Examination of the resonance contributions to dileptonic rare B decays

Mohammad R. Ahmady
Department of Physics, Ochanomizu University, 1-1 Otsuka 2, Bunkyo-ku, Tokyo 112, Japan
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We analyze the long-distance contribution to the $B \to X_s \ell^+\ell^-$ differential decay rate when the momentum dependence of the ψ and ψ'-γ conversion strength is taken into account. The results indicate that the resonance to nonresonance interference in the dilepton invariant mass distribution is substantially reduced.

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Rare B decays have been the focus of extensive experimantal and theoretical investigations. The observation of the inclusive $b \to s\gamma$ decay and the exclusive mode $B \to K^*\gamma$ by CLEO Collaboration [1] has raised the hope that other flavor-changing neutral current (FCNC) transitions, such as $b \to s\ell^+\ell^-$ ($\ell = e$, μ), for instance, will be within experimental reach in the near future. The measurement of these processes can serve, among other things, to shed light on some less-known Cabibbo-Kobayashi-Maskowa (CKM) matrix elements, as well as imposing constraints on "new physics" beyond the standard model (SM). However, in order to make any reliable conclusion on these from the experimantal measurements of rare B decays, we must improve our understanding of the theoretical uncertainties in the calculation of these processes.

One of the main background sources for rare B decays is the long-distance (LD) contributions to these processes. A lot of theoretical attention has been focused on this subject [2–7], due to the fact that without a reliable estimate of the LD contributions one cannot draw accurate conclusions from experimental results.

The CKM favored resonance contributions to the $b \to s$ transition are due to the conversion of the intermediate $\psi(NS)$ vector mesons to real ($b \to s\gamma$ decay) or virtual ($b \to s\ell^+\ell^-$ decay) photons. The momentum dependence of the ψ-γ conversion strength was investigated a long time ago in [8] in order to explain the data on the ψ leptonic width and photoproduction simultaneously. Recently, this has been pointed out again in [5,6], where a large suppression of the ψ-γ transition on the photon mass shell is argued to indicate that the LD contribution to $b \to s\gamma$ decay could be substantially smaller than previous estimates. For $b \to s\ell^+\ell^-$ decay, however, the momentum dependence of the $\psi - \gamma$ conversion strength has not been taken into account up to now.

In this paper, we analyze the resonance contribution to the inclusive dileptonic rare B decays using a momentum-dependent ψ-γ conversion strength. We show that the dileptonic mass distribution is indeed sensitive to the short-distance (SD) contributions for a broader q^2 (q^2 is the invariant dileptonic mass).

We start with the low-energy effective Lagrangian for $b \to s\ell^+\ell^-$ [9,10]:

$$L_{\rm eff} = \frac{G_F}{\sqrt{2}}\left(\frac{\alpha}{4\pi s_W^2}\right) V_{ts}^* V_{tb}(A\bar{s}L_\mu b\bar{\ell}L^\mu\ell + B\bar{s}L_\mu b\bar{\ell}R^\mu\ell + 2m_b s_W^2 F\bar{s}T_\mu b\bar{\ell}\gamma^\mu\ell), \qquad (1)$$

where

$$L_\mu = \gamma_\mu(1-\gamma_5), \quad R_\mu = \gamma_\mu(1+\gamma_5),$$

and

$$T_\mu = -i\sigma_{\mu\nu}(1+\gamma_5)q^\nu/q^2.$$

V_{ij} are the Cabibbo-Kobayashi-Maskawa matrix elements, $s_W^2 = \sin^2\theta_W \approx 0.23$ (θ_W is the weak angle), G_F is the Fermi constant and q is the total momentum of the final $\ell^+\ell^-$ pair.

The SD parts of A and B, denoted by $A^{\rm SD}$ and $B^{\rm SD}$, arise from W box diagrams and penguin diagrams with Z gauge boson and photon coupled to the $\ell^+\ell^-$ pair. For these coefficients and $C = s_W^2 F$ we have

$$\begin{aligned} A^{\rm SD} =&\ \bar{C}^{\rm box}(x_t) + \bar{C}^Z(x_t) + B^{\rm SD}, \\ B^{\rm SD} =&\ -s_W^2\Big[F_1^{(s)}(x_t) + 2\bar{C}^Z(x_t) - \tfrac{4}{9}(\ln x_t + 1) \\ &+ \tfrac{4\pi}{\alpha_s(M_W)}\Big\{-\tfrac{4}{33}(1-\eta^{-11/23}) \\ &+ \tfrac{8}{87}(1-\eta^{-29/23})\Big\}C_2(M_W)\Big], \end{aligned}$$

$$C = -s_W^2\Big\{\eta^{-16/23}\Big[F_2(x_t) - \frac{116}{135}(\eta^{10/23}-1)C_2(M_W) - \frac{58}{189}(\eta^{28/23}-1)C_2(M_W)\Big]\Big\}, \qquad (2)$$

where $x_t = m_t^2/M_W^2$, $\eta = \alpha_s(m_b)/\alpha_s(M_W)$, and [11]

$$\begin{aligned} \bar{C}^Z(x) &= \frac{1}{4}x + \frac{3}{8}\frac{x}{1-x} + \frac{3}{8}\frac{2x^2-x^2}{(1-x)}\ln(x), \\ \bar{C}^{\rm box}(x) + \bar{C}^Z(x) &= \frac{1}{4}x + \frac{3}{4}\frac{x}{1-x} + \frac{3}{4}\Big(\frac{x}{1-x}\Big)^2\ln(x), \\ F_1^{(s)}(x) &= \frac{63x - 151x^2 + 82x^3}{36(1-x)^3} \\ &\quad + \frac{63x - 138x^2 + 59x^3 + 10x^4}{36(1-x)^4}\ln(x), \end{aligned}$$

$$F_2(x) = \frac{7x - 5x^2 - 8x^3}{12(1-x)^3} + \frac{2x^2 - 3x^3}{2(1-x)^4}\ln(x). \qquad (3)$$

C_2 is the Wilson coefficient of the four-fermion operator with $C_2(M_W) = -1$. For our numerical evaluation we use $m_t = 180$ GeV (which is the weighted average of the recent Collider Detector at Fermilab (CDF) and D0 results [12]) and $\Lambda_{\rm QCD} = 100$ MeV so that $\alpha_s(M_W) = 0.12$ and $\eta = 1.75$ (for $m_b = 4.5$ GeV). As a result, we obtain

$$\begin{aligned} A^{\rm SD} &= 2.020, \\ B^{\rm SD} &= -0.173, \\ C &= -0.146. \end{aligned} \tag{4}$$

The LD contributions enter A and B coefficients through charm-quark loop ($c\bar{c}$ continuum), and the resonance contributions from ψ and ψ':

$$A^{\rm LD} = B^{\rm LD} = -s_W^2\left[3C_1(m_b) + C_2(m_b)\right](\tau^{\rm cont} + \tau^{\rm res}). \tag{5}$$

$C_1(m_b)$ and $C_2(m_b)$ are the QCD corrected Wilson coefficients:

$$\begin{aligned} C_1(m_b) &= \tfrac{1}{2}\left(\eta^{-6/23} - \eta^{12/23}\right)C_2(M_W), \\ C_2(m_b) &= \tfrac{1}{2}\left(\eta^{-6/23} + \eta^{12/23}\right)C_2(M_W). \end{aligned} \tag{6}$$

The $c\bar{c}$ continuum contribution is obtained from the electromagnetic penguin diagrams [10]

$$\tau^{\rm cont} = g\left(\frac{m_c}{m_b}\, ,\, z\right), \tag{7}$$

where $z = q^2/m_b^2$ and

$$g(y\, ,\, z) = \begin{cases} -\left[\dfrac{4}{9}\ln(y^2) - \dfrac{8}{27} - \dfrac{16}{9}\dfrac{y^2}{z} + \dfrac{2}{9}\sqrt{1-\dfrac{4y^2}{z}}\left(2+\dfrac{4y^2}{z}\right)\left(\ln\dfrac{|1+\sqrt{1-\frac{4y^2}{z}}|}{|1-\sqrt{1-\frac{4y^2}{z}}|} + i\pi\right)\right], & z \geq 4y^2, \\ -\left[\dfrac{4}{9}\ln(y^2) - \dfrac{8}{27} - \dfrac{16}{9}\dfrac{y^2}{z} + \dfrac{4}{9}\sqrt{\dfrac{4y^2}{z}-1}\left(2+\dfrac{4y^2}{z}\right)\arctan\dfrac{1}{\sqrt{\frac{4y^2}{z}-1}}\right], & z \leq 4y^2. \end{cases} \tag{8}$$

On the other hand, the resonance contributions from ψ and ψ' can be incorporated by using a Breit-Wigner form for the resonance propagator [3,4]:

$$\tau^{\rm res} = \frac{16\pi^2}{9}\left(\frac{f_\psi^2(q^2)/m_\psi^2}{m_\psi^2 - q^2 - im_\psi\Gamma_\psi} + (\psi \to \psi')\right)e^{i\phi}. \tag{9}$$

The relative phase ϕ that determines the sign between $\tau^{\rm cont}$ and $\tau^{\rm res}$ is chosen to be zero due to unitarity constraint [13].

The Wilson coefficient sum $3C_1(\mu) + C_2(\mu)$ is very sensitive to QCD scale parameter $\Lambda_{\rm QCD}$ as well as the renormalization point μ [14]. Inserting $\eta = 1.75$, one would obtain $3C_1(m_b) + C_2(m_b) = -0.389$. However, if we treat this combination of the Wilson coefficients as a phenomenological parameter, a value

$$|3C_1(m_b) + C_2(m_b)| = 0.72$$

fits the data on the semi-inclusive $B \to X_s\psi$ [5]. We use this phenomenological value throughout our calculations.

On the other hand, from (9) we observe that $\tau^{\rm res}$ depends quadratically on $f_V(q^2)(V = \psi\, ,\, \psi'\,)$ defined as

$$\langle 0|\bar{c}\gamma_\mu c|V(q)\rangle = f_V(q^2)\epsilon_\mu, \tag{10}$$

where ϵ_μ is the polarization vector of the vector meson V. As we mentioned earlier, it has been pointed out recently that in the context of vector meson dominance, data on photoproduction of ψ indicates a large suppression of $f_\psi(0)$ compare to $f_\psi(m_\psi^2)$ [5]. This has been confirmed independently in [6] by constraining the dominant LD contribution to $s \to d\gamma$ using the present upper bound on the $\Omega^- \to \Xi^-\gamma$ decay rate. In fact, it is argued that this large suppression results in a much smaller LD contribution to $b \to s\gamma$ transition.

In the dileptonic rare B decays, however, the momentum dependence of f_V (or equivalently, the ψ-γ transition) has not been taken into account up to now, and $f_V(q^2)$ is normally replaced with the decay constant $f_V(m_V^2)$ obtained from the leptonic width of ψ and ψ':

$$\Gamma(V \to \ell^+\ell^-) = \frac{16\pi\alpha^2}{27m_V^3}f_V^2(m_V^2).$$

The spectrum obtained this way is dominated by the resonance interference for a broad range of q^2, as already noted in the literature [7,9].

In this work, however, we use a momentum dependent $f_V(q^2)(V = \psi\, ,\, \psi')$ in $\tau^{\rm res}$ (as in [5], we assume that the same suppression occurs for ψ'). Of course, there is no significant change in the total branching ratio which is dominated by ψ and ψ' resonance contributions which in turn is due to the fact that the dileptonic mass spectrum has peaks at $q^2 = m_\psi^2, m_{\psi'}^2$. However, as we demonstrate later on, as a result, the resonance to nonresonance interference is substantially reduced, leaving a broader region of invariant mass spectrum sensitive to a large extent to SD physics.

We use the momentum dependent $f_V(q^2)(V = \psi\, ,\, \psi')$ derived in [8] based on the intermediate quark and antiquark state:

$$f_V(q^2) = f_V(0)\left(1 + \frac{q^2}{c_V}\left[d_V - h(q^2)\right]\right), \tag{11}$$

where $c_\psi = 0.54, c_{\psi'} = 0.77$, and $d_\psi = d_{\psi'} = 0.043$. $h(q^2)$ is obtained from a dispersion relation involving the imaginary part of the quark-loop diagram:

$$h(q^2) = \frac{1}{16\pi^2 r}\left\{-4 - \frac{20r}{3} + 4(1+2r)\sqrt{1-\frac{1}{r}}\arctan\frac{1}{\sqrt{1-\frac{1}{r}}}\right\} \quad (12)$$

with $r = q^2/4m_q^2$ for $0 \le q^2 \le 4m_q^2$. m_q is the effective quark mass and assuming that the vector mesons are weakly bound systems of a quark and an antiquark, we take $m_q \approx m_V/2$. As a result, Eq. (11), defined for $0 \le q^2 \le m_V^2$, is an interpolation of f_V from the experimental data on $f_V(0)$ (from photoproduction) and $f_V(m_V^2)$ (from leptonic width) based on a quark-loop diagram. We assume $f_V(q^2) = f_V(m_V^2)$ for $q^2 > m_V^2$ mainly due to the fact that the behavior of the ψ-γ conversion strength is not clear in this region. In any case, our focus will be on the invariant mass spectrum region below m_ψ^2 where the effect of the momentum-dependent ψ-γ transition is more significant.

The differential decay rate for $b \to X_s\ell^+\ell^-$, taken as the free quark decay $b \to s\ell^+\ell^-$, can be written as

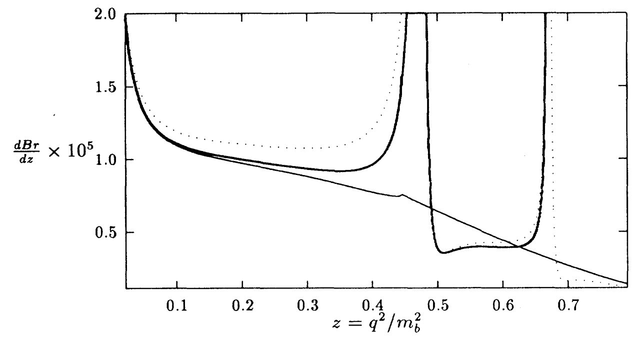

FIG. 1. The dileptonic invariant mass spectrum for the decay $b \to s\ell^+\ell^-$. The thin, dotted, and bold lines correspond to a spectrum without resonances, with resonances but constant V-γ conversion strength, and with resonances having momentum-dependent V-γ transition, respectively. For $q^2 > m_\psi^2$, where the latter two curves coincide, only the dotted curve is shown.

$$\frac{1}{\Gamma(B \to X_c e\bar{\nu})}\frac{d\Gamma}{dz}(B \to X_s\ell^+\ell^-) = \left(\frac{\alpha}{4\pi s_W^2}\right)^2 \frac{2}{f(m_c/m_b)}\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}(1-z)^2 \times\Big((|A|^2+|B|^2)(1+2z) + 2|C|^2(1+2/z) + 6\mathrm{Re}[(A+B)^*C]\Big), \quad (13)$$

where

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4\ln(x).$$

By normalizing to the semileptonic rate in (13), the strong dependence on the b-quark mass cancels out. In Fig. 1, we show the invariant dilepton mass spectrum corresponding to Eq. (13) for cases when (i) the resonance term τ^{res} is not included, (ii) τ^{res} is included but $f_V(q^2)$ in Eq. (9) is replaced with constant $f_V(m_V^2)$ and finally, (iii) τ^{res} with $f_V(q^2)$ inserted from Eq. (11) is included. From Fig. 1 we observe that the resonance to nonresonance interference in the invariant mass spectrum, which is measured by the deviation from the nonresonant spectrum (thin line), is suppressed considerably due to momentum dependence of the ψ-γ conversion strength. For example, at $q^2/m_b^2 \approx 0.3$, the resonance interference amounts to around 2% of the differential branching ratio as compared to 20% in the case where fixed $f_V(m_V^2)$ is used. As a result, we believe, contrary to previous conclusions [7], dilepton mass spectrum can be used for testing SD physics without significant resonance interference.

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