Oscillatory behavior of cumulant moments in e^+e^- collisions and stochastic processes

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Cumulant moments of negatively charged and charged particles observed in e^+e^- collisions are analyzed by the modified negative binomial distribution (MNBD). The cumulant moment of observed negatively charged particles shows an oscillatory behavior more regularly than that of the charged particles as the rank of the moment increases. Those behaviors are well described by the MNBD. The characteristics of the MNBD are discussed comparing them with those of the negative binomial distribution.

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INTRODUCTION

Recently it was shown that the cumulant moments of the QCD multiplicity distribution possess oscillating behavior [1]. Furthermore, analyses of the cumulant moments in hadron-hadron (hh) and e^+e^- collisions show that the jth rank normalized cumulant moment of observed charged multiplicity distributions oscillates irregularly around the zero with increasing the rank j [2]. However, the oscillatory behavior of the data cannot be described by the formula obtained from QCD.

Multiplicity distributions in hh and e^+e^- collisions are well described by the solutions of stochastic processes, for example, the negative binomial distribution (NBD) and the Peřina-McGill (PM) formula in Refs. [3] and [4]. The jth rank normalized cumulant moments of the NBD and the PM formula are positive, and decrease with increasing the rank j. Therefore the behavior of the cumulant moments obtained from the experimental data puts a new constraint on the model of multiplicity distributions.

On the other hand, the multiplicity distribution derived from the pure birth process under the initial condition of the binomial distribution is applied to the analyses of the multiplicity distributions in Refs. [4] and [5]. The distribution is called the modified negative binomial distribution (MNBD) [5]. In Ref. [6], cumulant moments of negatively charged particles in e^+e^- collisions are analyzed by the NBD and the MNBD up to the fourth rank. It is suggested in [6] that the cumulant moments calculated from the data change signs and cumulant moments calculated from the MNBD well reproduce the qualitative behavior of the data.

In this paper, the formula for the cumulant moments of the MNBD is obtained from the generating function. The possibility is pointed out that the jth rank cumulant moment of the MNBD changes the sign whether the rank j is odd or even. We also analyze the cumulant moments of negatively charged particles and of charged particles up to the 12th rank, and investigate whether the oscillatory behavior of the cumulant moments of the observed multiplicity distributions is described by the MNBD.

The generating function $\Pi(z)$ of the multiplicity distribution $P(n)$ is defined by

$$\Pi(z)=\sum_{n=0} P(n)z^n. \tag{1}$$

The factorial moment is given by Eq. (1) as

$$f_j=\langle n(n-1)\cdots(n-j+1)\rangle=\left.\frac{d^j\Pi(z)}{dz^j}\right|_{z=1}.$$

The cumulant moment is given by

$$\kappa_j=\left.\frac{d^jH(z)}{dz^j}\right|_{z=1}, \tag{2}$$

where

$$H(z)=\ln\Pi(z). \tag{3}$$

From Eq. (3), we have the relation between the normalized cumulant moments and the normalized factorial moments:

$$K_1=F_1=1,$$

$$K_j=F_j-\sum_{m=1}^{j-1} {}_{j-1}C_{m-1}F_{j-m}K_m, \quad j=2,3,\ \ldots, \tag{4}$$

where

$$K_j=\frac{\kappa_j}{\langle n\rangle^j}, \quad F_j=\frac{\langle(n(n-1)\cdots(n-j+1)\rangle}{\langle n\rangle^j}.$$

For example, the cumulant moments are written explicitly up to the third rank:

$$\kappa_1=f_1=\langle n\rangle,$$

$$\kappa_2=f_2-\langle n\rangle^2=\langle n(n-1)\rangle-\langle n\rangle^2,$$

$$\kappa_3=f_3-2f_2\langle n\rangle+\langle n\rangle^3.$$

CUMULANT MOMENT OF THE MNBD

The MNBD is derived from the pure birth process with the binomial distribution as the initial condition [4]. The generating function of the MNBD is written as

$$\Pi(z)=\left(\frac{1-r_1(z-1)}{1-r_2(z-1)}\right)^N, \tag{5}$$

where N is an integer, r_1 is real, and $r_2>0$. (See Refs. [4] and [5].)

The MNBD is obtained from $\Pi(z)$ as

$$P(0)=\Pi(0)=\left(\frac{1+r_1}{1+r_2}\right)^N,$$

$$P(n)=\frac{1}{n!}\left.\frac{\partial^n\Pi(z)}{\partial z^n}\right|_{z=0}$$
$$=\frac{1}{n!}\left(\frac{r_1}{r_2}\right)^N\sum_{j=1}^{N}{}_NC_j\frac{\Gamma(n+j)}{\Gamma(j)}\left(\frac{r_2-r_1}{r_1}\right)^j\frac{r_2^n}{(1+r_2)^{n+j}},$$
$$n=1,2,\ldots. \quad (6)$$

Using the equation

$$H(z)=\ln\Pi(z)=N\ \ln[1-r_1(z-1)]-N\ \ln[1-r_2(z-1)], \quad (7)$$

we have the jth rank cumulant moment,

$$\kappa_j=\left.\frac{\partial^jH(z)}{\partial z^j}\right|_{z=1}=-(j-1)!Nr_1^j+(j-1)!Nr_2^j,$$
$$j=1,2,\ldots. \quad (8)$$

If $r_1<0$, κ_j given by Eq. (8) shows oscillatory behavior with increasing the rank j. The first and the second cumulant moments are given from Eq. (8) by

$$\kappa_1=\langle n\rangle=N(r_2-r_1),$$
$$\kappa_2=\langle n(n-1)\rangle-\langle n\rangle^2=Nr_2^2-Nr_1^2. \quad (9)$$

Then, the parameters r_1 and r_2 are expressed as

$$r_1=\frac{1}{2}\left(F_2-1-\frac{1}{N}\right)\langle n\rangle,$$
$$r_2=\frac{1}{2}\left(F_2-1+\frac{1}{N}\right)\langle n\rangle. \quad (10)$$

The equations obtained in the above discussions are applied for the analysis of negatively charged particles.

The generating function $H_c(w)$ for cumulant moments of charged particles is given by that of negatively charged particles as $H_c(w)=H(w^2)$. Using this identity, we have the relation for cumulant moments κ_j^{c} $(j=1,2,\ldots)$ of charged particles with those of negatively charged particles as

$$\kappa_1^{\mathrm{c}}=2\kappa_1,$$
$$\kappa_2^{\mathrm{c}}=2(2\kappa_2+\kappa_1),$$
$$\kappa_3^{\mathrm{c}}=4(2\kappa_3+3\kappa_2),$$
$$\kappa_4^{\mathrm{c}}=4(4\kappa_4+12\kappa_3+3\kappa_2),$$
$$\kappa_5^{\mathrm{c}}=8(4\kappa_5+20\kappa_4+15\kappa_3).$$

TABLE I. The parameters of the MNBD used in the analysis of the cumulant moments in e^+e^- collisions.

	$\sqrt{s}$ (GeV)	N	r_1	r_2	$n_{\max}$	χ^2
TASSO	34.8	7	−0.6692	0.3005	18	14
DELPHI	91.0	7	−0.7745	0.7378	26	30
OPAL	91.0	7	−0.7166	0.6695	26	5.8
SLD	91.0	8	−0.6785	0.6115	27	

In general, the above equations are obtained from the recurrence equation

$$\kappa_j^c=\sum_{m=0}^{[j/2]}a_{j,j-m}\kappa_{j-m},\quad j=1,2,\ldots,$$
$$a_{1,1}=2,$$
$$a_{j+1,j+1}=2a_{j,j},$$
$$a_{j+1,j-m}=(j-2m)a_{j,j-m}+2a_{j,j-m-1}, \quad (11)$$

where $[j/2]$ is the largest integer not greater than $j/2$.

ANALYSIS OF THE EXPERIMENTAL DATA

The cumulant moments of observed negatively charged particles and charged particles are analyzed in this paper. The parameters N, r_1, and r_2 used in the analysis are shown in Table I. Those are determined to get the best fit of the MNBD to the observed multiplicity distributions of negatively charged particles. The values of parameter r_1 cited in Table I are always negative; then, we can expect from Eq. (8), the oscillatory behavior of cumulant moments of negatively charged particles is reproduced by the MNBD.

At first, using Eq. (8) and the parameters in Table I, we calculate the cumulant moments for negatively charged particles. The results show oscillatory behavior around zero as the rank of the moment increases because $r_1<0$ and $|r_1|>r_2$; if the rank is even (odd), the moment is positive (negative). Qualitative features of the data are described by the calculations. However, calculated results by Eqs. (8) and (11) for the charged particles are positive and decrease monotonously with increasing the rank. Therefore, the oscillatory behavior of the data cannot be reproduced by the calculations with Eq. (8).

Next, the factorial moments of negatively charged particles are calculated by the use of Eq. (6) as

$$f_j=\sum_{n}^{n_{\max}}n(n-1)\cdots(n-j+1)P(n),\quad j=1,2,\ldots, \quad (12)$$

where $n_{\max}$ is the maximum multiplicity of observed negatively charged particles. Then, cumulant moments of negatively charged particles are calculated from Eq. (4). The cumulant moments of charged particles are calculated from those of negatively charged particles from Eq. (11). It should be noted that Eq. (4) is also applicable for charged particles, and that cumulant moments of charged particles can be also calculated from

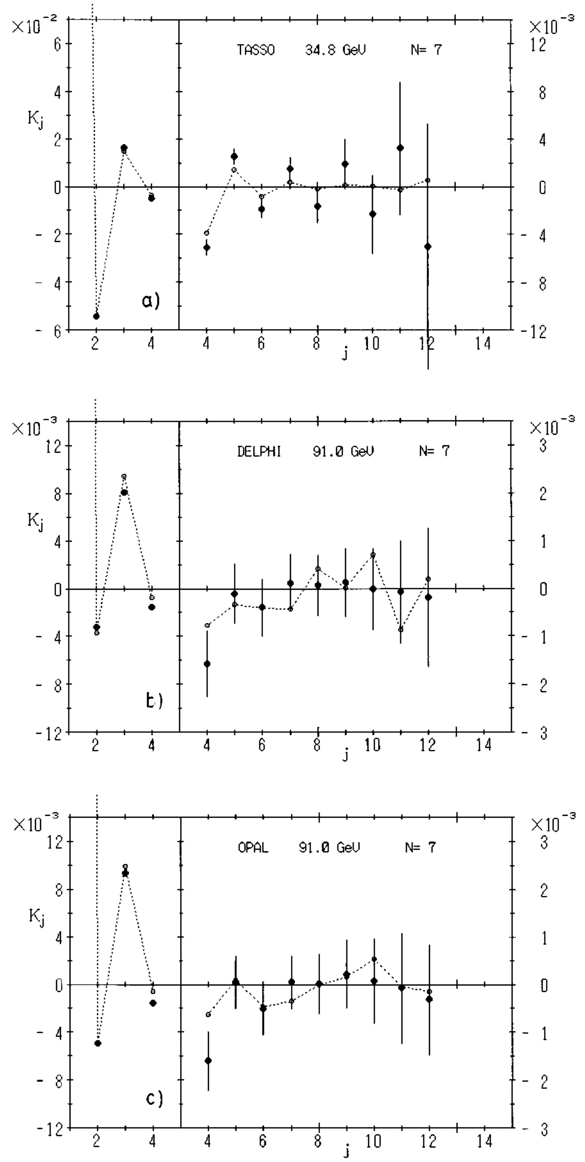

FIG. 1. The normalized cumulant moments of negatively charged particles in e^+e^- collisions. The full circles are calculated from the data in Refs. [7], [8], and [9]. The dashed lines are our calculations. (a) Data in [7] are analyzed. (b) The same as (a) but [8]. (c) The same as (a) but [9].

$$f_j^{\mathrm{ch}}=\sum_{n}^{n_{\max}} 2n(2n-1)\cdots(2n-j+1)P(n)$$

and Eq. (4).

The cumulant moments calculated from the MNBD are compared with those of negatively charged particles for the TASSO [7], the DELPHI [8], and the OPAL [9] Collaborations, respectively, in Figs. 1(a), 1(b), and 1(c). Calculated results well reproduce the data. We find that both the data and our calculations oscillate regularly up to the 7th rank; the value of the even rank moments are smaller than two adjacent odd rank moments. In Fig. 2, calculated cumulant moments for charged particles are compared with those obtained from the data. Our calculations for charged particles oscillate irregularly around zero, and reproduce the gross features of the data.

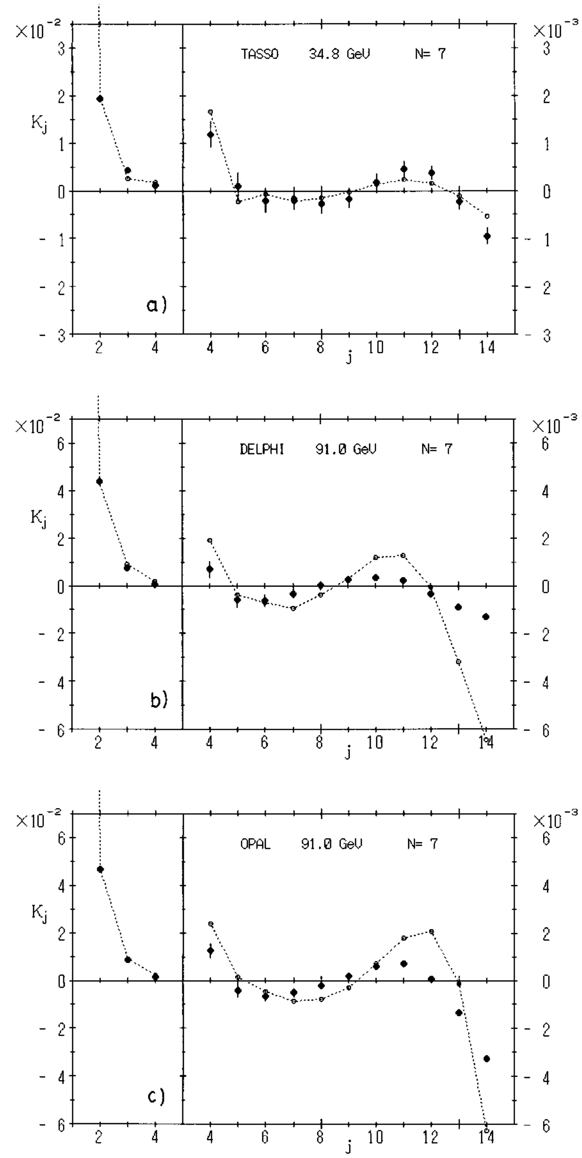

FIG. 2. The normalized cumulant moments of charged particles in e^+e^- collisions. The full circles are calculated from the data in Refs. [7], [8], and [9]. The dashed lines are our calculations. (a) Data in [7] are analyzed. (b) The same as (a) but [8]. (c) The same as (a) but [9].

CONCLUDING REMARKS

The cumulant moments of observed multiplicity distributions of negatively charged particles and charged particles in e^+e^- collisions are analyzed by the MNBD. The cumulant moments of observed negatively charged particles oscillate rather regularly as the rank of the moments increases. Its behavior is well described by the results calculated by means of the MNBD. The cumulant moments of observed charged particles also oscillate around zero as the rank increases, and the gross features of the data are also reproduced by the MNBD.

Very recently, the SLD collaboration has reported the new data on cumulant moments of charged particles in e^+e^- collisions [10]. In Fig. 3, the data of the cumulant moment H_j, which is defined by $H_j=K_j/F_j$ $(j=1,2,\ldots)$, are compared with our calculations. Parameters are also listed in

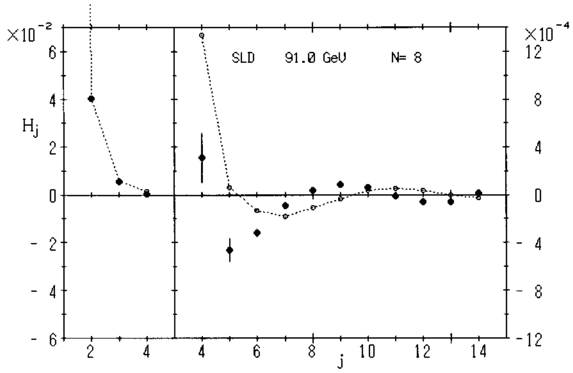

FIG. 3. The cumulant moments $H_j = K_j / F_j$ $(j = 1,2, \ldots)$ of charged particles in e^+e^- collisions. The full circles are obtained from the data [10]. The dashed line is calculated obtained from our calculations.

Table I. Parameter N is taken as $N=8$. Parameters r_1 and r_2 are determined by the observed average multiplicity and the observed second cumulant moment with Eq. (10). The oscillatory behavior of H_j moment is also described by the MNBD at least qualitatively.

It is pointed out in Ref. [11] that if the NBD is truncated at the maximum value of the observed multiplicities, cumulant moments calculated from it also show oscillatory behavior. However, the effect of truncation in e^+e^- collisions seems to be negligibly small compared to the data.

Here we discuss the MNBD, comparing with the NBD. These two distributions can be interpreted in the cluster picture. The NBD is obtained from the birth process with the immigration under the initial condition $P(n,t=0) = \delta_{n0}$ [4]. It is interpreted that the number distribution of clusters is given by Poisson distribution, and the distribution of particles from a cluster is given by Fisher's logarithmic distribution.

On the other hand, the MNBD is derived from the pure birth process with the binomial distribution as the initial condition; the number distribution of clusters is the binomial distribution, and particles are emitted under the geometric distribution from each cluster [4].

In the NBD, the maximum number of clusters becomes infinite, which should violate the energy-momentum conservation. However, in the MNBD, the maximum number of clusters is limited up to the finite number N, and the energy-momentum conservation is effectively taken into account.

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