

String theory and inflation

Thibault Damour

*Institut des Hautes Etudes Scientifiques, F-91440, Bures sur Yvette, France
and DARC, CNRS-Observatoire de Paris, F-92195 Meudon, France*

Alexander Vilenkin

*Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155
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String theory abounds with light scalar fields (the dilaton and various moduli) which create a host of observational problems, and notably some serious cosmological difficulties similar to the ones associated with the Polonyi field in the earliest versions of spontaneously broken supergravity. We show that all these problems are naturally avoided if a recently introduced mechanism for fixing the vacuum expectation values of the dilaton and/or moduli is at work. We study both the classical evolution and the quantum fluctuations of such scalar fields during a primordial inflationary era and find that the results are naturally compatible with observational facts. In this model, dilatons or moduli within a very wide range of masses (which includes the SUSY-breaking favored value ~ 1 TeV and extends up to the Planck scale) qualify to define a novel type of essentially stable ultraweakly interacting massive particles able to provide enough mass density to close the universe.

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I. INTRODUCTION

Superstring unification [1] and the inflationary scenario [2] have been, arguably, the most influential ideas in particle physics and cosmology over the last decade. Attempts at combining these ideas and constructing superstring-based inflationary models have introduced some interesting new ideas [3], but have encountered serious difficulties [4–7]. A specific source of difficulties is the existence of massless scalar fields (the moduli) having only gravitational strength couplings to ordinary matter. Among the moduli fields, the dilaton Φ , which invariably accompanies the graviton in all superstring models, plays a special role. In the presence of a (tree-level-coupled) dilaton, the cosmological evolution is drastically different from that in Einstein's gravity. In particular, instead of driving an exponential inflationary expansion, a constant vacuum energy drives the dilaton towards large negative values (corresponding to weak couplings), while the universe expands only as a small power of time. Even apart from inflation, a massless dilaton, or moduli, field (for simplicity, as these fields share many characteristics, we shall refer to any of them as “the dilaton” and denote it as Φ) can cause a host of other cosmological problems. During the most recent, matter-dominated epoch of the universe, such a field is necessarily time dependent. The masses of elementary particles and their couplings all depend on Φ , and the cosmological time variation of the dilaton runs into a violent conflict with observations, not to mention unacceptably large violations of the equivalence principle.

These problems are usually addressed by assuming that the dilaton develops a potential, so that one recovers the standard Einstein gravity after Φ settles at the minimum of the potential. The potential could originate from nonperturbative effects, such as gaugino condensation. The existence of a potential for Φ entails, however, new difficulties. On the one hand, it has been argued [7] that the minimum of the

nonperturbative potential is too shallow to confine the dilaton without fine-tuning the initial conditions (see, however, [8]). On the other hand, any potential $V(\Phi)$ for a weakly coupled field resurrects the Polonyi problem [9–12]: either the energy stored in the coherent oscillations of the vacuum expectation value (VEV) of Φ does not dissipate before now and exceeds the critical density needed to close the universe, or the field decays before now, and thereby generically produces an excessive amount of entropy. In the case of the nonperturbative potentials suggested by current supersymmetry- (SUSY-) breaking models in string theory, the slow decay rate of the moduli fields leaves the universe in a radiation-dominated era at a temperature which is generically (i.e., under naturalness assumptions for the couplings) much too low to be consistent with primordial nucleosynthesis [13–15].

In the present paper, we show that all the above difficulties associated with the dilaton (and moduli) fields are naturally avoided if the new mechanism introduced in Ref. [16] for fixing Φ is at work [17]. The main idea of [16] was to exploit the fact that string loop effects, associated with world sheets of arbitrary genus in intermediate string states, naturally generate some nonmonotonic dependence upon Φ of the various couplings of Φ to the other fields. Under the assumption that the different coupling functions $B_a(\Phi)$ have extrema at some common value of $\Phi = \Phi_0$, it has been shown that interactions with massive particles in an expanding universe drive the dilaton towards the value Φ_0 at which it *decouples* from matter. All deviations from general relativity in this model have been estimated to be extremely small at the present cosmological epoch, thereby naturally reconciling a massless dilaton with existing observational data. The assumption of coincident maxima can be satisfied in a technically natural way if there exists a discrete symmetry, e.g., $\Phi \rightarrow -\Phi$. Such a symmetry would guarantee that all couplings have extrema at $\Phi_0 = 0$ (the only additional necessary

assumption being that these extrema lead to minima rather than maxima of the masses as functions of Φ). We note that precisely the discrete symmetry $\Phi \rightarrow -\Phi$ (or $e^\Phi \rightarrow 1/e^\Phi$) is known to hold for some of the moduli fields (T duality), and has been conjectured to hold for the dilaton proper (S duality): $g_s \rightarrow 1/g_s$, where $g_s = e^\Phi$ is the string coupling).

Here we extend the analysis of Ref. [16] to inflationary models. It will be shown that inflation is extremely efficient in driving a homogeneous field Φ to Φ_0 (Sec. II). At the same time, inflation is known to generate significant quantum fluctuations in gravitational and other fields with a very wide range of wavelengths. Fluctuations of the dilaton on comoving scales smaller than the present horizon are potentially dangerous because they are not damped by the mechanism of Ref. [16]. One might worry that quantum fluctuations could reintroduce the Polonyi-moduli problem in a different form. However, in Sec. III we shall calculate the fluctuation spectrum and show that the predicted dilaton fluctuations are well below the observational constraints. We next consider, in Sec. IV, the possibility that, in addition to nontrivial matter-coupling functions with extrema at $\Phi = \Phi_0$, the dilaton also has a potential with a minimum at Φ_0 . (This may be enforced by the same discrete symmetry.) In this case the dilatons are massive, and can potentially run into conflict with observations, e.g., by overclosing the universe or by generating an excessive flux of γ rays due to dilaton decays. We shall see, however, that in our model the constraints on the dilaton mass derived in [11–13,18] can be substantially relaxed, due to the very weak couplings of the dilaton. We find that dilatons with a very large range of masses (which includes the suggested SUSY-breaking value ~ 1 TeV and extends up to the Planck scale) can qualify to define a new type of (essentially) stable weakly interacting massive particle (WIMP) able to provide enough mass density to close the universe.

II. THE EFFECTIVE ACTION

Up till now, most of the analysis of superstring cosmology has been based on the tree-level effective action, corresponding to the lowest order in the string loop expansion:

$$S_{\text{tree}} = \int d^4x \sqrt{\hat{g}} e^{-2\Phi} \{ (\alpha')^{-1} \hat{R} + 4 \hat{\nabla}^2 \Phi - 4 (\hat{\nabla} \Phi)^2 \} + \mathcal{L}_{\text{matter}} \}. \quad (2.1)$$

Here, Φ is the dilaton, and the matter Lagrangian includes fermions, gauge, and Higgs fields, and in particular the “inflaton” scalar field $\hat{\chi}$ whose potential $\hat{V}(\hat{\chi})$ drives the inflation:

$$\mathcal{L}_{\text{matter}} = -\frac{k}{4} \hat{F}^2 - \bar{\psi} \hat{D} \psi - \frac{1}{2} (\hat{\nabla} \hat{\chi})^2 - \hat{V}(\hat{\chi}) + \dots \quad (2.2)$$

Carets in Eqs. (2.1) and (2.2) indicate that the corresponding fields are taken in the “string frame,” that is, in the σ -model formulation of string theory.

The string coupling g_s , which plays the role of the expansion parameter in the string loop expansion, is determined by the expectation value of the dilaton, $g_s = e^\Phi$. The tree-level action (2.1) is proportional to g_s^{-2} , resulting in a universal,

multiplicative coupling of the dilaton to all other fields. With higher orders in the loop expansion taken into account, we expect the common factor $e^{-2\Phi}$ to be replaced by several coupling functions $B_a(\Phi)$ multiplying different terms in (2.1) and (2.2). In particular, the effective action for the graviton-dilaton-inflation sector is expected to be of the form

$$S = \int d^4x \sqrt{\hat{g}} \left\{ \frac{B_g(\Phi)}{\alpha'} \hat{R} + \frac{B_\Phi(\Phi)}{\alpha'} [4 \hat{\nabla}^2 \Phi - 4 (\hat{\nabla} \Phi)^2] - \frac{1}{2} B_\chi(\Phi) (\hat{\nabla} \hat{\chi})^2 - \hat{V}(\hat{\chi}, \Phi) \right\}, \quad (2.3)$$

where the functions $B_a(\Phi)$ admit a series expansion

$$B_a(\Phi) = e^{-2\Phi} + c_0^{(a)} + c_1^{(a)} e^{2\Phi} + \dots, \quad (2.4)$$

and a similar expansion for $\hat{V}(\hat{\chi}, \Phi)$. In the case of the moduli fields (by contrast with the four-dimensional dilaton proper) the effective action has also the generic form (2.3), the only difference being that the nontrivial Φ dependence is absent at tree level, and arises at one loop and beyond.

A more convenient form of the action can be obtained by a conformal transformation from the “string-frame” metric $\hat{g}_{\mu\nu}$ to the “Einstein-frame” metric

$$g_{\mu\nu} = C B_g(\Phi) \hat{g}_{\mu\nu}, \quad (2.5)$$

and by replacing the dilaton field Φ by the variable

$$\varphi = \int d\Phi \left[\frac{3}{4} \left(\frac{B'_g}{B_g} \right)^2 + 2 \frac{B'_\Phi}{B_g} + 2 \frac{B_\Phi}{B_g} \right]^{1/2}. \quad (2.6)$$

This gives

$$S = \int d^4x \sqrt{g} \left[\frac{1}{4q} R - \frac{1}{2q} (\nabla \varphi)^2 - \frac{1}{2} F(\varphi) (\nabla \chi)^2 - V(\chi, \varphi) \right], \quad (2.7)$$

where we have defined

$$\chi = C^{-1/2} \hat{\chi}, \quad (2.8a)$$

$$F(\varphi) = B_\chi(\Phi) / B_g(\Phi), \quad (2.8b)$$

$$V(\chi, \varphi) = C^{-2} B_g^{-2}(\Phi) \hat{V}(\hat{\chi}, \Phi). \quad (2.8c)$$

The constant C in Eq. (2.5) is chosen so that the string units coincide with Einstein units at the present cosmological epoch, $C B_g(\Phi_0) = 1$, and the constant q in (2.7) is related to the bare gravitational constant \bar{G} , $q = 4\pi\bar{G} = C\alpha'/4$. As shown in [16], \bar{G} is numerically nearly identical to the observed Newtonian gravitational constant so that $q = 4\pi/m_p^2$ with $m_p = 1.22 \times 10^{19}$ GeV.

The minimal condition required for the mechanism of Ref. [16] to work is that all $B_a(\Phi)$ should have an extremum at the same $\Phi = \Phi_0$. When formulated within the context of inflationary models and in terms of the rescaled fields φ and χ , this leads to requiring that the potential $V(\chi, \varphi)$ in Eq. (2.7) has a minimum (as a function of φ) at $\varphi = \varphi_0$ for any fixed value of χ . Here, $\varphi_0 = \varphi(\Phi_0)$. A simple toy model which sat-

isfies this condition is the case where higher loops are supposed to respect the tree-level universality of the dilaton couplings. In this case all functions $B_a(\Phi)$ in (2.3) are identical:

$$B_a(\Phi) = B(\Phi), \quad (2.9a)$$

and

$$\hat{V}(\hat{\chi}, \Phi) = B(\Phi) \hat{V}(\hat{\chi}). \quad (2.9b)$$

Then, in the Einstein-frame action (2.7), $F(\varphi)=1$ and the potential has a factorized form:

$$S = \int d^4x \sqrt{g} \left[\frac{1}{4q} R - \frac{1}{2q} (\nabla\varphi)^2 - \frac{1}{2} (\nabla\chi)^2 - B^{-1}(\varphi) V(\chi) \right]. \quad (2.10)$$

In this model σ is attracted [during both inflation and the subsequent matter-dominated era(s)] toward the maxima of $B(\varphi)$. Although the strong universality condition (2.9) may be too restrictive, it is useful to have in mind the action (2.10) which provides a very simple model containing most of the essential physics of the more general model (2.7).

III. CLASSICAL EVOLUTION

We shall assume, as is usually done in inflationary models, that the potential $V(\chi, \varphi)$ in Eq. (2.7) has a minimum with $V(\chi_0, \varphi_0) \approx 0$ and that it has a ‘‘slow-roll’’ region where it is a slowly varying function of χ :

$$V'_\chi{}^2 \ll 12qV^2, \quad (3.1a)$$

$$(\sqrt{V})''_{\chi\chi} \ll 3q\sqrt{V}, \quad (3.1b)$$

where we recall that $q = 4\pi/m_p^2$. At the same time, the coupling functions B_a and the potential V are not expected to be slowly varying functions of φ . For example, in the strong universality model (2.9) we expect $B(\varphi_0) \sim 1$ and

$$\kappa \equiv -B''(\varphi_0)/B(\varphi_0) \sim 1 \quad (3.2)$$

at the maximum of $B(\varphi)$. Note that, contrary to χ which has the usual dimension of mass, φ is a dimensionless variable whose expected range of variation is of order unity.

For the initial conditions of the universe, we shall assume that the fields φ and χ are displaced from the minimum of $V(\chi, \varphi)$ at $\{\chi_0, \varphi_0\}$ but are in its basin of attraction, at least in some parts of the universe [19]. Then it is easy to see, qualitatively, how the cosmological evolution will proceed. The field φ , which corresponds to the steep direction in $V(\chi, \varphi)$, will evolve on a much faster time scale than χ . It will start oscillating about $\varphi = \varphi_0$, these oscillations will be damped by the expansion of the universe, and the universe will quickly settle into a quasiexponential inflation driven by the potential energy of the field χ , $\hat{V}(\chi) = V(\chi, \varphi_0)$. The damping of φ oscillations during inflation is very efficient, and we expect that by the end of inflation the dilaton will be very close to φ_0 .

To describe this quantitatively, let us consider the field equations for χ and φ :

$$\nabla(F(\varphi)\nabla\chi) - \frac{\partial}{\partial\chi} V(\chi, \varphi) = 0, \quad (3.3)$$

$$\nabla^2\varphi - \frac{1}{2}q \frac{\partial F(\varphi)}{\partial\varphi} (\nabla\chi)^2 - q \frac{\partial}{\partial\varphi} V(\chi, \varphi) = 0. \quad (3.4)$$

During the slow-roll phase of inflation, the universe can be locally described by a flat Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t)dx^2, \quad (3.5)$$

with the expansion rate $H = \dot{a}/a$ given by

$$H^2 = \frac{1}{3}[2qV(\chi, \varphi) + \dot{\varphi}^2 + qF(\varphi)\dot{\chi}^2] \approx \frac{2}{3}qV(\chi, \varphi_0). \quad (3.6)$$

The spacetime is approximately de Sitter, with the curvature

$$R \approx 12H^2 \approx 8qV(\chi, \varphi_0). \quad (3.7)$$

To study the approach of φ to φ_0 , we expand $V(\chi, \varphi)$ in powers of $(\varphi - \varphi_0)$. For notational consistency with [16], it is convenient to denote by β_i the (positive) dimensionless parameter measuring the curvature (with respect to φ) of the inflationary mass scale around the minimum φ_0 :

$$\Lambda_i(\chi, \varphi) \equiv V^{1/4}(\chi, \varphi) \approx \Lambda_i(\chi, \varphi_0) [1 + \frac{1}{2}\beta_i(\varphi - \varphi_0)^2]. \quad (3.8)$$

Here, the index i stands for inflation. In the simple model (2.9), in which the potential is factorized, $V(\chi, \varphi) = B^{-1}(\varphi)V(\chi)$, one has $\beta_i = \kappa/4$, where κ was introduced in Eq. (3.2) above. In the general case, the value of β_i depends on the physics determining the mass scale Λ_i . As discussed in Ref. [16] [see Eq. (4.6b) there], if the hierarchy $\Lambda_i \ll m_p$ (which seems necessary in inflationary models) is due to nonperturbative effects, one expects to have $\beta_i \sim \ln(\Lambda_{\text{string}}/\Lambda_i) \geq 1$.

Substituting this in Eq. (3.4) and using (3.6) and (3.7), we obtain

$$(\nabla^2 - \xi_i R) \delta\varphi = 0, \quad (3.9)$$

where $\delta\varphi = \varphi - \varphi_0$ and

$$\xi_i = \frac{1}{2}\beta_i. \quad (3.10)$$

We note that Eq. (3.9) has exactly the form of that for a massless, nonminimally coupled field. The curvature R is a slowly varying function of time during inflation, and the effect of the curvature term in (3.9) is similar to that of a positive mass squared term $m^2 \sim H^2$ for the fluctuations of φ . Inflation is followed by a χ -dominated expansion when the inflaton field χ oscillates about the minimum of its potential [say $V \approx \frac{1}{2}m_\chi^2(\varphi)\chi^2$]. During this χ -dominated period, $\delta\varphi$ will approximately satisfy, after taking an average over the χ oscillations, an equation of the same form as (3.9) but with a different value of $\xi_\chi \equiv \frac{1}{2}\beta_\chi$ (here $\beta_\chi \equiv [\partial^2 \ln m_\chi(\varphi)/\partial\varphi^2]_{\varphi=\varphi_0}$ measures the φ curvature of the mass of the χ field; see the Appendix for a more exact equation satisfied by $\delta\varphi$).

Neglecting the spatial gradients of φ (which are rapidly suppressed by the cosmological expansion) we can finally rewrite (3.9) in the form

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + 12\xi_i H^2 \delta\varphi = 0. \quad (3.11)$$

During the slow-roll period, the field $\varphi(t)$ changes at a much faster rate than $H(t)$, and we can solve (3.11) using a WKB-type ansatz,

$$\delta\varphi = e^{W(t)}, \quad (3.12)$$

and assuming $|\ddot{W}| \ll \dot{W}^2$. Substitution of (3.12) into (3.11) gives a quadratic equation for \dot{W} with the solution

$$\dot{W}(t) = \left(-\frac{3}{2} \pm \sqrt{\frac{9}{4} - 12\xi}\right) H(t). \quad (3.13)$$

Alternatively we can follow the method of Ref. [20] and write the equation describing the evolution of φ in terms of the parameter p measuring the number of e -foldings:

$$p = \ln \frac{a(t)}{a(t_i)} = \int_{t_i}^t H dt, \quad (3.14)$$

where t_i denotes the time at the onset of inflation. The latter equation can be written in all eras of interest, inflation, χ dominated (in the approximation where one averages over the χ oscillations), radiation dominated, and matter dominated. When neglecting $(\varphi'_p)^2$ with respect to 1, this equation reads

$$\frac{2}{3}\varphi''_{pp} + (1-\lambda)\varphi'_p = -(1-3\lambda)\beta(\varphi - \varphi_0), \quad (3.15)$$

where λ is the ratio between pressure and energy density (i.e., -1 , 0 , or $\frac{1}{3}$ in vacuum-, χ - or matter-, and radiation-dominated eras, respectively), and where β is the parameter measuring the curvature of the φ -dependent mass scale driving the evolution of φ in the corresponding era (e.g., β_i in the inflationary era and β_χ in the χ -oscillation-dominated era).

The equation (3.15) is that of a damped harmonic oscillator. The critical value of β separating the overdamped-type solution from the damped-oscillation-type one is $\beta_c = \frac{3}{8}$ (in both the vacuum-dominated and the matter-dominated cases). During inflation, the approach of φ toward φ_0 is oscillatory if $\beta_i > \beta_c$, i.e., $\nu^2 < 0$, where we define

$$\nu^2 \equiv 6(\beta_c - \beta_i) = \frac{9}{4} - 12\xi_i. \quad (3.16)$$

From what we said above, we expect to be in this regime ($\nu = i|\nu|$). Then

$$\delta\varphi = A e^{-3p/2} \cos(|\nu|p + \delta), \quad (3.17)$$

where A and δ are constants. The number of e -foldings during inflation is $p_i \geq 65$, so that if initially $|\delta\varphi| \sim 1$, i.e., $A \sim 1$, then, by the end of inflation,

$$|\delta\varphi| \leq e^{-3p_i/2} \leq 10^{-42}. \quad (3.18)$$

During χ domination the average pressure of the oscillating χ field is nearly zero, the expansion law is $a(t) \propto t^{2/3}$, and the approach of φ toward φ_0 is described by Eq. (3.15) with $\lambda=0$ and $\beta=\beta_\chi$. This yields, when $\beta_\chi > \beta_c$, an additional factor of attraction of $\delta\varphi$ toward zero of order $e^{-3p_\chi/4}$, where p_χ is the number of e -foldings during χ domination. [Note that in the simple model (2.10) one has $\beta_\chi = \frac{1}{2}\kappa = 2\beta_i$.] The energy of χ oscillations eventually thermalizes, and the uni-

verse enters the radiation era with $a(t) \propto t^{1/2}$. Since $\lambda = \frac{1}{3}$ or $R=0$ then, we see either from (3.15) or from (3.9) that $\delta\varphi$ essentially stops evolving during the radiation era independently of the value of β . During the subsequent matter-dominated era, $\delta\varphi$ will be further attracted toward φ_0 by an additional factor $e^{-3p_m/4}$, where $p_m \sim 9$ is the number of e -foldings separating us from the end of the radiation era. Finally, the present value of $\delta\varphi$ is expected to be

$$|\delta\varphi| \leq 10^{-49}. \quad (3.19)$$

Note that the estimate (3.19) is independent of the precise values of β_i and β_χ as long as they are both $> \frac{3}{8}$. A nonzero value of $\delta\varphi$ causes a number of potentially observable deviations from general relativity [16]. However, all observable non-Einsteinian effects are proportional to the square of $\delta\varphi$. The present observational bounds (from equivalence principle tests) are, within the context of the model (2.9),

$$\kappa |\delta\varphi|_{\text{obs}} \leq 5 \times 10^{-6}. \quad (3.20)$$

Even if we were to assume that β_i , or equivalently ξ_i , is anomalously smaller than unity, it would only be in the extreme case where $\xi_i \leq 10^{-2}$ that the attraction factor due to inflation $\sim e^{-4\xi_i p_i}$ would not be much smaller than unity. We see that inflation is extremely efficient in driving a homogeneous, classical field φ to φ_0 . In the case considered below, where the field φ has a potential [sharing the discrete symmetry of the coupling functions $B_a(\varphi)$], we conclude that we have here a natural, non-fine-tuned, solution to the Polonyi-moduli problem as the VEV of φ is left, at the end of inflation, very precisely at the place where it stores no potential energy. However, as the change in the equation of state at the end of inflation can result in copious creation of dilatons we must consider whether this can regenerate a nontrivial quasiclassical VEV for φ . This will be discussed in the next section.

IV. QUANTUM CREATION OF DILATONS

Particle creation during and shortly after inflation can be studied using the standard methods of quantum field theory in curved spacetime [21]. For a massless scalar field with coupling to the curvature, as in Eq. (3.9), this has been done by Ford [22] in the limit where the coupling to the curvature is nearly conformal, $|\xi_i - \frac{1}{6}| \ll 1$. He assumed also that the χ -dominated period is very short, so that inflation is followed by thermalization in about one Hubble time. He found that the energy density of created particles at the end of inflation ($t = t_*$) is

$$\rho_\varphi(t_*) \sim 10^{-2} (6\xi_i - 1)^2 H_*^4, \quad (4.1)$$

where H_* is the expansion rate at $t = t_*$.

We have calculated the energy density and the spectrum of dilatons without assuming $(\xi_i - \frac{1}{6})$ to be small and without assuming that the χ -dominated period is necessarily short. We found that the matching to a χ -dominated expansion brings several qualitatively new features but does not change drastically the overall quantitative results of a matching to a radiation-dominated era. To keep things simple we discuss in the text only the latter case. The details of our calculation are

given in the Appendix (which contains also a brief discussion of the matching to a χ -dominated expansion), and here we shall only state the results.

The spectrum is expressed in terms of the comoving wave number k , which is equal to the physical momentum of the wave at $t=t_*$ [we set $a(t_*)=1$]. At later times the momentum is $p(t)=k/a(t)$, and the wavelength is $\lambda(t)=(2\pi/k)a(t)$. We find that the energy spectrum of dilatons is peaked at $k\sim H_*$ and is exponentially suppressed for $k\gg H_*$. In the limit of long wavelengths, $k\ll H_*$,

$$d\rho_\varphi(t) = \frac{\Gamma^2(\nu)}{32\pi^3 a^4(t)} (\nu - \frac{1}{2})^2 \left(\frac{2H_*}{k}\right)^{2\nu+1} k^3 dk \quad (4.2)$$

for ν real [i.e., $\xi_i < \frac{3}{16}$ in Eq. (3.16)] and

$$d\rho_\varphi(t) = \frac{H_*}{8\pi^2 |\nu| a^4(t)} (|\nu|^2 + \frac{1}{4}) k^2 dk \quad (4.3)$$

for $\nu=i|\nu|$, where in the latter case we assumed for simplicity that $\exp(-\pi|\nu|)\ll 1$. [Note that $\xi_i=\frac{1}{6}$ corresponds to $\nu=\frac{1}{2}$, for which Eq. (4.2) indeed predicts no particle production.]

When inflation is followed by χ domination, the dilaton spectrum contains a second peak at $k\sim m_\chi$ which contributes roughly the same number density as the peak at $k\sim H_*$ and an energy density differing by a factor $\sim m_\chi/H_*$. By integrating the spectra (4.2) or (4.3) up to $k_{\max}\sim H_*$, we find (roughly independently of ν as long as $|\nu-\frac{1}{2}|\geq 1$) that the total dilaton energy density is of order

$$\rho_\varphi(t) \sim 10^{-2} H_*^4 a^{-4}(t). \quad (4.4)$$

In other words, we find that when $6\xi_i-1$ becomes ≥ 1 , the factor $(6\xi_i-1)^2$ present in the nearly conformal case (4.1) saturates to something of order unity.

Using the approximate conservation of the comoving entropy $\propto \mathcal{N} T^3 a^3$ and Hubble's law at thermalization ($H_* \sim \mathcal{N}_*^{1/2} T_*^2/m_P$) we can write, for the present energy density in massless dilatons,

$$\Omega_\varphi \equiv \frac{\rho_\varphi}{\rho_c} \sim 10^{-2} \left(\frac{H_*}{m_P}\right)^2 \left(\frac{\mathcal{N}}{\mathcal{N}_*}\right)^{1/3} \Omega_r. \quad (4.5)$$

Here ρ_c is the critical density, \mathcal{N} is the number of spin degrees of freedom in radiation and relativistic particles (at present), \mathcal{N}_* is its value at t_* (i.e., at thermalization), $\Omega_r = \rho_r/\rho_c = (4 \times 10^{-5}) h^{-2}$, ρ_r is the radiation density, $h \sim \frac{1}{2}$ is the Hubble parameter, and m_P is the Planck mass. The characteristic wavelength of dilatons for the peak at $k\sim H_*$ (when $\xi_i \gtrsim \frac{1}{12}$ [23]) is

$$\lambda_c \sim 2\pi H_*^{-1} Z_* \sim 4(m_P/H_*)^{1/2} \text{ mm}, \quad (4.6)$$

where $Z_* \sim \mathcal{N}_*^{1/12} \mathcal{N}^{-1/3} (H_* m_P)^{1/2}/T$, with $T=2.74 \text{ K}=(0.83 \text{ mm})^{-1}$, is the redshift at $t=t_*$. The presence of a χ -oscillatory era would add a (possibly overlapping) second peak with characteristic wavelength differing by a factor $\sim H_*/m_\chi$.

Waves of wavelength smaller than the Hubble length, $\lambda(t) < t$, can be treated classically, and their amplitude can be estimated from

$$\varphi_k(t) \sim a(t) \left[\frac{q}{k} \frac{d\rho_\varphi(t)}{dk} \right]^{1/2}, \quad (4.7)$$

and at the present time we find, using Eqs. (4.2) and (4.3),

$$\varphi_k \sim 0.1 q^{1/2} Z_*^{-1} k (H_*/k)^{\nu+1/2}, \quad \nu > 0, \quad (4.8)$$

$$\varphi_k \sim 0.1 Z_*^{-1} (q H_* k)^{1/2}, \quad \nu = i|\nu|. \quad (4.9)$$

For $k\sim H_*$, corresponding to wavelengths $\sim \lambda_c$, both of these equations give

$$\varphi_{H_*} \sim (T/m_P) (H_*/m_P)^{1/2} \sim 10^{-32} (H_*/m_P)^{1/2} \leq 10^{-34}, \quad (4.10)$$

where we have used the bound [2] $H_* \leq 10^{-5} m_P$ on the rate of inflation. (Larger values of H_* result in an excessive amount of relic gravitational waves). In the case of $\nu^2 < \frac{1}{4}$ ($\xi_i > \frac{1}{6}$) (including $\nu^2 < 0$, i.e., $\xi_i > \frac{3}{16}$) the dilaton amplitude $\varphi_k \propto |k|^{1/2-\nu}$ decreases towards longer wavelengths. This means that for the wavelengths and time scales of relevance to laboratory experiments the quantum-regenerated $\delta\varphi$ is many orders of magnitude below the observational bounds (3.20).

A different behavior is obtained for $\nu > \frac{1}{2}$ ($\xi_i < \frac{1}{6}$), when φ_k grows towards longer wavelengths. The largest growth occurs for nearly minimal coupling, $\xi_i \approx 0$, $\nu \approx \frac{3}{2}$, when $\varphi_k \propto k^{-1+4\xi_i}$, and on the present Hubble scale $k\sim H a \sim H Z_*$ with $H^{-1} \sim 10^{28} \text{ cm}$,

$$\begin{aligned} \delta\varphi_{\max} &\sim \Omega_r^{1/2} (H/T)^{4\xi_i} (H_*/m_P)^{1-2\xi_i} \\ &\sim 10^{-7} 10^{-106\xi_i} (10^5 H_*/m_P)^{1-2\xi_i}. \end{aligned} \quad (4.11)$$

Even for anomalously small curvature couplings $\xi_i \leq 10^{-2}$, and the maximal allowed value of H_* , the dilaton amplitude is smaller than $\delta\varphi \sim 10^{-7}$, and therefore smaller than the observational limits (3.20), which become less stringent when $\kappa=8\xi_i$ is itself small, i.e., when the interaction of dilatons with matter is suppressed due to a small value of ξ_i .

It should be noted that Eqs. (4.7)–(4.9) cannot be used for wavelengths longer than the Hubble length, where real particles cannot yet be distinguished from vacuum polarization effects. In particular, we cannot conclude from (4.8) that φ_k can become arbitrarily large in the limit of long wavelengths. To estimate the dispersion of the dilaton field on superhorizon scales, we calculated the quantum expectation value $\langle \varphi^2 \rangle$. The calculation is outlined in the Appendix, and the result is that for $\nu > \frac{1}{2}$ we have $\langle \varphi^2 \rangle(t) \sim \varphi_{k_H}^2$, where k_H corresponds to the Hubble scale at time t . This indicates that superhorizon wavelengths do not significantly contribute to $\delta\varphi$.

V. MASSIVE DILATONS

Up till now we have been considering the case where the dilaton remains exactly massless at low energy. However, as we remarked above, under the assumption of a discrete φ symmetry [or some other universality feature such as the one built into the model (2.9)] the existence of a mass term for φ does not create the usual Polonyi problem because, after inflation, the VEV of φ is left very precisely pinned at the

minimum of its potential. We must, however, investigate what constraints on the dilaton mass m_φ are obtained by requiring that the present mass density of the quantum-generated dilatons does not exceed the critical density ρ_c . To simplify the discussion, we shall consider only the case of $v^2 < \frac{1}{4}$, when long-wavelength modes with $k \ll H_*$ are unimportant, and the particle interpretation of the field $\delta\varphi$ becomes valid shortly after t_* . The mass m_φ and the dilaton number density n_φ should then satisfy the condition

$$\Omega_\varphi \equiv \frac{n_\varphi m_\varphi}{\rho_c} \lesssim 1. \tag{5.1}$$

To estimate n_φ , we note that the ratio

$$r = n_\varphi / n_r, \tag{5.2}$$

where n_r is the density of particles with masses smaller than the temperature, remains approximately constant in the course of cosmological evolution (assuming that the dilaton lifetime exceeds the age of the universe, see below). At the end of inflation, we find from integrating the number density spectrum (see the Appendix)

$$n_\varphi(t_*) \sim 10^{-2} H_*^3. \tag{5.3}$$

Using the other equations

$$n_r(t_*) \sim \mathcal{N}_* T_*^3, \tag{5.4}$$

$$H_* \sim \mathcal{N}_*^{1/2} T_*^2 / m_P, \tag{5.5}$$

where T_* is the thermalization temperature, we get

$$r \sim 10^{-2} \mathcal{N}_*^{-1/4} (H_* / m_P)^{3/2}. \tag{5.6}$$

Inserting this in (5.1) we obtain

$$\Omega_\varphi \sim 10^{-2} \mathcal{N}_*^{-1/4} (H_* / m_P)^{3/2} (m_\varphi / T) \Omega_r, \tag{5.7a}$$

where, as above, $\Omega_r = \rho_r / \rho_c = (4 \times 10^{-5} h^{-2})$. Numerically this reads

$$\Omega_\varphi \sim \left(10^5 \frac{H_*}{m_P} \right)^{3/2} \frac{m_\varphi}{10 \text{ GeV}}. \tag{5.7b}$$

When the equality $\Omega_\varphi = 1$ is satisfied, dilatons dominate the mass density of the universe. This can happen for the whole range of masses $m_\varphi \geq 10 \text{ GeV}$. Let us note in particular that the value $m_\varphi \sim 1 \text{ TeV}$ suggested by many current SUSY-breaking models [13,14] is allowed and corresponds to $H_* \sim 10^{-7} m_P$, i.e., $T_* \sim 10^{15} \text{ GeV}$. We remark also that the value $m_\varphi \sim m_P$ [24] corresponds to a ‘‘weak scale inflation’’ $H_* \sim 100 \text{ GeV}$ [17] [i.e., to an intermediate scale reheating temperature $T_* \sim 3 \times 10^{10} \text{ GeV} \sim (m_W m_P)^{1/2}$]. Other authors have suggested the possibility that dilatons may provide the dark matter of the universe [18,25]. An important difference of our model is that our allowed mass range is $m_\varphi \geq 10 \text{ GeV}$ (which contains, notably, $m_\varphi \sim 1 \text{ TeV}$) and still corresponds to essentially *stable* dilatons, with a decay time much larger than the age of the universe (as is discussed next). [Usually [18], stable dilatons exist only for $m_\varphi \leq 100 \text{ MeV}$ because of Eq. (5.9) below.]

Let us finally examine whether additional constraints on the dilaton mass follow from an eventual flux of γ rays resulting from the dilaton decay, $\varphi \rightarrow \gamma\gamma$. This decay is described by the term

$$\mathcal{L}_{\text{int}} \propto (\varphi - \varphi_0) F_{\mu\nu}^2 \tag{5.8}$$

in the effective Lagrangian. When expressed in terms of a canonically normalized scalar field $\varphi_{\text{can}} = \varphi / \sqrt{q}$, Eq. (5.8) contains a coupling constant $\propto 1/m_P$. By dimensional analysis, if the dimensionless coefficient in front of (5.8) is of order unity, the corresponding lifetime is (see also [11,12,18])

$$\tau_\varphi \sim m_P^2 / m_\varphi^3. \tag{5.9}$$

Under the same assumptions (dimensionless coefficients of order unity) one gets a comparable lifetime for the decay of the dilaton into two light bosons. On the other hand, the decay into two light fermions is suppressed by a factor $(m_\psi / m_\varphi)^2$ because one can absorb any coupling function $B(\varphi)$ multiplying the fermion kinetic term $\bar{\psi} D \psi$ into a φ -dependent rescaling of ψ , leaving only a coupling proportional to the fermion mass: $B_\psi(\varphi) m_\psi \bar{\psi} \psi$ [26]. Let us also note that the term (5.8), when present with a coupling coefficient of order unity, opens an energy exchange channel between the dilaton and any electromagnetic or Yang-Mills condensate [27].

In our case, however, all coupling functions are expected to have extrema at $\varphi = \varphi_0$, and thus the dimensionless coefficient in front of (5.8), as well as analogous coefficients entering other (bosonic or fermionic) couplings, is $\propto \delta\varphi$, i.e., is exceedingly small. Then γ rays can only be produced in binary collisions $\varphi\varphi \rightarrow \gamma\gamma$. The corresponding interaction term is

$$\mathcal{L}_{\text{int}} \propto (\varphi - \varphi_0)^2 F_{\mu\nu}^2, \tag{5.10}$$

with a dimensionless coefficient of order unity. The corresponding annihilation cross section is extremely small [28]:

$$\sigma \sim m_\varphi^2 / v m_P^4, \tag{5.11}$$

where v is the average velocity of dilatons. The lifetime τ_φ can be found from $n_\varphi \sigma v \tau_\varphi \sim 1$, which gives

$$\tau_\varphi \sim m_P^4 / n_\varphi m_\varphi^2. \tag{5.12}$$

The annihilation rate per unit spacetime volume is

$$\frac{n_\varphi}{\tau_\varphi} \sim \frac{n_\varphi^2 m_\varphi^2}{m_P^4} \lesssim \frac{\rho_c^2}{m_P^4} \sim t_0^{-4}, \tag{5.13}$$

where t_0 is the present age of the universe. Hence there is no more than one annihilation in the entire visible universe during its whole lifetime.

Note that as all coupling functions have extrema at $\varphi = \varphi_0$, the type of dark matter our mechanism leads to has only exceedingly weak interactions with ordinary matter. Basically, its presence can be felt only through the gravitational effect of its mass. This leaves little hope of detecting it in laboratory experiments. For instance, for a macroscopically

sizable mass $m_\phi \sim m_p \sim 2 \times 10^{-5}$ g, the average present cosmological density of dilatons would be at most $n_\phi \sim 10^{-24}$ cm^{-3} .

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APPENDIX

In this Appendix we study the spectrum of created dilatons in simple models in which de Sitter inflation is followed either by a radiation-dominated expansion or by a χ -dominated one. We consider first the transition to a radiation-dominated era. Using the conformal time coordinate $d\eta = dt/a(t)$, the corresponding metric can be written as

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^2), \quad (\text{A1})$$

$$a(\eta) = \begin{cases} -(H_* \eta)^{-1}, & \eta < \eta_*, \\ H_*(\eta - \bar{\eta}), & \eta > \eta_*. \end{cases} \quad (\text{A2a})$$

$$a(\eta) = \begin{cases} -(H_* \eta)^{-1}, & \eta < \eta_*, \\ H_*(\eta - \bar{\eta}), & \eta > \eta_*. \end{cases} \quad (\text{A2b})$$

Here, $H_* = \text{const}$ is the expansion rate during inflation, $\eta_* < 0$ is the thermalization time when inflation ends, and $\bar{\eta} = \eta_* + (H_*^2 \eta_*)^{-1}$. It will be convenient to set $\eta_* = -H_*^{-1}$, so that $a(\eta_*) = 1$. The calculation in this Appendix follows the standard techniques reviewed in [14].

The field operator $\delta\hat{\phi}(x) = \hat{\phi}(x) - \phi_0$ satisfies a massless, nonminimally coupled field equation (3.9),

$$(\nabla^2 - \xi_i R) \delta\hat{\phi}(x) = 0, \quad (\text{A3})$$

and can be expanded in terms of creation and annihilation operators:

$$\delta\hat{\phi}(x) = \frac{q^{1/2}}{a(\eta)} \int \frac{d^3k}{(2\pi)^{3/2}} [\hat{a}_{\mathbf{k}} \psi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}]. \quad (\text{A4})$$

Here, carets indicate operator quantities and should not be confused with the notation of Sec. II, where they indicate quantities in the ‘‘string frame.’’ The mode functions $\psi_k(\eta)$ satisfy the normalization condition

$$\psi'_k \psi_k^* - \psi_k \psi_k^{*\prime} = -i, \quad (\text{A5})$$

corresponding to $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$. We shall assume that the quantum state of the dilaton field during the inflationary period $\eta < \eta_*$ is the de Sitter-invariant Bunch-Davis vacuum [i.e., that $\psi_k(\eta) \sim e^{-ik\eta/\sqrt{2k}}$ when $\eta \rightarrow -\infty$]. The corresponding mode functions are

$$\psi_k(\eta) = A(-\eta)^{1/2} H_\nu^{(1)}(-k\eta), \quad (\text{A6})$$

where

$$A = (\pi/4)^{1/2} e^{i\pi/4} e^{i\pi\nu/2}, \quad (\text{A7})$$

$H_\nu^{(1)}(z)$ are Hankel functions, and ν is given by Eq. (3.16).

The mode functions for the radiation-dominated period $\eta > \eta_*$ are

$$\psi_k(\eta) = \frac{1}{(2k)^{1/2}} [\alpha_k e^{-ik(\eta - \bar{\eta})} + \beta_k e^{ik(\eta - \bar{\eta})}], \quad (\text{A8})$$

and Eq. (A5) gives a normalization condition for α_k and β_k :

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \quad (\text{A9})$$

The coefficients α_k and β_k can be determined by matching the mode functions (A6) and (A8) and their derivatives at $\eta = \eta_*$. The dilaton spectrum (in number density and energy density) can then be found from

$$dn_\phi = \frac{1}{2\pi^2 a^3(t)} |\beta_k|^2 k^2 dk, \quad (\text{A10a})$$

$$d\rho_\phi = \frac{1}{2\pi^2 a^4(t)} |\beta_k|^2 k^3 dk. \quad (\text{A10b})$$

The instantaneous thermalization model (A2) is not adequate for modes with wavelengths shorter than the de Sitter horizon at η_* , $k \gtrsim H_*$. In a more realistic model, the transition from the vacuum to the radiation equation of state takes at least a Hubble time, and particle creation in such modes is exponentially suppressed, $\beta(k \gtrsim H_*) \approx 0$. In the opposite limit, $k \ll H_*$, we can find α_k and β_k using the asymptotic form of the Hankel functions for small values of the argument:

$$H_\nu^{(1)}(z) \approx \frac{i}{\sin(\nu\pi)} \left[\frac{e^{-i\nu\pi}}{\Gamma(1+\nu)} \left(\frac{z}{2}\right)^\nu - \frac{1}{\Gamma(1-\nu)} \left(\frac{z}{2}\right)^{-\nu} \right]. \quad (\text{A11})$$

For $\nu > 0$, the first term in (A11) is negligible, and we find $\alpha_k \approx -\beta_k$ and

$$|\beta_k|^2 = \frac{1}{16\pi} \left(\nu - \frac{1}{2} \right)^2 \Gamma^2(\nu) \left(\frac{2H_*}{k} \right)^{2\nu+1}. \quad (\text{A12})$$

In deriving (A12) we have used the identity

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}. \quad (\text{A13})$$

In the case of imaginary ν , $\nu = i|\nu|$, we shall assume for simplicity that $\exp(-\pi|\nu|) \ll 1$. Then again the first term in (A11) can be neglected, and we obtain $\alpha_k \approx -\beta_k$ with

$$|\beta_k|^2 \approx \frac{H_*}{4k|\nu|} (|\nu|^2 + \frac{1}{4}). \quad (\text{A14})$$

Combining (A12) and (A14) with (A10), we obtain Eqs. (4.2) and (4.3). Let us note in passing that the spectrum (A14) is formally of the Rayleigh-Jeans form $|\beta_k|_{\text{RJ}}^2 = T_{\text{RJ}}/k$, that is, the limit of $(e^{k/T_{\text{RJ}}} - 1)^{-1}$ when $k \ll T_{\text{RJ}}$. (Similar spectra also come out of the superinflationary scenario of Refs. [3], [18].) By contrast, we shall see below that when

inflation is followed by χ domination one does not get a Rayleigh-Jeans-type spectrum.

The dispersion of the dilaton field on superhorizon scales, $k \ll \eta^{-1}$, can be estimated by calculating the expectation value

$$\langle (\delta\hat{\varphi})^2 \rangle = (2\pi)^{-3} a^{-2} q \int |\psi_k(\eta)|^2 d^3k. \quad (\text{A15})$$

The contribution of long wavelengths ($k \ll H_*$) to this integral can be found using Eqs. (A8), (A12), and (A14) with $\alpha_k \approx -\beta_k$. This gives

$$\begin{aligned} \langle (\delta\hat{\varphi})^2 \rangle &\approx \frac{q}{4\pi^3 H_*^2 (\eta - \bar{\eta})^2} \int d^3k k^{-1} |\beta_k|^2 \\ &\times \sin^2[k(\eta - \bar{\eta})], \end{aligned} \quad (\text{A16})$$

and for $\nu > 0$ we find

$$\begin{aligned} \langle (\delta\hat{\varphi})^2 \rangle &\sim q \frac{(\nu - 1/2)^2 \Gamma^2(\nu)}{16\pi^3 H_*^2 (\eta - \bar{\eta})^2} \\ &\times (2H_*)^{2\nu+1} \int_0^{\sim H_*} dk k^{-2\nu} \sin^2[k(\eta - \bar{\eta})]. \end{aligned} \quad (\text{A17})$$

It is easily seen that for $\nu < \frac{1}{2}$ the integral is dominated by the upper limit ($k \sim H_*$), while for $\frac{1}{2} < \nu < \frac{3}{2}$ the dominant contribution is given by $k \sim \eta^{-1}$, that is, by the modes of wavelength comparable to the Hubble length. In the latter case the upper limit of integration can be extended to infinity, and we obtain a somewhat unwieldy expression

$$\begin{aligned} \langle (\delta\hat{\varphi})^2 \rangle &\sim -2\pi^{-3} \sin(\nu\pi) \Gamma(1-2\nu) \Gamma^2(\nu) \\ &\times (\nu - \frac{1}{2})^2 q H_*^2 [4a(\eta)]^{2\nu-3}, \end{aligned} \quad (\text{A18})$$

where $a(\eta)$ is given by (A2b). Disregarding numerical factors, this gives

$$\langle (\delta\hat{\varphi})^2 \rangle \sim q H_*^2 [a(\eta)]^{2\nu-3}. \quad (\text{A19})$$

This is to be compared with the dilaton amplitude φ_k which can be found from Eqs. (4.7) and (4.2):

$$\varphi_k^2(t) \sim \frac{qk^2}{a^2(t)} \left(\frac{H_*}{k} \right)^{2\nu+1}. \quad (\text{A20})$$

It is easily seen that $\langle (\delta\hat{\varphi})^2 \rangle \sim \varphi_k^2$ for $k \sim \eta^{-1}$.

Let us finally briefly indicate the peculiarities of the more general case where the inflationary evolution (A2a) is followed by a χ -dominated expansion with an averaged scale factor [we assume $\Gamma_\chi \ll H(\eta) \ll m_\chi$ away from $\eta = \eta_*$ and average over the oscillations at frequency m_χ]:

$$\langle a(\eta) \rangle = [1 + \frac{1}{2} H_*(\eta - \eta_*)]^2 \quad (\eta > \eta_*). \quad (\text{A21})$$

During this period, $\delta\varphi$ couples to the χ matter through the φ dependence of the potential energy $V(\chi, \varphi) \approx \frac{1}{2} m_\chi^2(\varphi) \chi^2 \approx \frac{1}{2} m_\chi^2(\varphi_0) \chi^2 [1 + \frac{1}{2} \beta_\chi \delta\varphi^2]^2$, which yields

$$\nabla^2 \varphi - q V''_{\varphi\varphi}(\chi, \varphi_0) \delta\varphi = \nabla^2 \varphi - 2\beta_\chi q V(\chi, \varphi_0) \delta\varphi = 0. \quad (\text{A22})$$

Then the Fourier modes $\psi_k(\eta)$ of Eq. (A4) satisfy the conformal-time evolution equation

$$\partial_\eta^2 \psi_k(\eta) + [k^2 - U(\eta)] \psi_k(\eta) = 0, \quad (\text{A23})$$

$$U(\eta) \equiv \frac{a''\eta}{a} - q a^2 V''_{\varphi\varphi}(\chi, \varphi_0) = \frac{a''\eta}{a} - 2q\beta_\chi a^2 V(\chi, \varphi_0). \quad (\text{A24})$$

The first forms of Eqs. (A22) and (A24) are valid in both the inflationary period [where $V''_{\varphi\varphi}(\chi, \varphi_0) = 4\beta_i V(\chi, \varphi_0)$] and the χ -dominated one. During inflation the effective potential reads $U(\eta) = 2(1 - 3\beta_i) \eta^{-2}$, while during χ domination

$$\begin{aligned} U(\eta) &= \frac{2}{(\eta - 3\eta_*)^2} \{1 - 3\beta_\chi + 3(1 - \beta_\chi) \\ &\times \cos[2m_\chi(t - t_*)]\} \quad (\eta > \eta_*). \end{aligned} \quad (\text{A25})$$

The effective potential (A25) contains two distinct spectral features: a monotonic piece $\propto (\eta - 3\eta_*)^{-2}$ varying on the (averaged) Hubble time scale, and an oscillatory piece involving $\cos[2m_\chi(t - t_*)]$. Correspondingly, the spectrum of quantum fluctuations generated by solving the Schrödinger-like equation (A23) will have two peaks: one at $k \sim H_*$ and one at $k \sim m_\chi$. (Depending upon the inflationary scenario considered, these peaks might be separated or might overlap). One can compute the Bogolyubov coefficient β_k of the first peak by matching at $\eta = \eta_*$ the exact solutions of Eq. (A23) when dropping the cosine term in Eq. (A25). As above, these are Hankel functions with $\nu = \nu_i \equiv i\sqrt{6(\beta_i - 3/8)}$ in the inflationary period and $\nu = \nu_\chi \equiv i\sqrt{6(\beta_\chi - 3/8)}$ during the χ -dominated one. (We assume here for definiteness that both are imaginary.) This yields, for the ‘‘Hubble time scale’’ peak in the long wavelength limit $k \ll H_*$,

$$|\beta_k|_H^2 = \frac{1}{2} \frac{|\nu_\chi|}{|\nu_i|} \left[\left(\frac{1}{2} - \frac{|\nu_i|}{|\nu_\chi|} \right)^2 + \left(\frac{3}{4|\nu_\chi|} \right)^2 \right]. \quad (\text{A26})$$

Note that this $|\beta_k|_H^2$ is independent of k , and therefore different from the Rayleigh-Jeans-type spectrum (A14). Integrating (A26) up to some effective cutoff $k_{\max} = \kappa H_*$ leads to

$$n_\varphi^H \sim \frac{1}{48\pi^2} \kappa^3 \frac{H_*^3}{a^3}, \quad (\text{A27a})$$

$$\rho_\varphi^H \sim \frac{1}{64\pi^2} \kappa^4 \frac{H_*^4}{a^4}. \quad (\text{A27b})$$

The value of κ depends upon the details of the transition between inflation and χ domination. We expect that $\kappa \lesssim 1$. Assuming $\kappa \sim 1$, we get the total energy density (4.4) roughly independently of whether inflation is followed by a radiation-dominated era or a χ -oscillatory one.

Finally, the ‘‘oscillatory’’ peak $k \sim m_\chi$ can be estimated by applying Born perturbation theory to Eq. (A23): namely, $\beta_k^{\text{osc}} \approx (2ik)^{-1} \int U^{\text{osc}}(\eta) e^{-2ik\eta} d\eta$. This yields (θ denoting the Heaviside step function)

$$|\beta_k|_{\text{osc}}^2 = \left(\frac{3}{8}\right)^2 \pi (\beta_\chi - 1)^2 \frac{H_*^3 m_\chi^{3/2}}{k^{9/2}} \theta(k - m_\chi), \quad (\text{A28})$$

and, for instance,

$$n_\varphi^{\text{osc}} = \frac{3}{64\pi} (\beta_\chi - 1)^2 \frac{H_*^3}{a^3}. \quad (\text{A29})$$

[These estimates are accurate when $(\beta_\chi - 1)H_*^2 \ll m_\chi^2$ and should give the right order of magnitude when $\beta_\chi - 1 \sim 1$.] In order of magnitude the number density (A29) is comparable to (A27a). The corresponding energy density would differ from (A27b) by a factor $\sim m_\chi/H_*$.

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- [24] Our calculation of n_φ in Sec. IV is valid only for $m_\varphi^2 \ll 12\xi_i H_*^2$. However, in the opposite limit the fractional change in $(m_\varphi^2 + \xi_i R)$ at the end of inflation is small, and the production of φ particles should be suppressed. Hence we expect that the bound on m_φ in this regime should be even weaker.
- [25] Instead of using dilatons as WIMP’s, other authors have mentioned that the Polonyi-type energy of oscillation of the VEV of φ around the minimum of its potential could close the universe. See, e.g., Ref. [13] and a recent paper by P. J. Steinhardt and C. M. Will, Phys. Rev. D **52**, 628 (1995).
- [26] Note that the coupling (5.8) is absent (at the classical level) in the case where φ , instead of being the dilaton in the sense of string theory, is a ‘‘dilaton’’ in the older sense of being coupled to the trace of the energy-momentum tensor, i.e., when φ is a Jordan-Fierz-Brans-Dicke (JFBD) field. In that case, the decay $\varphi \rightarrow \gamma\gamma$ can proceed only via triangular Feynman graphs involving fermions [see M. Yoshimura, Phys. Rev. Lett. **66**, 1559 (1991)]. Beware that the estimate of the latter reference for the bosonic decays of a JFBD field is incorrect. Such decays generically dominate the fermionic ones by a factor $\sim (m_\varphi/m)^2$, where m is the mass of the decay products, and give lifetimes comparable to (5.9).
- [27] M. C. Bento and O. Bertolami, Phys. Lett. B **336**, 6 (1994).
- [28] Equation (5.11) can be understood on dimensional grounds if we note that each factor of $(\varphi - \varphi_0)$ in the interaction Lagrangian gives a factor of m_p^{-1} in the scattering amplitude, and thus we should have $\sigma \propto m_p^{-4}$. The factor of v^{-1} is a kinematic factor appearing in all low-energy cross sections.