Extended technicolor model with QCD-like symmetry breaking

Thomas Appelquist and Nick Evans
Department of Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520
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We present a one-doublet extended technicolor model, with all fermions in fundamental representations. The bare Lagrangian has no explicit mass terms but generates masses through gauge symmetry breaking by purely QCD-like dynamics. The model generates three families of quarks and leptons and can accommodate the observed third family mass spectrum (including a large top quark mass and light neutrinos). In addition, we show how the model may be extended to incorporate a top color driven top quark mass without the need for a strong U(1) interaction. We discuss the compatibility of the model with experimental constraints and its possible predicitive power with respect to first and second family masses.

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I. INTRODUCTION

The Higgs model of electroweak symmetry (EWS) breaking is less than satisfying because it offers no understanding of fermion masses and is plagued by a technical hierarchy problem with respect to the Higgs boson mass. Technicolor models [1] break EWS by the formation of fermion condensates in a strongly interacting theory patterned after QCD. There are no fundamental scalars and therefore no Higgs boson mass hierarchy problem. It has been proposed that the fermion (quark and lepton) masses could be generated in technicolor models by extending the gauge sector so that the fermions and the technifermions are unified above the EWS-breaking scale. In such extended technicolor (ETC) [2] models, the hierarchy of fermion masses is generated by a hierarchy of breaking scales of the unified gauge group. The problem of the origin of the fermion masses is replaced by the problem of the origin of the ETC symmetry-breaking scales.

A number of proposals has been made for the origin of the ETC symmetry-breaking scales. The ETC symmetries may be broken by including Higgs scalars [3] in appropriate representations of the ETC group. This approach, however, is usually assumed to be a low energy approximation to an even higher scale dynamics since it reintroduces the technical hierarchy problem that technicolor is designed to solve. It has so far not pointed the way to an understanding of fermion mass.

A more audacious explanation of the ETC symmetry breaking is that the ETC group(s) break themselves by becoming strong at high scales and forming fermion condensates which are not singlets under the ETC group. This is the tumbling mechanism [4]. It is appealing in its economy, but the desired symmetry-breaking patterns require placing the fermions and technifermions in unusual, nonfundamental representations, chosen to achieve the desired breaking pattern. Furthermore, tumbling models have so far relied on speculative most-attractive-channel (MAC) analyses to determine the condensates that form at each scale.

In this paper, we explore an alternative approach to the ETC symmetry-breaking scales which is purely dynamical (no fundamental scalars and no bare mass terms in the Lagrangian), which puts fermions only in the fundamental representation and which employs only QCD-like dynamics. It thus avoids the use of MAC analyses as well as nonfundamental representations. Instead, the breaking pattern (the pattern of quark and lepton masses) is arranged here by the choice of groups into which new fermions are placed and the coupling strengths of these groups. Time will tell whether this holds the key to a deeper understanding of the quark and lepton masses.

Dynamical models with fermions in only the fundamental representation of the gauge groups have also been proposed in Refs. [5,6]. However, they generate the light fermion masses by means of couplings to new fermions with mass terms containing the observed mass structure and were intended to demonstrate that flavor dynamics could be separated from EW-scale physics in ETC models.

The model presented has one doublet of technifermions and involves Pati-Salam unification [7] at high scales. It gives a relatively small contribution to the electroweak parameter S [8,9] and gives rise to no pseudo Goldstone bosons at the technicolor scale. Within this model we are able to dynamically generate three family scales, flavor breaking within each family, a large top mass, and light neutrinos. The dynamics responsible for these features do not generate flavor-changing neutral currents (FCNC's). FCNC's induced by Cabibbo-Kobayashi-Maskawa (CKM) mixing, the origin of which we do not address here, can be suppressed by small mixing angles or the familiar walking [10] and strong ETC [11] solutions to the problem.

The model as presented contains global symmetries above the highest ETC symmetry-breaking scale (typically of order 1000 TeV) that, when dynamically broken, generate exactly massless, physical Goldstone bosons. They couple to ordinary matter through ETC interactions or the standard model (SM) interactions of their constituent fermions. These interactions are suppressed by the ETC scale and are not visible in current laboratory experiments. Astrophysical constraints [13] from stellar lifetimes do, however, rule out light Goldstone bosons with SM couplings. We anticipate that yet higher scale unifications than those discussed here may generate masses for these Goldstone bosons which are above the astophysical constraints.

ETC models that generate the large top quark mass tend to give rise to contributions to the $\Delta\rho$ $(=\alpha T)$ [8,9] param-

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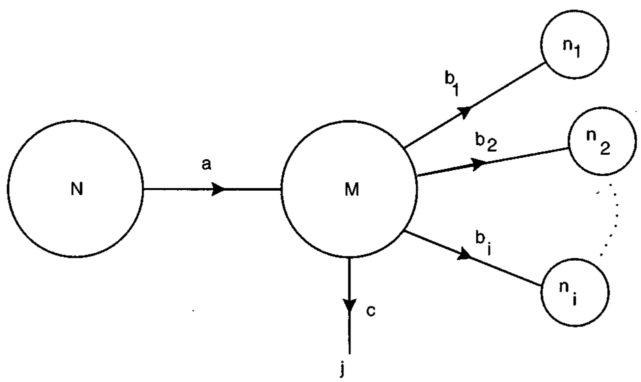

FIG. 1. A model of gauge symmetry breaking.

eter that are near or beyond the experimental bound. The $\Delta\rho$ parameter may be reduced in top-color-assisted technicolor models [14] in which the top mass is generated by a close to critical top self-interaction. We show how an alternative model of top color may be included simply in our ETC model. Unlike in the orginal top color model, the isospin breaking that splits the top and bottom masses is the result of chiral non-Abelian color groups rather than a strong U(1) gauge group.

In Sec. II, we describe the basic QCD-like mechanism for breaking gauge symmetries. We apply this dynamic in the case of a one-doublet model in Sec. III. We discuss both family symmetry breaking leading to different mass scales for each of the three quark-lepton families and flavor symmetry breaking within each family. Phenomenological aspects of the model are also discussed. In Sec. IV we comment on the possibility of employing strong ETC interactions at the lowest ETC scale and in Sec. V we discuss how the model may be extended to include a variation on top-color-assisted technicolor. In Sec. VI, we summarize the work and present some conclusions.

II. GAUGE SYMMETRY BREAKING WITH QCD-LIKE DYNAMICS

In this section, we describe our breaking mechanism using a simple model in which an SU(N) gauge group is broken to i gauged subgroups and an SU(j) global symmetry group using purely QCD-like dynamics. The driving force is an additional SU(M) gauge interaction which becomes strongly interacting at a scale Λ_M. The model contains the essential dynamics used to break the ETC symmetries in the following sections. There, the SU(N) group will be the ETC group, with quarks, leptons, and technifermions in its fundamental representation. There will also be particles transforming according to the fundamental representation of both the SU (N) and SU(M) groups, which will play an active role in the ETC symmetry breaking. In this section, only the latter particles will be included for simplicity.

In Fig. 1 we show the model in moose notation [5] with

$$n_1+n_2+\cdots+n_i+j=N. \tag{1}$$

A circled number N corresponds to an SU(N) gauge symmetry and directional lines represent left-handed Weyl fermions that transform according to the fundamental representation of the gauge groups they connect. A line leaving (entering) a circle with a number N inside represents a fermion transforming under the N ($\bar{N}$) representation of that group. Lines labeled by a number j that is not circled correspond to j copies of the representation of the gauge group and hence have a global symmetry $SU(j)\otimes U(1)$.

The fermion content of the model pictured in Fig. 1 is therefore

	SU(N)	SU(M)	SU(n_1)	SU(n_2)	$\cdots$	SU(n_i)
a	N	$\bar{M}$	0	0	$\cdots$	0
b_1	0	M	n_1	0	$\cdots$	0
b_2	0	M	0	n_2	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
b_i	0	M	0	0	$\cdots$	n_i
c	0	M	0	0	$\cdots$	0,

(2)

where the index a runs over the j flavors of the c fermions. This model is not anomaly free as shown but we shall assume that the additional degrees of freedom required to make SU(N) and the SU(n_i) gauge groups anomaly free do not transform under the SU(M), which is anomaly free with the constraint of Eq. (1). The SU(M) will be the only strongly interacting gauge group at its confinement scale Λ_M.

At this scale, the confining SU(M) interaction leads to the formation of the condensates

$$\langle \bar{a}^{1\cdots n_1} b_1\rangle \neq 0, \quad \langle \bar{a}^{n_1+1\cdots n_1+n_2} b_2\rangle \neq 0, \quad \cdots,$$
$$\langle \bar{a}^{n_1+n_2+\cdots+n_i+1\cdots N} c\rangle \neq 0. \tag{3}$$

With the other gauge interactions neglected, the global symmetry on the fermions a, b, and c would be $SU(N)_L\otimes SU(N)_R$. The condensates break this symmetry in the usual pattern

$$SU(N)_L\otimes SU(N)_R \rightarrow SU(N)_V. \tag{4}$$

In the presence of the other gauge interactions, the gauged SU(N) group is therefore broken to

$$SU(n_1)\otimes SU(n_2)\otimes\cdots\otimes SU(n_i), \tag{5}$$

where the gauge field and gauge coupling for each group are a linear combination of the fields and couplings of Fig. 1. We note that all N^2-1 Goldstone bosons associated with the broken symmetry are absorbed by the N^2-1 gauge bosons that acquire a mass (of order Λ_M).

This symmetry-breaking mechanism is of course reminiscent of technicolor itself. Here, as there, the symmetry breaking is driven by an additional, strongly coupled gauge interaction, and the breaking pattern is being imposed by the choice of the SU(n_i) gauge groups. In each case, this is to be compared with the choice of scalar representation in the Higgs mechanism. For ETC symmetry breaking, it can also be compared with the choice of fermion representations in tumbling models.

III. ONE DOUBLET TECHNICOLOR

As an example of ETC symmetry breaking using the above mechanism, we construct an ETC model with a single doublet of technifermions U and D [1,6]:

$$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L, \quad Q_R = \begin{pmatrix} U \\ D \end{pmatrix}_R. \tag{6}$$

The quarks and leptons must be unified in a single ETC multiplet with the technifermion doublet. The simplest realization of this unification is a Pati-Salam [7] $\mathrm{SU}(N+12)$ symmetry where the technicolor group is $\mathrm{SU}(N)_{\mathrm{TC}}$ and where the SM fermions and technidoublet form the multiplets

$$\mathscr{U}_R = (U, t, \nu_\tau, c, \nu_\mu, u, \nu_e)_R,$$

$$\Psi_L = \left(\begin{pmatrix} U \\ D \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \nu_e \\ e \end{pmatrix} \right)_L,$$

$$\mathscr{D}_R = (D, b, \tau, s, \mu, d, e)_R. \tag{7}$$

A. Family structure

1. Single family model

To introduce the model we restrict attention to the technidoublet and the third family quark and leptons only. The ETC group is then $\mathrm{SU}(N+4)$. The model is shown in moose notation in Fig. 2.

The $\mathrm{SU}(M)$ gauge groups become strong in the order A and then X (at scales Λ_A and Λ_X, both of order a few TeV), triggering the breaking of the ETC group to $\mathrm{SU}(N)_{\mathrm{TC}}$. Consider the highest of these two scales, Λ_A. The fermions transforming under $\mathrm{SU}(M)_A$ also transform according to the fundamental representations of the gauged $\mathrm{SU}(N+4) \otimes \mathrm{SU}(N) \otimes \mathrm{SU}(3)$. The $\mathrm{SU}(3)$ gauge group is present in order to leave an unbroken $\mathrm{SU}(3)$ subgroup of $\mathrm{SU}(N+4)$, which will become QCD, acting on the third family of quarks. The strong $\mathrm{SU}(M)_A$ interactions form condensates

$$\langle \bar{a}^{1 \cdots N} b \rangle \neq 0, \quad \langle \bar{a}^{N+1 \cdots N+3} c \rangle \neq 0, \quad \langle \bar{a}^{N+4} d \rangle \neq 0, \tag{8}$$

breaking the gauged $\mathrm{SU}(N+4) \otimes \mathrm{SU}(N) \otimes \mathrm{SU}(3)$ symmetry to $\mathrm{SU}(N) \otimes \mathrm{SU}(3)_{\mathrm{QCD}}$. The multiplets in Eq. (7) are broken, with the SU(3) subgroup corresponding to the t and b quarks with QCD interactions, the singlet to the third family lepton doublet and the $\mathrm{SU}(N)$ subgroup to the unbroken technicolor gauge group. All $(N+4)^2-1$ Goldstone bosons generated at this first stage of breaking are absorbed by gauge bosons which acquire masses of order of the confinement scale.

The ETC gauge bosons corresponding to generators broken at Λ_A acquire masses of order F_A, the decay constant of the Goldstone bosons formed at Λ_A that are absorbed by the gauge bosons ($F_A^2 \simeq M \Lambda_A^2 / 4\pi^2$). Below the technicolor scale, where the technifermions condense, these gauge bosons will generate masses for the third family quarks and leptons given by

$$m_f \simeq \frac{\langle \bar{Q} Q \rangle}{F_A^2}, \tag{9}$$

where we have assumed that the ETC coupling is perturbative and have used the four fermion approximation for the ETC gauge boson. The ETC gauge boson mass is proportional to its coupling ($M_{\mathrm{ETC}}^2 \simeq g^2 F_A^2$) and hence the ETC coupling cancels in the quark and lepton masses. In this simple model the quarks and leptons are degenerate. We shall address generating flavor breaking within each family in Sec. III B.

To cancel anomalies in the model, the additional fermions e, f, g, and h, transforming under the $\mathrm{SU}(M)_X$ gauge group, have been introduced. The $SU(M)_X$ group confines these new fermions to remove them from the physical spectrum at low energies. We assume that this confinement scale Λ_X lies between the technicolor scale and the $\mathrm{SU}(M)_A$ confinement scale. At the scale Λ_X there is a global $\mathrm{SU}(N+4)_L \otimes \mathrm{SU}(N+4)_R$ symmetry acting on the fermions transforming under $SU(M)_X$.The preferred vacuum alignment is that no gauge interactions are broken at this extra breaking scale so there are $(N+4)^2-1$ Goldstone bosons which are not absorbed. The Goldstone bosons that transform under the adjoint or fundamental representations of technicolor or QCD acquire masses governed by the scale Λ_X. The remaining two Goldstones are massless and we leave discussion of them to Sec. III F.

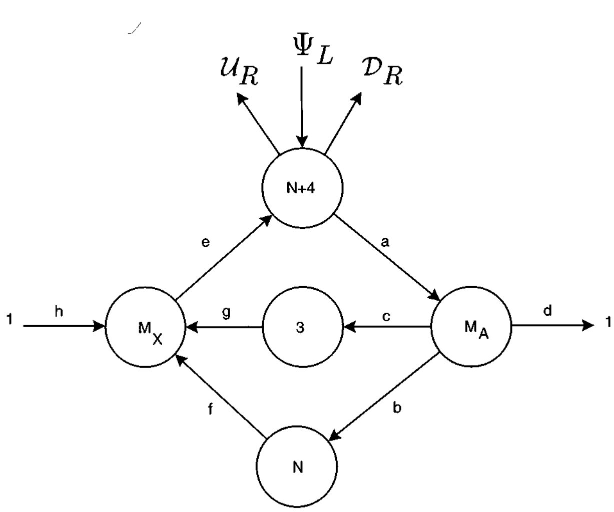

FIG. 2. A one-family, one-technidoublet ETC model.

2. Three families

The model can be generalized to include three families of quarks and leptons as shown in Fig. 3. The ETC symmetry $\mathrm{SU}(N+12)$ is broken to $\mathrm{SU}(N)_{\mathrm{TC}} \otimes \mathrm{SU}(3)_{\mathrm{QCD}}$ at three separate scales. There is a separate $\mathrm{SU}(M)$ group to trigger the breaking at each scale. Each is assumed to become strongly interacting in the order A (at a scale of order a few hundreds of TeV), B (at a scale of order a few tens of TeV), and finally

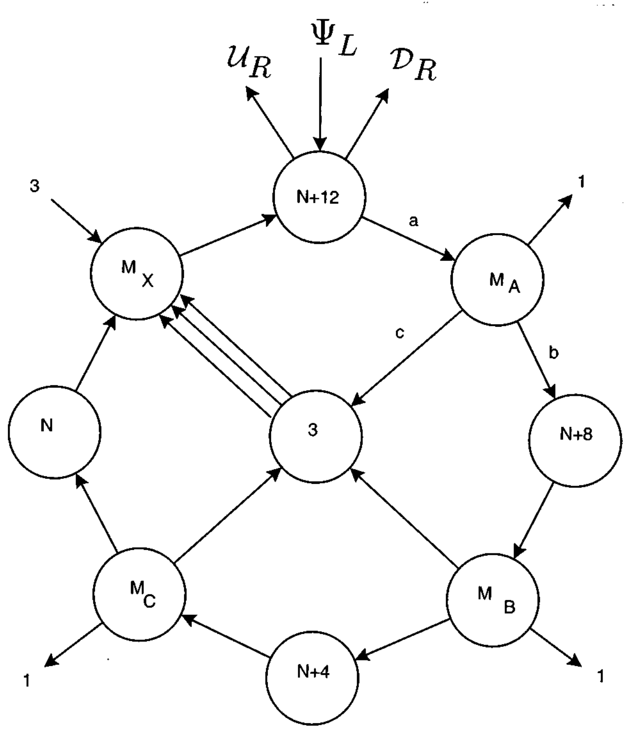

FIG. 3. A one-doublet ETC model with three family scales.

C (at a scale of order a few TeV). At each scale the breaking pattern is the same as that discussed in the one-family model; at Λ_A the ETC symmetry SU$(N+12)$ is broken to SU$(N+8)\otimes$SU(3). This breaking pattern is then repeated by the groups B and C. At the scale Λ_B it is the SU(3) containing the SU(3) subgroup of SU$(N+12)$ that is broken at the scale Λ_A, and an SU(3) subgroup of the SU$(N+8)$ group that break together to an SU(3) group that finally at the lowest breaking scale becomes QCD. The QCD interactions are finally shared by all quarks in the model. The broken gauge bosons of the ETC group now divide into three sets: those with masses of order F_A connecting the first family of SM fermions to more massive generations, those with masses of order F_B connecting the second family to more massive generations, and those with masses of order F_C connecting the third family to technifermions. This hierarchy of ETC gauge boson masses will generate the hierarchy of quark and lepton family masses below the technicolor scale.

Anomalies are again canceled in the model by the fermions transforming under the extra SU$(M)_X$ gauge group that confines these fermions between the technicolor and lowest ETC scale. In the enlarged model there are six Goldstone bosons that have no gauge interactions and are hence massless.

B. Flavor symmetry breaking

The model in Fig. 3 has an SU(8) flavor symmetry within each family, broken only by the weak SM interactions. To generate the observed quark and lepton masses we must introduce quark-lepton symmetry-breaking interactions and isospin symmetry-breaking interactions for both the quarks and leptons. For ease of understanding let us discuss a model of just the third family and the technidoublet.

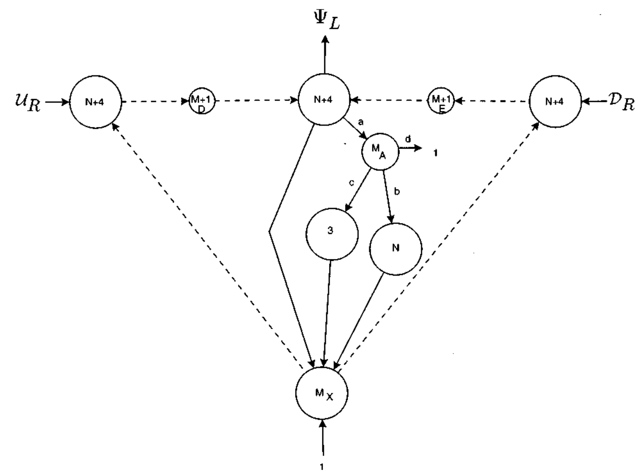

FIG. 4. Isospin breaking in the model of the third family.

1. Isospin breaking

We shall break isospin degeneracy by making the ETC gauge group chiral [15]. We take it to be SU$(N+4)_L$ $\otimes$SU$(N+4)_{\mathscr{U}_R}\otimes$SU$(N+4)_{\mathscr{D}_R}$, as shown in the model in Fig. 4. The one family model in Fig. 2 is shown by the solid lines in Fig. 4, with the additional sectors discussed in this section shown as dashed lines.

The SU$(M)_A$ gauge group forms condensates at Λ_A and breaks the SU$(N+4)_L$ ETC group to SU$(N)\otimes$SU(3) as in the simplest model. The two gauge groups SU$(M+1)_D$ and SU$(M+1)_E$ then become strong between the scale Λ_A and the technicolor scale (for the purposes of making estimates we shall take $\Lambda_A\simeq\Lambda_D\simeq\Lambda_E$), breaking the chiral ETC groups to the vector SU$(N)_{\rm TC}\otimes$SU$(3)_{\rm QCD}$. At each of these breakings, all Goldstone modes are absorbed by gauge bosons associated with broken generators.

There are now three degrees of coupling freedom associated with the interactions of the quarks and leptons: the SU$(N+4)_L$ coupling g_L, the SU$(N+4)_{\mathscr{U}_R}$ coupling $g_{\mathscr{U}_R}$, and the SU$(N+4)_{\mathscr{D}_R}$ coupling $g_{\mathscr{D}_R}$. The couplings that enter into the quark and lepton masses are these running couplings evaluated at the breaking scale of the ETC interactions and they will in general break the isospin symmetry of the model. The left- and right-handed ETC gauge bosons mix through loops of the fermions transforming under SU$(M+1)_D$ and SU$(M+1)_E$ that have condensed at $\Lambda_{D,E}$ as shown in Fig. 5. We shall use these extra degrees of freedom to generate the top-bottom quark mass splitting. The two extra parameters will not be sufficient to explain quark-lepton mass differences which we leave to the next section.

If we assume that the ETC gauge bosons coupling to the top have couplings g_L and $g_{\mathscr{U}_R}$ of order one or greater (but less than 4π at which the ETC gauge bosons become strongly coupled), these gauge bosons will have masses of order $F_A\simeq F_D$ or larger. We may approximate them at the technicolor scale by four-fermion interactions. The ETC couplings cancel as in Eq. (9) and the top quark mass can be estimated to be roughly

$$m_t\simeq\frac{N}{4\pi^2}\,\frac{\Sigma(0)^2}{F_A^2}\Sigma(0), \qquad (10)$$

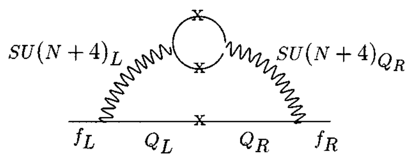

FIG. 5. Generation of third family fermion f mass from the technifermion Q condensate.

where $\Sigma(p)$ is the dynamical technifermion mass. A simple Pagels-Stokar [16] estimate, compatible with QCD, gives $v^2 \equiv (250\ \text{GeV})^2 \simeq N/4\pi^2 \Sigma(0)^2$ and hence $\Sigma(0) \simeq 1$ TeV for $N \simeq 2$. To generate m_t in the required range we therefore need $F_A \leqslant 800$ GeV. Although F_A must, therefore, be approximately at the technicolor scale, the scale $\Lambda_A \simeq 2\pi F_A/\sqrt{M}$ will be larger, as in QCD, and hence there is some running space for the technicolor coupling to evolve from its value at the breaking scale to its critical value at the technicolor scale. The estimates above are clearly naive approximations to the full nonperturbative technicolor dynamics and are not to be trusted to more than factors of 2.

If the ETC coupling is raised close to its critical value (the value of the ETC coupling at which the ETC interactions alone would break the chiral symmetry of the quark and leptons) at the ETC symmetry breaking scale, then the approximations above are not valid and the ETC coupling will not cancel from the top quark mass. A 175 GeV top mass may be generated, though how close the ETC coupling must be to its critical coupling is unclear. We shall assume in the discussions below that the ETC interactions are perturbative, leaving the possibility that they might be strong and near critical to those in Sec. IV.

To generate a smaller bottom quark mass we take the coupling $g_{\mathscr{D}_R}$ to be less than 1. The ETC gauge bosons associated with $\text{SU}(N+4)_{\mathscr{D}_R}$ therefore acquire a mass $g_{\mathscr{D}_R} F_E$, which are light relative to $F_{D,E} \simeq F_A \simeq \Sigma(0)$. Referring again to Fig. 5, the bottom quark mass is given approximately by

$$m_b \simeq \frac{N}{4\pi^2} \int dk^2 k^2 \frac{\Sigma(k)}{k^2+\Sigma(k)^2} \frac{g_{\mathscr{D}_R}^2}{k^2+g_{\mathscr{D}_R}^2 F_A^2}, \tag{11}$$

where we have taken $F_E = F_A$ and set the external momentum to zero. With $g_{\mathscr{D}_R}^2 F_A^2 < \Sigma(0)^2$, the integral can be estimated to give roughly

$$m_b \simeq \frac{N}{4\pi^2} g_{\mathscr{D}_R}^2 \Sigma(0), \tag{12}$$

where we have again neglected interactions between the ETC gauge boson and the technicolor gauge bosons. The bottom quark mass is thus suppressed relative to the top quark mass by $g_{\mathscr{D}_R}^2$. The choice $g_{\mathscr{D}_R} \simeq 1/6$ gives a realistic value for m_b and leads to a mass of order 200–300 GeV for the $\text{SU}(N+4)_{\mathscr{D}_R}$ ETC gauge boson.

The technifermion mass splitting $\Delta\Sigma(p) \equiv \Sigma_U(p) - \Sigma_D(p)$ can also be estimated perturbatively in the ETC interactions. The main contribution in the model is from the isospin-violating, massive gauge bosons that transform under the adjoint representation of $\text{SU}(N)$. The splitting can be estimated to be roughly

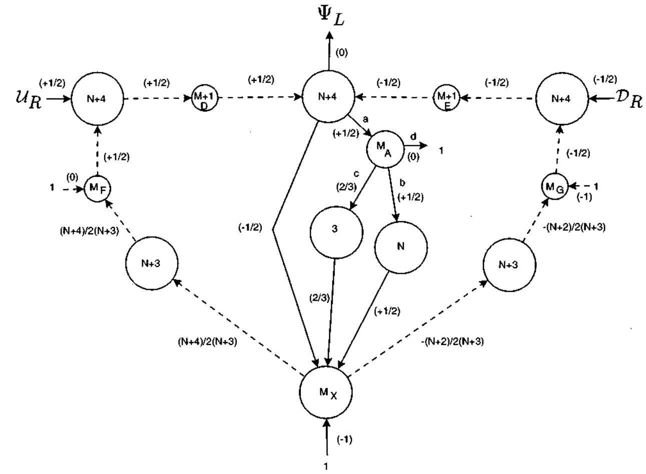

FIG. 6. Quark lepton mass splitting in the model of the third family. The fermion lines are labeled by their U(1) hypercharges.

$$\Delta\Sigma \simeq \frac{N}{4\pi^2} \frac{\Sigma(0)^3}{F_A^2} \simeq m_t. \tag{13}$$

We discuss the implications of this mass splitting for the $\Delta\rho \equiv \alpha T$ parameter in Sec. III E below.

2. Lepton Masses

In the model in Fig. 4, the lepton interactions are only split from their quark isospin partners by SM interactions. Although QCD interactions may be enhanced if the ETC interactions are close to critical (a possibility we discuss below) and hence could possibly explain the τ-bottom-quark mass splitting, they cannot explain why the τ neutrino is so light or massless. In order to give a fully perturbative ETC model we shall generate the τ–bottom-quark and τ-neutrino–top-quark mass splittings by further ETC symmetry breaking dynamics at new scales.

The extra sectors are shown in Fig. 6. The $\text{SU}(M)_F$ gauge group becomes strongly interacting at the scale Λ_F and breaks a single gauge color from the $\text{SU}(N+4)_{\mathscr{U}_R}$ gauge group. The corresponding broken eigenstate of the multiplet in (3.2) will become the neutrino with mass

$$m_{\nu_\tau} \simeq \frac{N}{4\pi} \frac{F_D^2}{F_A^2} \frac{\Sigma(0)^3}{F_F^2}, \tag{14}$$

with $F_A \sim F_D$ and with a suitably high choice of F_F ($\geqslant 100$ TeV) the τ neutrino mass may be suppressed below the experimental bound of roughly 30 MeV.

The gauge group $\text{SU}(M)_G$ plays the same role for the τ lepton, suppressing its mass relative to the bottom quark by F_E^2/F_G^2 from which we learn that F_G must be of order of a few TeV to reproduce the observed τ–bottom-quark mass splitting.

C. First and second families

The lightest two families of quarks and leptons may be incorporated in the model following the discussion in Sec. III A 2 and will have mass scales set by the higher two ETC symmetry-breaking scales. The top-bottom quark mass splitting will feed down to the lightest two-family quarks, generating isospin breaking that could explain the charm-strange mass splitting. The three right-handed neutrinos could all be broken from their ETC multiplet at the scale Λ_F. The neutrino masses would then be suppressed relative to the charged lepton masses by $(F_D/F_F)^2$. The single scale Λ_F could thus serve to explain the lightness of all three neutrinos. The quark-lepton mass splittings, however, can probably not be generated from the third family in perturbative ETC models, since the bottom quark and τ contributions to, for example, the strange and muon masses are small in comparison with the feeddown from the technifermion self-energies. If neccessary, extra breaking scales may be introduced to explain the splittings using the dynamics discussed above. Similarly the ETC gauge groups acting on the right-handed up and down quarks may be broken at additional scales, providing the freedom to accommodate the up-down quark mass inversion. The symmetry breaking patterns presented here are not capable of producing the CKM mixing angles in the quark sector since the families correspond to distinct ETC gauge eigenstates broken at different scales. We leave the generation of the mixing angles for future work.

D. U(1) embedding

Hypercharge may be embedded in the moose model of Fig. 6 by assigning each particle the U(1) charge indicated on the fermionic lines. The final hypercharge group is a subgroup of the $U(1)_R$ group of the quarks, leptons, and technifermions and the broken diagonal generators of the $SU(N+4)$ ETC group. To achieve the correct breaking pattern the condensates formed by $SU(M)_A$ must be invariant to $U(1)_Y$. Since the $SU(N+4)$ symmetry of the fermions transforming as an $\bar{M}_A$ is explicitly broken, their U(1) charges must correspond to the relevant subgroup of their $SU(N+4)\otimes U(1)$ symmetry.

E. Phenomenology

Since there is only one technidoublet in the model, there are no pseudo Goldstone bosons generated at the technicolor scale. The single doublet will also generate only a small contribution to the S parameter [8,9], $S\sim 0.1N$, which we expect to be compatible with the current experimental two-standard-deviation upper limit $S<0.4$.

The isospin-violating ETC interactions will, of course, give rise to a contribution to the $\Delta\rho(=\alpha T)$ parameter. The W and Z masses are generated by techifermion condensation and deviations from the $\Delta\rho$ parameter from corrections to the relevant diagrams due to exchange of isospin-violating ETC gauge bosons. At first order in the ETC interactions [20] the largest contribution will be generated by the exchange of the massive gauge bosons transforming under the adjoint of SU(N) across the techifermion loop. We roughly estimate this "direct" contribution to be

$$\Delta\rho \simeq \frac{v^2}{4F_A^2}, \tag{15}$$

which is of the order of a few percent.

The isospin violation of the ETC interactions will also feed into the technidoublet, giving rise to mass splitting between the techniup and technidown [estimated in Eq. (13)]. There is thus an "indirect" contribution to the $\Delta\rho$ parameter from loops of nondegenerate technifermions which is second order in ETC gauge boson exchange. Roughly estimating the contribution using the peturbative result for $\Delta\rho$ [8] and the estimate of $\Delta\Sigma$ in Eq. (13) we find

$$\Delta\rho \simeq \frac{N\Delta\Sigma^2}{12\pi^2 v^2} \simeq \frac{v^4}{3F_A^4}. \tag{16}$$

These estimates of $\Delta\rho$ are of course naive, ignoring the effects of the strong technicolor dynamics between the technifermion loops and neglecting a complete analysis of the many massive ETC gauge bosons. If they are accurate, they could be difficult to reconcile with the experimental constraint $\Delta\rho \lesssim 0.3\%$ (see also the estimates in Ref. [21]). We leave a more detailed computation of $\Delta\rho$ to a subsequent paper. In any case, in Sec. IV below we present a variation of the model that will not overly infect $\Delta\rho$.

The model will also give rise to corrections to the $Zb\bar{b}$ vertex. These arise from both the exchange of the *sideways* gauge boson [17], coupling technifermions to the bottom, across the Zbb vertex, and from mixing of the Z with the diagonal broken ETC generator [18]. Each of these contributions can be as large as a few percent for an ETC scale of order 1 TeV but have opposite signs. The magnitude and sign of the combined correction have been shown to be compatible with the experimental measurement for some models (the exact correction is dependent on N and the relative sizes of g_L and $g_{\mathcal{U}_R}$).

As presented the model does not give rise to quark- or lepton-number-changing FCNC's since each family quark and lepton number is a conserved ETC charge in the model. Of course the most stringent FCNC constraints on ETC models come from $K^0\bar{K}^0$ mixing through the CKM mixing angles which break quark number within each family to a single subgroup. Since we have not addressed the generation of these mixing angles in this paper, we cannot address this constraint. We note though that these FCNC's may be suppressed in several ways, by small mixing angles in the up-type quark sector or by a walking technicolor theory or strong ETC interactions that enhance the ETC scales.

F. Massless Goldstone bosons

Massless Goldstone bosons are generated in the model at the scale Λ_X as discussed above. These Goldstone bosons carry no charge under any of the gauge groups in the model. However, their constituents are charged, and so they can be produced by gluon or photon fusion or in the decay of the Z [12]. They can also be produced through the exchange of the heavy ETC gauge bosons. The amplitude in each case is proportional to $1/F_X$ where $F_X\simeq 1$ TeV, so that the production rate is down by at least an order of magnitude relative to the production of the Goldstone bosons composed of technifermions that arise in a one-family technicolor model. The rate is below current laboratory limits. With the Goldstone bosons massless or very light, however, their production by the above mechanisms is a major energy loss mechanism for stars [13], and is ruled out by stellar abundances.

The Goldstone bosons are thus troublesome but may acquire masses from further unifications above the scales discussed in the model so far. In the spontaneous breaking at Λ_X, the Goldstone bosons complete an adjoint representation of the unbroken SU$(N+12)$ vector global symmetry group (in the three family model). If at some higher scale this group is gauged (corresponding for example to gauging the full chiral symmetry group in Fig. 3), then all the Goldstone bosons will acquire masses given by

$$M_A^2 \simeq 4\pi F_X^4/\Lambda_{\mathrm{new}}^2, \tag{17}$$

which is potentially sufficient to ensure that the Goldstone bosons will not be a source of energy loss in stellar interiors.

IV. STRONG ETC

The model presented so far appears capable of producing a 175 GeV top quark mass, treating the ETC interactions perturbatively. However, the contributions of the isospin-violating ETC gauge bosons to the $\Delta\rho$ parameter are close to or above experimental limits. The direct contributions [Eq. (15)], which scale as $1/M_{\mathrm{ETC}}^2$, can be reduced by increasing the lowest ETC scale, but at the expense of having to tune the ETC coupling close to its critical value from below to generate the large top quark mass. A near-critical ETC interaction for the third family would also enhance the QCD corrections to the third family quark masses and could potentially explain the bottom-quark–mass splitting without the need for the extra ETC symmetry-breaking scale Λ_G discussed in Sec. III B 2. Finally increasing the lowest ETC scale would allow us to increase the scale Λ_X and hence generate larger masses for the Goldstone bosons formed at that scale.

Although near-critical ETC interactions with a larger ETC scale may suppress the direct contribution to $\Delta\rho$, the indirect contribution will remain roughly the same size (it of course may no longer be considered second order). This follows from a gap equation analysis which suggests that the technifermion mass splitting will remain of order m_t.

Therefore if the large top quark mass is the result of either perturbative or strongly interacting sideways ETC interactions, the contribution to the $\Delta\rho$ parameter may conflict with the experimental limit.

V. CHIRAL TOP COLOR

Recently Hill [14] has renewed interest in the idea that the large top quark mass may be generated by a near-critical top self-interaction [19]. The usual sideways ETC interaction is therefore weak in these models, generating a contribution to the top quark mass of only approximately m_b [the scale F_A in Eq. (10) is therefore of order 5–10 TeV].

The ETC gauge boson with the large isospin-violating coupling responsible for the bulk of the top quark mass does not couple to the technifermions. Direct contributions from the mixing of this ETC gauge boson to the Z will only occur through loops of top and bottom quarks and hence will be suppressed by $[m_t/\Sigma(0)]^2$ relative to the models discussed above. The direct contribution will at worst be a tenth of a percent. The feedback of the top-bottom quark mass splitting by the sideways gauge bosons will be suppressed by the small sideways coupling and from Eq. (13) we have $\Delta\Sigma \simeq 5$ GeV. The indirect contribution to the $\Delta\rho$ parameter will therefore also be small.

Hill generates the top self-interaction by assuming that at ETC scales there is a separate $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Y$ gauge group acting on the third family that is near critical when broken to the SM gauge groups.

We can extend our model to include a top color interaction as shown in Fig. 7. The new $\mathrm{SU}(M)_H$ group becomes strongly interacting at Λ_H, breaking the $\mathrm{SU}(N+3)_{\mathscr{U}_R}$ group, left after the right-handed neutrino has decoupled, to $\mathrm{SU}(N)\otimes\mathrm{SU}(3)$. The right-handed SU(3) color group coupling will run independently of the technicolor coupling below this breaking scale [we require that Λ_H be large enough that there is enough running time for the SU(3) and SU(N) group couplings to significantly diverge] and this interaction of the top quark will be assumed to be near critical when broken to the vector QCD subgroup at Λ_D. Unlike in Hill's model in which the top-bottom quark mass splitting is the result of a strongly coupled $\mathrm{U}(1)_Y$ gauge interaction (with the associated problem of its coupling being close to its Landau pole) here the isospin splitting is provided by chiral, asymptotically free, *non-Abelian* gauge groups.

VI. SUMMARY AND CONCLUSIONS

We have presented a one-doublet technicolor model in which the ETC gauge symmetries are broken by purely QCD-like dynamics. All fermions transform only under the fundamental representation of gauge groups. The model has chiral ETC gauge groups, explicitly breaking custodial symmetry, and Pati-Salam unification at high scales. Its main features are the following.

Three families of quarks and leptons are incorporated, with a hierarchy of three family symmetry-breaking scales. Within the third family, the full spectrum of masses can be accommodated. In particular, we argue that with a third family ETC scale on the order of 1 TeV, it may be possible to generate both the top and bottom quark masses through perturbative ETC interactions. A light τ neutrino mass can be achieved by breaking the ETC group for right-handed isospin $+1/2$ fermions at a high scale. To place m_{ν_τ} below the current limit of roughly 30 MeV, this scale must be above about 100 TeV.

Since the model contains a single doublet of technifermions, no pseudo Goldstone bosons are formed at the electroweak scale and the S parameter can be kept relatively small. The weak custodial isospin symmetry breaking built into the model leads to a so-called "direct" contribution [20] to $\Delta\rho \equiv \alpha T$, which is first order in the ETC interaction. Our naive estimate suggests that this contribution may be as large as a few percent and hence possibly above the experimental limit.

A more detailed analysis of this contribution (and that to the $Zb\bar{b}$ vertex) will be given in a succeeding paper. The

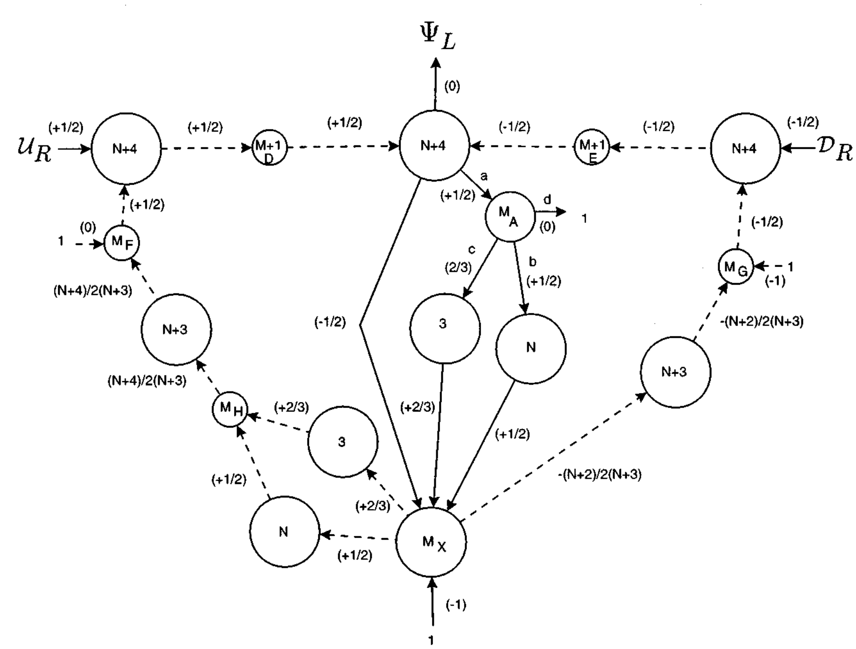

FIG. 7. Chiral top color in the model of the third family.

"indirect" contribution, arising from loops of nondegenerate technifermions, is second order in ETC interactions and is small relative to the direct contribution when the ETC interactions are perturbative.

The model contains global symmetries at the ETC scales, whose spontaneous breaking leads to massless Goldstone bosons. They can couple to ordinary matter through SM interactions and are ruled out by stellar energy loss constraints [13]. They can, however, be given phenomenologically acceptable masses by further unifications above the ETC scales, which break the global symmetries.

Some of the mass splittings within the first two families will be fed down naturally from the third family. We have argued that the charm-strange mass splitting may be a result of the top-bottom quark mass splitting. The suppression of all three generations of neutrino masses may be explained by a single ETC breaking scale. We have not discussed the origin of quark mixing angles in this work, though it will clearly be important to address this point in the future.

We have also demonstrated that a large top quark mass can be generated dynamically in technicolor by a near-critical top color interaction without the need for a strong U(1) interaction. This variant of the model is compatible with the experimental value of $\Delta\rho$.

The model presented here illustrates that ETC symmetries can be broken using only QCD-like dynamics and fermions in fundamental representations. The requisite number of quark-lepton and isospin symmetry-violating parameters may be introduced to accomodate the third family spectrum. It remains to be seen whether this approach leads to an explanation of quark and lepton masses and CKM mixing angles.

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