behavior, in Ref. [9] authors claim that the observed "intermittent-like" behavior of moments of multiplicity distributions can be understood as an effect driven by quantum statistical properties of the particle emitting system and it does not necessarily imply evidence for intermittency.

In Ref. [10] the EHS/NA22 Collaboration shows that Bose-Einstein correlations (with exponential parametrization of the $\mathscr{F}_{12}$ in the FRITIOF Monte Carlo event generator) introduced to the model calculations using the HBT picture described above can give very good reproduction of the like-charged two-particle correlator. The very careful analysis of different effects which can influence the particle correlation measurement shows that the NA22 data need no additional physical factors to achieve an agreement (within some reasonable accuracy limits) with the standard, local Bose-Einstein picture. Even "the cumulants of higher orders are strongly overestimated, especially at the smallest values of Q^2." It is interesting to note that the Dalitz decays and undetected γ conversion (estimated as $\sim 25\%$) can play an important role in a low order genuine correlator measurements for unlike-charged particles.

The exponential Bose-Einstein effect parametrization in Eq. (4) used in Ref. [10] originates from some relativistic string fragmentation model estimations (see, e.g., Refs. [11,2,12]). It is hard to interpret such a form on the basis of a HBT picture as a Fourier transform of the source space-time distribution and of course the meaning of the parameter is no longer the measure of the particle creation source size. Because of the different behavior at low four-momentum transfer values than the Goldhaber Gaussian parametrization the comparison with the power-law-like (intermittent) data point dependence favors of course the exponential picture. Anyhow, even if the exponential parametrization looks similar to the power-law picture of intermittency in a Q^2 region seen experimentally at present, it should differ for the smaller bin sizes. The question arises if it is possible at all to lower the bin size by about an order of magnitude to settle that problem. Anyhow, the title of the Ref. [13] "Has intermittency been observed in multi-particle production?" is a really good question still.

As it has been said, the existing data give the possibility to study intermittent (power-law) behavior of factorial moments in more than two decades wide phase-space distance measure (however it will be defined: rapidity, momentum or four-momentum difference, box volume, etc.). The data show more or less definite power-law-like dependence on the bin size. However there is also a very clear signal about the like and unlike charge difference of the correlation strength which suggests its Bose-Einstein origin. The possibility to achieve an agreement between those two, on the first sight contradicting, experimental facts will be discussed in the frame of common quantum physics.

It should be remembered that Eq. (2) was obtained in the case when only two particles were emitted from the source. That situation is of course different when one is dealing with a multiparticle source (it was mentioned by Cocconi in 1973 [14]) [15]. In some particular cases (when there are really a small number of particles emitted in the large phase-space volume) the two-particle correlator given by Eq. (2) still can be used as at least a first approximation. But when one wants to look closely at the high multiplicity events or to study multiparticle correlations Eq. (2) has to be modified.

When n identical bosons are emitted the probability of the particular momenta configuration $\{p_i\}$ is given by

$$P_{\{n\}} \sim \sum_{\sigma} \mathscr{F}_{1\sigma(1)} \mathscr{F}_{2\sigma(2)} \cdots \mathscr{F}_{n\sigma(n)}, \tag{6}$$

where σ is a permutation of a sequence $\{1,2,\ldots,n\}$, $\sigma(i)$ is the ith element of this permutation, and the sum is over all $n!$ permutations.

To see what real difference is introduced by such a complete treatment, the two-particle correlation function such as that in Eq. (2) in the case of a three-particle emitting source is written explicitly below:

$$P_3 \sim 1 + |\mathscr{F}_{12}|^2 + |\mathscr{F}_{13}|^2 + |\mathscr{F}_{23}|^2 + 2|\mathscr{F}_{12}||\mathscr{F}_{13}||\mathscr{F}_{23}|. \tag{7}$$

If all three particles are very close to each other the statistical weight of such events tends to $6 = n!$. The limit for a two-particle correlator in an n-particle emitting source is $n!$ not 2 as it comes from Eq. (2). The same limit was obtained in Ref. [9] but it was interpreted as a limit of an nth factorial moment. The multiplicity distribution in the very small phase-space bin tends to the geometrical one which, on the other side, can be treated as a Bose-Einstein statistics driven multiplicity distribution while $n \to \infty$, $\delta \to 0$ with $n\,\delta = \text{const}$.

A quite different approach to the Bose-Einstein phenomenon is discussed in Ref. [16]. The authors argued for the local nature of the Bose-Einstein effect. In general, their treatments lead to the weighing procedure with the event probability proportional to

$$P'_{\{n\}} \sim \sum_{\text{all pairs}} (1 + |\mathscr{F}'_{ij}|^2). \tag{8}$$

The definition of $\mathscr{F}'$ in Eq. (8) is not given by Eq. (4) but is based on a string fragmentation picture. However the difference is rather in the physical interpretation than in the general behavior. It should be noted that Eq. (8) overestimates the very close particle limit. There it is equal to $2^{n(n-1)/2}$. The arguments for such a treatment are discussed in Ref. [16] (a similar attempt is presented in Ref. [12]) and will not be discussed here. One of the arguments not given there but of practical importance is that the above idea can be easily incorporated into the Monte Carlo event generator. It was, in fact, done in the LUBOEI subroutine which is a part of the Lund hadronization scheme JETSET 7.3. The general difference between the Ref. [16] strategy and that proposed in the present paper is in the fact that the sum in Eq. (8) is performed over permutations of the particle ensemble in which only two particles are exchanged (locality of Bose-Einstein interaction) while in our treatment all event permutations can give a contribution to the event weight (global Bose-Einstein approach). The importance of many-particle exchange contributions will be discussed later on.

The problem with the complete weighing procedure is also a practical one. The sum over $n!$ elements can be performed easily for about ten particles or less. For higher multiplicities the calculation time rises tremendously. But it is quite clear that for the two very distant particle exchanges the contribution coming from all permutations concerning that particular exchange is negligible. The algorithm was invented to omit all the negligible permutations and calculations of the weights according to Eq. (6) became possible also for larger multiplicities. In the present paper only the data from the NA22 experiment will be analyzed. The mean charged particle multiplicity is of order of 8 and the largest like-type boson multiplicity (in one chain, as will be discussed later) in the sample of about 500 000 of our Monte Carlo (MC) generated events do not exceed 15.

To study the influence of the Bose-Einstein weighing method on the shape of the fluctuations in small bins the sample of events in the "world of absence of Bose-Einstein correlation" is needed. There is a number of Monte Carlo generators which can be used to get this. In the present work, the one called the geometrical two-chain was used. It is described in detail in Ref. [17]. The important difference between that generator and the other is that the hadronization is not a branching process. The advantage of that generator is the minimum of correlations introduced there. The ones existing are due to the conservation requirements (charge, baryon number, strangeness, momentum, and energy), the resonance production, and the large scale clustering due to chain mass distribution in the model. There are also correlations connected with the hadronization procedure adapted: the transverse momentum is conserved locally in the fragmenting chains so the subsequent hadrons are inclined to have the negatively correlated momenta perpendicular to the interaction axis. Our chain fragmentation picture also leads to ordering in rapidity of subsequently produced hadrons. All those features are present in most of the models working on the partonic level. The last but very well seen specially for large bin sizes is a contribution related to non-Poissonian multiplicity distribution in the multiparticle production.

The main interaction characteristics are very well reproducible by the generator as was shown in Ref. [17].

About 500 000 nonsingle-diffractive events for π^+ and K^+ interactions with proton at laboratory momentum of 250 GeV/c were generated and combined to get the reference sample without Bose-Einstein correlations included. Then for each event the weight was calculated according to Eqs. (4) and (6). In principle the Bose-Einstein weighing procedure could change the multiplicity distribution (which was one of the arguments against global treatment of Bose-Einstein correlation in Ref. [16]). To avoid this the weights were renormalized to get the average value of the weights for n identical bosons equal to 1 and these were used afterward. The detailed comparison with the experimental data leads to the conclusion that if the Bose-Einstein symmetrization were performed for the whole events then the correlations are too strong for very small bin sizes. In our model there is only one parameter to be adjusted, correlation radius R_0, while in the standard HBT procedure there is also the incoherence parameter which allows one to make softer the correlation strength. In the geometrical two-chain model particles are produced by the fragmentation of two well-defined chains so there is a natural subdivision of all secondary particles to two distinct classes. To make the correlation weaker there is the possibility of symmetrizing amplitudes Ψ not over all particle exchanges but only over the exchanges of the particles produced from the same chain.

The very convenient variable to study the two-particle correlation is the differential form of the second factorial moment as was used in Ref. [18]. The definition using the density integral method [19] is

$$D_2(Q^2)=\frac{1}{\text{norm}}\,2\sum_{i<j}\,\Theta(Q^2-Q_{ij}^2)\Theta(Q_{ij}^2-Q^2+\delta),\quad (9)$$

where Θ is the Heaviside unit step function and norm is a normalization term defined by the so-called "mixed events" technique. The particles used for the normalization were chosen randomly from the all event "pool" of the large number of generated interactions, ensuring that they belong to different real events. To avoid in the reference sample the correlations due to a non-Poissonian multiplicity distribution in hadronic interactions, the multiplicity in the mixed events was taken from the Poisson distribution with the average value the same as in MC generated events.

In the particular NA22 experimental data which we want to compare with the rapidity cut $|y|<2$ has been used. Thus in all the calculations the same cut is applied. In the experimental procedure it was also not possible, in general, to determine the particle masses, so all the particles (except low energy proton and very energetic particles in K^+-induced interactions $p_{\text{lab}}>150$ GeV/c) were treated as pions. The same procedure has been used in our analysis of the Monte Carlo events. The experimental accuracy of the particle four-momentum difference determination described in Ref. [20] was taken into account in the calculations as well. The calculations of D_2 were performed for all charged particles as well as for like and unlike charge combinations. The results are presented in Fig. 1 by the solid line. The remaining correlations produced in the geometrical two-chain model, which were indicated above, lead to the outcome depicted by the dotted line. It represents the result of the correlation calculations without Bose-Einstein weighing.

It is seen that the power-law-like behavior of $D_2(q^2)$ is quite well reproduced by our weighing method. The small overestimation of the unlike particle correlator at a four-momentum difference of about $Q^2\sim10^{-2}$–10^{-1} (GeV/c)^2 is a consequence of the strict ordering in rapidity of the chain fragmentation products which always introduce between close (in rapidity) like type charged hadron the one with the opposite sign. The four-momentum difference of that unlike charged pair is determined by transverse momentum distributions so the effect does not influence the very small bin size analysis.

However, the main argument for intermittency comes from the analysis of the higher multiplicity correlation measurements. To study this effect the correlation measures have to be defined for three- and more particle systems. The most commonly used variables are the factorial moments. For practical purposes the best method of factorial moment calculations is again the one proposed in the Ref. [19] density integral method:

$$F_q(Q^2)=\frac{1}{\text{norm}}q!\sum_{i(1)<i(2)<\cdots<i(q)}\ \prod_{\text{all pairs }(i(k1),i(k2))}\times\Theta(Q^2-Q_{i(k1)i(k2)}^2),\quad (10)$$

with the normalization by the mixed event technique again. Results of our calculations are presented in Fig. 2. The power-law-like increase of factorial moments with decreasing bin size is again quite well reproduced in the whole range of Q^2 measured experimentally.

In Figs. 1 and 2 the results of event weighing defined by Eqs. (4) and (6) with the sum over permutations with only one particle pair exchange are also presented (by the dashed line). As it is seen the effect [for the same value of the R_0 parameter in Eq. (4)] is much weaker. This illustrates the

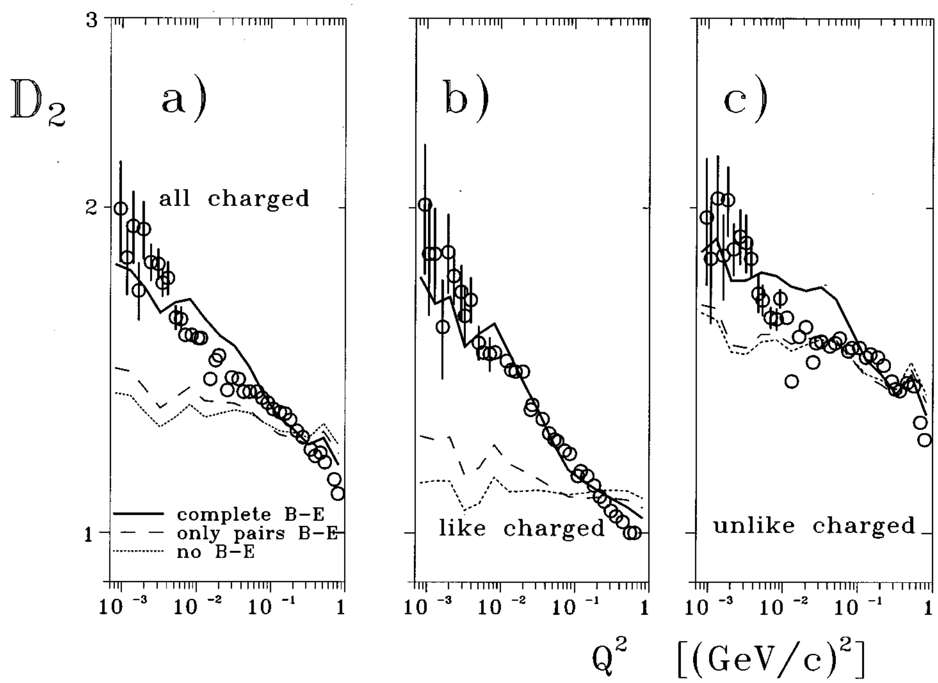

FIG. 1. The differential second factorial moments for (a) all charged, (b) like-charged, and (c) unlike-charged pairs as a function of four-momentum difference. The data points are from the NA22 experiment. The solid line represents the result of our complete Bose-Einstein weight method, the dotted line shows the correlations used in the geometrical two-chain model. The dashed line is for the sum in Eq. (6) over only one pair of boson exchanges.

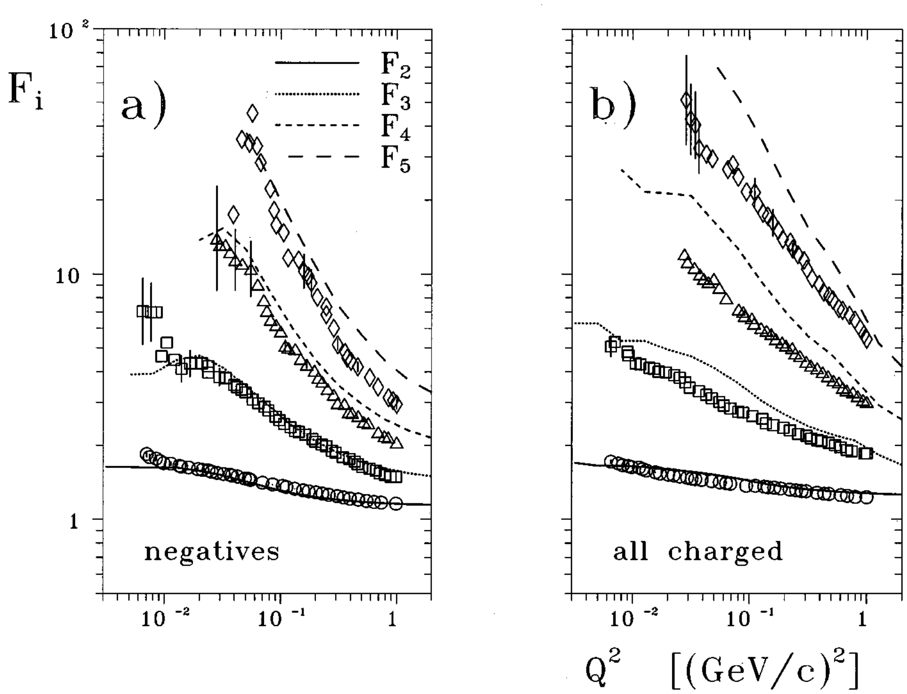

FIG. 2. The factorial moments (a) for negatives and (b) for all charged particles as a function of the four-momentum difference. The data points are from the NA22 experiment.

importance of global treatment of the Bose-Einstein correlation. The introduction to the sum of the weights of very many relatively small terms leads to a really great increase of the effect.

To reproduce the shape of $D_2(Q^2)$ and $F_2(Q^2)$ dependencies measured by the NA22 experiment the value of the parameter R_0 in Eq. (4) had to be adjusted. The large statistical fluctuations of the weights influence the estimation of the source size parameter so the accuracy achieved is not higher than 10%. In Ref. [18] the source size was found using the standard technique of the HBT effect [Eq. (5)]. The value found there was (0.82 ± 0.02) fm. Our complete weighing procedure gives stronger correlations (even after weights renormalization) so the value of R_0 used to obtain the results given in Figs. 1 and 2 is about 50% higher, which gives a source radius of about 1.25 fm in the Gaussian approximation [Eq. (4)] interpretation.

To summarize, the importance of the global treatment of the Bose-Einstein correlation has been shown. The symmetrization over all permutations leads to the power-law-like behavior of factorial moments in the four-momentum difference regions where they are measured experimentally. However it is still an open question whether a working local treatment is possible, since NA22 can obtain a comparable description of their data with a local relativistic string fragmentation inspired treatment of Bose-Einstein effects [10]. The more detailed analysis is in progress and the results will be presented elsewhere.

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