How many new worlds are inside a black hole?

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We propose a possible internal structure for a Schwarzschild black hole resulting from the creation of multiple de Sitter universes with lightlike boundaries when the curvature reaches Planckian values. The intersection of the boundaries is studied and a scenario leading to disconnected de Sitter universes is proposed. The application to the information loss problem is then discussed.

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I. INTRODUCTION

The internal structure of black holes and their final state are two intriguing problems of black-hole physics. Both of these problems require knowledge of physics at Planckian scales for their solution. It is generally believed that only a union of quantum mechanics and gravity can provide us with a proper theory. Until now such a theory, quantum gravity, has not been constructed. It seems that one cannot overcome its main problem, nonrenormalizability, without unifying gravity with other physical fields. Superstring theory is one of the most promising approaches in this direction. But in spite of the impressive development of superstring theory we are still very far from understanding physics at Planckian scales.

Under these circumstances it is natural to use the following approach. One might assume that the notion of a quantum average of a metric $g=\langle\hat{g}\rangle$ is still valid in the regions under consideration, and the average metric g obeys some effective equations. We do not know these equations at the moment, but we might assume that these equations and their solutions obey some general properties and restrictions. For example, it is natural to require that the effective equations for g in the low-curvature limit reduce to the Einstein equations with possible higher-curvature corrections. It is also possible to assume that the future theory of quantum gravity would solve the problem of the singularities of classical general relativity. One of the possibilities is that the equations of the complete theory would simply not allow dynamically infinite growth of the curvature, so that the effective curvature $\mathscr{R}$ of g is bounded by the Planckian value $\approx 1/l_{\rm Pl}^2$. The principle of a limiting curvature was proposed by Markov [1,2]. This principle excludes curvature singularity formation, so that the global properties of the solutions must change.

A special form of the gravitational action for cosmological models providing the limiting-curvature principle was considered in Ref. [3]. It was shown that a collapsing homogeneous isotropic universe must stop its contraction and begin expansion, while during the transition phase its evolution is described by a metric close to the de Sitter one. Mukhanov and Brandenberger [4] proposed a general nonlinear gravitational action which allows only regular homogeneous isotropic solutions. Polchinski [5] proposed a simple realization of the limiting-curvature principle by modifying the action and inserting inequality constraints into it, restricting the growth of curvature. In the case of the collapse of an inhomogeneous universe, formation of a few ''daughter universes'' can be expected [2].

In the application to the black-hole-interior problem the limiting-curvature principle means that the singularity which, according to the classical theory exists inside a black hole, must be removed in the complete quantum theory, so that the global structure of spacetime would be essentially modified. We cannot hope to derive this result without knowledge of the theory, but we may at least discuss and classify possibilities. Such a ''zoological'' approach is a natural first step and it was used in a number of publications. One of the first models of a spherically symmetric black hole without singularities was proposed by Frolov and Vilkovisky [6]. In this model the apparent horizon does not cross $r=0$, so that $r=0$ is a regular timelike line. Two cases are logically possible. (1) Inner and external parts of the apparent horizon remain disparate. In this case a black hole does not evaporate completely and a permanent black-hole remnant remains. (2) The apparent horizon is closed. In this case, there is no event horizon (and hence, strictly speaking, no black hole), but practically all the observable properties of a black hole would be present until late times, when the apparent horizon disappears. This model was discussed later in Ref. [7] and, recently, both types of these singularity-free models of a black hole were used in the discussion of the final state of an evaporating black hole and information loss problem (see, e.g., [8]).

Another logically possible singularity-free model of a black-hole interior was proposed by Frolov, Markov, and Mukhanov (FMM) [9,10]. According to this model, inside a black hole there exists a closed universe instead of a singu-

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larity . The metric is obtained by gluing the Schwarzschild metric to the de Sitter metric through a surface $r=r_0=$const located inside the black hole. The parameter r_0 is chosen in such a way that the value of a curvature calculated at r_0 coincides with a limiting curvature, which is assumed to be of Planckian order. In this approach a fast transition between regimes is assumed and the transition region required for change of the regimes is approximated by a thin spacelike shell. In the FMM model [11] the spacetime passes through the deflation stage and instead of the singularity a new inflating universe is created. Morgan [12] showed that a similar result (formation of a contracting closed de Sitter-like universe with its further inflation) can be obtained in the framework of the Polchinski approach to the limiting-curvature principle. Different aspects of the model of a singularity-free black-hole interior with an inner de Sitter-like universe were also studied in [5,12–14].

One of the assumptions of the FMM and other similar models is that a "phase transition" to the de Sitter-like phase takes place at the homogeneous spacelike surface $r=r_0$. The presence of perturbations and quantum fluctuations, growing as $r \to 0$, could spoil the homogeneity. The bubbles of the new de Sitter-like phase could be formed independently at points separated by spacelike distances. For these reasons one could expect that different parts of a black-hole interior can create spatially disconnected worlds. The aim of this paper is to consider a simple model which could describe possible features of this process. Namely, we suppose that spherical bubbles of the new de Sitter-like phase which are formed independently are separated from the old (Schwarzschild) phase by relativistically moving boundaries. Under this assumption one can reduce the problem to the study of the evolution of light shells representing the boundaries and their intersection. The general theory of lightlike shells was developed by Barrabes and Israel [15] (see also [16]). This approach is purely kinematic in the following sense. It allows one to take into account the conservation of energy and momenta during the process of nucleation and the further evolution of the boundaries, including possible intersection of the boundaries of two different bubbles. But it certainly does not answer questions concerning probability of bubble formation or the structure of the transition regions between phases. One of the interesting results of the model is that it does not exclude creation of a large number of new-born universes. This fact could have an interesting application to the information-loss puzzle.

The paper is organized as follows. Section II contains the discussion of the model and gives the conditions for the nucleation of a de Sitter bubble inside a Schwarzschild black hole. The creation of multiple de Sitter bubbles is considered in Sec. III, and the interaction between the boundaries of newly created de Sitter bubbles is discussed in Sec. IV. In Sec. V we discuss the possible application of the process of multiple universe formation to the information-loss puzzle. Some of the main properties of timelike and lightlike shells which are used in our model are summarized in the Appendix.

II. DESCRIPTION OF THE MODEL

A. Model

The Schwarzschild metric

$$ds^2=-F^{-1}dr^2+Fdt^2+r^2d\Omega^2, \tag{1}$$

$$F=r_+/r-1, \quad d\Omega^2=d\theta^2+\sin^2\theta d\phi^2 \tag{2}$$

inside the gravitational radius ($r<r_+$) describes the contracting homogeneous Kasner-like universe with the isometry group $R(1)\times O(3)$. The section of fixed time $r=$ const$<r_+$ is a homogeneous spacelike surface Σ with topology $R^1\times S^2$. The square of the curvature $\mathcal{R}^2\equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}=12r_+^2/r^6$. The value of the curvature $\mathcal{R}$ is constant along Σ and it is of order of the Planckian curvature $l_{\rm Pl}^{-2}$ at $r\sim r_0=(12)^{1/6}(r_+/l_{\rm Pl})^{1/3}l_{\rm Pl}$. In the FMM model it is assumed that as soon as the spacetime curvature reaches some critical value $\mathcal{R}=1/l^2$ a new de Sitter-like phase is formed. In the application to the unperturbed Schwarzschild metric the change to a new phase occurs everywhere simultaneously [at the spacelike surface $r=r_0\sim(r_+l^2)^{1/3}$]. Another assumption of the FMM model is that the transition takes a short time, so that the transition region can be approximated by a thin shell.

For a black hole formed by the collapse of a body with small deviations from spherical symmetry the metric at a finite radius r tends to the Schwarzschild metric (1) at large distance from the collapsing body [17]. On the other hand perturbations existing in the black-hole exterior and propagating inwards grow infinitely near the singularity. Quantum fluctuations of metric also become important when the spacetime curvature reaches the limiting (Planckian) value l^{-2}. The "phase transition" into the new de Sitter-like phase happens independently in different spatially separated parts of the black-hole interior. It is also plausible that due to the fluctuations of the new-phase bubble formation there is a dispersion in the times of bubble formation. Under these conditions the assumption of spatial homogeneity used in the FMM model is rather restrictive and it is necessary to consider a generalization of this model. Our purpose is to generalize the FMM model to the case where the transition to the de Sitter-like phase occurs independently in spatially separated regions. In order to describe the possible structure of spacetime we consider a toy model in which spherical symmetry is preserved. We assume that, near the singularity of a Schwarzchild black hole, the transition to a new "de Sitter phase" takes place at two-spheres S. We preserve another assumption of the FMM model, namely, that the transition takes a short time and the transition region can be approximated by a thin shell. In our generalization of the FMM model we assume that the boundary between the two phases (Schwarzschild and de Sitter) is composed of two null hypersurfaces lying to the future of S and intersecting at S. Using this simple model we discuss different possibilities of nucleation of bubbles with de Sitter-like interiors and the interaction between the newly created bubbles.

B. Lightlike shells

Lightlike shells separating two regions of a spacetime with different characteristics have proved to be a convenient way of dealing with various physical or mathematical problems in general relativity [15]. This happens because the dynamics of lightlike shells is simple and it is directly related with geometrical properties at the junction of the two spacetime regions. A good example illustrating this is the collision of two null shells. It has been shown, first in the restricted

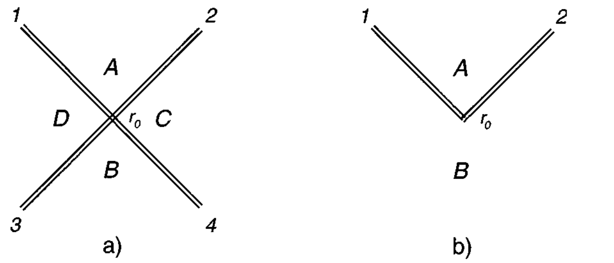

FIG. 1. The intersection of two lightlike shells (a). The incoming lightlike shells 3 and 4 after intersection at the two-dimensional surface r_0 propagate further as lightlike shells 1 and 2 with parameters, different from the original ones. A special case of this process is shown in (b): The masses of the incoming shells 3 and 4 vanish, so that outgoing shells 1 and 2 are created from "nothing."

case of spherical symmetry [18,19] and later in more general situations [20], that the geometries of the four spacetime domains bounded by the ingoing and outgoing shells are matched at the collision by only two remarkably simple algebraic relations. In the particular case of two concentric spherical shells moving radially toward each other with the velocity of light one of the matching conditions is trivially satisfied while the other takes the form

$$f_A(r_0) f_B(r_0) = f_C(r_0) f_D(r_0). \tag{3}$$

Here r_0 is the radius of the collision sphere, and the functions $f_A, f_B, \ldots$ for the spacetime domains $A, B, \ldots$ [see Fig. 1(a)] are defined by $f = g^{\alpha\beta}\partial_\alpha r \partial_\beta r = g^{rr}$. In what follows we consider a spherically symmetric spacetime with the metric

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2. \tag{4}$$

Both Schwarzschild and de Sitter metrics are of this form [21].

As a special case formula (3) describes the creation of a pair of lightlike shells from "nothing" [see Fig. 1(b)]. When two lightlike shells are created at the sphere of radius r_0 the geometries of the three spacetime domains B, C, D are identical and Eq. (3) becomes

$$f_B(r_0)[f_A(r_0) - f_B(r_0)] = 0.$$

If none of the shells coincides with the horizon of the region B [$f_B(r_0) \neq 0$], the matching relation (3) reduces to [22]

$$f_A(r_0) = f_B(r_0). \tag{5}$$

The consistency between the geometrical formula (5) and the conservation laws which have to be satisfied at the moment of creation of two shells can be checked.

In a spherically symmetric spacetime and in the absence of energy fluxes the surface stress-energy tensor of the null shell is uniquely determined by the surface energy density $\sigma(r)$, which is given by

$$4\pi r^2 \sigma(r) = \zeta r [f_+(r) - f_-(r)]/2. \tag{6}$$

Here $\zeta = +1(-1)$ if r increases (decreases) in the direction of the future-directed null generators $n = \zeta \partial/\partial r$ and f_+ (f_-) refers to the future (past) side of the shell (for more details see Ref. [16] and the Appendix). Relations (5) and (6) show that the energy surface density of the lightlike shells must vanish at the moment of creation [$\sigma_1(r_0) = \sigma_2(r_0) = 0$]. After the creation the shells possess nonvanishing surface energy densities $\sigma_1(r)$ and $\sigma_2(r)$ given by (6). The relative signs of $\sigma_1(r)$ and $\sigma_2(r)$ depend on the values of the ζ's and the jump of the function f.

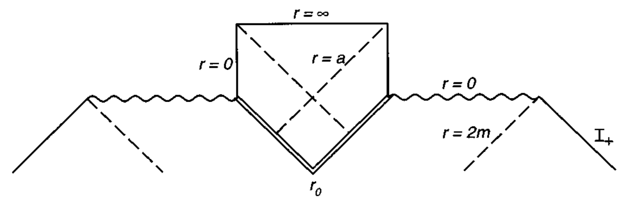

FIG. 2. A single de Sitter phase bubble formation.

C. de Sitter phase bubble creation in the black-hole interior

We consider at first the formation of a single de Sitter phase bubble. Denote by r_0 the radius of the sphere S where a single bubble is nucleated. We call S the *vertex sphere*. According to our assumption the creation of the bubble is accompanied by the creation of two lightlike shells separating a newly formed de Sitter phase from the "old" Schwarzschild one. The corresponding Penrose-Carter diagram is shown in Fig. 2.

Both shells converge towards $r = 0$. For this reason their creation can only occur in the region $r > a$ of the de Sitter spacetime where all future-directed light rays contract. Introducing $f_A(r) = 1 - r^2/a^2$ and $f_B(r) = 1 - 2m/r$ in the matching equation (5) one gets

$$r_0^3 = 2ma^2. \tag{7}$$

Here a is the radius of the de Sitter horizon, $a^2 = 3/\Lambda = 3/8\pi\rho$ (ρ being the false-vacuum energy density). We assume that the value of a is a fixed parameter of our model and that $a \ll r_0 \ll 2m$.

The two lightlike shells bounding the de Sitter universe behave identically and both converge towards $r = 0$. Their surface energy densities are thus equal $\sigma_1(r) = \sigma_2(r)$, and from (7) they are given by

$$4\pi r^2 \sigma_1(r) = -m\left(1 - \frac{r^3}{r_0^3}\right). \tag{8}$$

This relation shows that the surface energy density is negative and the value of the negative mass of the shells grows to $-m$ as their size r goes to zero.

In comparison with the FMM model, where the transition between the Schwarzschild and de Sitter spacetimes occurs instantaneously along a spacelike shell, we now have a situation which is no longer homogeneous as one moves along hypersurfaces $r = \text{const} < r_0$. This inhomogeneity can even be enhanced if several bubbles are created and if their boundaries intersect.

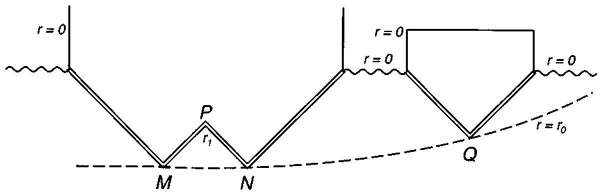

FIG. 3. Multiple bubbles of the de Sitter-like phase creation.

III. CREATION OF MULTIPLE de SITTER BUBBLES

A. Conditions for the intersection of bubbles

Consider now the creation of multiple bubbles with a de Sitter-like phase interior. If a couple of bubbles is created not far from one another, their lightlike boundaries may intersect before reaching the singularity $r=0$. Let us obtain the conditions when it occurs.

The Schwarzschild metric (1) near $r=0$ can be approximated by

$$ds^2=-\frac{r}{2m}dr^2+\frac{2m}{r}dt^2+r^2d\Omega^2.$$

Introducing the proper time coordinate $d\tau=-(r/2m)^{1/2}dr$ we get

$$ds^2=-d\tau^2+\left(-\frac{3\tau}{4m}\right)^{-\frac{2}{3}}dt^2+\left(-\frac{3\tau}{4m}\right)^{\frac{4}{3}}(2m)^2d\Omega^2. \tag{9}$$

The radius r and the proper time τ are related as $r^3=9m\tau^2/2$ and r decreases as τ increases.

Consider a couple of de Sitter bubbles created at the vertex spheres M and N of the same radius r_0 and intersecting at the sphere P of the radius r_1 (see Fig. 3). What happens after the collision of the lightlike shells at the sphere P is for the moment left unspecified and will be discussed later.

Using the metric (9) it is easy to show that the proper distance l between two vertex spheres M and N expressed as the function of the intersection radius r_1 is

$$l=r_0\left(1-\frac{r_1^2}{r_0^2}\right)\left(\frac{r_0}{2m}\right)^{\frac{1}{2}}=a\left(1-\frac{r_1^2}{r_0^2}\right), \tag{10}$$

while the coordinate t distance is

$$\Delta t=\frac{r_0^2-r_1^2}{2m}=\frac{a^2}{r_0^3}(r_0^2-r_1^2). \tag{11}$$

The distance l reaches its maximum value $l_{\max}$ when $r_1=0$. One has

$$l_{\max}=r_0\left(\frac{r_0}{2m}\right)^{\frac{1}{2}}=a, \quad \Delta t_{\max}=\frac{r_0^2}{2m}=\frac{a^2}{r_0}. \tag{12}$$

Two de Sitter universes created at the vertex spheres N and Q and separated by a proper distance l which is larger than $l_{\max}$ remain completely disconnected. For $l<l_{\max}$ the boundaries of the two bubbles intersect before they reach the singularity. What happens after the intersection depends upon the assumptions made at the collision of the two lightlike shells. In this section we consider the simplest possible scenario, when the two shells crossing one another have only gravitational interaction and remain lightlike after their intersection. The second logically possible and more complicated case of merging shells will be discussed in the next section.

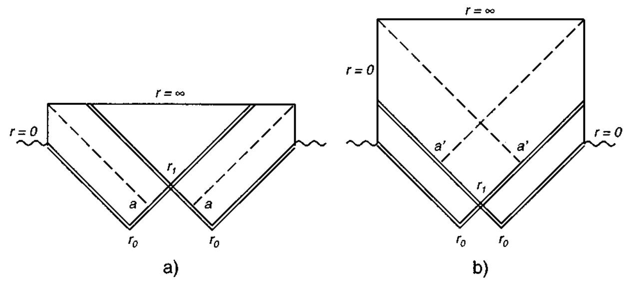

FIG. 4. Free intersection of lightlike boundaries of two bubbles.

B. Crossing of the boundaries

Consider at first the case when the incoming lightlike shells pass through each other and produce two outgoing lightlike shells. We assume that after the collision a new de Sitter universe, with a different horizon a', is formed. By using the matching relation (3) we get

$$\left(1-\frac{2m}{r_1}\right)\left(1-\frac{r_1^2}{a'^2}\right)=\left(1-\frac{r_1^2}{a^2}\right)^2, \tag{13}$$

where r_1 is the radius of the intersection sphere. Because the intersection takes place inside the gravitational radius of the black-hole ($r_1<2m$), the collision can only occur if r_1 is larger than a'.

Using the dimensionless variables

$$z=\frac{r_0}{a}=\left(\frac{2m}{a}\right)^{\frac{1}{3}}, \quad x=\frac{r_1}{a}, \tag{14}$$

one can rewrite Eq. (13) in the form

$$\left(\frac{a}{a'}\right)^2=\frac{1}{x^2}+\frac{(1-x^2)^2}{x(z^3-x)}. \tag{15}$$

According to our assumptions $0\leqslant x\leqslant z$. For $a=a'$ one has $x=1$. Equation (15) shows that $x>1$ for $a<a'$ and $x<1$ for $a>a'$. In the former case we have $a'<r_1<a$, while in the latter case $a<a'<r_1$. These two cases correspond to two different ways of gluing the two de Sitter spacetimes shown respectively in Figs. 4(a) and 4(b). A new bubble of false-vacuum with a different energy density appears between the two initially created false-vacuum bubbles. The new bubble either coexists indefinitely with two others [Fig. 4(a)] or finally occupies the whole space [Fig. 4(b)]. In the former case the initial false-vacuum bubbles must be nucleated close enough to one another. As the limiting value of r_1 separating the two cases is such that $r_1=a=a'$, it follows from (10) that this occurs whenever the proper distance between the

vertex spheres is smaller than $a(1-a^2/r_0^2)$. In the latter case, shown in Fig. 4(b), the new de Sitter spacetime is flatter than the original ones.

For symmetry reasons the equations of motion of both shells as well as their surface energy density are identical, $\sigma_1'(r)=\sigma_2'(r)$. The surface energy density of the shells after their intersection can be obtained from Eq. (6) and it is of the form

$$4\pi\sigma_1'(r)=\frac{\zeta r}{2}\left(\frac{1}{a^2}-\frac{1}{a'^2}\right). \tag{16}$$

This relation shows that the surface energy density of the shells after their intersection is a linear function of the radius r. In the case where $a'<r_1<a$, we have $\zeta=1$ and the lightlike shells which were initially contracting bounce at the collision and expand to infinity [see Fig. 4(a)]. In the second case [Fig. 4(b)] we have $\zeta=-1$ and the lightlike boundaries contract to zero radius. In both cases $\sigma_1'(r)$ is negative as it is expected to be from the law of conservation of energy at the collision (the ingoing shells have negative energies). For the case shown in Fig. 4(b) the mass of the shells vanishes at the point when the shells cross $r=0$.

IV. INTERACTION OF THE LIGHTLIKE BOUNDARIES OF TWO de SITTER UNIVERSES

A. Merging of the boundaries

A different situation which may occur at the collision of the two false-vacuum bubbles is when their lightlike boundaries interact strongly and merge into a single timelike shell (this process is in some sense analogous to the creation of a massive particle from two colliding photons). In that case the two bubbles remain attached after the collision through a spherical surface layer moving with subluminal velocity (see Fig. 5).

Both incoming lightlike shells are contracting, so that the radius of collision r_1 is smaller than r_0. The metrics in the regions A and B shown in Fig. 5 are de Sitter metrics, while the metric in the region C is the Schwarzschild one. The corresponding metric functions $f(r)$ are

$$f_A(r)=f_B(r)=1-\frac{r^2}{a^2}, \quad f_C(r)=1-\frac{2m}{r}. \tag{17}$$

Let us obtain the expression for the parameters of the timelike shell in terms of the parameters for colliding null shells. For this purpose we use the matching condition (A15):

$$[\dot r_1+\varepsilon_A\sqrt{f_A(r_1)+\dot r_1^2}][\dot r_1-\varepsilon_B\sqrt{f_B(r_1)+\dot r_1^2}]=-f_C(r_1). \tag{18}$$

(This relation as well as other useful formulas for moving and colliding shells are collected in the Appendix.) Here r_1 is the radius of the collision sphere, and the parameter ε which enters this expression is defined as $\varepsilon=\mathrm{sgn}(n^\alpha\partial_\alpha r)$. Since the spacetime domains bordering the timelike shell are identical, we must take $\varepsilon_A=-\varepsilon_B$; otherwise, there is no shell [see Eq. (A5) of the Appendix]. Equation (18) implies

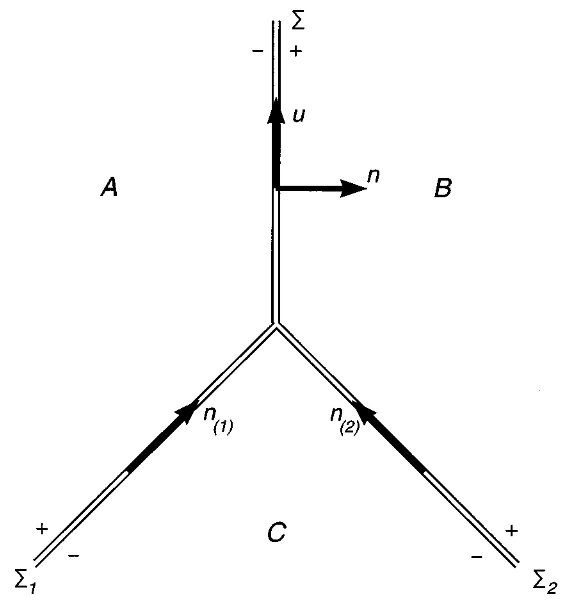

FIG. 5. Merging of two lightlike shells into one timelike shell.

$$\dot r_1^2=-\frac{[f_A(r_1)+f_C(r_1)]^2}{4f_C(r_1)}. \tag{19}$$

On the other hand for the timelike shell one has [see Eq. (A5)]

$$\frac{M(r)}{r}=2\varepsilon_A[f_A(r)+\dot r^2]^{1/2}, \tag{20}$$

where $M(r)=4\pi r^2\sigma(r)$ is the inertial mass of the timelike shell and $\sigma(r)$ its energy surface density. By combining relations (19) and (20) we get the initial mass $M(r_1)$ of the timelike shell at the moment of the collision:

$$M(r_1)=\frac{2M_1(r_1)}{|f_C(r_1)|^{1/2}}. \tag{21}$$

Here $M_1(r_1)=4\pi r_1^2\sigma_1(r_1)$ is the mass of the lightlike shells at the collision. As expected from the conservation of energy, it follows from (21) that the masses of the ingoing lightlike shells and of the outgoing timelike shell have the same negative sign (recall that the lightlike shells have negative energy densities).

The further evolution of the timelike shell is given by Eq. (20). It can be rewritten in the form

$$\dot r^2+V(r)=-1, \tag{22}$$

where $V(r)$ is an effective potential given by

$$V(r)=-\frac{r^2}{a^2}-4\pi^2r^2\sigma^2(r). \tag{23}$$

Equation (22) shows that the motion is only possible when $V(r)\leqslant-1$.

To study this equation it is convenient to use the dimensionless radius $y=r/a$ and time $T=\tau/a$ as well as the variables x and z already defined by (14). (x is an initial value of y and $dy/dT=\dot{r}$.) In these variables the equation of motion (22) takes the form

$$\left(\frac{dy}{dT}\right)^2+V(y,x,z)=-1, \tag{24}$$

where z is a given quantity and x a free parameter such that $z\gg 1$ and $x\in[0,z]$. The initial mass (21) of the timelike shell can be rewritten as

$$M(r_1)=aF(x,z), \tag{25}$$

with

$$F^2(x,z)=\frac{(z^3-x^3)^2x}{z^3-x}. \tag{26}$$

B. Evolution of a merging boundary

The motion of the timelike shell depends on the equation of state for the matter forming the shell. For simplicity we assume a dustlike equation of state. Under this assumption $\sigma r^2=$ const<0 along the shell and the mass of the shell is conserved and coincides with $M(r_1)$ given by Eqs. (25) and (26).

For a dustlike equation of state the potential V is

$$V(r)=-\frac{r^2}{a^2}-\frac{M^2(r_1)}{4r^2},$$

and written in terms of the dimensionless variables it takes the form

$$V(y,x,z)=-y^2-\frac{F^2(x,z)}{4y^2}. \tag{27}$$

If the initial condition is chosen so that $T=0$ when $y=x$, the solution of Eq. (24) is given in the implicit form

$$T=\frac{1}{2}\ln\left|\frac{2y^2-1+(4y^4-4y^2+F^2)^{1/2}}{2x^2-1+(4x^4-4x^2+F^2)^{1/2}}\right|. \tag{28}$$

The solution depends on x (the initial value of y) and on the parameter z. We consider a black hole of large mass, so that we have $z\gg 1$.

Motion of the timelike shell is possible only in the region where $V(y,x,z)\leqslant -1$. Denote by y_m the value of y where the potential V reaches its maximum value $V_m=V(y_m)$. Equation (27) implies that

$$y_m=\frac{F^{1/2}(x,z)}{\sqrt{2}},\quad V_m=-F(x,z). \tag{29}$$

It is easy to show that there exist two values of x (x_1 and x_2) for which $V_m=-1$. For $z\gg 1$ one has

$$x_1=\frac{1}{z^3}+o\left(\frac{1}{z^7}\right),\quad x_2=z-\frac{1}{3z}+o\left(\frac{1}{z^5}\right). \tag{30}$$

$V_m<-1$ for $x\in[x_1,x_2]$ [curve B in the Fig. 6] and $-1<V_m<0$ for $x\in[0,x_1]$ or $x\in[x_2,z]$ (curves A and C in the Fig. 6).

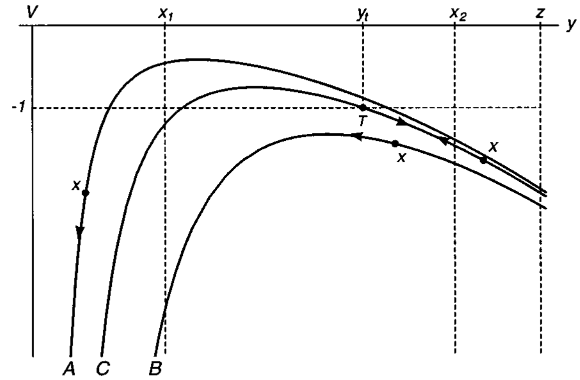

FIG. 6. The potential $V(y,x,z)$ for different values of the parameter x.

For a fixed parameter z the evolution of the shell depends on the initial condition x (the initial radius of the timelike shell). A timelike shell is created at a stage of contraction, so that $(dy/dT)_{T=0}=\dot{r}_1<0$. Under this condition only two qualitatively different types of motion are possible.

(1) $0<x\leqslant x_2$. In this case the timelike shell monotonically contracts from its initial radius r_1 down to zero [23]. The two de Sitter universes which were initially attached through the timelike shell become disconnected after the time interval $T_{\rm sep}$:

$$T_{\rm sep}=\frac{1}{2}\ln\left|\frac{F-1}{2x^2-1+(4x^4-4x^2+F^2)^{1/2}}\right|. \tag{31}$$

For $x\in[0,x_1]$ (curve A in Fig. 6) the separation time is less than $z^{-6}a$ and the bubbles separate almost immediately after their intersection. For $x\in[x_1,x_2]$ (curve B of Fig. 6), the time of the separation can reach values of the order of a.

(2) $x_2<x<z$. The trajectory admits a turning point $V(y_t)=-1$ at the value $y=y_t$:

$$y_t^2=\frac{1}{2}[1+(1-F^2)^{1/2}]. \tag{32}$$

In that case the timelike shell first contracts, bounces at y_t, and then expands to infinity (curve C in Fig. 6). The two de Sitter universes remain connected through a spherical shell which has a radius that bounces and increases to infinity. The time which is needed for the shell to bounce from its initial radius can be estimated as $T\sim \ln z$.

The conformal Penrose-Carter diagrams for these two qualitatively different cases are shown in Fig. 7. Figure 7(a) illustrates the evolution of a timelike shell with the initial condition $y=x\in[0,x_2]$ at the moment of its formation. The contraction of the timelike shell results in the separation of two de Sitter-like universes at the moment P. Figure 7(b) illustrates the evolution of a timelike shell with the initial condition $y=x\in[x_2,z]$. The timelike shell changes its contraction into expansion. The two de Sitter-like universes remain connected.

The condition $x_2<x<z$ which guarantees that two bubbles form the same de Sitter-like universe after merging

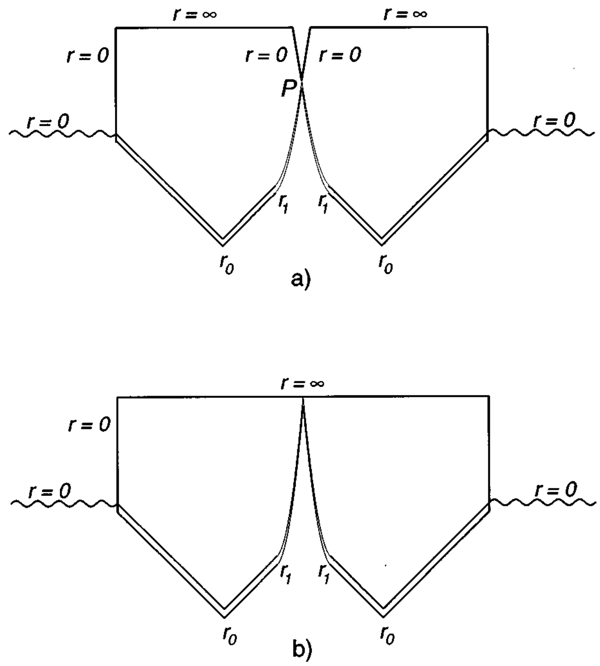

FIG. 7. Conformal Penrose-Carter diagrams for a spacetime with two de Sitter-like phase bubbles with merging boundaries for different distances between bubbles at the moment of creation. In both figures two double lines beginning at the radius r_1 and lying to the future represent the same timelike shell and must be identified.

of their boundaries can be rewritten in terms of the separation between the vertex spheres of the bubble nucleation. First we remark that this condition, written in dimensional form, is $r_0 - a^2/3r_0 < r_1 < r_0$. [We use here the relation (30) valid for $z \gg 1$.] Using relations (10) and (11) we get that this condition is equivalent to

$$0 < l < \frac{a}{3}\left(\frac{a}{r_0}\right)^2, \quad 0 < \Delta t < \frac{a}{3}\left(\frac{a}{r_0}\right)^3. \tag{33}$$

Since $a/r_0 \ll 1$, two bubbles form a unique de Sitter universe only if their vertices are extremely close one to another. In other words, in the framework of the chosen model the creation of multiple disconnected de Sitter-like universes is the most plausible process.

V. CONCLUDING REMARKS

We discuss now some physical consequences of the possible multiple de Sitter-like universe formation inside a black-hole. The above consideration indicates that the nucleation of bubbles of the de Sitter-like phase might result in the formation of disconnected worlds as soon as $\Delta t \gtrsim a^4/r_0^3$. It means that during the time $t_{\rm evap} \sim t_{\rm Pl}(m/\mu_{\rm Pl})^3$ of the quantum evaporation of a black hole of mass m there might be formed as many as

$$N_{\max} \sim \left(\frac{l_{\rm Pl}}{a}\right)^2 \left(\frac{m}{\mu_{\rm Pl}}\right)^4 \tag{34}$$

new de Sitter-like disconnected universes. ($t_{\rm Pl}$, $l_{\rm Pl}$, and $\mu_{\rm Pl}$ are Planckian time, length, and mass, respectively.) For $a \sim l_{\rm Pl}$ when the curvature of a newly created de Sitter world is Planckian, $N_{\max} \sim (m/\mu_{\rm Pl})^4$. Remarkably this quantity is much larger than the dimensionless Bekenstein-Hawking entropy of a black-hole $S^{\rm BH}$ or the number of emitted quanta of the Hawking radiation $N \sim (m/\mu_{\rm Pl})^2$ (both are of the same order of magnitude).

The number of different possibilities for the distribution of N identical particles in $N_{\max}$ "boxes" (representing newly created worlds) is $N_{\max}!/(N_{\max}-N)!N!$. If these possibilities are equally likely, then the corresponding entropy S is

$$S = \ln[N_{\max}!/(N_{\max}-N)!N!]. \tag{35}$$

Using the relation $\ln N! \approx N \ln(N/e)$ we get, for $N \ll N_{\max}$

$$S \approx N \ln(N_{\max}/N). \tag{36}$$

In other words for $N_{\max} \gg N$ the entropy connected with all possible distributions of the created Hawking quanta over the newly born worlds inside a black hole is greater than the entropy of these quanta. It means that entropy considerations in principle do not exclude the situation where the internal state of each newly created universe is pure while the ensemble of universes is described by a density matrix. This remark might have quite interesting applications to the information-loss puzzle.

To summarize, we proposed a simple model for the black-hole interior. In the framework of this model we showed that at least kinematically the creation of many separated universes is the most probable process. Certainly the model itself contains a lot of simplifications. But the obtained results indicate that multiple universe formation in the black-hole interior should be taken seriously in the discussion and classification of different nonsingular black-hole models.

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APPENDIX: TIMELIKE AND LIGHTLIKE SHELLS IN A SPHERICALLY SYMMETRIC SPACETIME

In this appendix we derive the matching relations which have to be fulfilled when two ingoing lightlike shells merge into a single timelike shell. We only consider the case of concentric spherical shells that move radially. The three spacetime domains bounded by the shells are static and spherically symmetric with line elements of the form

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{A1}$$

We call A, B, C the three spacetimes, Σ_1, Σ_2 the lightlike shells, and Σ the timelike shell (see Fig. 5).

Before deriving the matching relation at the intersection let us recall the main equations describing timelike and lightlike shells (for more details see [16]).

We call $u=d/d\tau$ the normalized four-velocity and n the unit normal along the timelike surface Σ. With the line element (A1) the components of these vectors are

$$u^{\alpha}=(\varepsilon_1 f^{-1}(r)\sqrt{f(r)+\dot{r}^2},\dot{r},0,0), \tag{A2}$$

$$n^{\alpha}=(\varepsilon_2\dot{r}f(r),\varepsilon\sqrt{f(r)+\dot{r}^2},0,0), \tag{A3}$$

with $\varepsilon_1,\varepsilon_2=\pm 1$, $\varepsilon=\varepsilon_1\varepsilon_2=$ sgn$(n^{\alpha}\partial_{\alpha}r)$, and $\dot{r}=d/d\tau$. The induced metric of Σ is equal to

$$ds^2=-d\tau^2+r^2(\tau)(d\theta^2+\sin^2\theta d\phi^2). \tag{A4}$$

A spherically symmetric timelike shell is a two-dimensional perfect fluid with surface energy σ and surface pressure p. Let us call $\mathscr{A}=4\pi r^2$ the area and $M=\sigma\mathscr{A}$ the internal mass. Spherical symmetry reduces the number of independent equations describing the motion of the shell to the following two:

$$-\frac{M}{r}=[\varepsilon\sqrt{f(r)+\dot{r}^2}], \tag{A5}$$

$$\frac{dM}{d\tau}=-p\frac{d\mathscr{A}}{d\tau}+[T_{\alpha\beta}u^{\alpha}n^{\beta}]. \tag{A6}$$

Here $T_{\alpha\beta}$ is the stress-energy tensor of the external medium and the square brackets represent the jump of the enclosed quantity across the surface, i.e., $[F]=F_+-F_-$. It is assumed that the normal n is directed toward the $+$ side.

Equation (A5) is the equation of motion of the shell and Eq. (A6) the equation of conservation of energy. To specify the problem one needs also to give an equation of state. If, for instance, one takes $p=(\alpha-1)\sigma$ with $\alpha=$ const and if $[T_{\alpha\beta}u^{\alpha}n^{\beta}]=0$, one gets, from (A6),

$$\sigma r^{2\alpha}=\text{ const.} \tag{A7}$$

This relation allows one to define $M=M(r)$ which enters Eq. (A5).

The description of a null shell is very different from a timelike one and in some sense simpler because the equation of motion is fixed. What makes a null hypersurface so peculiar is that its normal vector is at the same time tangent to it and that its induced metric is degenerate.

Let us call Σ_i with $i=1,2$ the two ingoing null shells. The basis vectors $e_{(A)}=\partial/\partial\xi^A$ where $\xi^A=(\theta,\phi)$ are tangent to the shells and one takes $n_{(i)}=\zeta_i\partial/\partial r$ as the null vector tangent to the null generators of Σ_i. Here $\zeta_i=+1(-1)$ whenever r increases (decreases) toward the future along the null generators. The "extrinsic" curvature is defined by

$$K_{iAB}=-n_{(i)}\frac{\delta e_{(A)}}{\delta\xi^B}, \tag{A8}$$

where $\delta/\delta\xi^B$ is the four-dimensional covariant derivative. As $n_{(i)}$ is tangent to Σ_i, J_{iAB} is an intrinsic property of the shell which actually describes the behavior of its null generators. For instance, the trace $K_i=g^{AB}K_{iAB}$ represents their expansion rate and is equal to

$$K_i=\frac{2}{r}n^{\alpha}_{(i)}\partial_{\alpha}r. \tag{A9}$$

Finally when no energy is transferred to the shell, i.e., $[T_{\alpha\beta}n^{\alpha}_{(i)}n^{\beta}_{(i)}]=0$, the surface stress-energy tensor of a spherical null shell is characterized only by a surface energy density $\sigma_i(r)$, which is given by

$$4\pi r^2\sigma_1(r)=\frac{\zeta_1 r}{2}[f_A(r)-f_C(r)], \tag{A10}$$

$$4\pi r^2\sigma_2(r)=\frac{\zeta_2 r}{2}[f_B(r)-f_C(r)]. \tag{A11}$$

Let us now find the matching relations at the intersection of the shells. This intersection is in fact a two-sphere S with radius r_1 and at any point of S we can write the decompositions

$$n^{\alpha}_{(1)}=(u^{\alpha}+n^{\alpha})/\eta_1\sqrt{2}, \tag{A12}$$

$$n^{\alpha}_{(2)}=(u^{\alpha}-n^{\alpha})/\eta_2\sqrt{2}, \tag{A13}$$

where η_1 and η_2 are arbitrary positive scalars. Furthermore, the four-vectors $(n_{(i)},e_{(A)})$ form a basis at any point of S and we have the completeness relation

$$g^{\alpha\beta}=g^{AB}e^{\alpha}_{(A)}e^{\beta}_{(B)}-\frac{2n^{(\alpha}_{(1)}n^{\beta)}_{(2)}}{n_{(1)}\cdot n_{(2)}}. \tag{A14}$$

In order to get the matching relation we use the fact that K_i are intrinsic quantities and express the product K_1K_2 in two different manners using the spacetime domains A, B, and C. First using (A2), (A3), (A9), and (A12) one gets

$$K_1K_2=\frac{2}{r_1^2\eta_1\eta_2}[\dot{r}_1+\varepsilon_A\sqrt{f_A(r_1)+\dot{r}_1^2}][\dot{r}_1-\varepsilon_B\sqrt{f_B(r_1)+\dot{r}_1^2}].$$

Second, using (A9) and (A14) in sector C one gets

$$K_1K_2=-\frac{2n_{(1)}\cdot n_{(2)}}{r_1^2}f_C(r_1).$$

One then immediately derives the matching relation

$$[\dot{r}_1+\varepsilon_A\sqrt{f_A(r_1)+\dot{r}_1^2}][\dot{r}_1-\varepsilon_B\sqrt{f_B(r_1)+\dot{r}_1^2}]=-f_C(r_1). \tag{A15}$$

This equation gives the initial velocity $\dot{r}_1$ of the timelike shell after the merging of the lightlike shells. When this result is inserted in (A5) one gets the initial mass of the timelike shell.

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[23] This is evident for $x_1 < x < x_2$ because there are no turning points. For $0 < x < x_1$ there are turning points, but the contraction begins at the initial value $y = x$ lying to the left of the turning points.