Gravitational wave emission from a binary black hole system in the presence of an accretion disk

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We study the time evolution and gravitational wave emission properties of a black hole orbiting *inside* an accretion disk surrounding a massive black hole. We simultaneously solve the structure equations of the accretion disk in the presence of heating, cooling, and viscosity as well as the equations governing the companion orbit. The deviation from the Keplerian distribution of the angular momentum of the disk due to pressure and advection effects causes a significant exchange of angular momentum between the disk and the companion. This significantly affects the gravitational wave emission properties from the binary system. We show that when the companion is light, the effect is extremely important and must be taken into account while interpreting gravitational wave signals from such systems.

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I. INTRODUCTION

The study of gravitational wave emission from binary systems has received a significant boost in recent years because of the realization that the detection of gravitational waves would directly identify compact and strongly gravitating bodies, such as neutron stars and black holes. The laser Interferometric Gravitational Wave Observatory (LIGO) and Laser Interferometer Space Antenna (LISA) project instruments are being constructed to achieve these goals [1,2]. In order to be able to obtain as accurate information about the radiating compact bodies as possible, efforts are being made to obtain correct forms of quadrupole radiation from a binary system [3–6]. In binary systems composed of only neutron stars and stellar black holes, these computations are adequate. However, when studying effects around a massive black hole, which is assumed to be present in centers of many galaxies, one needs to consider an additional effect, the effect of an accretion disk. It is widely believed that galactic centers are endowed with massive black holes [7,8], and in order to explain the observed luminosity from a galactic core, one needs to supply matters ranging from a few hundredths to a few solar masses per year, some of which may be in the form of stars [9]. Some of the stars could be compact, namely, neutron stars and stellar mass black holes which orbit the massive ones at the same time gradually spiraling in towards the center due to loss of angular momentum by gravitational waves.

Chakrabarti [10] pointed out that the accretion disks close to the black hole need not be Keplerian and it would affect the gravitational wave properties. The radiation or gas pressure-dominated disks are likely to be super-Keplerian which would transfer angular momentum to the orbiting companion and, in some extreme situations, can even stabilize its orbit from coalescing any further. This was later verified by time-dependent numerical simulations [11]. When one considers the more general solutions of viscous, transonic, accretion disks [12,13], one finds that the angular momentum distribution close to the black hole could be sub-Keplerian as well, depending upon the viscosity and the angular momentum at the inner boundary of the disk. The disk becomes Keplerian roughly in a distance of $x_{\rm Kep}\sim(M^2/\alpha v)^2$ from the black hole, where $M=v/a$ is the Mach number of the flow, v and a are the radial and sound velocities, and $\alpha\lesssim 1$ is a constant describing the viscosity [14]. Assume that a companion of mass M_2 is in an instantaneous circular Keplerian orbit around a central black hole of mass M_1. This assumption is justified, especially when the orbital radius is larger than a few Schwarzchild radii where the energy loss per orbit is very negligible compared to the binding energy of the orbit. The rate of loss of energy, dE/dt, in this binary system with an orbital period P (in hours) is given by [15,16]

$$\frac{dE}{dt}=3\times 10^{33}\left(\frac{\mu}{M_\odot}\right)^2\left(\frac{M_{\rm tot}}{M_\odot}\right)^{4/3}\left(\frac{P}{1\ {\rm h}}\right)^{-10/3}\ \ \text{ergs sec}^{-1}, \tag{1}$$

where

$$\mu=\frac{M_1M_2}{M_1+M_2}$$

and

$$M_{\rm tot}=M_1+M_2\,.$$

The orbital angular momentum loss rate would be

$$R_{\rm GW}=\left.\frac{dL}{dt}\right|_{\rm GW}=\frac{1}{\Omega}\,\frac{dE}{dt}, \tag{2}$$

where $\Omega=\sqrt{GM_1/r^3}$ is the Keplerian angular velocity of the secondary black hole with mean orbiting radius r. The subscript "GW" signifies that the rate is due to gravitational wave emission. In the presence of an accretion disk coplaner with the orbiting companion, matter from the disk [with local specific angular momentum $l(r)$] will be accreted onto the companion at a rate close to its Bondi accretion rate [17,18],

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$$\dot{M}_2 = \frac{4\pi\overline{\lambda}\rho(GM_2)^2}{(v_{\rm rel}^2 + a^2)^{3/2}}, \tag{3}$$

where ρ is the density of disk matter, $\overline{\lambda}$ is a constant of order unity (which we choose it to be 1/2 for the rest of the paper), and $v_{\rm rel} = v_{\rm disk} - v_{\rm Kep}$ is the relative velocity of matter between the disk and the orbiting companion. The rate at which the angular momentum of the companion will be changed due to Bondi accretion will be [10]

$$R_{\rm disk} = \left.\frac{dL}{dt}\right|_{\rm disk} = \dot{M}_2[l_{\rm Kep}(x) - l_{\rm disk}(x)]. \tag{4}$$

Here, $l_{\rm Kep}$ and $l_{\rm disk}$ are the local Keplerian and disk angular momenta, respectively. The subscript on the left-hand side signifies that the effect is due to the disk. If some region of the disk is sub-Keplerian ($l_{\rm disk} < l_{\rm Kep}$), the effect of the disk would be to reduce the angular momentum of the companion further and hasten coalescence. If some region of the disk is super-Keplerian, the companion will gain angular momentum due to accretion, and the coalescence is slowed down. In a *thin* disk with a high accretion rate, the Bondi accretion rate could be very high and the latter effect could, in principle, stop the coalescence completely [10].

In order to appreciate the effect due to the intervention of the disk, we consider a special case where $M_2 \ll M_1$ and $l_{\rm disk} \ll l_{\rm Kep}$. In this case, $\mu \sim M_2$ and $M_{\rm tot} \sim M_1$. The ratio R of these two rates is

$$R = \frac{R_{\rm disk}}{R_{\rm GW}} = 1.518 \times 10^{-7} \; \frac{\rho_{10}}{T_{10}^{3/2}} \; x^4 M_8^2. \tag{5}$$

Here, x is the companion orbit radius in units of the Schwarzschild radius of the primary, M_8 is in units of $10^8 M_\odot$, ρ_{10} is the density in units of 10^{-10} g cm^{-3}, and T_{10} is the temperature of the disk in units of 10^{10} K. It is clear that, for instance, at $x = 10$ and $M_8 = 10$, the ratio $R \sim 0.015$, suggesting that the effect of the disk could be a significant correction term to the general relativistic loss of angular momentum. In the above example, both the disk and the gravitational wave work in the same direction in reducing the angular momentum of the secondary. Alternatively, when $l_{\rm disk} > l_{\rm Kep}$ they act in opposite directions and may slow down the loss of angular momentum [10]. In either case, the ratio R is independent of the mass of the companion black hole, as long as $M_2 \ll M_1$.

In what follows, we present equations governing the disk and the companion (Sec. II). In Sec. III, we solve these equations simultaneously along with Eqs. (2) and (4) in a few typical cases which show varied nature of the disk structure and evolution of the companion. These disks are the generalization of the viscous, isothermal disks obtained earlier [12]. We also present some interesting observations on nonaxisymmetric disks containing spiral shocks. Finally, in Sec. IV, we make concluding remarks.

II. GOVERNING EQUATIONS

To simplify the equations governing the accretion disks, we make the usual assumption that the disk is thin, so that the vertical averaging of density, pressure, and viscous stress could be done and the vertical velocity component can be ignored. We assume the disk to be axisymmetric, an assumption relaxed in studying nonaxisymmetric disks. Instead of solving fully general relativistic equations, we assume the Paczyński-Wiita [19] potential approach which is sufficiently accurate to describe physical quantities around a Schwarzschild black hole, provided one is not too close to the horizon ($r \sim r_g = 2GM_1/c^2$ is the Schwarzschild radius of the primary black hole). In this approach, the potential of the central body $\Phi(r) = -GM_1/r$ is replaced by a pseudo-Newtonian potential $\Phi_{\rm PN}(r) = -GM_1/(r - r_g)$. The effect we are presently discussing, namely, the transport of angular momentum from the disk to the companion, is not of general-relativistic origin. Hence the major conclusions should not be affected by our simplified approach.

The steady-state accretion disk equations [12] are (a) the radial momentum equation

$$v\frac{dv}{dx} + \frac{1}{\rho}\frac{dP}{d\rho} + \frac{l_{\rm Kep}^2 - l^2}{x^3} = 0, \tag{6a}$$

(b) the continuity equation

$$\frac{d}{dx}(\rho x h v) = 0, \tag{6b}$$

(c) the azimuthal momentum equation

$$v\frac{dl(x)}{dx} - \frac{1}{\rho x h}\frac{d}{dx}\left(\frac{\alpha P x^3 h}{\Omega_{\rm Kep}}\frac{d\Omega}{dx}\right) = 0, \tag{6c}$$

and (d) the entropy equation

$$\Sigma v T\frac{ds}{dx} = Q^+ - Q^-. \tag{6d}$$

Here $l_{\rm Kep}$ and $\Omega_{\rm Kep}$ are the Keplerian angular momentum and Keplerian angular velocity, respectively, Σ is the density ρ vertically integrated, $h = h(x)$ is the half-thickness of the disk at radial distance x, v is the radial velocity, s is the entropy density of the flow, and Q^+ and Q^- are the heat gained and lost by the flow. We compute $h(x)$ assuming the disk is in a hydrostatic balance equation in the vertical direction. $l(x)$ is the angular momentum distribution of the disk matter. Here, we have chosen geometric units; thus, $x = r/r_g$ is distance in units of the Schwarzschild radius, $l(x)$ is in units of $2GM_1/c$, and velocities are in units of the velocity of light. We have implicitly assumed $M_2 \ll M_1$ so that the gravitational effects due to the companion in shaping the disk could be ignored. However, locally, the companion is capable of exerting its effect to accrete matter from the disk. $\alpha \lesssim 1$ in the above equation is the viscosity parameter of Shakura and Sunyaev [14], which is widely used to describe the viscous stress: $w_{r\phi} = -\alpha P$. This stress transports angular momentum from the inner to the outer region of the disk. We choose total pressure (thermal plus ram) in this prescription in order that the angular momentum remain continuous across shock waves as well [13].

The equation governing the companion, treated as a test particle in the field of the massive black hole, is simply

$$\left(\frac{dx}{dt}\right)^2 = -\frac{1}{x-1} + \frac{l_{\mathrm{Kep}}^2}{x^2}. \tag{7}$$

In the following section, we present simultaneous solutions of the four sets of equations: (2), (4), (6a)–(6d), and (7) obtained by a very accurate fourth-order Runge-Kutta method [20].

We note from the estimate of the ratio R in (5) that it is independent of the mass of the companion. However, the evolution time scale $L/(dL/dt)$ of the companion orbit depends inversely upon its mass M_2. Thus, even when the effect is very small, $R \ll 1$, the smaller companion will evolve so slowly that the number of cycles will be significantly affected. While integrating the above equations, however, we had to consider the computing ability of our machines. This constrained us to study cases only with a faster evolution time scale: We chose $M_1 = 10^8 M_\odot$ and $M_2 = 10^6 M_\odot$. Second, we note from (5) that the effect is directly proportional to the density of the gas in the disk, which in turn depends upon the accretion rate $\dot{M}_1$ of the primary. It is customary to express accretion rates in astrophysics in units of Eddington rates, $\dot{M}_{\mathrm{Edd}} = 4\pi G M_1 m_p / \sigma_T \sim 0.2 M_\odot M_8\ \mathrm{yr}^{-1}$, where m_p is the proton mass and σ_T is the Thomson cross section. Unlike the accretion process onto ordinary stars radiating from its surface, a black hole accretion process need not be limited by its Eddington rate. Since at the most $\eta = 0.06$ fraction of the rest mass energy is released by accreting matter on a Schwarzschild black hole [18], a critical rate of $\dot{M}_{\mathrm{crit}} \sim \dot{M}_{\mathrm{Edd}}/\eta \sim 16 \dot{M}_{\mathrm{Edd}}$ is very reasonable to choose. For concreteness, we assume $\dot{M}_1 \sim 70 \dot{M}_{\mathrm{crit}}$ (i.e., $\dot{M}_1 \sim 1000 \dot{M}_{\mathrm{Edd}}$). The effects we describe will be proportionately weaker when smaller rates are used.

To keep the problem simple enough, we have considered the companion orbit to be circular. The general elliptic orbit is easily studied by including the evolution of the azimuthal coordinate in conjunction with Eq. (7). This will be done later. Second, we assume that the companion orbit is coplaner with the disk, so that the companion is always immersed inside the disk. When it is not so, one has to include the fraction of time the companion is exchanging angular momentum with the disk and the effect would be proportionately reduced as well. In the case of lighter companions, the time scale of evolution is very long, and it is not unlikely to imagine that the orbits originally away from the equatorial plane will gradually lose the momentum component normal to the disk by repeated interaction [21] and eventually come to the plane of the disk much before our effects become important. In the case of massive companions, the evolution due to gravitational waves could be very rapid, and they may remain inclined to the plane, as is possibly the case for OJ287 [22].

III. SIMULTANEOUS SOLUTIONS OF THE GOVERNING EQUATIONS

Before we present the results of our investigation, we discuss briefly what type of accretion disk solutions are expected. From (6c) (also see Chakrabarti [12]) one observes that a weakly viscous disk starts deviating from Keplerian distribution very far away, whereas the strongly viscous disk remains Keplerian until close to the black hole. The disk cannot remain Keplerian very close to the black hole, as the velocity increases and the advection term [first term in (6a)] becomes important. Similarly, when the accretion rate is high or very low, the radiation pressure [19] or the gas pressure [23] becomes important and the pressure term [second term in (6b)] cannot be ignored. Both of these terms were ignored in the study of Keplerian disks [14,24]. Also, in a Keplerian disk, angular momentum $l(r) = l_{\mathrm{Kep}}$ is used independent of viscosity prescription. But generally, for some ranges of viscosity and accretion rates, this need not be true. Since our effects are nonzero only in non-Keplerian disks [Eq. (4)], it is essential that we include these effects. Near $x = x_{\mathrm{Kep}}$ where $l(x) \sim l_{\mathrm{Kep}}$, the distribution rapidly deviates from Keplerian to highly sub-Keplerian. This causes a "microburst" of the gravitational wave emission as we shall show below.

An important class of stable solutions of (6a)–(6d) involves shock waves [12,13] where the centrifugal barrier of the flow brakes the radial motion of the disk before the disk can continue through the sonic point to become supersonic, thus satisfying the boundary condition on the horizon. At the shock wave, the density, velocity, and temperature change discontinously and the effect we are considering is expected to be discontinuous as well. This causes a "microglitch" in the gravitational wave. In the case of nonaxisymmetric disks, the spiral shocks cause microglitches to appear repeatedly depending on the number of spiral arms. This will be discussed towards the end of this section.

A few cases of the solutions are presented here, which cover all possible types of solutions. All the disk solutions are characterized by three parameters (instead of four, since the flow passes through one or more sonic points [12]). These parameters are l_{in} [angular momentum at the inner edge of the disk, the integration constant of Eq. (6c)], $\dot{M}_1$ [accretion rate on the primary, the integration constant of Eq. (6b)], and x_{in} (the location of the inner sonic point; this defines specific energy of the flow at a given point). Alternatively, we could choose x_{Kep}, the location where the disk starts deviating from the Keplerian distribution, but we prefer to choose the sonic point location for convenience. We also choose an α, the unknown viscosity parameter, the polytropic index of the gas γ which defines the specific internal energy of an ideal gas: $e = (\gamma - 1)^{-1} P/\rho$ and the $Q_+ - Q_-$, the relative importance of cooling and heating [Eq. (6d)]. These are not completely independent parameters, but to obtain them one requires to include other equations in the list (6a)–(6d) to describe the viscous mechanism (such as poorly understood turbulence and convections) and cooling processes (such as the Compton effect, bremsstrahlung, pair creations and annihilations, etc.). Instead of bringing in these equations we choose reasonable values for these quantities. Since the ratio R [Eq. (5)] does not depend on M_2 (but the orbital evolution time does), we consider only the case of $M_1 = 10^8 M_\odot$ and $M_2 = 10^6 M_\odot$ in order to hasten the evolution of the companion orbit.

Case A. Figures 1(a)–1(c) show results where the disk always remains sub-Keplerian after deviating from the Keplerian disk at $x_{\mathrm{Kep}} \sim 90 r_g$. The disk smoothly passes through the inner sonic point at $x_{\mathrm{in}} = 2.3$. Other parameters are $l_{\mathrm{in}} = 1.7$, $\dot{M}_1 = 1000 \dot{M}_{\mathrm{Edd}}$, $\gamma = 5/3$, $\alpha = 0.02$, and $Q_- = Q_+$. In Fig. 1(a), we notice that the flow quickly becomes highly sub-Keplerian first. However, before entering

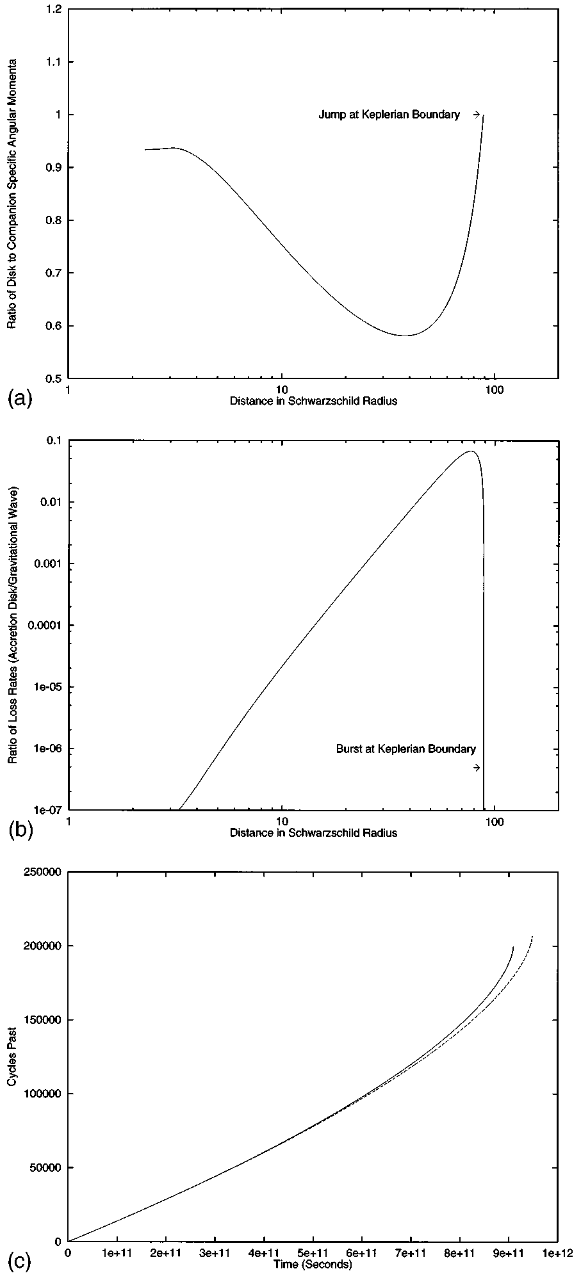

FIG. 1. (a) Ratio of disk angular momentum distribution to Keplerian distribution of a disk which is entirely sub-Keplerian for $x<x_{\mathrm{Kep}}=90$. See text for flow parameters. (b) Ratio of the rates of change of angular momentum of the companion due to exchange with the disk and due to gravitational wave emission. The ratio is highest in regions closer to the Keplerian boundary. (c) Comparison of the number of orbital cycles in a binary with a disk (solid line) and without a disk (dashed line) as time passes since the companion enters the sub-Keplerian region of the disk. The companion falls faster when the disk is present.

the black hole it becomes only moderately sub-Keplerian. Figure 1(b) shows the ratio $R=R_{\mathrm{disk}}/R_{\mathrm{GW}}$. The ratio R jumps to almost 0.1 around $x=80$ before decreasing to a very small value close to a black hole. Figure 1(c) shows the number of times the companion orbits the primary (twice the number of full gravitational waves emitted). The solid curve is drawn including the effect of the accretion disk, while the dashed curve is drawn considering the usual binary orbit evolution [Eq. (2)] without the presence of the disk. Time in the abscissa denotes time passed since the companion entered the sub-Keplerian disk. Two effects are clear: (a) The binary coalescence takes place roughly 10% times faster and (b) the number of orbital cycles is also about 10% times higher at the time of coalescence. If an accretion rate of $\dot{M}_1=\dot{M}_{\mathrm{Edd}}$ were chosen instead, the effect would be reduced by a factor of 1000. If a lighter black hole of $M_2\sim M_\odot$ was chosen instead, a longer orbital evolution time due to weaker gravitation wave loss gives rise to the same effect.

Case B. In this case we choose a solution with a standing shock wave. The disk parameters are $l_{\mathrm{in}}=1.6$, $x_{\mathrm{in}}=2.87$, $\dot{M}_1=1000\dot{M}_{\mathrm{Edd}}$, $\gamma=4/3$, $\alpha=0.05$, and $Q_-=0.5Q_+$. Figure 2(a) shows the Mach number variation as a function of distance from the black hole. The arrowed curve is followed by the disk after it deviates from Keplerian disk at $x_{\mathrm{Kep}}=480r_g$. The disk first passes through the outer sonic point (located at $x_{\mathrm{out}}=50$), then through the shock at $x_s=13.9$, and finally enters the black hole through the inner sonic point at $x_{\mathrm{in}}=2.87$. The shock location or the location of the outer sonic point is not a free parameter, but is self-consistently determined from the Rankine-Hugoniot relation [25,12]. The shock solution is always chosen if it is available to the disk, since the entropy at the inner sonic point is higher compared to its value at the outer sonic point, and the required entropy must be generated at the shock. Figure 2(b) shows the ratio of disk to Keplerian angular momentum distributions. Figure 2(c) shows the ratio R as a function of the distance. The ratio becomes almost 5 (5×10^{-3} for $\dot{M}_1=\dot{M}_{\mathrm{Edd}}$), a microburst of a sort, around $x=400r_g$. There is also a glitch at the shock location. In cases with a stronger shock wave the glitch would be stronger.

Case C. In this case we choose disk parameters so as to obtain a super-Keplerian region in the disk. We choose $l_{\mathrm{in}}=1.88$, $x_c=2.2$, $\dot{M}_1=1000\dot{M}_{\mathrm{Edd}}$, $\alpha=0.005$, $\gamma=4/3$, and $Q_-=Q_+$. The disk deviated from Keplerian disk at $x_{\mathrm{Kep}}=7.5r_g$. Figure 3(a) shows the ratio of disk to Keplerian distributions which clearly shows the sub-Keplerian as well as super-Keplerian regions. Figure 3(b) shows the ratio R [Eq. (5)]. The fractional change in orbital cycle number with and without the disk is $\delta N/N\sim R$. Thus, $\delta N\sim1$ only when

$$N\sim R^{-1}\sim\frac{1}{p}\frac{l(x_{\mathrm{Kep}})}{dl/dt|_{\mathrm{GW}}}.$$

Thus, even if R is small, lighter companions should survive long enough to feel the effect of angular momentum exchange. Chakrabarti [10] considered a thin disk where the density of the disk was high enough to stabilize the companion orbit in the super-Keplerian region.

Case D. In this case we solve nonaxisymmetric disk equations [26] instead of Eqs. (6a)–(6d). Here spiral shocks formed would produce repeated glitches in the gravitational waves pattern. The simplest solutions of the nonaxisymmetric shocks are obtained by assuming self-similarity in x and all the disk velocity components vary as $q_i(\phi)x^{-1/2}$ and the density of the disk varies as $q_\rho(\phi)x^{-3/2}$. Here, azimuthally varying coefficients $q_i(\phi)$ and $q_\rho(\phi)$ are to be determined from boundary conditions. Figure 4(a) shows a typical solution for the velocity coefficients when a two-armed spiral

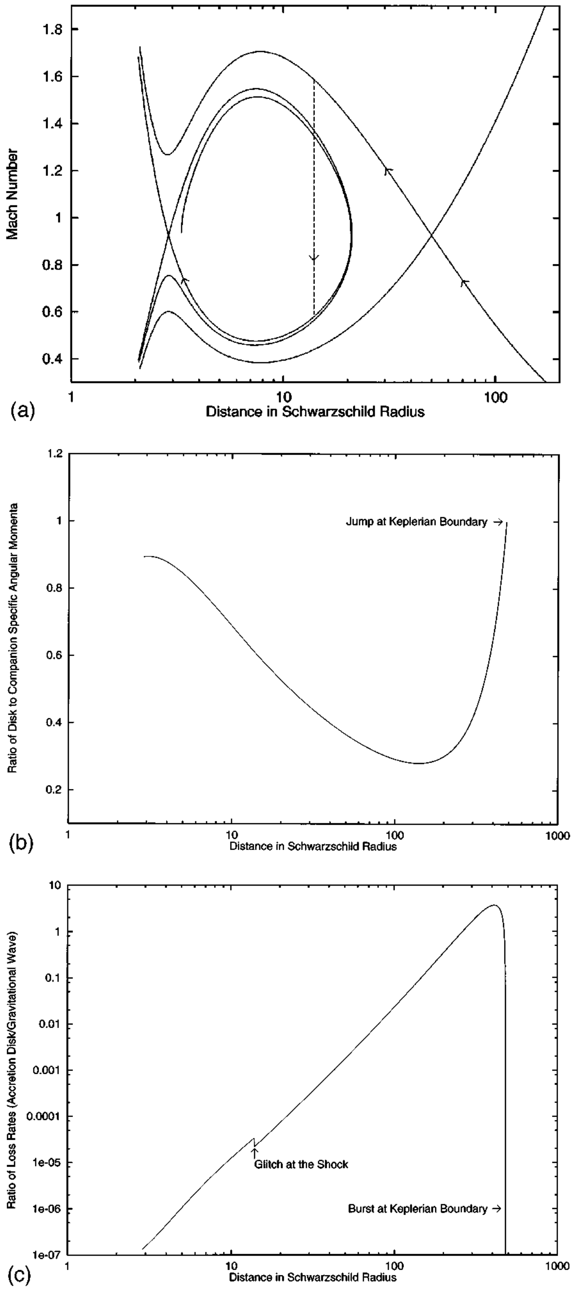

FIG. 2. (a) Variation of the Mach number of the disk which includes a shock wave at $x_s \sim 13.9$. The arrowed curves are the solutions chosen by the flow. See text for flow parameters. (b) Ratio of disk angular momentum distribution to Keplerian distribution of the disk with a shock which is entirely sub-Keplerian for $x < x_{\rm Kep} = 480$. (c) Ratio of the rates of change of angular momentum of the companion due to exchange with the disk and due to gravitational wave emission. The ratio is highest in regions closer to the Keplerian boundary. Note the glitch at the shock location which could be very high for stronger shocks.

shock solution is considered. The solid, long-dashed, and short-dashed curves show the the radial, azimuthal, and sound velocity coefficients and the dotted curves show the density coefficients as they vary with the azimuthal angle. The shocks are located at $\phi = 0$ and $\phi = \pi$. In a Keplerian disk, the azimuthal velocity coefficient would be unity throughout the disk. In this example, the azimuthal velocity coefficient varies from 92% Keplerian to 26% Keplerian as

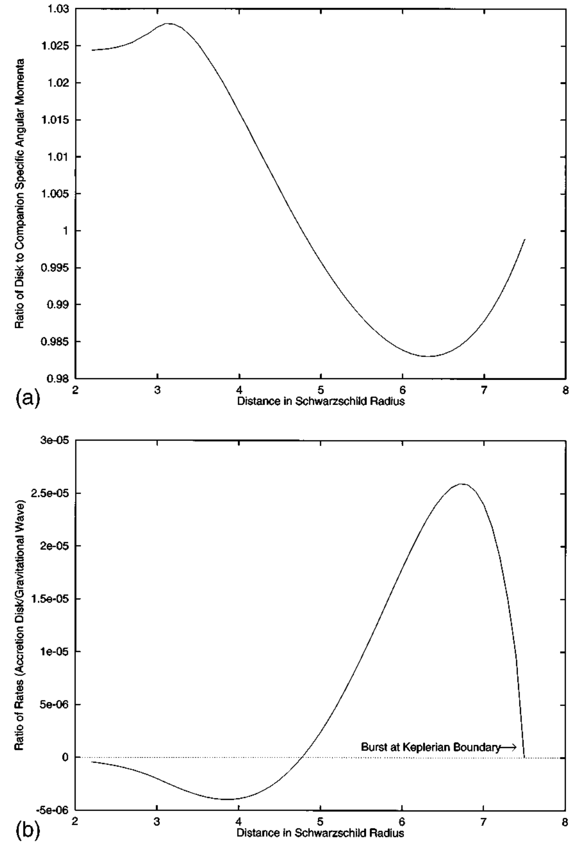

FIG. 3. (a) Ratio of disk angular momentum distribution to Keplerian distribution of the disk where the sub-Keplerian disk below $x < x_{\rm Kep} = 7.5$ becomes super-Keplerian close to the black hole. See text for flow parameters. (b) Ratio of the rates of change of angular momentum of the companion due to exchange with the disk and due to gravitational wave emission. The ratio is highest in regions closer to the Keplerian boundary. Note the change in sign of the ratio as the companion enters the super-Keplerian region.

the flow crosses the shock front. Other components also suffer a jump. In a single circular orbit, the companion thus passes twice through these jumps. Figure 4(b) shows (in arbitrary units) the glitches in the ratio R in a single orbit. In an axisymmetric disk, the glitch appears only once, but in a disk with spiral shocks the effect occurs repeatedly and cumulative effect becomes important due to the repeated passage of the companion through the shock. This could significantly modify the shape of the gravity wave signals.

IV. CONCLUDING REMARKS

It is widely recognized that accurate templates of possible signals may be essential to determine the nature of radiating compact bodies [1]. In this paper, we have discussed several important ways a gravitational wave signal from a binary companion could be modified in the presence of an accretion disk. We find that even under very normal circumstances, the effects will be sufficiently significant and our effect may influence the templates constructed assuming the absence of accretion disks.

In a binary system containing lighter mass black hole components, the accretion disk need not be present. Systems involving massive black holes at the galactic center may nec-

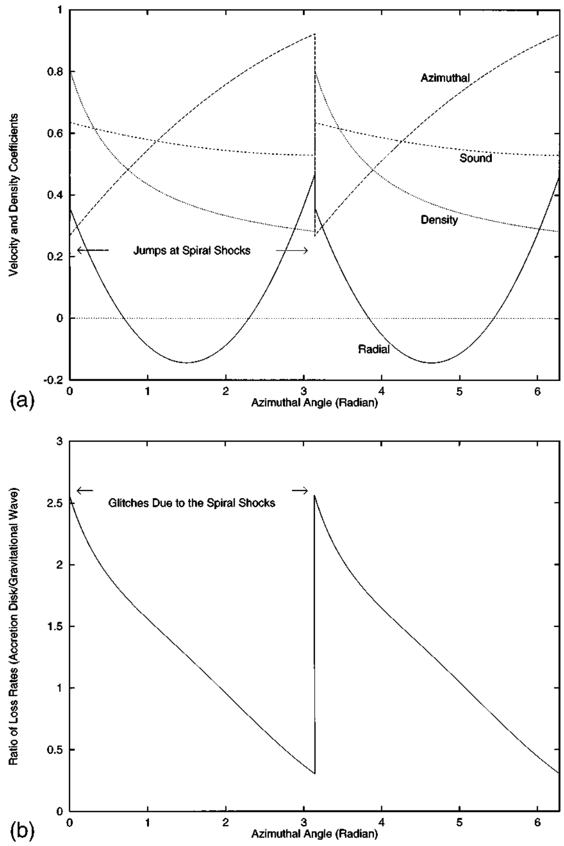

FIG. 4. (a) Velocity and density variations with azimuthal angle in a nonaxisymmetric disk with a two-armed spiral shock waves located at $\phi=0$ and $\phi=\pi$. The velocities are in units of local Keplerian velocity, while the density is in an arbitrary unit. (b) Ratio of the rates of change of angular momentum of the companion due to exchange with the disk and due to gravitational wave emission. The jump in the ratio at the spiral shocks produces glitches twice per orbital cycle (once per gravitational wave signal).

essarily contain accretion disks and lighter black hole companions. The frequency of the gravitational wave $f_{\mathrm{GW}}=2.25\times10^{-4}x^{-3/2}M_8^{-1}$ is well outside LIGO sensitivity, but could be well within LISA sensitivity [2,27]. By self-consistently solving the equations governing the accretion disk structure and the evolution of the binary orbit, we first showed that accretion disks close to the black hole are in general *non-Keplerian*. In the sub-Keplerian region of the disk, the residence time of the companion inside a disk and the probability of its observation would be reduced. On the other hand, the super-Keplerian region enhances the residence time and the probability of observing these systems is higher. We also find that the orbital evolution may be faster away from the black holes where the disk angular momentum distribution starts deviating from Keplerian distribution. The population density of compact stars close to galactic nuclei should be affected by their interaction with the disk. These effects should be taken into consideration while determining the band of maximum sensitivity of future instruments for gravitational wave astronomy.

The discussions made in this paper involving a companion black hole are valid even when a neutron star is chosen instead and may approximately remain valid when an ordinary star orbits the central black hole. In the latter case, the angular velocity of the disk changes significantly along a radial direction across the star. This would cause some angular momentum of the disk to spin up or spin down the star itself rather than changing its orbital angular momentum. Furthermore, the star may lose angular momentum through winds. Therefore, our result need not be strictly valid in these systems. These effects are negligible if the companion is a black hole or a neutron star because of its small size and the absence of winds.

The assumption of a thin disk in vertical equilibrium (namely, that the vertical velocity is negligle compared to the radial or azimuthal velocity) enabled us to integrate the governing equations. Numerical simulations of fully three-dimensional disks [28] indicate that the assumption of the vertical equilibrium adequately describes the disk properties and therefore we do not believe that the conclusions drawn in the present paper are affected by this assumption. Another implicit assumption has been that the disk remains continuous (and does not break apart in the form of rings as in the case of orbiting matter around Saturn) even in the presence of an orbiting companion. The formation of gaps in the disk is possible only if the instantaneous gap is not filled in by the accreting matter through radial pressure or viscous forces [21,29]. This implies that either the Roche radius (R_L) of the star orbiting at radius r is greater than the disk thickness, $R_L\sim(M_2/M_1)^{1/3}r\gtrsim h\sim ar^{3/2}\sim0.5r$ (here, the sound speed $a\sim1/\sqrt{3}r^{-1/2}$), or the viscosity parameter is so small that the angular momentum transfer rate by tidal coupling through the satellite is higher than that by viscosity: $\alpha<1/40(M_2/M_1)^2(r/h)^5\sim3/8(M_2/M_1)^2$. It is clear that for the cases we are interested in, namely, for $M_2/M_1\lesssim10^{-6}$ and $\alpha\gtrsim10^{-3}$, neither of these conditions would be satisfied. Thus, we do not think that gaps would be formed by orbiting black holes or neutron stars.

Our goal in this paper has been to indicate a new physical effect which may change the gravitational wave pattern significantly. Construction of accurate templates for inferring component masses of the gravitating systems is beyond the scope of the present paper. It is possible that one could estimate the mass of the central black hole by comparing the observed optical or UV radiation spectra and hard or soft x rays with the theoretically derived spectra using these general disk models [13,30]. The accretion rate of the Keplerian disk could be inferred from the normalization of the optical or UV flux as well. Thus two parameters are easily eliminated. Viscosity is not a well-understood process in the context of accretion phenomena, but typical values of the parameters have been presented in the literature from time to time [31,32] which we have considered here for simplicity. Since whether the flow is Keplerian or non-Keplerian depends very crucially upon the viscosity parameter [13], undoubtedly, a complete resolution of the present problem hinges upon a better understanding of the viscosity of the accretion disk.

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[1] K.S. Thorne, in *Three Hundred Years of Gravitation*, edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987).
[2] K. Danzmann *et al.*, in ''Laser Interferometer Space Antenna for Gravitational Wave Measurements,'' ESA (Max-Planck-Institute für Quantenoptic report, 1993).
[3] E. Poisson, Phys. Rev. D **47**, 2198 (1993).
[4] C. Cutler, L.S. Finn, E. Poisson, and G.J. Sussman, Phys. Rev. D **47**, 1511 (1993).
[5] A.G. Wiseman, Phys. Rev. D **48**, 4757 (1993).
[6] L. Blanchet, T. Damour, B.R. Iyer, C.M. Will, and A.G. Wiseman, Phys. Rev. Lett. **74**, 3515 (1995).
[7] R.J. Harms *et al.*, Astrophys. J. **435**, L35 (1994); S.K. Chakrabarti, *ibid.* **441**, 576 (1995).
[8] M. Miyoshi *et al.*, Nature (London) **373**, 127 (1995).
[9] M.J. Rees, Annu. Rev. Astron. Astrophys. **22**, 471 (1984).
[10] S.K. Chakrabarti, Astrophys. J. **411**, 610 (1993).
[11] D. Molteni, G. Gerardi, and S.K. Chakrabarti **436**, 249 (1994).
[12] S.K. Chakrabarti, Mon. Not. R. Astron. Soc. **243**, 610 (1990); *Theory of Transonic Astrophysical Flows* (World Scientific, Singapore, 1990); Astrophys. J. (to be published).
[13] S.K. Chakrabarti, Phys. Rep. **266**, 238 (1996).
[14] N.I. Shakura and R.A. Sunyaev, Astron. Astrophys. **24**, 337 (1973).
[15] P.C. Peters and J. Matthews, Phys. Rev. **131**, 435 (1963).
[16] K.R. Lang, *Astrophysical Formula* (Springer-Verlag, New York, 1980).
[17] H. Bondi, Mon. Not. R. Astron. Soc. **112**, 195 (1952).
[18] S.L. Shapiro and S.A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars — the Physics of Compact Objects* (Wiley, New York, 1983).
[19] B. Paczyński and P.J. Wiita, Astron. Astrophys. **88**, 23 (1980).
[20] W.H. Press *et al.*, *Numerical Recipes* (Cambridge University Press, Cambridge, England, 1992).
[21] D. Syer, C.J. Clarke, and M.J. Rees, Mon. Not. R. Astron. Soc. **250**, 505 (1991).
[22] A. Sillanpää, S. Haarala, M.J. Valtonen, B. Sundelius, and G.G. Byrd, Astrophys. J. **325**, 628 (1988).
[23] M.J. Rees, M.C. Begelman, R.D. Blandford, and E.S. Phinney, Nature (London) **295**, 17 (1982).
[24] I. Novikov and K.S. Thorne, in *Black Holes*, edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1973).
[25] S.K. Chakrabarti, Astrophys. J. **347**, 365 (1989).
[26] S.K. Chakrabarti, Astrophys. J. **362**, 275 (1990).
[27] D. Hils and Peter L. Bender, Astrophys. J. Lett. **445**, L7 (1995).
[28] D. Molteni, G. Lanzafame, and S.K. Chakrabarti, Astrophys. J. **425**, 161 (1994).
[29] J.C.B. Papaloizou and D.C. Lin, Astrophys. J. **285**, 818 (1994).
[30] S.K. Chakrabarti and L.G. Titarchuk, Astrophys. J. **455**, 623 (1995).
[31] E.T. Vishniac and P. Diamond, Astrophys. J. **347**, 435 (1989).
[32] S.A. Balbus and J. F. Hawley, Astrophys. J. **376**, 214 (1991).