Diffractive light vector meson production at large momentum transfers

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The diffractive process $\gamma^*(Q^2)p \to V+X$ (where $V=\rho^0,\omega,\phi$ are the vector mesons, consisting of light quarks, and X represents the hadrons to which a proton dissociates) is studied. We consider the region of large momentum transfers $|t| \gg \Lambda^2_{\mathrm{QCD}}$, and large energies s. In the leading logarithm approximation of perturbative QCD (using the BFKL equation) the asymptotic behavior of the cross section in the limit $s\to\infty, s\gg|t|,Q^2$ is obtained. We compare the results derived from the BFKL equation with that obtained in the lowest order of QCD (two-gluon exchange in the t channel). The possibility to investigate these reactions at DESY HERA is discussed.

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I. INTRODUCTION

The processes of diffractive production of light neutral mesons in photon-proton collisions have attracted the attention of both theorists and experimenters for a long time. In particular, this interest has increased recently, in connection with the possibility to study these processes at the DESY *ep* collider HERA in the region of larger center-of-mass energies and virtualities of incoming photons as compared to the previous fixed target experiments.

At sufficiently large energies, diffractive interactions are usually considered in the frame of Regge theory [1]. It is expected that the diffractive production of neutral mesons can be described in terms of the exchange of a Pomeron, the object with the quantum numbers of the vacuum. However, the nature of a Pomeron now is not clear. As was established, the phenomenology based on the exchange of the Pomeron with the intercept of 1.08 (the soft Pomeron) describes well the data on the energy dependence of hadron-hadron total cross sections and the variety of data on soft diffractive processes. However, there is another picture of the Pomeron [the hard or Balitskii-Fadin-Kuraev-Lipatov (BFKL) Pomeron], which was derived in perturbative QCD (*p*QCD), see Refs. [2–5]. Modern experiments at HERA support the idea that the physics of hard diffractive interactions can be understood in the frame of the hard Pomeron concept. Therefore, to clarify the nature of the Pomeron, it seems important to obtain the predictions of perturbative QCD (*p*QCD) for various diffractive processes and to compare them with the experiment.

In this paper we study in perturbative QCD the inelastic diffractive process of light vector meson production ($V=\rho^0,\omega,\phi$) in $\gamma^*(Q^2)p$ interactions:

$$\gamma^* p \to VX, \tag{1}$$

where the meson V and the hadrons X to which the proton dissociates are divided by a large rapidity gap. Q^2 is the virtuality of the photon. The process (1) will be considered in the region of large momentum transfers, $|t|\gg\Lambda^2_{\mathrm{QCD}}$, and sufficiently large center-of-mass energies, $s\gg|t|,Q^2$. We will discuss both the diffractive photoproduction [process (1) at $Q^2=0$] and the diffractive process of meson production in the deep inelastic scattering for an arbitrary relation between Q^2 and $|t|$.

The common interest, as a rule, is related to the elastic process $\gamma^* p\to Vp$ in the region of low momentum transfers initiated by real or virtual photon. It was studied in various frameworks in Refs. [6–10]. Recently, the results of investigation of this process at HERA were reported for the case of photoproduction of the ρ^0 meson [11] as well as for the case of ρ^0 production in deep inelastic scattering [12]. The most impressive result of the experiment [12] is the observed strong rise with increasing energy of the cross section of ρ^0 meson production initiated by virtual photon, when compared with the data at lower energy. This behavior cannot be explained by production through soft Pomeron exchange.

The cross section of the elastic process $\gamma^* p\to Vp$ decreases strongly with the growth of transferred momentum. In the region of large momentum transfers the inelastic diffractive process (1) is more probable than the elastic one. Our estimates give not too small cross sections for process (1), and we hope that it can be really measured in the future at HERA. The investigation of the process (1) in a wide range of parameters $(s,|t|,Q^2)$ should give us important insight into the nature of hard diffractive interactions; it opens the possibility to examine the hard Pomeron trajectory away from $t=0$.

We reason that the transferred momentum (or transferred momentum and virtuality of photon) is large enough and therefore the applicability of perturbative QCD to the description of the process (1) is beyond doubt. However, perturbative expansion for the hard scattering process can be not trivial. The perturbative series in the discussed kinematical region contain two parameters $\alpha_s(\Lambda^2)\ll 1$ and $\alpha_s(\Lambda^2)\ln(s/\Lambda^2)$. Here, $\Lambda^2\sim|t|,Q^2$ is the hard scale for process (1). Since the parameter $\alpha_s \ln(s/\Lambda^2)$ can be not small, there is a necessity of perturbative series summation.

Process (1) has the vacuum quantum numbers in the t channel. The summation of leading logarithms $[\alpha_s \ln(s)]$ in the vacuum channel leads to the famous BFKL equation, which describes the perturbative (hard) QCD Pomeron in the leading logarithm approximation (LLA); see Refs. [2–5].

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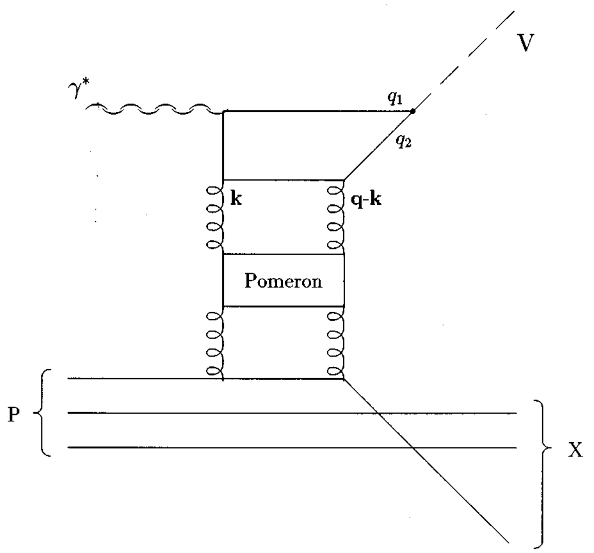

FIG. 1. One of the diagrams that describes the process of diffractive vector meson production at large momentum transfer. Other diagrams are obtained by all the possible transpositions of the gluon's lines.

Using the solution of the BFKL equation, we will obtain the asymptotic behavior of the cross section of the process considered at $s\to\infty$ and nonzero value of momentum transfer.

The process (1) in the same kinematics was considered in Ref. [13], where, for the description of the transition of the quark pair to the meson, consisting of heavy quarks $(V=J/\psi,\Upsilon)$, the nonrelativistic approximation was used. The process of light meson production needs to be considered separately, since here the nonrelativistic approximation is not applicable. It is necessary to take into account the nontrivial longitudinal motion of quarks in a light meson in accordance with the standard leading twist approach to the hard exclusive processes; see [14,15]. Besides, there is a substantial difference in the polarization state of the meson produced in reaction (1). For heavy meson production there is an approximate helicity conservation; the helicity of the meson coincides with the helicity of the initial photon $\lambda_M=\lambda_\gamma=\pm1$; see [16]. In contrast, the light mesons, as a consequence of helicity conservation at each vertex in the chiral limit of QCD, are produced in a state with helicity $\lambda_M=0$, irrespective of the helicity state of the initial photon. The amplitudes of production of light and heavy mesons have a different structure; compare Eqs. (9) and (12) with (13) and (14). The results for the amplitude of the J/ψ production process cannot be directly (by substitution $m_{J/\psi}\to m_{\rho^0}$) transferred to light meson production. Therefore, the conclusion of Ref. [13] (which is based on this substitution), about a larger yield at $|t|\gg m_V^2$ of heavy vector mesons than that of the lighter ones, seems to us an arbitrary one.

The diagram for the process (1) is presented in Fig. 1. At sufficiently large momentum transfer $(-t\gg\Lambda^2_{\rm QCD})$ the lower part of Fig. 1, which describes the disintegration of the proton after the absorption of the colorless t-channel gluon system, can be described in the frame of the parton model. The gluons in the t channel can be considered as a probe of the parton content of the proton, just as the virtual photon probes the proton in ordinary deep inelastic processes. The essential difference between the virtual photon and the two-gluon colorless system as the proton's probes is that the latter has coupling with both the quarks and the gluons. Therefore the cross section of process (1) may be represented as the product of "hard scattering" cross section and parton distribution functions in a proton:

$$\frac{d\sigma(\gamma^*p\to VX)}{dtdx}=\left(\frac{81}{16}G(x,t)+\sum_f\,[q(x,t)+\bar{q}(x,t)]\right)\times\frac{d\sigma(\gamma^*q\to Vq)}{dt}. \tag{2}$$

The coupling of the two-gluon colorless system with the gluon is $\frac{9}{4}$ times stronger than the coupling with the quark; therefore the coefficient $\frac{81}{16}$ in Eq. (2) is present. In Ref. [13] some arguments are presented suggesting that, at HERA energies and at $|t|\geqslant2\ \mathrm{GeV}^2$, the deviation from this simple parton picture (the possibility to connect t-channel gluons with the different partons in the proton) does not exceed 15% for events in which the invariant mass of the products of the photon's disintegration is not too large, $M_X^2\sim-t$.

The signature of the hard diffractive process (1) is the large empty rapidity interval (rapidity gap $\eta\sim$a few units) between the produced meson and other hadrons X to which the proton disintegrates. Variable x in Eq. (2) and the rapidity gap between meson and scattered quark in Fig. 1 are related by the simple equation $x=(4|t|/s)\cosh^2(\eta/2)$. The hadronization process of the scattered quarks can lead to some reduction of this rapidity gap. The mean value of this reduction was estimated in Ref. [17]; it is ~0.7. To obtain the estimates for the cross sections, below we will ignore this complexity and simply integrate over the region $x\geqslant x_0$.

In the Born approximation (two-gluon exchange in the t channel) the process of light vector meson production on the quark $\gamma^*(Q^2)q\to Vq$ was considered in detail earlier [18]. In the next section we obtain the results for this process in LLA. In the closing section the comparison of results of the LLA and the Born approximation will be done; we also give estimates for the cross sections of hard diffractive process (1) and discuss the possibility to study it at HERA. In the Appendix we will discuss the approximate method for the calculation of amplitudes in the LLA based on the result of [19] for the amplitude of quark-quark scattering.

II. PROCESS $\gamma^*Q\to VQ$ IN LLA

First we recall general LLA results of perturbative QCD and the expressions which describe the photon to meson transition. In what follows we present results obtained for the process considered separately for the cases of real and virtual photons in the initial state.

A. General LLA expressions

In the LLA, as was established in Refs. [2–4], the amplitude of the (quasi)elastic scattering of two colorless objects may be written as

$$A(s,t)=is\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{\Lambda^2}\right)^{\omega}f_\omega(\mathbf{q}^2),\quad t=-\mathbf{q}^2,$$

$$f_\omega(q^2)=\int d^2\mathbf{k}_1 d^2\mathbf{k}_2\Phi^1(\mathbf{k}_1,\mathbf{q})\Phi^2(\mathbf{k}_2,\mathbf{q})f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}),\tag{3}$$

where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the two-dimensional momenta of the t- channel gluons in Fig. 1, $f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})$ is the partial wave amplitude of gluon-gluon scattering in the color singlet channel, and the impact factors $\Phi^{1,2}(\mathbf{k},\mathbf{q})$ depend on the internal structure of the scattering objects. Λ^2 is the hard scale of the process. In the case when process (1) is initiated by real photons, the only hard scale is the momentum transfer. Therefore in this case $\Lambda^2\sim -t$. Throughout the paper all momentum vectors are two-dimensional vectors in the plane orthogonal to the axis of reaction.

The impact factors of colorless particles tend to zero when the momenta of the gluons in the t channel vanish,

$$\Phi^{1,2}(\mathbf{k},\mathbf{q})|_{\mathbf{k}=\mathbf{0}}=\Phi^{1,2}(\mathbf{k},\mathbf{q})|_{\mathbf{k}-\mathbf{q}=\mathbf{0}}=0.\tag{4}$$

The partial wave amplitude $f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})$ is the solution of the BFKL integral equation [2]. In Ref. [4] Lipatov has obtained the result for $f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})$ in the important case when the momentum transfer is nonzero, $\mathbf{q}\neq\mathbf{0}$. The high energy asymptotic of the partial wave amplitude can be written as

$$f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})=\frac{1}{64\pi^6}\int\frac{d\nu}{(\nu^2+\frac{1}{4})^2}\times\frac{\nu^2}{\omega-\omega(0,\nu)}I_\nu^*(\mathbf{k}_1,\mathbf{q})I_\nu(\mathbf{k}_2,\mathbf{q}),\tag{5}$$

$$I_\nu(\mathbf{k},\mathbf{q})=\int d^2\rho_1 d^2\rho_2\left(\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2}{\rho_1^2\rho_2^2}\right)^{\frac{1}{2}+i\nu}\times\exp[i\mathbf{k}\boldsymbol{\rho}_1+i(\mathbf{q}-\mathbf{k})\boldsymbol{\rho}_2].\tag{6}$$

In the derivation of Eqs. (5) and (6) we keep only the leading $(n=0)$ term in Eq. (29) of Ref. [4], and perform simple rearrangements similar to those done in Ref. [19].

In Ref. [19] the LLA result for the differential cross section of quark-quark elastic scattering was derived. In Eq. (5),

$$\omega(n,\nu)=\frac{6\alpha_s}{\pi}\chi(n,\nu),$$

$$\chi(n,\nu)=\mathrm{Re}\left[\psi(1)-\psi\left(\frac{|n|+1}{2}+i\nu\right)\right],\tag{7}$$

where ψ is a standard logarithmic derivative of the Γ function.

It is necessary to define the impact factors in Eq. (3). The impact factor of the quark is

$$\Phi_{q\to q}=\alpha_s\frac{\delta^{ab}}{N};\tag{8}$$

here $N=3$ is the number of colors. Below, for the sake of simplicity, we will discuss the production of ρ^0 mesons; the results for ω and ψ production can be trivially obtained by the change of coupling constants. The impact factor of the $\gamma\to\rho^0$ transition, which describes the upper block of Fig. 1, was derived in Ref. [20]:

$$\Phi_{\gamma\to\rho^0}(\mathbf{k},\mathbf{q})=-e\alpha_s\frac{\delta^{ab}}{2N}f_\rho Q_\rho\int_{-1}^{+1}d\xi\varphi_\rho(\xi)\xi(\mathbf{Q}\cdot\mathbf{e}).\tag{9}$$

According to the QCD leading twist approach to the hard exclusive reactions [14,15] this impact factor has the form of a convolution (the integral over ξ) of the hard amplitude of the photon to collinear quark pair transition and the process independent meson distribution amplitude $\varphi(\xi,\mu)$. The variable ξ describes the relative longitudinal motion of quarks in a meson and μ is a factorization scale. For the photoproduction process it is natural to choose the value of momentum transfer as the factorization scale, $\mu^2=-t$. The information about the distribution amplitude can be obtained from analysis of other exclusive processes or is derived by nonperturbative QCD methods (QCD sum rules, calculations on a lattice). Further, we will use the simple parametrization for the ρ^0 meson distribution amplitude:

$$\varphi_\rho(\xi)=\frac{3}{4}(1-\xi^2)\left(1-\frac{b_V}{5}+b_V\xi^2\right).\tag{10}$$

The value of the parameter b_V was estimated by the QCD sum rules method in Ref. [15]. At $\mu\simeq0.5-1.0$ GeV, $b_V=1.5$. The dependence of the distribution amplitude on the factorization scale is predictable in perturbative QCD, to the lowest order:

$$b_V(\mu_2)=b_V(\mu_1)[\alpha_s(\mu_2)/\alpha_s(\mu_1)]^{50/9b_0},\quad b_0=11-\frac{2}{3}n_{\mathrm{fl}}.\tag{11}$$

In Eq. (9) δ^{ab}/N is the color factor, which appears due to the projection on the colorless state of the quark pair. $\mathbf{e}$ is the polarization vector of the photon and $f_\rho=210$ MeV is the dimensional coupling constant. $Q_\rho=1/\sqrt{2}$ is connected with the charge content of the ρ^0 meson, $|\rho^0\rangle=(|u\bar{u}\rangle-|d\bar{d}\rangle)/\sqrt{2}$. Vector $\mathbf{Q}$ in Eq. (9) has the form

$$\mathbf{Q}=\left[\frac{\mathbf{q}}{\mathbf{q}^2(1+\xi)/2}+\frac{\mathbf{k}-\mathbf{q}(1+\xi)/2}{[\mathbf{k}-\mathbf{q}(1+\xi)/2]^2}\right]-[\xi\leftrightarrow-\xi].\tag{12}$$

The terms that do not depend on $\mathbf{k}$ in Eq. (12) describe those diagrams as depicted in Fig. 1 when the t-channel gluons link to the same quark (or antiquark) line in the upper block of Fig. 1. Terms in Eq. (12) depending on $\mathbf{k}$ describe the diagrams where the qluons link to various quark lines.

Note that Eq. (9) describes the transition of the photon to the longitudinal polarized meson. The impact factor of production of a meson in a state with helicity $\lambda=\pm1$ is

$$\Phi_{\gamma\to\rho^0}(\mathbf{k},\mathbf{q})=e\alpha_s\frac{\delta^{ab}}{2N}f_\rho Q_\rho\int_{-1}^{+1}d\xi\varphi_\rho(\xi)mR(\mathbf{e}\cdot\mathbf{e}_\mathbf{M}^*);\tag{13}$$

here m is the quark mass of which the meson consists and $\mathbf{e}_\mathbf{M}$ is the polarization vector of the meson. The scalar R is symmetric about the permutation of the quark's momenta:

$$R=\left[\frac{1}{[\mathbf{q}(1+\xi)/2]^2}+\frac{1}{[\mathbf{k}-\mathbf{q}(1+\xi)/2]^2}\right]+[\xi\leftrightarrow-\xi]. \qquad (14)$$

To produce the meson in a state with helicity $\lambda=\pm1$ it is necessary to flip the quark's spin. It is the reason why amplitude (13) is proportional to m, the current mass of the quark. Therefore light mesons can be produced in a state with helicity $\lambda=0$ only.

In the high energy limit $\alpha_s \ln(s)\gg 1$, the region of small ν gives the main contribution to the amplitude in Eqs. (5) and (6). Therefore we can integrate over ν and represent the amplitude in the factorized form

$$A(s,t)=is\frac{\exp(\rho\ \ln 4)}{\pi[28\pi^3\zeta(3)\rho]^{3/2}}J_2^*(q)J_1(q),$$

$$\rho=\frac{6\alpha_s}{\pi}\ln(s/\Lambda^2). \qquad (15)$$

All dependence on the structure of particles, participating in the reaction, contains in the factors $J_{1,2}$

$$J_{1,2}(q)=\int d^2k\Phi^{1,2}(\mathbf{k},\mathbf{q})I_{\nu=0}(\mathbf{k},\mathbf{q}). \qquad (16)$$

The factor $J_2(q)$ appears in the calculation of quark-quark scattering in the LLA; it was found in Ref. [19]:

$$J_2(q)=-\frac{2\alpha_s}{3}\frac{(2\pi)^3}{q}\delta^{ab}. \qquad (17)$$

B. Real photon in initial state

Let us define the factor $J_1(q)$ that describes the upper part of Fig. 1, the transition of $\gamma\to\rho^0$. The integration over d^2k in Eq. (16) gives

$$\int d^2k\ \exp(i\mathbf{k}\cdot\boldsymbol{\rho})\Phi_{\gamma\to\rho^0}(\mathbf{k},\boldsymbol{\rho})=-\frac{e\alpha_s}{6}\delta^{ab}Q_\rho f_\rho\int_{-1}^{+1}d\xi\xi\varphi_\rho(\xi)\left[(2\pi)^2\left(\frac{-4\xi}{1-\xi^2}\right)\delta^2(\boldsymbol{\rho})\frac{\mathbf{e}\cdot\mathbf{q}}{q^2}+(2\pi i)\frac{\mathbf{e}\cdot\boldsymbol{\rho}}{\rho^2}\left\{\exp\left(i\mathbf{q}\cdot\boldsymbol{\rho}\frac{1+\xi}{2}\right)-\exp\left(i\mathbf{q}\cdot\boldsymbol{\rho}\frac{1-\xi}{2}\right)\right\}\right]. \qquad (18)$$

Note that the contribution of the first $\sim\delta^2(\boldsymbol{\rho})$ term in the right-hand side of Eq. (18) vanishes at the subsequent integration over variables ρ_1 and ρ_2. The contribution of the second term is

$$J_1(q)=-(4\pi i)\frac{e\alpha_s}{6}\delta^{ab}Q_\rho f_\rho\int_{-1}^{+1}\xi\varphi_\rho(\xi)d\xi\int d^2\rho_1 d^2\rho_2 \times\frac{\mathbf{e}\cdot(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{|\boldsymbol{\rho}_1||\boldsymbol{\rho}_2||\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|}\exp\left[i\mathbf{q}\left(\boldsymbol{\rho}_1\frac{1+\xi}{2}+\boldsymbol{\rho}_2\frac{1-\xi}{2}\right)\right]. \qquad (19)$$

Let us next write $J_1(q)$ as

$$J_1(q)=-(16\pi)\frac{e\alpha_s}{6q^2}\delta^{ab}Q_\rho f_\rho(\mathbf{e}\cdot\mathbf{n})\int_{-1}^{+1}\xi\varphi_\rho(\xi)d\xi\frac{d}{d\xi}I(\xi),$$

$$\mathbf{n}=\frac{\mathbf{q}}{q}, \qquad (20)$$

where the function $I(\xi)$ is the integral over dimensionless vectors $\mathbf{r}_{1,2}$,

$$I(\xi)=\int\frac{d^2\mathbf{r}_1 d^2\mathbf{r}_2}{|\mathbf{r}_1+\mathbf{r}_2||\mathbf{r}_1-\mathbf{r}_2|r_2}e^{i\mathbf{n}\cdot(\mathbf{r}_1+\xi\mathbf{r}_2)}. \qquad (21)$$

After simple rearrangements we obtain

$$I(\xi)=2\pi\int\frac{d^2\mathbf{r}}{|\mathbf{r}+\mathbf{n}||\mathbf{r}-\mathbf{n}||\mathbf{r}-\xi\mathbf{n}|}. \qquad (22)$$

Unfortunately, we have failed to obtain the analytical expression for $I(\xi)$. Therefore, in Eq. (20) the integral over ξ with the distribution amplitude of a meson was calculated first and then the integration over $d^2\mathbf{r}$ was performed.[1] As a result we obtain

$$J_1(q)=-\frac{\pi^5}{2}\frac{e\alpha_s\delta^{ab}Q_\rho f_\rho(\mathbf{e}\cdot\mathbf{n})}{q^2}\eta_V, \quad \eta_V=\left(1+b_V\frac{131}{320}\right). \qquad (23)$$

Equations (15), (17), and (23) define the amplitude of the meson photoproduction on quarks, $\gamma q\to\rho^0 q$. The cross section of this reaction may be expressed in terms of an experimentally measured quantity—the width of decay of the ρ^0 meson into an e^+e^- pair, $\Gamma^V_{e^+e^-}=4\pi\alpha^2 f_V^2 Q_V^2/3m_V$,

$$\frac{d\sigma^{\mathrm{BFKL}}}{dt}=\frac{4\pi^4\eta_V^2}{1029\zeta^3(3)}\frac{\alpha_s^4 m_V\Gamma^V_{e^+e^-}}{\alpha|t|^3}\frac{e^{(2\rho\ \ln 4)}}{\rho^3}(\mathbf{e}\cdot\mathbf{n})^2. \qquad (24)$$

C. Virtual photon in initial state

In the previous subsection we have obtained the analytical results for the process (1) in the case of photoproduction, when $Q^2=0$. Let us go to the consideration of meson production by off-shell photons.

Note again that the helicity conservation discussed above holds for both real and virtual photons in the initial state.

[1] At this stage a transition to elliptic coordinates was used.

Perturbative QCD thus predicts the definite helicity of the produced mesons, $\lambda=0$.

The impact factors of the $\gamma^*(Q^2)\to\rho^0$ transition were derived earlier [18]. The off-shell photon can be longitudinally (S) or transversely (T) polarized. First we consider a process initiated by the T photon. The impact factor of the $\gamma^{(T)}\to\rho^0$ transition has the same form as the impact factor for a real photon (9). The inclusion of the photon's virtuality leads to modification of the structure $\mathbf{Q}$:

$$\mathbf{Q}\to\mathbf{Q}=\left[\frac{\mathbf{q}(1+\xi)/2}{(1-\xi^2)Q^2/4+[\mathbf{q}(1+\xi)/2]^2}+\frac{\mathbf{k}-\mathbf{q}\frac{1+\xi}{2}}{(1-\xi^2)Q^2/4+[\mathbf{k}-\mathbf{q}(1+\xi)/2]^2}\right]-[\xi\leftrightarrow-\xi]. \tag{25}$$

Now the factor J_1 depends on two variables (on the momentum transfer q and on the virtuality of the photon Q). After simple rearrangements it can be brought to the form

$$J_1^{(T)}=-(16\pi)\frac{e\alpha_s}{6q^2}\delta^{ab}Q_\rho f_\rho(\mathbf{e}\cdot\mathbf{n})\int_{-1}^{+1}\xi\varphi_\rho(\xi)\times L^{(T)}(\eta,\xi)d\xi,\quad \eta=\frac{Q}{q}\sqrt{(1-\xi^2)}; \tag{26}$$

here $L^{(T)}(\eta,\xi)$ is expressed in terms of Bessel functions of the first kind J_1 and K_1:

$$L^{(T)}(\eta,\xi)=\int\frac{d^2k_1d^2k_2(\mathbf{k}_1\cdot\mathbf{k}_2)}{k_1|\mathbf{k}_1+\mathbf{k}_2(1+\xi)||\mathbf{k}_1-\mathbf{k}_2(1-\xi)|}\times J_1(k_1)\eta K_1(\eta k_2). \tag{27}$$

It can be shown that in the limit $Q^2\ll q^2$ the obtained result reduces to that derived in the previous subsection for real photons, $L^{(T)}(\eta,\xi)\to(d/d\xi)I(\xi)$; see Eqs. (20), (21), and (23).

When the virtuality of the photon is high, $Q^2\gg q^2$, the main contribution to integral (27) originates from the region of small k_2, $k_2\leqslant 1/\eta$. Therefore,

$$L^{(T)}(\eta,\xi)|_{\eta\gg1}\simeq\int\frac{d^2k_1d^2k_2(\mathbf{k}_1\cdot\mathbf{k}_2)}{k_1^3[1+2\xi(\mathbf{k}_1\cdot\mathbf{k}_2)/k_1^2]}J_1(k_1)\eta K_1(\eta k_2)\simeq-2\pi\xi\int\frac{k_2^2dk_1d^2k_2}{k_1^2}J_1(k_1)\eta K_1(\eta k_2). \tag{28}$$

This expression is logarithmically divergent at the lower limit of the integration on k_1. This singularity slows down at $k_1\sim 1/\eta$. Hence the result with logarithmic accuracy can be written as

$$L^{(T)}(\eta,\xi)|_{\eta\gg1}\simeq-\pi\xi\ln(\eta)\int d^2k_2k_2^2\eta K_1(\eta k_2)=-3\pi^3\xi\frac{\ln(\eta)}{\eta^3}. \tag{29}$$

Performing the integration over ξ in Eq. (26) we obtain

$$J_1^{(T)}{}_{Q^2\gg q^2}\approx3\pi^5e\alpha_s\delta^{ab}\frac{Q_\rho f_\rho(\mathbf{e}\cdot\mathbf{n})q}{Q^3}\left[\ln\frac{Q}{q}\left(1+\frac{11}{20}b_V\right)-\frac{(1+\ln4)}{2}-\frac{b_V}{2}\left(\frac{74}{80}+\frac{44}{80}\ln4\right)\right]. \tag{30}$$

Let us turn to the case of longitudinal polarization of the initial photon. The impact factor of the $\gamma^{(S)}\to\rho^0$ transition has the form

$$\Phi_{\gamma\to\rho^0}(\mathbf{k},\mathbf{q})=-e\alpha_s\frac{\delta^{ab}}{2N}f_\rho Q_\rho\times\int_{-1}^{+1}d\xi\varphi_\rho(\xi)\frac{1-\xi^2}{2}\sqrt{Q^2}R(Q). \tag{31}$$

The amplitude of the (S) photon to meson transition is proportional to the virtuality of the photon. In addition, the dependence on the photon's virtuality is contained in the structure $R(Q)$:

$$R(Q)=\left[\frac{1}{m^2+[(1-\xi^2)/4]Q^2+[\mathbf{q}(1+\xi)/2]^2}-\frac{1}{m^2+[(1-\xi^2)/4]Q^2+[\mathbf{k}-\mathbf{q}(1+\xi)/2]^2}\right]+[\xi\leftrightarrow-\xi]. \tag{32}$$

For the subsequent discussion of the heavy mesons production we keep the quark mass in Eq. (32). The factor J_1 for the scalar photon is

$$J_1^{(S)}=(16\pi)\frac{e\alpha_sQ}{6q^3}\delta^{ab}Q_\rho f_\rho\int_{-1}^{+1}(1-\xi^2)\varphi_\rho(\xi)L^{(S)}(\eta,\xi)d\xi, \tag{33}$$

where $L^{(S)}(\eta,\xi)$ is expressed in terms of Bessel functions of the zeroth kind:

$$L^{(S)}(\eta,\xi)=\int d^2k_1d^2k_2\frac{k_2J_0(k_1)K_0(\eta k_2)}{|\mathbf{k}_1+\mathbf{k}_2(1+\xi)||\mathbf{k}_1-\mathbf{k}_2(1-\xi)|}. \tag{34}$$

In the limiting case $Q^2\gg q^2$,

$$L^{(S)}(\eta,\xi)|_{\eta\gg1}\simeq2\pi^3\frac{\ln(\eta)}{\eta^3}. \tag{35}$$

Substituting (35) in Eq. (33) and performing the integration over ξ we obtain

$$J_1^{(S)}(q)|_{Q^2\gg q^2} \approx 2\pi^5 e\alpha_s \delta^{ab} \frac{Q_\rho f_\rho}{Q^2}\left[\ln\frac{Q}{q}\left(1+\frac{b_V}{20}\right) + \frac{(1-\ln 4)}{2} - b_V\left(\frac{3}{80}+\frac{\ln 4}{40}\right)\right]. \quad (36)$$

The asymptotic expression (36) for the amplitude of a process initiated by (S) photon becomes valid at lower virtuality of the photon (at fixed q) in comparison with the Q^2 for which the asymptotic expression (30) for the amplitude of a process initiated by (T) photon comes into play. Compared to the impact factor of the $\gamma^{(T)}\to\rho^0$ transition, the impact factor of $\gamma^{(S)}\to\rho^0$ contains the additional factor $1-\xi^2$, which in the subsequent integration over ξ in Eq. (33) suppresses the contributions of the end point regions $\xi\to\pm1$. Thus the mean η in the integration in Eq. (33) is $\sim Q/q$ and the constant, which in Eq. (36) is subtracted from the term $\sim\ln(Q/q)$, is small. The same constant in the asymptotic expression (30) for the amplitude of the process initiated by (T) photon is substantially larger.

In Ref. [13] integral (34) was calculated numerically at $\xi=0$ for various values of η. It turns out that the asymptotic expression (35) differs from the numerical result by less than 10% when $\eta\geqslant3$. Therefore, we will anticipate that the asymptotic expression (36) can give a result that is close to the true one when $Q^2/q^2\geqslant10$, in so far as the end point regions (when $\xi\to\pm1$) do not give a substantial contribution in the subsequent integration over ξ in Eq. (33). Further, we will use Eq. (36) for estimates of the cross section at HERA energies. The inaccuracy connected with using the asymptotic Eq. (36) rather than the exact results (33) and (34) cannot exceed 10% when $Q^2/q^2\geqslant10$.

III. DISCUSSION

In the previous part we derived in the LLA the cross section of vector meson photoproduction on quarks at $s/|t|\to\infty$. Let us compare (24) with the result obtained in Ref. [20] for the cross section in the Born approximation (two-gluon exchange in the t channel):

$$\frac{d\sigma^{2G}}{dt} = \frac{64\pi}{3}\,\frac{\alpha_s^4 m_V \Gamma^V_{e^+e^-}}{\alpha|t|^3}(\mathbf{e}\cdot\mathbf{n})^2\upsilon_V^2, \quad \upsilon_V=(1+\tfrac{7}{15}b_V). \quad (37)$$

In the Born approximation the cross section does not depend on the collision energy. In the LLA the cross section increases as a power of energy. Nevertheless, the ratio

$$\frac{d\sigma^{\rm BFKL}}{d\sigma^{2G}} = \frac{\pi^3}{5488\zeta^3(3)}\,\frac{\eta_V^2}{\upsilon_V^2}\,\frac{\exp(2\rho\ \ln 4)}{\rho^3} \simeq 0.003\times\frac{\exp(2\rho\ \ln 4)}{\rho^3} \quad (38)$$

exceeds unity only at the sufficiently large values of parameter $\rho=(6\alpha_s/\pi)\ln(s/\Lambda^2)$, when $\rho\geqslant3.43$. For the process (1) at $Q^2=0$ it is natural to choose the square of momentum transfer as Λ^2 and the normalization point of the strong interaction constant $\Lambda^2=\mathbf{q}^2$, $\alpha_s(\mathbf{q}^2)$. In Table I we give the values of (c.m.) energies at which the ratio (38) is equal to unity depending on momentum transfer.

TABLE I. Values of center-of-mass energies at which the ratio (38) is equal to unity depending on momentum transfer.

$\mathbf{q}$ (GeV)	3	4	5	6	7
$\sqrt{s}$ (GeV)	104	201	335	510	720

In the region of large momentum transfers, $q\geqslant4-5$ GeV, the high energy asymptotic of the LLA exceeds the Born predictions only when $\sqrt{s_{\gamma p}}>200-340$ GeV. These energies are too high even for the HERA collider. At these transfers and energies available at HERA, $\sqrt{s_{\gamma p}}\sim50-200$ GeV, the expressions obtained in the high energy limit of the LLA are not applicable, since they give a result which is less than that obtained in lowest order. In that instance the more accurate analysis of the BFKL series is needed. One can try to calculate a few first items of the expansion in α_s, by means of iterating the BFKL equation for partial amplitude. In the region under consideration the parameter ρ is sufficiently large, $\rho\sim2-3.4$. Since the required number of iterations is $\sim\rho$, this analysis will be complicated.

Nevertheless, it is natural in our opinion that the Born result can give a lower bound for the cross section. The value of cross section obtained in the two-gluon approximation is not too small. According to Eqs. (2) and (37) when $x>0.01$ and $|t|\geqslant16$ GeV^2 the total cross section of hard diffractive production of ρ^0 mesons $\sigma^{2G}(\gamma p\to\rho^0X)\approx4.6$ nb; when $x>0.01$ and $|t|\geqslant25$ GeV^2 this cross section reduces to ≈1.1 nb.[2]

At not too large transfers, q$\sim3-4$ GeV, the energy region where the BFKL result is larger than the Born one, $\sqrt{s_{\gamma p}}>100-200$ GeV, is available at HERA. However, these transfers most likely are not sufficiently high for applicability of perturbative QCD to the description of the photon to meson transition. In the paper [18] some arguments are made in favor of the boundary value of momentum transfer, from which the asymptotic (in $1/q$) equations of perturbative QCD (9) and (13) become applicable, being $\sim4-5$ GeV for the Born amplitude.[3]

Investigating processes (1) over a wide range of transfers, it would be possible to determine the range of validity of perturbative QCD. Important information about the region of validity of perturbative QCD can give a measurement of the meson's polarization. Recall that perturbative QCD predicts the definite helicity of the produced meson, $\lambda=0$. On the contrary, in the region of small transfers one would expect, in

[2]In the integration of the differential cross section (37) the dependence of α_s on transfer momentum was taken into account.

[3]The Born amplitude gets a sizable contribution in the regions where one of the virtual quarks in the upper block of Fig. 1 is near mass shell. For the self-consistency of the perturbative approach it is necessary that the relative value of the contribution of nonperturbative regions, where the virtuality of the quark $\leqslant\Lambda^2_{\rm QCD}$, is small. According to Ref. [18], this criterion is fulfilled beginning with sufficiently large transfer momenta $q\geqslant4-5$ GeV.

the spirit of the vector dominance model (VDM), a helicity conservation in the transition $\gamma^* \to \rho^0$.

We show that in the process of hard diffractive photoproduction of light vector mesons the BFKL behavior is attained only at sufficiently large values of s/t; see Table I. The origin of the anomalously small numerical coefficient (~ 0.003) in Eq. (38) is connected *mainly* with the small numerical coefficient in the general equation for the amplitude in the high energy limit of the LLA; see Eq. (15). In addition, it is known [19,13,21] that the high energy asymptotic of the solution of the BFKL equation displays a smaller degree of infrared singularity as compared with that of the sum of diagrams that describe the process in the lowest order. This leads to the appearance of the additional infrared logarithm $\ln q_1^2/q_2^2$ in the integration of the Born amplitude over $d^2\mathbf{k}$ (over the momenta of gluons in the t channel), where $q_{1,2}$ are the quark's momenta in Fig. 1. This additional logarithm leads in the integration over the relative longitudinal momentum of the quarks to a larger numerical coefficient in the Born amplitude as compared to that in the LLA amplitude.

Let us discuss the diffractive production of ρ^0 mesons initiated by off-shell photons. In the previous section we derived expressions for the amplitudes in terms of integrals: (26) and (27) for transverse polarization and (33) and (34) for longitudinal polarization of the photon. In fact, the amplitudes are determined by three-dimensional integrals and can be evaluated numerically. But this calculation is difficult enough, because the integrand expression oscillates. Instead, we will analyze the process (1) using the analytical expressions (30) and (36) obtained above for the asymptotic of the amplitudes in the limit $Q^2 \gg q^2$.

Let us compare the LLA asymptotic expressions and those obtained in the Born approximation.

For transverse polarization of photons,

$$\left.\frac{d\sigma^{\mathrm{BFKL}}}{d\sigma^{2G}}\right|_{Q^2 \gg q^2} \approx 0.03 \times \frac{Q^2}{q^2 \ln(Q/q)^2}\,\frac{\exp(2\rho\,\ln 4)}{\rho^3}. \tag{39}$$

For longitudinal polarization of photons,

$$\left.\frac{d\sigma^{\mathrm{BFKL}}}{d\sigma^{2G}}\right|_{Q^2 \gg q^2} \approx 0.01 \times \frac{Q^2}{q^2}\,\frac{\exp(2\rho\,\ln 4)}{\rho^3}. \tag{40}$$

In composing relations (39) and (40), we use results of Ref. [18] for the amplitude of the process $\gamma^*(Q^2) q \to \rho^0 q$ in the Born approximation.

In the limit of large virtualities of photons the asymptotic result of the LLA for the amplitude exceeds substantially that obtained in the two-gluon approximation. Note the additional enhancing factor Q^2/q^2 in Eqs. (39) and (40) in comparison with Eq. (38). When $Q^2 \gg q^2$, the relevant scale of the process is determined in the Born approximation by the transverse distance between quarks in the $q\bar{q}$ pair with which the two-gluon system is connected. This distance $\rho \sim 1/Q$ and does not depend essentially on the value of momentum transfer q. The BFKL amplitude, at $Q^2 \gg q^2$, is characterized by two different scales: by the size of the quark pair, $\rho_1 \sim 1/Q$, and by the intrinsic distance between gluons in the BFKL ladder, $\rho_2 \sim 1/q$. In the course of development of the gluon ladder the transition from scale $\sim 1/Q$ to the larger one $\sim 1/q$ occurs. Therefore, the BFKL amplitude, as distinct from the Born one, depends on both the virtuality of the photon and the momentum transfer at $Q^2 \gg q^2$.

Let us estimate the value of the cross section for process (1) integrated over the region of transfers 2 GeV$^2 \leqslant q^2 \leqslant 3$ GeV^2 at $Q^2 = 25$ GeV^2 and HERA energies. First of all, note that in this region the amplitude of the process initiated by scalar photon $A^{(S)}$ is larger than that initiated by transversely polarized photon $A^{(T)}$.[4] Therefore we will estimate the value of the cross section of process (1) for the case of longitudinal polarization of the initial photon. We believe that in the considered region (where $Q^2/q^2 \sim 10$) the asymptotic expression (36) for the amplitude will give a result differing from the exact one by a value which does not exceed 10–20%. According to Eqs. (36), (15), and (17),

$$\left.\frac{d\sigma^{\mathrm{BFKL}}_{\gamma^{(S)} q \to \rho^0 q}}{dt}\right|_{Q^2/q^2 \gg 1} \approx 0.96 \frac{\alpha_s^4}{\alpha}\,\frac{m_V \Gamma^V_{e^+e^-}}{Q^4 q^2} \times \left[\ln\frac{Q^2}{q^2} - 0.505\right]^2 \frac{\exp(2\rho\,\ln 4)}{\rho^3}. \tag{41}$$

The dependence of the amplitude of the process $\gamma^{(S)} q \to \rho^0 q$ on the shape of the meson distribution amplitude is not too strong. Therefore we present result (41) for the distribution amplitude with the parameter $b_V = 1.0$. According to Eq. (11), this value of b_V is accepted when $\mu \sim 3-5$ GeV.

Contrary to the photoproduction process discussed above, where it is natural to choose as the normalization point of the strong interaction constant the transferred momentum q as well as to put $\Lambda^2 = q^2$, there are two scales ($Q^2 = 25$ GeV^2 and $q^2 \sim 2-3$ GeV^2) in the kinematical region under consideration. If we choose as the hard scale of the process $\Lambda^2 = Q^2$ and use the same value as the argument of α_s in Eq. (41), then integrated over the region 2 GeV$^2 \leqslant q^2 \leqslant 3$ GeV^2, $x \geqslant 0.1$, the cross section $\sigma^{\mathrm{BFKL}}(\gamma^{(S)}(Q^2 = 25\ \mathrm{GeV}^2) p \to \rho^0 X)$ constitutes ≈ 0.19 nb at $\sqrt{s_{\gamma p}} = 100$ GeV and ≈ 0.48 nb at $\sqrt{s_{\gamma p}} = 200$ GeV.

We point out that the results of the calculation are extremely sensitive to the choice of the normalization point for α_s and Λ^2 in Eq. (41). If as Λ^2 and the normalization point for the strong interaction constant we take $0.8Q^2$, then the results for cross sections increase more than 1.5 times. This uncertainty is connected with the use of the LLA. Unfortunately, the results for the QCD Pomeron in the next to leading logarithm approximation are not derived up to now. However, work in this way is in progress now; see [22–24]. Therefore, in the complicated case when there are two strongly differing scales ($1/Q$ and $1/q$), we cannot make a justified choice of the normalization point for the strong con-

[4] In the Born approximation [18] these amplitudes coincide at $Q^2 = q^2$. With further growth of photon's virtuality the amplitude $A^{(S)}$ decreases more slowly than $A^{(T)}$.

stant and the parameter Λ^2. For estimating the cross section in the previous paragraph, we chose the lowest scale, $1/Q$. We hope that numbers obtained therewith are estimates for the cross section from below.

The estimates obtained above for cross sections are not too small. Therefore there is a hope to investigate the process (1) at HERA in the region of large photon virtualities, $Q^2 \gg q^2 \sim 2-3$ GeV^2. In this region we predict a fast increase of the cross section with energy growth of the $\gamma^* p$ system. At $Q^2 = 25$ GeV^2 and $q^2 \sim 2-3$ GeV^2 we expect a growth of the cross section more than 2.5 times when $\sqrt{s_{\gamma p}}$ increases from 100 to 200 GeV. It will be interesting to handle experimental data using Eqs. (41) and (2), varying herewith the normalization point of α_s and the parameter Λ^2.

CONCLUSIONS

In the high energy limit of the LLA the process of diffractive light vector meson production was studied in the region of large momentum transfers with arbitrary virtuality of the initial photon. In an analytical form the results for photoproduction (when $Q^2 = 0$) and for the limiting case of large virtuality ($Q^2 \gg q^2$) are derived.

It turns out that the LLA amplitude of process (1) does not exceed at $Q^2 = 0$ the Born one even at the energies of the collider HERA.

When $Q^2 \gg q^2 \gg \Lambda^2_{\rm QCD}$ the LLA result is larger than the Born one. The corresponding cross section therewith is not too small. In this region we expect fast growth of the cross section of process (1) with the energy of $\gamma^* p$ collisions, as is typical for the BFKL Pomeron.

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APPENDIX

Inserting (13) and (32) in Eq. (15) we obtain the amplitude of hard diffractive production of the mesons consisting of heavy quarks, J/ψ, Υ. The distribution amplitude of heavy mesons has a small width $\sim v/c \ll 1$, where v is the velocity of quarks in the meson. If the approximation $\varphi(\xi) = \delta(\xi)$ is used, the calculation is simplified drastically and reduced to the case discussed above of meson production by scalar photon; see Eqs. (9) and (13). Relevant results may be obtained by the simple replacement in Eqs. (33) and (34) $-\sqrt{Q^2}/2 \to m(\mathbf{e}\cdot\mathbf{e}_M^*)$ and $\eta \to \eta' = \sqrt{(4m^2+Q^2)/q^2}$. They are in agreement with that obtained in Ref. [13].

At the same time, the analytical result of Ref. [13], which describes the asymptotic of the amplitude in the limit $q^2 \gg (4m^2+Q^2)$, contains an uncertainty. According to Eq. (11) of Ref. [13], at $\rho \gg 1/Q$,

$$\int d^2R J_0(QR) \frac{|\rho|}{|R+\rho/2||R-\rho/2|} \approx J_0\left(\frac{1}{2}QR\right)\frac{4\pi}{Q}. \qquad \text{(A1)}$$

This result is obtained if we anticipate that the main contribution to the integral gives the small $\Delta R \sim 1/Q$ regions in the vicinity of the points $\mathbf{R} = \pm \boldsymbol{\rho}/2$. The authors of Ref. [13] advocate that (A1) confirms the ''natural'' conjecture that in the limit $q^2 \gg (4m^2+Q^2)$ the contribution of the diagrams in which the t-channel gluons connect with the various quarks in a meson can be neglected and one can use the prescription of Ref. [19] for the amplitude of quark-quark scattering to calculate the contribution of diagrams in which both the gluons in the t channel link to the same quark in a meson.

Note that there are other integration regions that give a contribution to (A1) of the same order as the small region in the vicinity of the singular points discussed above. The contribution of the region $0 \le R \le \rho' \lesssim \rho/2$ can be estimated as $\approx (8\pi\rho'/\rho Q) J_1(Q\rho')$. The region $\rho' 2 \lesssim \rho'' \le R \le \infty$ gives the contribution $\approx -(2\pi\rho/\rho'' Q) J_1(Q\rho'')$. Because the function J_1 oscillates quickly when $Q\rho \gg 1$, one cannot hold that $J_1(Q\rho') \approx J_1(Q\rho'')$. Hence the contributions of these regions do not compensate each other exactly and give a result of the same order as the contribution that gives the small regions in the vicinity of the points $\mathbf{R} = \pm\boldsymbol{\rho}/2$. Therefore the asymptotic result of Ref. [13] for the amplitude in the limit of high transfers $q^2 \gg (4m^2+Q^2)$ is only parametrically valid. The numerical coefficient in the asymptotic is incorrect. Nevertheless, the difference of the true numerical coefficient and that derived in Ref. [13] is presumably modest. Hence, the authors of Ref. [13] do not detect this inaccuracy in the comparison of the asymptotic expression with the results of numerical calculations.

An integral of type (A1) appears in the analysis of any exclusive reaction in the frame of the BFKL theory. As was pointed out in Ref. [21], the reduction of the full integral to the contribution of small regions near singular points is equivalent to the approximation when we take into account only diagrams in which the t-channel gluons link to the same parton, and use the prescription of Ref. [19] for the amplitude of quark-quark scattering. We discussed in the above paragraph that in the region of high transfers this approximation [Mueller-Tang (MT) approximation] gives the right power behavior of the amplitude, but it is not sufficient to calculate exactly the numerical coefficient. Nevertheless, as the MT approximation significantly simplifies the calculations, it can be used for the estimate of an amplitude in a complicated situation, when the exact calculation encounters substantial difficulties. This estimate can give a result which does not differ tangibly from the exact one. For instance, for the process $\gamma q \to \rho^0 q$ discussed above the MT approximation gives the answer

$$A^{\rm MT}(s,t) = \frac{32}{3\pi^2}\left(1+\frac{2}{5}b_V\right) \Bigg/ \left(1+\frac{131}{320}b_V\right) A(s,t), \qquad \text{(A2)}$$

which exceeds the exact one (23) by less than 10% practically at any b_V.

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