Fermion condensates of massless two-dimensional QED at finite density in nontrivial topological sectors

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Vacuum expectation values of products of local bilinears $\bar{\psi}\psi$ are computed in massless two-dimensional QED at finite density. It is shown that chiral condensates exhibit an oscillatory inhomogeneous behavior depending on the chemical potential. The use of a path integral approach clarifies the connection of this phenomenon with the topological structure of the theory.

I. INTRODUCTION

Fermion condensates play an important role in particle physics and cosmology. In particular, they are relevant in connection with chiral symmetry breaking [1], a phenomenon related to the structure of the QCD vacuum [2].

Recently, it has been found in the large N_C limit [3], that the order parameter for chiral symmetry, $\langle\bar{\psi}\psi\rangle$, is at high fermion densities inhomogeneous and anisotropic so that the ground state of quark matter has the structure of a standing wave with respect to the order parameter.

At present, it is well understood that the fermion condensate $\langle\bar{\psi}(x)\psi(x)\rangle$ picks contributions from nontrivial topological sectors. Then, in order to decide whether the breakdown of chiral symmetry occurs, one should consider instanton effects on the fermion condensate.

Two-dimensional models such as two-dimensional QED (QED$_2$) and QCD$_2$ provide a natural ground to study these phenomena since, although simplified, the basic features (chiral symmetry breaking, nontrivial topological sectors, etc.) are still present and exact calculations can be in many cases performed. In this context, we calculate in the present work vacuum expectation values of products of local bilinears, $\bar{\psi}\psi$, at finite density for the Schwinger model. We employ a path integral approach which leads in a very simple way to exact results and has shown to be very adequate for studying non-Abelian extensions [4–7]. Our conclusions go in the same direction as those in [3] and extend the results obtained in [8]. In particular, we are able to show that multipoint chiral condensates exhibit an oscillatory inhomogeneous behavior depending on the chemical potential, as it is the case in the $1/N_C$ results for QCD$_4$ [3]. The lack of translation invariance manifests through a dependence on differences as well as sums of spacial coordinates. This last result corrects the oscillatory behavior computed approximately in [8] for the two-point correlator in the Schwinger model.

Let us start by observing that quantum field theories at finite fermion density can be studied by introducing a chemical potential leading to a quantum theory in the presence of a classical background charge distribution [9–11]. Concerning topological contributions, Bardakci and Crescimanno [12] proposed a natural way to take into account topologically nontrivial configurations, which is suitable for the study of the fermion condensates within the path integral formulation. In this approach, one decomposes a given gauge field belonging to the nth topological sector in the form

$$A_\mu(x) = A_\mu^{(n)}(x) + a_\mu(x) \tag{1}$$

with $A_\mu^{(n)}$ a fixed configuration carrying the (whole) topological charge n and a_μ the path integral variable, which accounts for quantum fluctuations and belongs to the trivial topological sector. In this way, calculations easily workable in the $n = 0$ background (as the evaluation of Fujikawa Jacobians after a fermionic change of variables using techniques requiring a compact manifold) can be handled without problems and, at the same time, the contribution of topologically nontrivial sectors is properly taken into account.

II. THE MODEL

We start from the two-dimensional (Euclidean) Lagrangian for massless QED$_2$:

$$\mathcal{L} = \bar{\psi}(i\slashed{\partial} + \slashed{A})\psi - \frac{1}{4e^2}F_{\mu\nu}^2, \tag{2}$$

where our γ matrices are taken as the Pauli matrices, $\gamma_0 = \sigma_1$, $\gamma_1 = -\sigma_2$ so that $\gamma_\mu\gamma_5 = i\epsilon_{\mu\nu}\gamma_\nu$.

The partition function $\mathcal{Z}$ for the model is

$$\mathcal{Z} = \int D\bar{\psi}D\psi DA_\mu \exp\left(-\int d^2x \mathcal{L}\right). \tag{3}$$

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In order to include fermion density effects, it will be convenient to add still another vector field A_μ^b which will allow for the introduction of the chemical potential in a simple way. This field describes an *external* charge density acting on the quantum system. Whenever A_μ^b is taken as i times the chemical potential [11], it will represent a uniform charge background. If one first considers a finite length $(2L)$ distribution and then takes the $L \to \infty$ limit, translation symmetry breaking becomes apparent and at the same time ambiguities in the definition of the finite density theory are avoided (see Ref. [10]).

Now we can write the gauge field in Lagrangian (2) in the form

$$A_\mu(x) = A_\mu^{(n)}(x) + A_\mu^b(x) + a_\mu(x). \tag{4}$$

Hence, the Lagrangian in the nth topological sector reads

$$\mathcal{L}^{(n)} = \bar{\psi}(i\partial\!\!\!/ + a\!\!\!/ + A\!\!\!/^{(n)} + A\!\!\!/^b)\psi - \frac{1}{4e^2}[f_{\mu\nu}^2 + F_{\mu\nu}^{(n)2}$$

$$+ 2f_{\mu\nu}F_{\mu\nu}^{(n)} + (F_{\mu\nu}^b)^2 + 2f_{\mu\nu}F_{\mu\nu}^b$$

$$+ 2F_{\mu\nu}^{(n)}F_{\mu\nu}^b] + \mathcal{L}_c. \tag{5}$$

In Eq. (5) we have added a counterterm $\mathcal{L}_c$ which will be conveniently chosen to cancel out a divergency arising when the background A_μ^b is taken as a constant to introduce the chemical potential (see below).

The partition function now reads

$$\mathcal{Z} = \sum_n \int D\bar{\psi}D\psi Da_\mu \exp\left(-\int d^2x \mathcal{L}^{(n)}\right), \tag{6}$$

where we have written the path integral as a sum over all topological sectors.

As is well known, in two dimensions fermions can be completely decoupled from gauge fields by an appropriate chiral and gauge transformations of the fermion fields. At the quantum level, the corresponding change in the fermionic variables is accompanied by a nontrivial Fujikawa Jacobian [13]. As it can be inferred from the connection between the chiral anomaly (and the index theorem for the Dirac operator) with the change in the fermionic path integral measure, calculation of the Jacobian can be more easily performed in compact space-time manifolds. The latter implies that only decouplings from topologically trivial gauge fields should be considered. With this in mind, we perform the following change in the fermionic variables:

$$\psi = \exp(\gamma_5\phi + i\eta + \gamma_5\phi^b + i\eta^b) \chi,$$
$$\bar{\psi} = \bar{\chi} \exp(\gamma_5\phi - i\eta + \gamma_5\phi^b - i\eta^b) \tag{7}$$

according to a standard decomposition of the vector fields

$$a_\mu = -\epsilon_{\mu\nu}\partial_\nu\phi + \partial_\mu\eta \tag{8}$$

and

$$A_\mu^b = -\epsilon_{\mu\nu}\partial_\nu\phi^b + \partial_\mu\eta^b. \tag{9}$$

With this change of variables, the only field that remains coupled to the fermions is $A_\mu^{(n)}$, a classical configuration carrying all the topological charge. In terms of the new variables, Lagrangian (5) becomes

$$\mathcal{L}^{(n)} = \bar{\chi}(i\partial\!\!\!/ + A\!\!\!/^{(n)})\chi - \frac{1}{4e^2}\left(F_{\mu\nu}^{(n)}\right)^2$$

$$- \frac{1}{2e^2}\left((\Box\phi)^2 + \epsilon_{\mu\nu}F_{\mu\nu}^{(n)}\Box\phi\right) + \mathcal{L}_c, \tag{10}$$

where we have already taken A_μ^b as a constant, in order to introduce the chemical potential.

Equation (10) describes the effect of the change of variables merely at the classical level. At the quantum level, within the path integral approach, one has to take into account the change in the fermionic measure, i.e., the Fujikawa Jacobian associated with transformations (7). The evaluation of this Jacobian is standard and we just quote the result [4]

$$J = \exp\left(\frac{1}{2\pi}\int d^2x \; \phi \Box \left(\phi + 2\phi^{(n)}\right)\right)$$

$$\times \exp\left(\frac{1}{2\pi}\int d^2x \; \phi_b \Box \left(\phi + 2\phi^{(n)}\right)\right), \tag{11}$$

where $\phi^{(n)}$ is accordingly defined by

$$A_\mu^{(n)} = -\epsilon_{\mu\nu}\partial_\nu\phi^{(n)}. \tag{12}$$

Concerning the Jacobian associated with the change of the bosonic variables [Eqs. (4) and (8)],

$$Da_\mu = J_{\text{bos}}D\phi D\eta, \tag{13}$$

it can be identified with the Faddeev-Popov determinant for the $\eta = 0$ (Lorentz) gauge, $J_{\text{bos}} = \det\Box$, and it will be ignored in what follows.

To relate A_μ^b with the chemical potential μ, note that fixing the fermion number density through a term

$$\mathcal{L}_{\text{chem}} = -i\mu\bar{\psi}\gamma_0\psi \tag{14}$$

corresponds to the choice

$$A_\nu^b = -i\mu\delta_{\nu 0} \tag{15}$$

or, equivalently,

$$\phi_b = i\mu x_1, \quad \eta_b = 0. \tag{16}$$

Notice that, as has been thoroughly analyzed by Actor [11], A_μ^b does not correspond to a pure gauge. Were it not so, the introduction of a chemical potential would not have physical consequences. For the same reason, one cannot gauge away $\mathcal{L}_{\text{chem}}$ from the Lagrangian by means of the alternative choice: $\eta_b = -i\mu x_0$, $\phi_b = 0$. In fact, this transformation would correspond to an unbounded (in the temporal axis) gauge one. Although transformation (16) is unbounded as well, as mentioned above, one can handle this problem as is usually done: putting the

system in a spatial box, introducing counterterms, and taking the infinite limit for its length at the end of the calculations. In this way, the usual divergency associated to the electromagnetic energy carried by fermions can be eliminated by an appropriate counterterm [10]. In our approach the divergency manifests through the term $\phi_b \Box \phi^{(n)}$ in Eq. (11). As stated above, this infinite contribution is canceled out by an appropriate choice of $\mathcal{L}_c$. This counterterm is the Lagrangian counterpart of the one usually employed in the Hamiltonian approach to handle this problem [8,10]. In the canonical QFT, this is tantamount to a redefinition of creation and annihilation operators which is equivalent to a shift in the scale used to measure excitations.

Putting all this together, the partition function of the model can be written in the form

$$\mathcal{Z} = \mathcal{N} \sum_n \int D\bar{\chi} D\chi D\phi \, \exp(-S_{\text{eff}}^{(n)}), \qquad (17)$$

where $S_{\text{eff}}^{(n)}$ is the effective action in each topological sector,

$$S_{\text{eff}}^{(n)} = \int d^2x \, \bar{\chi}(i\partial\!\!\!/ + A\!\!\!/^{(n)})\chi$$

$$- \frac{1}{2e^2} \int d^2x \left((\Box \phi)^2 + \epsilon_{\mu\nu} F_{\mu\nu}^{(n)} \Box \phi \right)$$

$$- \frac{1}{4e^2} \int d^2x (F_{\mu\nu}^{(n)})^2 - \frac{1}{2\pi} \int d^2x \, \phi \Box (\phi + 2\phi^{(n)}). \qquad (18)$$

Note that with the choice of $\mathcal{L}_c$ discussed above, the effective action written in terms of the "decoupled" fermions does not depend on the chemical potential μ. Nevertheless, μ reappears when computing correlation functions of fermion fields, once $\bar{\psi}$ and ψ are written in terms of the decoupled fields $\bar{\chi}$ and χ through Eq. (7).

Of course, the fermionic integral in Eq. (17) is the determinant of the Dirac operator in the background of a gauge field carrying topological charge n. Now, as is well known [12], the Dirac operator has, for $n > 0$ $(n < 0)$, n positive (negative) chirality zero modes so that actually none but the $n = 0$ sector does contribute to $\mathcal{Z}$. However, in computing vacuum expectation values (VEV's) of products of fermion bilinears, Grassman coefficients accompanying zero modes render nontrivial certain path integrals in a given topological sector. This happens according to the number of bilinears appearing in the VEV, and then only this sector will contribute (see next section).

III. CORRELATION FUNCTIONS OF FERMION BILINEARS

We are now ready to study the behavior of chiral condensates and their dependence on the chemical potential

μ. To this end, let us define the chiral charge-changing correlators

$$s_+(w) = \bar{\psi}_+ \psi_+(w), \qquad (19)$$

$$s_-(w) = \bar{\psi}_- \psi_-(w), \qquad (20)$$

where ψ_+ and ψ_- are the right-handed and left-handed components, respectively, of the Dirac spinors. Thus, the fermion condensate $\langle \bar{\psi}\psi(w) \rangle$ is the sum of the VEV's of the composites defined in Eqs. (19) and (20).

Notice that this correlator could be naively expected to vanish as a direct consequence of the chiral invariance of massless QED$_2$. However, as explained above, when one is to compute $\langle s_+(w) \rangle$ $[\langle s_-(w) \rangle]$, using the partition function given by Eqs. (17) and (18), a nonzero value is found. In fact, only the $n = 1$ $(n = -1)$ sector will contribute to the sum over topologically nontrivial sectors. One then has

$$\langle s_\pm(w) \rangle = \frac{1}{\mathcal{Z}_0} \int D\bar{\chi} D\chi D\phi \, \bar{\chi}_\pm(w)\chi_\pm(w)$$

$$\times \exp\{\pm[\phi(w) + \phi_b(w)]\} \exp\left(-S_{\text{eff}}^{(\pm 1)}\right), \qquad (21)$$

where $\mathcal{Z}_0$, the only nonvanishing contribution to $\mathcal{Z}$, arises from the $n = 0$ sector. After some algebra, Eq. (21) becomes

$$\langle s_\pm(w) \rangle = \frac{1}{\mathcal{Z}_F} \int D\bar{\chi} D\chi \, \bar{\chi}_\pm(w)\chi_\pm(w)$$

$$\times \exp\left(-\int d^2x \, \bar{\chi}(i\partial\!\!\!/ + A\!\!\!/^{(\pm 1)})\chi\right) \frac{1}{\mathcal{Z}_B} \int D\phi$$

$$\times \exp\{\pm 2[\phi(w) + i\mu w_1]\} \exp\left\{-\frac{1}{4e^2}(F_{\mu\nu}^{(\pm 1)})^2\right\}$$

$$\times \exp\left[\frac{1}{2\pi} \int d^2x \left(\phi \Box \phi + 2\phi\epsilon_{\mu\nu}\partial_\mu A_\nu^{(\pm 1)}\right)\right] \qquad (22)$$

with

$$Z_F = \det i\partial\!\!\!/ \qquad (23)$$

and

$$Z_B = \det^{-1/2}\left[\Box\left(\Box - \frac{e^2}{4\pi}\right)\right]. \qquad (24)$$

It can be easily proved in the $\mu = 0$ case that $\langle s_+ \rangle_{+1}^{\mu=0}$ and $\langle s_- \rangle_{-1}^{\mu=0}$ coincide. This, in turn, implies that

$$\langle \bar{\psi}\psi(w) \rangle^{\mu=0} = 2\langle s_+ \rangle_{+1}^{\mu=0}. \qquad (25)$$

Now, from Eq. (22), we can see that

$$\langle s_\pm \rangle_{\pm 1} = \exp(\pm 2i\mu w_1)\langle s_\pm \rangle_{\pm 1}^{\mu=0} ; \qquad (26)$$

hence, one finally gets

$$\langle\bar{\psi}\psi(w)\rangle^{\mu\neq0} = \cos(2\mu w_1)\langle\bar{\psi}\psi(w)\rangle^{\mu=0}. \qquad (27)$$

In order to have an explicit formula for Eq. (27), let us recall that

$$\langle s_+\rangle^{\mu=0}_{+1} = -\frac{m}{4\pi}e^\gamma, \qquad (28)$$

where γ is the Euler constant and $m = e^2/\pi$ is the mass of the effective boson [14]. Equation (27) coincides with that presented in [8], using operator bosonization rules. Apart from the simplicity of our derivation in the path integral framework, it should be stressed that it is within this approach that the role of different topological sectors becomes apparent. This can be put in evidence in the calculation of N-point correlators which will be discussed in the remaining of this section.

According to the previous discussion, we consider the contribution of the nth topological sector to the N-point correlation function

$$\left\langle\prod_{i=1}^N s_\pm(w_i)\right\rangle^{\mu\neq0}_{\pm n} = \frac{1}{\mathcal{Z}_0}\int D\bar{\chi}D\chi D\phi\prod_{i=1}^N \bar{\chi}P_\pm\chi(w_i)$$
$$\times\exp\left(\pm2\sum_{i=1}^N\theta(w_i) - S^{(\pm n)}_{\text{eff}}\right). \quad (29)$$

Here $\langle\,\rangle_{\pm n}$ means that the VEV is computed in the $\pm n$th-topological sector; P_+ (P_-) is the projector on the right-handed (left-handed) subspace

$$P_\pm = \frac{1}{2}(1\pm\gamma_5) \qquad (30)$$

and we have defined

$$\theta(w) = \phi(w) + \phi^b(w). \qquad (31)$$

Now, because of Grassman integration rules, it can be easily proved that the only nonvanishing contribution to these correlators may arise when the number of insertions equals the absolute value of the topological charge, i.e., $|n| = N$ [15]. Performing the decoupling of fermions as before, we finally get the expression

$$\left\langle\prod_{i=1}^N s_\pm(w_i)\right\rangle^{\mu\neq0}_{\pm N} = F_{\pm N}(w_1, w_2, ..., w_N)$$

$$\times B_{\pm N}(w_1, w_2, ..., w_N). \qquad (32)$$

Here,

$$F_{\pm N} = Z_F^{-1}\int D\bar{\chi}D\chi\prod_{i=1}^N \bar{\chi}_\pm\chi_\pm(w_i)$$
$$\times\exp\left(-\int d^2x\,\bar{\chi}(i\not{\partial} + \not{A}^{(\pm N)})\chi\right) \qquad (33)$$

and

$$B_{\pm N} = Z_B^{-1}\int D\phi\,\exp\left(\pm2\sum_{i=1}^N\theta(w_i)\right)$$
$$\times\exp\left(\frac{1}{2e^2}\int d^2x\,\phi\,\square\,(\phi + 2\phi^{(\pm N)})\right)$$
$$\times\exp\left(\frac{1}{2e^2}\int d^2x[(\square\phi)^2\right.$$
$$\left. + 2\,\square\,\phi^{(\pm N)}\,\square\,\phi + (\square\phi^{(\pm N)})^2]\right). \qquad (34)$$

We thus see that the dependence of the correlator on the chemical potential factorizes, resulting in

$$\left\langle\prod_{i=1}^N s_\pm(w_i)\right\rangle^{\mu\neq0}_{\pm N} = \exp\left(\pm i\mu\sum_{i=1}^N w_{i1}\right)$$
$$\times\left\langle\prod_{i=1}^N s_\pm(w_i)\right\rangle^{\mu=0}_{\pm N}, \qquad (35)$$

where

$$\left\langle\prod_{i=1}^N s_\pm(w_i)\right\rangle^{\mu=0}_{\pm N} = (-me^\gamma/4\pi)^N$$
$$\times\exp\left(-2\sum_{i>j}K_0(m|w_i - w_j|)\right) \qquad (36)$$

are the general minimal correlation functions for $\mu = 0$ [15,16].

We are now ready to go further and study the so-called nonminimal [12,16,17] correlation functions which will be needed in order to compute multipoint vacuum condensates. We start considering the complete two-point composite

$$\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle = \langle s_+(x)s_+(y)\rangle + \langle s_-(x)s_+(y)\rangle$$
$$+\langle s_+(x)s_-(y)\rangle$$
$$+\langle s_-(x)s_-(y)\rangle. \qquad (37)$$

In the $n = 0$ topological sector, the unique contribution to Eq. (37) comes precisely from the simplest nonminimal VEV which, after fermion decoupling, becomes

$$\langle s_-(x)s_+(y)\rangle^{\mu\neq0} + \langle s_+(x)s_-(y)\rangle^{\mu\neq0} = \{\exp[-2i\mu(x_1 - y_1)]$$
$$\times\langle\bar{\chi}_+\chi_+(x)\bar{\chi}_-\chi_-(y)\rangle_F + \exp[2i\mu(x_1 - y_1)]\langle\bar{\chi}_-\chi_-(x)\bar{\chi}_+\chi_+(y)\rangle_F\}$$
$$\times\int D\phi\exp\left[\int d^2x\left(\frac{1}{2e^2}(\square\phi)^2 + \frac{1}{2\pi}(\phi\,\square\,\phi)\right)\right]. \qquad (38)$$

This gives a contribution to $\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle$ in the $n=0$ topological sector, of the form

$$\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle^{\mu\neq0}_{n=0} = \cos[2\mu(x_1 - y_1)]$$
$$\times\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle^{\mu=0}_{n=0}. \quad (39)$$

There are two remaining contributions arising from the $n=\pm2$ sectors. They follow directly from Eq. (35):

$$\langle s_+(x)s_+(y)\rangle^{\mu\neq0}_{+2} = \exp[-2i\mu(x_1 + y_1)]$$
$$\times\langle s_+(x)s_+(y)\rangle^{\mu=0}_{+2},$$
$$\langle s_-(x)s_-(y)\rangle^{\mu\neq0}_{-2} = \exp[2i\mu(x_1 + y_1)]$$
$$\times\langle s_-(x)s_-(y)\rangle^{\mu=0}_{-2}. \quad (40)$$

Again,

$$\langle s_+(x)s_+(y)\rangle^{\mu=0}_{+2} = \langle s_-(x)s_-(y)\rangle^{\mu=0}_{-2}$$
$$= \frac{1}{2}\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle^{\mu\neq0}_{|n|=2} \quad (41)$$

so that we finally have

$$\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle^{\mu\neq0}$$

$$= \cos[2\mu(x_1 - y_1)]\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle^{\mu=0}_{n=0}$$
$$+ \cos[2\mu(x_1 + y_1)]\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle^{\mu=0}_{|n|=2}. \quad (42)$$

At this point some remarks are in order. (i) Once again the topological structure of the theory manifests, making possible the discrimination of the contributions to fermion bilinear correlation functions from each topological sector; (ii) Eq.(42) exhausts *all* topological contributions; (iii) the lack of translation invariance (which is broken by the background charge distribution) becomes apparent, particularly through the last term in the right-hand side (RHS) of Eq. (42), which depends on the combination x_1+y_1; (iv) no clustering ansatz has been needed in order to obtain these results.

Let us finally note that if we put $y_1 = 0$ in Eq. (42), we get a compact formula analogous to Eq. (27):

$$\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle^{\mu\neq0} = \cos(2\mu x_1)\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\rangle^{\mu=0}. \quad (43)$$

This last result can be seen to coincide with that obtained in Ref. [8], using cluster decomposition. In fact, the result reported in [8] corresponds just to the trivial topological sector and does not reproduce the contribution of $n \neq 0$ sectors whenever $x_1 \neq y_1$.

Correlators of a larger number of bilinears can be very simply obtained following the same procedure as above. As an example one gets, for the three-point correlator,

$$\sum_n\langle\bar{\psi}\psi(x)\bar{\psi}\psi(y)\bar{\psi}\psi(z)\rangle^{\mu\neq0}_n = 2\cos[2\mu(x_1 + y_1 + z_1)]\langle s_+(x)s_+(y)s_+(z)\rangle^{\mu=0}_{n=3}$$
$$+2\cos[2\mu(x_1 + y_1 - z_1)]\langle s_+(x)s_+(y)s_-(z)\rangle^{\mu=0}_{n=1}$$
$$+2\cos[2\mu(x_1 - y_1 + z_1)]\langle s_+(x)s_-(y)s_+(z)\rangle^{\mu=0}_{n=1}$$
$$+2\cos[2\mu(-x_1 + y_1 + z_1)]\langle s_-(x)s_+(y)s_+(z)\rangle^{\mu=0}_{n=1} \quad (44)$$

[we have emphasized that the left-hand side of Eq. (44) exhausts all the topological contributions by explicitly showing the sum over n].

From the examples above, one can easily infer the structure for the general N-point correlator:

$$\langle\bar{\psi}\psi(w^1)\bar{\psi}\psi(w^2)\dots\bar{\psi}\psi(w^N)\rangle^{\mu\neq0}$$

$$= \sum_k\cos\left[2\mu\left(\sum_i w^i_1\right)\right]\langle s_+(w^1)s_+(w^2)\dots s_+(w^N)\rangle^{\mu=0}_{n=N} + \sum_k\cos\left[2\mu\left(\sum_{i\neq k} w^i_1 - w^k_1\right)\right]$$

$$\times\langle s_+(w^1)s_+(w^2)\dots s_+(w^{k-1})s_-(w^k)s_+(w^{k+1})\dots s_+(w^N)\rangle^{\mu=0}_{n=N-1}$$

$$+ \sum_{k,j}\cos\left[2\mu\left(\sum_{i\neq k,j} w^i_1 - w^k_1 - w^j_1\right)\right]\langle s_+(w^1)s_+(w^2)\dots s_+(w^{k-1})s_-(w^k)s_+(w^{k+1})\dots s_+(w^{j-1})s_-(w^j)$$

$$\times s_+(w^{j+1})\dots s_+(w^N)\rangle^{\mu=0}_{n=N-2} + \cdots . \quad (45)$$

A compact expression for the nonminimal correlation functions appearing in Eq. (45) is

$$\left\langle\prod_{i=1}^r s_+(w_i)\prod_{j=1}^s s_-(w_j)\right\rangle^{\mu=0}_{r-s} = (-me^\gamma/4\pi)^N \exp\left[-2\sum_{i>j} e_i e_j K_0(m|w_i - w_j|)\right], \quad (46)$$

where $r + s = N$ (for details see [16]).

In summary, we have presented the correlation functions of fermion bilinears in QED_2 at finite density, using a path integral approach, particularly adequate for identifying contributions arising from different topological sectors. We have been able to exactly compute correlation functions for an arbitrary number of bilinears, showing its dependence on the chemical potential. One of our motivations was a recent work by Deryagin, Grigoriev, and Rubakov [3], where it has been shown that in the large N_C limit, condensates in QCD are inhomogeneous and anisotropic at high fermion density.

The Schwinger model is a favorite laboratory for testing phenomena which are expected to occur in QCD_4. In fact, an oscillatory inhomogeneous behavior in $\langle \bar{\psi}\psi \rangle$ was discussed in the Schwinger model [8], using operator bosonization. We think that the path integral approach employed in the present paper is more appropriate to make apparent the crucial role that topological sectors play in the behavior of condensates (they are actually responsible for the nonvanishing of $\langle \bar{\psi}\psi \rangle$). In fact, our analysis implies that the phenomenon is not just a byproduct of two-dimensional peculiarities.

It is striking that the oscillatory behavior that we have found, exactly coincides (apart from the anisotropy that, of course, cannot be tested in one-spatial dimension) with that described in [3] for QCD_4. The structure of the N-point correlation functions, given by Eq. (45), shows a nontrivial dependence on spatial coordinates. This makes apparent that the ground state has, at finite density, an involved structure which is a superposition of standing waves with respect to the order parameter.

Several interesting issues are open for further investigation using our approach. One can, in particular, study in a very simple way the behavior of condensates at finite temperature. The chiral anomaly is independent of temperature and plays a central role in the behavior of condensates through its connection with the index theorem. Therefore, one should expect (as discussed in Ref. [8] for $\langle \bar{\psi}\psi \rangle$) that a formula such as (45) is valid also for $T > 0$. Of course, VEV's at $\mu = 0$ in the RHS of this equation should be replaced by those computed at finite temperature and hence the issue of zero modes in a toroidal manifold should be carefully examined (see, e.g., [16]).

Another extension which can be undertaken is related to the study of massless QCD_2. Indeed, the decoupling change of variables at the root of our approach, can be easily extended for non-Abelian gauge groups and has lead to deep insights in the properties of the model [5–7]. Finally, it should be worthwhile to consider massive fermions and compute fermion correlation functions via a perturbation expansion in the fermion mass following the approach of [18]. We hope to report on these problems in a future work.

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