PHYSICAL REVIEW D VOLUME 53, NUMBER 1 1 JANUARY 1996

B_c spectroscopy in a quantum-chromodynamic potential model

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(Received 1 August 1995)

We have investigated B_c spectroscopy with the use of a quantum-chromodynamic potential model which was recently used by us for the light-heavy quarkonia. We give our predictions for the energy levels and the $E1$ transition widths. We also find, rather surprisingly, that although B_c is not a light-heavy system, the heavy quark effective theory with the inclusion of the m_b^{-1} and $m_b^{-1}\ln m_b$ corrections is as successful for B_c as it is for B and B_s.

PACS number(s): 12.39.Pn, 12.39.Hg, 14.40.Lb, 14.40.Nd

I. INTRODUCTION

B_c spectroscopy has been investigated by several authors [1–4] in recent years by using different models and arriving at different predictions for this hitherto unobserved quarkonium. Although B_c consists of heavy quarks, its decay modes are not the same as those of $b\bar{b}$ and $c\bar{c}$. Indeed, because of flavor conservation in strong and electromagnetic interactions, the B_c ground state can only decay weakly, which makes it particularly interesting for the study of weak interactions.

We shall present our results for the B_c spectroscopy by using a quantum-chromodynamic potential model which was recently used by us for the light-heavy quarkonia [5]. An essential feature of our model is the inclusion of the one-loop radiative corrections in the quantum-chromodynamic potential, which is known to be responsible for the remarkable agreement between the theoretical and experimental results for spin splittings in the $b\bar{b}$ and $c\bar{c}$ spectra [6]. Another advantage of our model is that it is based on a nonsingular form of the quarkonium potential, and thus avoids the use of an illegitimate perturbative treatment.

The choice of potential parameters for B_c in the absence of experimental data will be discussed in Sec. II, while its spectrum and $E1$ transition widths will be given in Sec. III. We shall also demonstrate the rather surprising result that although B_c is not a light-heavy system, the heavy quark effective theory [7] with the inclusion of the m_b^{-1} and $m_b^{-1}\ln m_b$ corrections is as successful for B_c as it is for B and B_s.

II. B_c POTENTIAL PARAMETERS

Our model is based on the Hamiltonian

$$H = H_0 + V_p + V_c, \tag{1}$$

where

$$H_0 = (m_c^2 + \mathbf{p}^2)^{1/2} + (m_b^2 + \mathbf{p}^2)^{1/2} \tag{2}$$

is the relativistic kinetic energy, and V_p and V_c are nonsingular quasistatic perturbative and confining potentials, which are fully given in Ref. [5]. The perturbative potential with the one-loop corrections involves the parameters m_c, m_b, μ, and α_s, while the phenomenological scalar-vector exchange confining potential involves, besides the quark masses, the parameters A and B as well as an additive constant C.

We expect the dynamics of B_c to be largely dependent on the lighter quark c. Therefore, in the absence of experimental data, we assume that m_c, μ, α_s, A, and B for B_c have the same values as those for $c\bar{c}$, while m_b for B_c is obtainable from its value for $b\bar{b}$ by the QCD transformation relation. The constant C is usually fixed by the experimental value of the quarkonium ground state, but here we make the *ad hoc* assumption that C is equal to the average of its values for $c\bar{c}$ and $b\bar{b}$, so that

$$C_{b\bar{c}} = \tfrac{1}{2}\left(C_{c\bar{c}} + C_{b\bar{b}}\right). \tag{3}$$

We give in Tables I and II the spectra and parameter values for $c\bar{c}$ and $b\bar{b}$ by updating our earlier results [6] with the use of the latest experimental data provided by the Particle Data Group [8]. The values of α_s for $c\bar{c}$ and $b\bar{b}$ in these tables approximately satisfy the QCD transformation relation

$$\alpha_s' = \frac{\alpha_s}{1 + \beta_0(\alpha_s/4\pi)\ln(\mu'^2/\mu^2)}, \tag{4}$$

where $\beta_0 = 11 - \frac{2}{3}n_f$, $n_f = 3$. We also note that, according to the QCD transformation relation

$$m' = m\left(\frac{\alpha_s'}{\alpha_s}\right)^{2\gamma_0/\beta_0}, \tag{5}$$

with $\gamma_0 = 2$, the value of m_b in Table II for $\mu = \mu_{b\bar{b}}$ leads to

$$m_b = 5.453 \text{ GeV} \quad \text{for } \mu = \mu_{c\bar{c}}. \tag{6}$$

III. B_c SPECTRA AND $E1$ TRANSITIONS

We have calculated the B_c spectrum by using the potential parameters in Sec. II and following the same pro-

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TABLE I. $c\bar{c}$ spectrum and parameter values. The energy levels are given in MeV.

	Theory	Expt.
$1\ {}^1S_0\ (\eta_c)$	2979.1	2978.8±1.9
$1\ {}^3S_1\ (J/\psi)$	3096.9	3096.88±0.04
$2\ {}^1S_0\ (\eta_c')$	3617.9	
$2\ {}^3S_1\ (\psi')$	3685.9	3686.00±0.09
$1\ {}^3P_0\ (\chi_{c0})$	3415.2	3415.1±1
$1\ {}^3P_1\ (\chi_{c1})$	3510.8	3510.53±0.12
$1\ {}^3P_2\ (\chi_{c2})$	3556.5	3556.17±0.13
$1\ {}^1P_1\ (h_c)$	3526.4	3526.14±0.24
m_c (GeV)	2.212	
$\mu_{c\bar{c}}$ (GeV)	2.942	
α_s	0.306	
A (GeV^2)	0.181	
B	0.244	

TABLE II. $b\bar{b}$ spectrum and parameter values. The energy levels are given in MeV.

	Theory	Expt.
$1\ {}^1S_0\ (\eta_b)$	9407.6	
$1\ {}^3S_1\ (\Upsilon)$	9460.3	9460.37±0.21
$2\ {}^1S_0\ (\eta_b')$	9990.5	
$2\ {}^3S_1\ (\Upsilon')$	10016.1	10023.30±0.31
$3\ {}^1S_0\ (\eta_b'')$	10338.0	
$3\ {}^3S_1\ (\Upsilon'')$	10357.9	10355.3±0.5
$1\ {}^3P_0\ (\chi_{b0})$	9861.9	9859.8±1.3
$1\ {}^3P_1\ (\chi_{b1})$	9893.4	9891.9±0.7
$1\ {}^3P_2\ (\chi_{b2})$	9914.2	9913.2±0.6
$1\ {}^1P_1\ (h_b)$	9900.8	
$2\ {}^3P_0\ (\chi_{b0}')$	10228.8	10232.1±0.6
$2\ {}^3P_1\ (\chi_{b1}')$	10253.5	10255.2±0.5
$2\ {}^3P_2\ (\chi_{b2}')$	10269.8	10268.5±0.4
$2\ {}^1P_1\ (h_b')$	10259.4	
m_b (GeV)	5.406	
$\mu_{b\bar{b}}$ (GeV)	3.435	
α_s	0.283	
A (GeV^2)	0.184	
B	0.388	

TABLE III. B_c energy levels in MeV. Effective theory results are given with the m_b^{-1} and $m_b^{-1}\ln m_b$ corrections as well as in the limit of $m_b \to \infty$.

	Theory	Effective theory	$m_b \to \infty$
$1\ {}^1S_0\ (B_c)$	6246.9	6246.9	6246.9
$1\ {}^3S_1\ (B_c^*)$	6308.0	6311.0	6246.9
$2\ {}^1S_0$	6852.8	6853.5	6828.6
$2\ {}^3S_1$	6885.9	6887.9	6828.6
$1\ {}^3P_0$	6688.6	6693.8	6716.8
$1\ {}^3P_1'$	6737.5	6737.9	6716.8
$1\ {}^1P_1'$	6757.3	6758.1	6752.3
$1\ {}^3P_2$	6773.2	6772.3	6752.3
θ	25.6°	28.8°	35.6°

TABLE IV. B_c and B_c^* energy levels in MeV in our model and some earlier potential models.

	This work	Chen-Kuang	Eichten-Quigg	Gershtein *et al.*
${}^1S_0\ (B_c)$	6247	6310	6264	6253
${}^3S_1\ (B_c^*)$	6308	6355	6337	6317
$B_c^* - B_c$	61	45	73	64

TABLE V. $E1$ transition widths for B_c.

Transition	Photon energy (MeV)	$\lvert r_{fi}\rvert$ (GeV^{-1})	Γ_{E1} (keV)
$1\,{}^3P_2 \to 1\,{}^3S_1$	449	1.30	73.6
$1\,{}^3P_1' \to 1\,{}^3S_1$	416	1.19	49.0
$1\,{}^3P_1' \to 1\,{}^1S_0$	473	0.57	16.6
$1\,{}^3P_0 \to 1\,{}^3S_1$	370	1.33	43.0
$1\,{}^1P_1' \to 1\,{}^1S_0$	491	1.08	66.6
$1\,{}^1P_1' \to 1\,{}^3S_1$	434	0.52	10.5
$2\,{}^3S_1 \to 1\,{}^3P_2$	112	1.91	4.0
$2\,{}^3S_1 \to 1\,{}^3P_1'$	147	1.56	3.6
$2\,{}^3S_1 \to 1\,{}^1P_1'$	127	0.80	0.6
$2\,{}^3S_1 \to 1\,{}^3P_0$	194	1.49	2.6
$2\,{}^3S_0 \to 1\,{}^1P_1'$	95	1.73	3.6
$2\,{}^3S_0 \to 1\,{}^3P_1'$	114	0.78	1.3

cedure as was applied to the light-heavy quarkonia in Ref. [5]. The theoretical results for the energy levels, together with the ${}^3P_1'$-${}^1P_1'$ mixing angle arising from the spin-orbit mixing terms, are given in Table III. In this table, one set of results corresponds to the direct use of our model, while the other two sets are obtained by means of heavy quark expansions of our potentials with the inclusion of the m_b^{-1} and $m_b^{-1}\ln m_b$ corrections as well as without these corrections. Our results numerically differ to varying degrees from those of Chen and Kuang [2], Eichten and Quigg [3], and Gershtein *et al.* [4], and a comparison of various results for the lowest S states is shown in Table IV.

It should be noted that only the energy differences among the energy levels are predicted by our potential model, while the absolute energy levels have been obtained by making use of the assumption (3). A variation of the parameter $C_{b\bar{c}}$ will cause a common shift of our energy levels in Tables III and IV.

In Table V, we give the results for the $E1$ transition widths for B_c by using the formulas

$$\begin{aligned}
\Gamma_{E1}({}^3S_1 \to {}^3P_J) &= \frac{4}{9}\frac{2J+1}{3}\alpha\langle e_Q\rangle^2 k_J^3 |r_{fi}|^2,\\
\Gamma_{E1}({}^3P_J \to {}^3S_1) &= \frac{4}{9}\alpha\langle e_Q\rangle^2 k_J^3 |r_{fi}|^2,\\
\Gamma_{E1}({}^1P_1 \to {}^1S_0) &= \frac{4}{9}\alpha\langle e_Q\rangle^2 k_J^3 |r_{fi}|^2,\\
\Gamma_{E1}({}^1S_0 \to {}^1P_1) &= \frac{4}{3}\alpha\langle e_Q\rangle^2 k_J^3 |r_{fi}|^2,
\end{aligned} \tag{7}$$

where the mean charge $\langle e_Q\rangle$ is given by [3]

$$\langle e_Q\rangle = \frac{m_b e_c - m_c e_{\bar{b}}}{m_b + m_c}. \tag{8}$$

The photon energies for the $E1$ transition widths have been obtained from the energy difference of the initial and final $b\bar{c}$ states by taking into account the recoil correction.

Apart from numerical differences, our results in Table V differ from those of Ref. [3] in two respects. In Ref. [3], the results for r_{fi} are the same for all $1P \to 1S$ transitions as well as for all $2S \to 1P$ transitions [9]. We have a different value for r_{fi} for each transition because our nonsingular potential allows us to include the spin-dependent terms in the unperturbed Hamiltonian. Furthermore, in Ref. [3] some of the widths for transitions involving the mixed P states are vanishingly small, while this is not the case in our treatment. This difference indicates that our potential gives rise to a larger spin-orbit mixing effect.

Finally, a comparison of our results for B_c in Table III with the corresponding results for B and B_s in Ref. [5] shows that the heavy quark expansion with the m_b^{-1} and $m_b^{-1}\ln m_b$ corrections is as successful for B_c as it is for B and B_s. This is rather surprising because the heavy quark effective theory has been generally applied to the light-heavy quarkonia.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-85ER40209.

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[9] We have denoted the lowest P states of quarkonia as $1P$ in accordance with the notation followed by the Particle Data Group [8]. This notation differs from that used in Ref. [3].