

## Flavor symmetry breaking in the $1/N_c$ expansion

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The breaking of flavor SU(3) symmetry in the axial vector couplings and magnetic moments of baryons is analyzed in the  $1/N_c$  expansion. A simple meson loop graph which is known to be of order  $\sqrt{m_s}$  and leading order in  $1/N_c$  correctly predicts the pattern of symmetry breaking in the magnetic moments. It is, however, not possible to use this graph to predict the magnitude of the breaking. The situation with the axial vector couplings is less clear. In this case the breakings are relatively small and do not appear to follow an obvious pattern. Nevertheless there is a clear indication that, with symmetry breaking taken into account, the  $F/D$  ratio [defined in the presence of SU(3) breaking] is considerably less than the SU(6) value of  $2/3$ . With sizable uncertainty, we find  $F/D \approx 0.44$ . The quantity  $3F - D$  which is relevant for the analysis of spin-dependent deep inelastic scattering is considerably smaller than the SU(6) value of unity. The new value  $3F - D = 0.28 \pm 0.09$  is consistent with vanishing strange-quark spin in the nucleon.

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### I. INTRODUCTION

The  $1/N_c$  expansion has proved to be quite useful in understanding the spin-flavor structure of baryons in QCD [1-5]. In the flavor symmetry limit, a structure similar to SU(6) emerges [1-9]. This is not, however, a statement that there is an SU(6) symmetry in the Lagrangian or that the quarks are nonrelativistic. Rather, it is the simplified dynamics [10,11] of QCD at large  $N_c$  that produces patterns similar, but not exactly equal to, the predictions of SU(6). In this paper we focus on flavor SU(3) breaking in the baryon magnetic moments and axial vector couplings in the  $1/N_c$  expansion. The mathematical groundwork, which involves a considerable amount of group theory, was set up in a previous paper [1] — here we will concentrate on the data and on the physical interpretation of results.

The pattern of symmetry breaking displayed by the baryon magnetic moments is in quite good agreement with that predicted by the simple meson loop diagram shown in Fig. 1 [12,13]. In the chiral limit, this diagram is of order  $\sqrt{m_s} N_c$ , i.e., it is of order  $\sqrt{m_s}$  rather than the naively expected  $m_s$ , and is leading order in  $N_c$ . However, a naive evaluation of the diagram in Fig. 1 predicts symmetry breaking which is, in magnitude, about a factor of 2 too large. As pointed out in Ref. [12], this is almost certainly due to the fact that the diagram involves meson momenta on the order of  $M_K$  which are too large for the validity of chiral perturbation theory and, consequently, the numerical value of the diagram cannot be taken seriously. Nevertheless, the group theoretic structure suggested by this diagram is in rather impressive agreement with the data.

For the axial vector couplings the situation is less clear. Here the data are the widths of the decuplet baryons, converted to axial vector couplings through the Goldberger-Treiman relation, and the values of  $g_A$  extracted from  $\beta$  decays of the octet baryons. In this case the SU(3)

breaking is actually rather small—the leading term is order  $m_s N_c$  rather than  $\sqrt{m_s} N_c$ —and the experimental errors are larger. While we do find what appears to be a stable fit with an interesting physical interpretation, we have some concerns about the results. Although we feel that we are not in a position to make precise and definitive statements about symmetry breaking in the axial vector couplings, some trends do emerge. The first is that the pattern of symmetry breaking is broadly consistent with the  $1/N_c$  expansion and the fact that chiral perturbation theory does not predict a dominant pattern. The second is the value of the  $F/D$  ratio, a quantity which in the presence of SU(3) breaking must be carefully defined. We parametrize the couplings in such a way that the matrix elements of the strangeness preserving  $\Delta S = 0$  currents (both isovector and isoscalar) between nucleon states are exactly given by  $D$  and  $F$ , at least through first order in SU(3) breaking. Using this definition we find  $F/D \approx 0.44$  with perhaps a 10% error. Although the error is significant, the central value for the  $F/D$  ratio is considerably smaller than the SU(6) value of  $2/3$ . Finally, the above definition of  $D$  and  $F$  is used to obtain a new value for the quantity  $3F - D$  which appears in the analysis of spin-dependent deep inelastic scattering [14-17]. Our new value,  $3F - D \approx 0.28 \pm 0.09$ , is considerably smaller than the SU(6) value of unity or the value of  $\approx 0.6$  obtained from a SU(3) symmetric fit to the data. The value of the strange-quark spin in the nucleon extracted from experimental data is significantly reduced

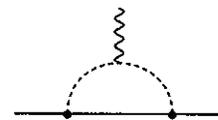


FIG. 1. Leading chiral correction to the baryon magnetic moments. The solid line is the baryon, and the dashed line is the meson. The graph is of order  $M_K \sim \sqrt{m_s}$ .

using the new value of  $3F - D$  and is consistent with zero. The uncertainty in the value of  $3F - D$  limits the accuracy of the extraction of the nucleon's strange-quark spin. Since one is in a region of parameter space where there is a large cancellation between  $3F$  and  $D$ , neither this analysis nor any other is likely to give a highly accurate value for  $3F - D$ .

We had originally hoped that studying the magnetic moments would help in the interpretation of the axial vector couplings — in the naive nonrelativistic quark model, magnetic moments and axial vector couplings tend to behave in the same way. However, as mentioned above, the actual symmetry breaking in the magnetic moments appears to come largely from a single diagram which has no analog in the axial vector couplings. The reason we can make some progress in the analysis of the axial couplings is that the  $1/N_c$  expansion predicts that some SU(3)-breaking operators are suppressed, and it relates the octet and decuplet axial vector couplings so that one has more experimental input to constrain the fit.

The baryon axial currents in the presence of SU(3) breaking also have been studied recently by Ehrnsperger and Schäfer [18] who assume that the SU(3) breaking is proportional to the baryon masses, and by Lichtenstadt and Lipkin [19] using a model. Both calculations give results which are similar to those obtained here: SU(3) breaking lowers the value of  $F/D$  and of  $3F - D$ .

The paper is organized as follows. In the next section the operator analysis of Ref. [1] is summarized, with some additional mathematical details given in an appendix. The operator analysis leads to a seven parameter fit to the hyperon  $\beta$  decays and decuplet pionic decays, with coefficients listed in Table I, and a seven parameter fit to the baryon magnetic moments with coefficients listed in Table II. The reader not interested in the technical details of the  $1/N_c$  expansion can skip directly to Secs. III and IV, which present the analysis of the axial vector couplings and the magnetic moments, respectively.

## II. OPERATOR ANALYSIS

Confining ourselves to the physically interesting case of three light flavors, the lowest-lying baryon states fall into a representation of the spin-flavor group SU(6). At the physical value  $N_c = 3$ , this is the familiar 56 dimensional representation of SU(6) while for larger  $N_c$  the representation becomes more complex, containing spins greater than  $3/2$  and SU(3) representations bigger than the 8 and 10. The complexity of these large representations of SU(6) makes a straightforward application of group theory very tedious at large  $N_c$ . It was pointed out in Refs. [1,7,8] that it is easier to focus on the operators than the states. The magnetic moments, for example, have an expansion in operators whose coefficients are inverse powers of  $N_c$ . Working to a given order in  $1/N_c$ , one can truncate the expansion and connect to physics by evaluating the matrix elements at  $N_c = 3$ . For any representation of SU(6), polynomials in the generators  $J^i$ ,  $T^a$ , and  $G^{ia}$  form a complete set of operators. For the particular representations relevant to the lowest-lying

baryons, it suffices to keep polynomials through order  $N_c$  and, in addition, there are a number of identities among the polynomials of order less than or equal to  $N_c$ . The problem of finding a complete and independent set of operators for the baryon representations was solved in Ref. [1].

The results of Ref. [1] can be summarized as follows. The basic building blocks are the operators  $J^i$ ,  $T^a$ , and  $G^{ia}$ , where  $J^i$  is the spin, the  $T^a$  are the generators of SU(3), and the  $G^{ia}$  are the remaining generators of SU(6). They satisfy the commutation relations

$$\begin{aligned} [J^i, J^j] &= i \epsilon^{ijk} J^k, \\ [T^a, T^b] &= i f^{abc} T^c, \\ [G^{ia}, G^{jb}] &= i \delta^{ij} f^{abc} T^c + i \epsilon^{ijk} (\delta^{ab} J^k + d^{abc} G^{kc}), \\ [T^a, G^{ib}] &= i f^{abc} G^{ic}, \\ [J^i, G^{ja}] &= i \epsilon^{ijk} G^{ka}, \end{aligned}$$

and can be defined in terms of the quarks as

$$\begin{aligned} J^i &= q^\dagger \frac{\sigma^i}{2} q, \\ T^a &= q^\dagger \frac{\lambda^a}{2} q, \\ G^{ia} &= q^\dagger \frac{\sigma^i \lambda^a}{4} q. \end{aligned}$$

A complete set of operators can be constructed from polynomials in the operators  $J$ ,  $T$ , and  $G$ . Because antisymmetric products can be reduced using the commutation relations, one need only consider products which are completely symmetric in noncommuting operators. Furthermore, it can be shown that all products of  $T$ 's and/or  $G$ 's in which two flavor indices are summed over or contracted with  $d$  or  $f$  symbols can be eliminated in terms of lower order products.

The way in which large  $N_c$  dynamics enters can best be seen through an example. Let  $P^{ia}$  be the operator whose matrix elements between SU(6) symmetric states gives the actual axial vector couplings of the baryons. It is spin-1, an octet under SU(3), and odd under time reversal. In the absence of SU(3) breaking and neglecting quartic and higher order polynomials,  $P^{ia}$  can be written as

$$\begin{aligned} \frac{1}{2} P^{ia} &= a G^{ia} + b J^i T^a + d \{J^2, G^{ia}\} \\ &+ e \{J^i, \{J^k, G^{ka}\}\}, \end{aligned} \quad (2.1)$$

where  $a, b, d$ , and  $e$  are unknown coefficients. By examining diagrams one can see that  $a$  is order  $N_c^0$ ,  $b$  is of order  $N_c^{-1}$  and  $d$  and  $e$  are of order  $N_c^{-2}$ . Hence  $P^{ia}$  can also be expressed as

$$\begin{aligned} \frac{1}{2} P^{ia} &= a' G^{ia} + b' \frac{1}{N_c} J^i T^a + d' \frac{1}{N_c^2} \{J^2, G^{ia}\} \\ &+ e' \frac{1}{N_c^2} \{J^i, \{J^k, G^{ka}\}\}, \end{aligned} \quad (2.2)$$

where the new coefficients are of order unity for large  $N_c$ . If we consider states whose spin remains fixed as  $N_c \rightarrow$

$\infty$ , then the matrix elements of  $J$  never become large. The operators  $T$  and  $G$  are more complicated — for some states in the representation  $G$  has matrix elements of order  $N_c$  while  $T$  has matrix elements of order unity, but for other states  $G$  has matrix elements of order unity while  $T$  has matrix elements of order  $N_c$ . There are other states for which both  $G$  and  $T$  have matrix elements of order  $\sqrt{N_c}$ . Nevertheless, it is clear that truncating  $P^{ia}$  at order  $N_c^{-1}$  is a consistent approximation; the remaining two terms multiplied by  $d$  and  $e$  are everywhere smaller than the first term. Dropping the second term is more problematic—there are states for which the matrix elements of  $J^i T^a/N_c$  are of the same order as the matrix elements of  $G^{ia}$ . Thus, in general, this term must be retained. Finally, note that keeping all four terms allows for arbitrary values of the four possible SU(3) symmetric couplings of pseudoscalar mesons to the octet and decuplet baryons. This is an example of the fact that for  $N_c = 3$  we never have to go beyond operator products of third order in the generators.

When first order SU(3) breaking is taken into account,  $P^{ia}$  contains pieces transforming according to all SU(3) representations contained in the product  $\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8}_A \oplus \mathbf{8}_S \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$ . We summarize the results of a more detailed analysis presented in Ref. [1]. The possible spin-1 singlets containing three or fewer generators are  $J^i$  and  $J^2 J^i$  with the second operator having a coefficient of order  $N_c^{-2}$  relative to that of the first. The only singlet term that we will keep is  $\delta^{a8} J^i$ . The octet operators were discussed in the previous paragraph. We will drop the  $N_c^{-2}$  terms, leaving  $d^{ab8} G^{ib}$  and  $d^{ab8} J^i T^b$  as the octet terms. (Similar terms with the  $d$  symbol replaced by an  $f$  symbol are ruled out by time reversal.) We do, however, have to keep the operator

$$[J^2, [T^8, G^{ia}]], \quad (2.3)$$

which can be reduced, by use of the commutation relations, to a sum of operators of the form  $\{J^k, G^{jb}\}$ —this particular sum cannot appear in the SU(3) limit by time reversal invariance but is allowed in the case of broken symmetry. This operator receives a coefficient of order  $N_c^{-1}$  and contributes only to processes which change both spin and strangeness. The leading operator containing a  $\mathbf{27}$  is

$$\{G^{ia}, T^8\} + \{G^{i8}, T^a\}, \quad (2.4)$$

which receives a coefficient of order  $N_c^{-1}$ . In the next order there are two operators,  $J^i \{T^a, T^8\}$  and  $\{J^k G^{ka}, G^{i8}\} + \{J^k G^{k8}, G^{ia}\}$  which have coefficients of order  $N_c^{-2}$ . The leading operator containing  $\mathbf{10} \oplus \overline{\mathbf{10}}$  is

$$\{G^{ia}, T^8\} - \{G^{i8}, T^a\} \quad (2.5)$$

with a coefficient of order  $N_c^{-1}$  and in the next order one has  $\{J^k G^{ka}, G^{i8}\} - \{J^k G^{k8}, G^{ia}\}$  with a coefficient of order  $N_c^{-2}$ . We will keep only the leading  $\mathbf{27}$  and  $\mathbf{10} \oplus \overline{\mathbf{10}}$  terms. In the Appendix, it is shown that matrix elements of the higher order terms are always down by a factor of at least  $N_c^{-1}$  relative to matrix elements of the leading terms.

In terms of  $N_s$  and  $J_s^i$ , the number of strange quarks and the strange quark spin, defined by

$$\begin{aligned} G^{i8} &= \frac{1}{2\sqrt{3}}(J^i - 3J_s^i), \\ T^8 &= \frac{1}{2\sqrt{3}}(N_c - 3N_s), \end{aligned} \quad (2.6)$$

the leading  $\mathbf{10} \oplus \overline{\mathbf{10}}$  and  $\mathbf{27}$  operators are  $\{G^{ia}, N_s/N_c\}$  and  $\{T^a, J_s^i/N_c\}$ , where we have dropped constants and terms that simply renormalize the symmetric couplings. A consistent truncation of  $P^{ia}$  valid to first order in SU(3) breaking is therefore

$$\begin{aligned} \frac{1}{2} P^{ia} &= (d'\delta^{ab} + c'_1 d^{ab8}) G^{ib} + (b'\delta^{ab} + c'_2 d^{ab8}) \frac{J^i T^b}{N_c} \\ &+ c'_3 \left\{ G^{ia}, \frac{N_s}{N_c} \right\} + c'_4 \left\{ T^a, \frac{J_s^i}{N_c} \right\} \\ &+ c'_5 \left[ J^2, \left[ \frac{N_s}{N_c}, G^{ia} \right] \right] + c'_6 \delta^{a8} J^i, \end{aligned} \quad (2.7)$$

where the  $c'_k$  are of order SU(3) breaking and the scaling with  $N_c$  is explicit. The large  $N_c$  expansion yields one further piece of information: as explained in Ref. [1] the coefficients are constrained by

$$3c'_6 = c'_1 + c'_2 \quad (2.8)$$

up to terms of order  $N_c^{-1}$ . Rearranging terms and absorbing factors of  $N_c^{-1}$  into the coefficients leads to the form that we will actually use in fitting data:

$$\begin{aligned} \frac{1}{2} P^{ia} &= a G^{ia} + b J^i T^a + \Delta^a (c_1 G^{ia} + c_2 J^i T^a) \\ &+ c_3 \{G^{ia}, N_s\} + c_4 \{T^a, J_s^i\} \\ &+ \frac{\delta^{a8}}{\sqrt{3}} W^i - \frac{d}{2} (\{J^2, G^{ia}\} \\ &- \frac{1}{2} \{J^i, \{J^j, G^{ja}\}\}), \end{aligned} \quad (2.9)$$

where

$$\begin{aligned} W^i &= (c_4 - 2c_1) J_s^i + (c_3 - 2c_2) N_s J^i \\ &- 3(c_3 + c_4) N_s J_s^i, \end{aligned} \quad (2.10)$$

$\Delta_a = 1$  for  $a = 4, 5, 6$ , or  $7$  and is zero otherwise, and the unprimed coefficients are linear combinations of the primed ones. The term involving  $[J^2, [T^8, G^{ia}]]$  which does not contribute to any observed decay has been dropped and a term proportional to  $d$  has been added to allow the SU(3) symmetric parameters [20]  $D$ ,  $F$ , and  $C$  to have arbitrary values. Our main interest is to study SU(3) breaking and adding this extra symmetrical term keeps symmetry breaking effects from being mixed up with  $1/N_c$  corrections in the symmetric couplings. The coefficient  $d$  is of order  $N_c^{-2}$  and is presumably comparable to some of the other coefficients, e.g.,  $c_2$  which is of order  $\epsilon/N_c$  where  $\epsilon \sim 0.3$  is the strength of SU(3) violation. Note that the couplings have been parametrized in such a way that only the symmetric parameters  $a$ ,  $b$ , and  $d$  contribute to processes which take place entirely

TABLE I. Coefficients for axial vector couplings.

	$a$	$b$	$d$	$c_1$	$c_2$	$c_3$	$c_4$
$\Delta N$	-2	0	9/2	0	0	0	0
$\Sigma^* \Lambda$	-2	0	9/2	0	0	-4	0
$\Sigma^* \Sigma$	-2	0	9/2	0	0	-4	8
$\Xi^* \Xi$	-2	0	9/2	0	0	-8	4
$np$	5/3	1	0	0	0	0	0
$\Sigma \Lambda$	$\sqrt{2/3}$	0	0	0	0	$\sqrt{8/3}$	0
$\Lambda p$	$-\sqrt{3/2}$	$-\sqrt{3/2}$	0	$-\sqrt{3/2}$	$-\sqrt{3/2}$	$-\sqrt{3/2}$	$-\sqrt{3/2}$
$\Sigma n$	1/3	-1	0	1/3	-1	1/3	1/3
$\Xi \Lambda$	$1/\sqrt{6}$	$\sqrt{3/2}$	0	$1/\sqrt{6}$	$\sqrt{3/2}$	$\sqrt{3/2}$	$7/\sqrt{6}$
$\Xi \Sigma$	$5/\sqrt{18}$	$1/\sqrt{2}$	0	$5/\sqrt{18}$	$1/\sqrt{2}$	$5/\sqrt{2}$	$1/\sqrt{2}$

in the strangeness zero sector. We define  $D$  and  $F$  in the presence of SU(3) breaking by

$$\begin{aligned} D &= a, \\ F &= 2a/3 + b. \end{aligned} \quad (2.11)$$

The  $\pi NN$  and  $\eta NN$  couplings are given exactly by the formulas in the SU(3) symmetry limit with these values of  $D$  and  $F$ .

For any given process, the matrix element of  $P^{ia}$  can be expressed as a sum of the seven parameters  $a, b, d, c_1, \dots, c_4$  times coefficients derived from the matrix elements of the operators. The coefficients for the axial vector couplings are tabulated in Table I and those for the magnetic moments are tabulated in Table II.

### III. THE AXIAL VECTOR COUPLINGS

In the large  $N_c$  limit  $P^{ia}$  gives the matrix elements of the space components of the axial vector currents between baryon states. For the octet baryons there is little ambiguity as to how to apply this to the real world of  $N_c = 3$ . We use the  $g_A$  parameters as conventionally defined in  $\beta$ -decay experiments with a normalization such that  $g_A \approx 1.26$  and  $g_V = 1$  for neutron decay. Experimentally, one measures the lifetimes which are proportional to  $1/(|g_V|^2 + 3|g_A|^2)$  and the asymmetry which gives  $g_A/g_V$ . The  $g_V$  parameters are taken from SU(3) because of the Ademollo-Gatto theorem which states that SU(3) violation in  $g_V$  occurs only in second order. When form factor effects<sup>1</sup> [21], radiative corrections [22,23], and weak magnetism are taken into account, the values of  $g_A$  obtained from the asymmetries and the rates are in general, although not spectacularly good, agreement with each other.<sup>2</sup> There is one exception: for  $\Xi^- \rightarrow \Lambda \beta$  de-

cay the two methods produce inconsistent values of  $g_A$ . This can be understood from the fact that for this decay  $g_A/g_V$  is unusually small, and the extraction of  $g_A$  from the rate is difficult both because of ambiguities in the rate and the various corrections and because the extracted value of  $g_A$  is likely to be sensitive to the (second order) SU(3) violations in  $g_V$ . For this decay we use only the value of  $g_A$  extracted from the asymmetry. In the case of  $\Xi^-$  to  $\Sigma^0$  decay, only the rate has been measured so we have no choice but to take  $g_A$  from the rate—fortunately  $|g_A/g_V|$  is fairly large for this decay. For the other decays, we have combined the values of  $g_A$  from the decay asymmetry and lifetimes using scaled errors, as recommended by the Particle Data Group [25]. The experimental values for  $g_A$  are listed in the fourth column of Table III. Apart from the  $\Xi^- \rightarrow \Sigma^0$  entry, they are essentially the same as the standard values derived from the asymmetry, except that in some cases the errors have been enlarged to account for discrepancies between the rates and the asymmetries.

Off-diagonal elements of  $P^{ia}$  connecting the decuplet baryons to the octet baryons can be extracted from the  $\pi$  decays of the decuplet. Here there is some ambiguity in how to handle the kinematics—in the  $N_c \rightarrow \infty$  limit the baryon masses become infinite and static kinematics apply, but in the real world we have to deal with finite masses. There is no definitive answer to this problem, but Peccei [26] has developed a formalism that is internally consistent and compatible with chiral symmetry. In his

TABLE II. Coefficients for magnetic moments.

	$a$	$b$	$d$	$c_1$	$c_2$	$c_3$	$c_4$
$p$	1	1	0	0	0	0	0
$n$	-2/3	0	0	0	0	0	0
$\Lambda$	-1/3	0	0	-2/3	-2/3	-2/3	0
$\Sigma^+$	1	1	0	2/9	-2/3	2	-2/3
$\Sigma^-$	-1/3	-1	0	2/9	-2/3	-2/3	2/3
$\Sigma^0 \Lambda$	$-1/\sqrt{3}$	0	0	0	0	$-2/\sqrt{3}$	0
$\Xi^0$	-2/3	0	0	-8/9	-4/3	-8/3	0
$\Xi^-$	-1/3	-1	0	-8/9	-4/3	-4/3	-8/3
$p\Delta^+$	$\sqrt{8/3}$	0	$-3/\sqrt{2}$	0	0	0	0
$\Omega^-$	-1	-3	0	-2	-6	-6	-6
$\Delta^{++}$	2	6	0	0	0	0	0

<sup>1</sup>We have used a dipole form for the axial and vector form factors, with masses  $M = 1.08$  GeV for the  $\Delta S = 0$  axial form factor,  $M = 1.25$  GeV for the  $\Delta S = 1$  axial form factor [24]. The corresponding masses for the vector form factor are  $M = 0.84$  GeV and  $M = 0.97$  GeV.

<sup>2</sup>In all cases, the value of  $|g_A|$  obtained from the lifetime is greater than that obtained from the asymmetry.

TABLE III. Axial vector coupling fits. The experimental values for the decay rates and asymmetries are taken from the Particle Data Group [25]. We have also included a new measurement of  $g_A$  for neutron  $\beta$  decay [29].

Decay	Experiment	$g_A$	Average	Fit A	Fit B	Fit C
$\Delta \rightarrow N\pi$			$-2.04 \pm 0.01$	-2.04	-2.04	-2.03
$\Sigma^* \rightarrow \Lambda\pi$			$-1.71 \pm 0.03$	-1.75	-1.73	-1.74
$\Sigma^* \rightarrow \Sigma\pi$			$-1.60 \pm 0.13$	-1.62	-1.73	-1.74
$\Xi^* \rightarrow \Xi\pi$			$-1.42 \pm 0.04$	-1.40	-1.42	-1.45
$n \rightarrow p\ell\nu$	$e$	$1.2711 \pm 0.002$	$1.2665 \pm 0.0043$	1.266	1.267	1.271
	$g_A/g_V$	$1.2573 \pm 0.0028$				
	$g_A/g_V$	$1.266 \pm 0.004$				
$\Sigma \rightarrow \Lambda\ell\nu$	$\Sigma^+ \rightarrow e$	$0.601 \pm 0.015$	$0.601 \pm 0.015$	0.601	0.596	0.590
	$\Sigma^- \rightarrow e$	$0.624 \pm 0.079$				
$\Lambda \rightarrow p\ell\nu$	$e$	$-0.906 \pm 0.024$	$-0.889 \pm 0.015$	-0.895	-0.899	-0.867
	$g_A/g_V$	$-0.879 \pm 0.018$				
	$\mu$	$-0.977 \pm 0.180$				
$\Sigma \rightarrow n\ell\nu$	$e$	$0.348 \pm 0.030$	$0.342 \pm 0.015$	0.340	0.342	0.352
	$g_A/g_V$	$0.340 \pm 0.017$				
	$\mu$	$0.309 \pm 0.071$				
$\Xi \rightarrow \Lambda\ell\nu$	$e$	$0.428 \pm 0.049$	$0.306 \pm 0.061$	0.219	0.177	0.158
	$g_A/g_V$	$0.306 \pm 0.061$				
	$\mu$	$1.010 \pm 0.776$				
$\Xi \rightarrow \Sigma\ell\nu$	$e$	$0.929 \pm 0.112$	$0.929 \pm 0.112$	0.718	0.717	0.703

formalism, which we will adopt, the width of a decuplet baryon  $B'$  decaying to a pion and an octet baryon  $B$  is

$$\Gamma_{B'} = \frac{g^2 C(B, B')^2 (E_B + M_B) \mathbf{q}^3}{24\pi f_\pi^2 M_{B'}}, \quad (3.1)$$

where  $E_B$  and  $\mathbf{q}$  are the octet baryon energy and the pion three-momentum in the rest frame of the decaying baryon,  $f_\pi$  is the pion decay constant equal to 93 MeV,  $g$  is the analog of  $g_A$  for this process, and  $C(B, B')$  is a Clebsch-Gordan coefficient  $\{1, 1/\sqrt{2}, 1/\sqrt{3}, 1/\sqrt{2}\}$  for  $\{\Delta \rightarrow N\pi, \Sigma^* \rightarrow \Lambda\pi, \Sigma^* \rightarrow \Sigma\pi, \Xi^* \rightarrow \Xi\pi\}$  chosen so that all the couplings become equal in the SU(3) symmetric limit. For each decay we take the widths of the different charged states and use the Particle Data Group [25] averaging procedure to determine  $g$ . These numbers are listed in the fourth column of Table III.

#### A. Fits to the experimental data

We will perform a number of different fits to the experimental data. All the fits have large  $\chi^2$ . In interpreting the results, it is important to keep in mind that the dominant error in all the fits is *theoretical*; the theoretical formulas are not as accurate as the experimental measurements. For example, the SU(3) symmetric fit to the hyperon  $\beta$  decays discussed below has  $\chi^2 = 13.1$  for four degrees of freedom. The large  $\chi^2$  is an indication that the experimental data show evidence for SU(3) breaking in the axial vector currents. The value of  $\chi^2$  can be used to estimate the amount of SU(3) breaking. If one includes, for example, a theoretical uncertainty of  $\pm 0.025$

(added in quadrature to the experimental errors) then  $\chi^2$  is reduced to 3.7 for four degrees of freedom. This indicates that the SU(3)-breaking part of the octet baryon  $g_A$  (which is of order unity) is of order 0.025. What is surprising is that SU(3) breaking in the hyperon  $\beta$  decays is so small.

#### 1. SU(3) symmetric fit

We first perform a two parameter fit to the experimental data on hyperon  $\beta$  decays alone using  $a$  and  $b$ . This is identical to a fit using only  $F$  and  $D$  neglecting all SU(3)-breaking effects. The results are  $D = 0.79 \pm 0.01$ ,  $F = 0.47 \pm 0.01$  (or equivalently  $a = 0.794 \pm 0.007$  and  $b = -0.060 \pm 0.011$ ), and  $3F - D = 0.62 \pm 0.03$ , with  $\chi^2 = 13.1$  for four degrees of freedom. The results are consistent with earlier fits [27], and the differences are due to different treatment of the inconsistent experimental values for  $g_A$ . As mentioned above, the large  $\chi^2$  is an indication of SU(3) breaking. It also indicates that the nominal errors on  $F$ ,  $D$ , and  $3F - D$  obtained from the fit are underestimates of the true error.

#### 2. $\Delta S = 0$ fit

Before presenting the results of a full fit, it is useful to make a preliminary investigation of the  $n \rightarrow p$  and  $\Sigma \rightarrow \Lambda$   $\beta$  decays and the strong decays  $\Delta \rightarrow N\pi$  and  $\Sigma^* \rightarrow \Lambda\pi$ . These  $\Delta S = 0$  decays depend only on the four parameters  $a$ ,  $b$ ,  $d$ , and  $c_3$  which can therefore be extracted from these data alone. The results,  $a = 0.90 \pm 0.02$ ,  $b = -0.24 \pm 0.04$ ,

$d = -0.05 \pm 0.01$ , and  $c_3 = -0.08 \pm 0.01$ , are consistent with expectations. The leading parameter  $a$  is of order unity,  $b \sim 1/N_c$  is small compared to  $a$ ,  $d \sim 1/N_c^2$  is quite small, and  $c_3 \sim \epsilon/N_c$  is consistent with a 30% SU(3)-breaking  $\epsilon$  divided by  $N_c = 3$ . The  $D$  and  $F$  couplings derived from this analysis are  $D = 0.90 \pm 0.02$  and  $F = 0.37 \pm 0.02$  with  $3F - D = 0.20 \pm 0.1$ .<sup>3</sup>

A variant of the above is to include all six  $\Delta S = 0$  decays, which can be fit using the five parameters  $a$ ,  $b$ ,  $d$ ,  $c_3$ , and  $c_4$ . The results are  $a = 0.893 \pm 0.023$ ,  $b = -0.222 \pm 0.039$ ,  $d = -0.056 \pm 0.010$ ,  $c_3 = -0.079 \pm 0.007$ , and  $c_4 = 0.002 \pm 0.014$ , with  $F = 0.37 \pm 0.02$ ,  $D = 0.89 \pm 0.02$ , and  $3F - D = 0.23 \pm 0.09$ , with  $\chi^2 = 1.1$  for one degree of freedom.

### 3. Global fits

Turning now to the global fit, we minimize  $\chi^2$  weighting each datum with its experimental error. The fit has  $\chi^2 = 7.9$  with three degrees of freedom, and the results, labeled as fit A, are summarized in Tables III and IV. We have tried weighting the data with theoretical errors of various forms<sup>4</sup> and, although  $\chi^2$  decreases, the general character of the fit changes very little. Note that in this fit  $c_1$  and  $c_4$  are very small, suggesting a fit where  $c_1$  and  $c_4$  are constrained to be zero. The results of the constrained fit, labeled as fit B, are also listed in Tables III and IV. Here  $\chi^2 = 9.9$  but there are now five degrees of freedom. Note that the values of  $a$ ,  $b$ ,  $d$ ,  $c_2$ , and  $c_3$  do not change much between the constrained and unconstrained fits.

Fit A is shown in Fig. 2. The first four points are the decuplet decays, followed by the hyperon  $\beta$  decays. Rather than plot the data points directly, we have subtracted the best SU(3) symmetric fit ( $a = 0.794$ ,  $b = -0.060$ ,  $d = -0.086$ ) from experiment and theory. The

TABLE IV. Best fit parameters for axial vector couplings.

	Fit A	Fit B	Fit C
$a$	$0.880 \pm 0.022$	$0.885 \pm 0.013$	$0.867 \pm 0.011$
$b$	$-0.201 \pm 0.037$	$-0.208 \pm 0.022$	$-0.176 \pm 0.019$
$d$	$-0.061 \pm 0.010$	$-0.058 \pm 0.006$	$-0.067 \pm 0.005$
$c_1$	$-0.021 \pm 0.025$	0	0
$c_2$	$0.128 \pm 0.039$	$0.135 \pm 0.020$	$-b/2$
$c_3$	$-0.072 \pm 0.006$	$-0.078 \pm 0.004$	$-0.073 \pm 0.004$
$c_4$	$0.016 \pm 0.012$	0	0
$F$	$0.39 \pm 0.02$	$0.38 \pm 0.014$	$0.40 \pm 0.01$
$D$	$0.88 \pm 0.02$	$0.89 \pm 0.013$	$0.87 \pm 0.01$
$3F - D$	$0.28 \pm 0.09$	$0.26 \pm 0.05$	$0.34 \pm 0.04$

<sup>3</sup>There are correlated errors on the parameters in all the fits, so that the error on  $F$ ,  $D$ , and  $3F - D$  is computed using the covariance matrix.

<sup>4</sup>For example, assuming that there is an additional theoretical uncertainty in the formulas for the  $\Delta S = 1$  decays and decuplet decays, because of the larger momentum transfer.

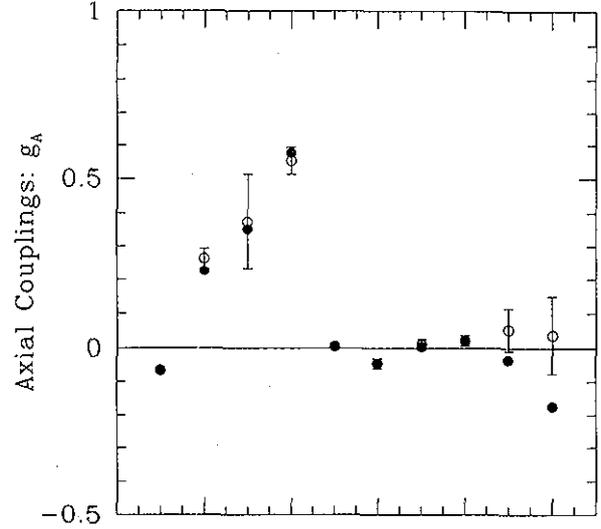


FIG. 2. Deviation of the axial vector couplings from the best SU(3)-symmetric fit. The open circles are the experimental data, and the filled circles are the values from fit A discussed in the text. The points plotted are (from left to right)  $\Delta \rightarrow N$ ,  $\Sigma^* \rightarrow \Lambda$ ,  $\Sigma^* \rightarrow \Sigma$ ,  $\Xi^* \rightarrow \Xi$ ,  $n \rightarrow p$ ,  $\Sigma \rightarrow \Lambda$ ,  $\Lambda \rightarrow p$ ,  $\Sigma \rightarrow n$ ,  $\Xi \rightarrow \Lambda$ , and  $\Xi \rightarrow \Sigma$ .

plot shows the deviations from SU(3) symmetry of the axial vector couplings and the best fit. It is clear from the plot that the SU(3) breaking in the decuplet decays is significantly larger than that in the hyperon  $\beta$  decays. The hyperon  $\beta$  decays show very little SU(3) breaking. Our large- $N_c$  fits indicate that the  $g_A$  values for  $\Xi$  decay should be smaller than the experimental central values in Table III. The fits A and B, as well as our  $\Delta S = 0$  fits all indicate that  $3F - D$  is approximately  $0.28 \pm 0.09$ , which is significantly smaller than the SU(3) symmetric fit value of  $0.62 \pm 0.03$ .

Graphically, the parameter  $b$  arises from diagrams where the quark line to which the current is attached has a spin-dependent interaction with another quark. The parameter  $c_2$  is a measure of how this interaction is modified when the current carrying quark line represents a strange quark. If  $c_2$  were exactly equal to  $-b/2$ , it would mean that this spin-dependent interaction is completely ineffective for the heavier strange quark. Since the fits do produce a value of  $c_2$  which is close to  $-b/2$ , we have done another fit with three constraints,  $c_1 = 0$ ,  $c_4 = 0$ , and  $c_2 = -b/2$ . The results of this fit, labeled as fit C, are also shown in Tables III and IV. By now  $\chi^2$  has increased to 17.3 for six degrees of freedom. Note that this doubly constrained fit has the same four parameters that appeared in our preliminary investigation plus the constraint  $c_2 = -b/2$ . The parameters turn out to be almost identical with  $a = 0.87$ ,  $b = -0.18$ ,  $d = -0.07$ ,  $c_3 = -0.07$ ,  $D = 0.87$ ,  $F = 0.40$ , and  $3F - D = 0.34$ . We believe that this fit is the closest to the physics. The physical interpretation of  $b$  and  $c_2$  has already been discussed. To interpret  $c_3$ , we note that in diagrams it corresponds to a nonstrange quark line carrying the current interacting, through a spin-independent gluon exchange, with strange quarks elsewhere in the baryon. Thus  $c_3$  can

be interpreted as the effect of the strange quark mass on the average, spin-independent color field through which the quarks propagate.

While we have achieved an apparently stable set of fits with an interesting interpretation, we are not entirely comfortable with these results. For one thing, we do not understand why  $c_4$  and especially  $c_1$  are so small. Also, the  $\Xi$  decays have been measured in only one experiment which causes some worry — note that in fit A  $\chi^2$  is dominated by the  $\Xi$  decays — and, except for the  $\Delta$ , there has been no recent work on the decuplet widths. While our fit may not turn out to be definitive, there is, nevertheless, a very clear trend. By all indications,  $3F - D \approx 0.28$  is small compared to the SU(3) symmetric value of 0.6, or the nonrelativistic quark model value of one. As a word of warning, however, we are in a parameter regime where there are large cancellations between  $3F$  and  $D$  and the fractional error in  $3F - D$  may be relatively large.

#### IV. MAGNETIC MOMENTS

In the large  $N_c$  limit, the baryon magnetic moments have the same kinematic properties as the axial vector couplings and can be expressed in terms of the same operators. The operator which gives the magnetic moments is  $M^i = P^{i3} + P^{i8}/\sqrt{3}$ . It is straightforward to determine the coefficients in  $M^i$ . We use the measured moments of the octet baryons and the  $\Omega^-$  as well as the  $\Sigma^0\Lambda$  and  $\Delta N$  transition moments. The  $\Delta^{++}$  magnetic moment is not included in the fits, because of the very large experimental error. As before, we minimize  $\chi^2$  but this time the data are accurate enough that we have to include a theoretical error in order to get a meaningful  $\chi^2$ . The dominant error is the theoretical error in the formulas used, not the experimental errors on the data. Guessing that the higher order [in SU(3) breaking and  $1/N_c$ ] effects are at the few percent level we arbitrarily add an extra error of 0.05 nuclear magnetons to each moment. The fit produces  $\chi^2 = 5.5$  with four degrees of freedom but this particular value is largely a reflection of our choice of theoretical error. The results of the fit are listed as fit A in Tables V and VI. The fit is quite good and sizes of

TABLE V. Magnetic moment fits. The experimental value for the  $\Omega^-$  magnetic moment is from Ref. [30]. The remaining values are taken from the Particle Data Group [25].

	Experiment	Fit A	Fit B
$p$	$2.793 \pm 0.000$	2.844	2.803
$n$	$-1.913 \pm 0.000$	-1.871	-1.915
$\Lambda$	$-0.613 \pm 0.004$	-0.588	-0.596
$\Sigma^+$	$2.458 \pm 0.010$	2.455	2.464
$\Sigma^-$	$-1.160 \pm 0.025$	-1.058	-1.123
$\Sigma^0\Lambda$	$-1.610 \pm 0.080$	-1.511	-1.509
$\Xi^0$	$-1.250 \pm 0.014$	-1.278	-1.257
$\Xi^-$	$-0.651 \pm 0.003$	-0.608	-0.629
$p\Delta^+$	$3.230 \pm 0.100$	3.530	3.530
$\Omega^-$	$-2.024 \pm 0.056$	-2.078	-2.057
$\Delta^{++}$	$5.6 \pm 1.9$	5.836	5.859

TABLE VI. Best fit parameters for magnetic moments.

	Fit A	Fit B
$a$	$2.806 \pm 0.061$	$2.784 \pm 0.058$
$b$	$0.037 \pm 0.058$	$0.078 \pm 0.053$
$d$	$-0.417 \pm 0.071$	$-0.427 \pm 0.071$
$c_1$	$-0.515 \pm 0.103$	$-0.535 \pm 0.099$
$c_2$	$0.089 \pm 0.041$	$0.033 \pm 0.040$
$\delta c_2$	$0.003 \pm 0.041$	$-0.056 \pm 0.044$
$c_3$	$-0.095 \pm 0.035$	$-0.085 \pm 0.035$
$\delta c_3$	$-0.009 \pm 0.048$	$0.004 \pm 0.047$
$c_4$	$0.038 \pm 0.037$	$0.041 \pm 0.036$
$\delta c_4$	$-0.048 \pm 0.030$	$-0.049 \pm 0.030$

the various output parameters are consistent with expectations. The plot of the deviations of the experimental data from the SU(3) symmetric values (using  $a = 2.602$ ,  $b = -0.071$ , and  $d = -0.508$ ) are shown in Fig. 3. The figure clearly shows that SU(3) breaking in the magnetic moments is significantly larger than in the axial vector current sector. The small value of  $b$  also shows that the  $F/D$  ratio for the baryon magnetic moments is very close to  $2/3$ .

As mentioned in the Introduction, the diagram in Fig. 1 should be the dominant source of SU(3) violation in the magnetic moments. Using the leading order coupling  $G^{ia}$  at the meson-baryon vertices and neglecting the baryon mass differences, this diagram can be written as

$$M_{\text{loop}}^i = \frac{1}{N_c} \epsilon^{ijk} G^{ja} G^{kb} I^{ab}, \quad (4.1)$$

where  $I^{ab}$  is an antisymmetric matrix which is the result of doing the loop integral. The Hermitian matrix

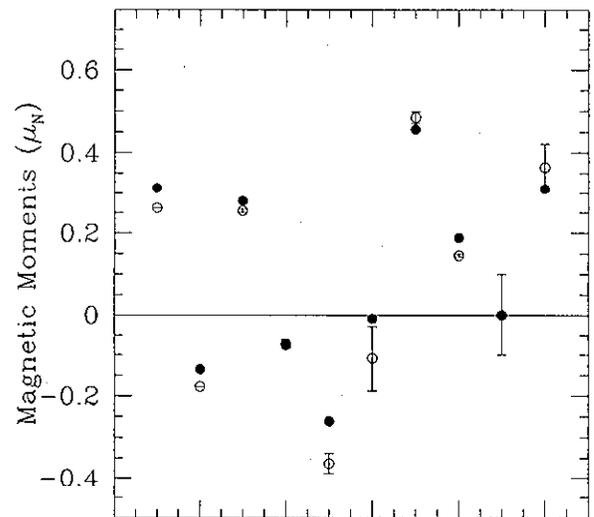


FIG. 3. Deviation of the magnetic moments from the best SU(3)-symmetric fit. The open circles are the experimental data, and the filled circles are the values from fit A discussed in the text. The order of the magnetic moments is the same as in Table V. The  $\Delta^{++}$  magnetic moment has not been plotted, since the experimental value has a very large error.

$iI^{ab}$  is diagonal in a basis corresponding to particles of definite quantum numbers. It has four zero eigenvalues corresponding to the four neutral mesons, two equal and opposite eigenvalues  $\pm I_K$  corresponding to  $K^+$  and  $K^-$  and two more equal and opposite eigenvalues  $\pm I_\pi$  corresponding to  $\pi^+$  and  $\pi^-$ . We can write

$$I^{ab} = \frac{I_\pi + 2I_K}{3} I_0^{ab} + \frac{I_\pi - I_K}{3} I_1^{ab} + \frac{I_\pi - I_K}{\sqrt{3}} J^{ab}, \quad (4.2)$$

where

$$\begin{aligned} I_0^{ab} &= f^{ab}Q, \\ I_1^{ab} &= f^{ab}\bar{Q}, \\ J^{ab} &= f^{ac}Q d^{bc8} - f^{bc}Q d^{ac8} - f^{abc}d^{cQ8}, \end{aligned} \quad (4.3)$$

and

$$\begin{aligned} T^Q &= T^3 + \frac{1}{\sqrt{3}}T^8 = \begin{bmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{bmatrix}, \\ T^{\bar{Q}} &= T^3 - \frac{1}{\sqrt{3}}T^8 = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & -2/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}. \end{aligned} \quad (4.4)$$

$I_0^{ab}$  is an SU(3) octet that transforms like the electric charge  $Q$  and gives baryon magnetic moments with  $F/D = 2/3$  that satisfy the Coleman-Glashow relations [28].  $I_1^{ab}$  is also an SU(3) octet, which transforms like the electric charge rotated by  $\pi$  in isospin space. Its contribution to the magnetic moments satisfies the Coleman-Glashow relations, but not  $F/D = 2/3$ .  $J^{ab}$  breaks SU(3) as a  $10 \oplus \bar{10}$ . The SU(3)-violating part of the diagram is, therefore, a constant times

$$\frac{1}{N_c} \epsilon^{ijk} G^{ja} G^{kb} J^{ab}, \quad (4.5)$$

where  $J^{ab}$  is completely independent of the dynamics. For example, if the diagram is modified by putting in form factors to cut off the kaon loops,  $I_\pi - I_K$  will change but  $J^{ab}$  will not. The identities of Ref. [1] can be used to express  $\epsilon^{ijk} G^{ja} G^{kb} J^{ab}$  in terms of the operators which occur in our parametrization of the magnetic moments. The coefficients have the correct scaling with  $N_c$  and for the physical case of  $N_c = 3$ , we find  $c_2 = -c_1/6$ ,  $c_3 = c_1/6$ , and  $c_4 = -c_1/6$  where  $c_1$  is arbitrary and reflects the overall scale of the diagram. Defining deviations from the coefficients predicted by the diagram,  $\delta c_2 = c_2 + c_1/6$ ,  $\delta c_3 = c_3 - c_1/6$ , and  $\delta c_4 = c_4 + c_1/6$ , the fit gives  $c_1 = -0.515 \pm 0.103$ ,  $\delta c_2 = 0.003 \pm 0.041$ ,  $\delta c_3 = -0.009 \pm 0.048$ , and  $\delta c_4 = -0.048 \pm 0.030$ . The fact that  $c_1$  is an order of magnitude larger than the  $\delta c_i$  is a striking indication that the symmetry breaking part of the magnetic moments is dominated by this one diagram. Actually, the current fit does not reflect the full content of the diagram. The expression in Eq. (4.5) fails to satisfy the relation  $3c'_6 = c'_1 + c'_2$  by a term which is explicitly of order  $1/N_c$  and such terms have been dropped in the above analysis. However, there is no harm in keeping this extra term and doing so is equivalent to adding

a term  $eJ^i$  to  $M^i$  where the diagram gives  $e = c_1/9$  at  $N_c = 3$ . If we add  $c_1 J^i/9$  to  $M^i$  and redo the fit,  $\chi^2$  drops from 5.5 to 1.3 and the new coefficients are listed as fit B in the tables. We take the drop in  $\chi^2$  associated with this extra term as a further indication that this single diagram is the dominant symmetry breaking effect in the magnetic moments. The deviation of the baryon magnetic moments from the SU(3) symmetric fit plus the loop graph of Fig. 1 is shown in Fig. 4. The residual SU(3) breaking is clearly quite small.

Despite the factor of  $1/N_c$  which comes from the pion couplings,  $M_{\text{loop}}^i$  is leading order in  $N_c$ —the expansion of  $\epsilon^{ijk} G^{ja} G^{kb}$  contains a term  $N_c G^{ic}$ . In the limit of small  $m_s$ , the symmetry breaking part of  $M_{\text{loop}}^i$  is also of order  $\sqrt{m_s}$ , so it should not be surprising that this diagram dominates. However, when one actually calculates the loop integrals, the numerical value of the diagram is about a factor of 2 too large. The resolution of this paradox is, no doubt, that the integrals contain virtual meson momenta up to roughly  $M_K$  which is too large for chiral perturbation theory to be valid and that some dynamical effect is cutting off the loop integrals at a lower value of the momenta.

## V. CONCLUSIONS

This work has produced two clear results. The first is that the pattern of symmetry breaking in the magnetic moments is in excellent agreement with the pattern associated with the meson loop shown in Fig. 1. The magnitude of the loop is, however, about a factor of 2 too big. The discrepancy is probably due to dynamical effects that cut off the loop integral at a momentum somewhat smaller than  $M_K$ . The second result is that the quantity  $3F - D \approx 0.28$  is considerably smaller than its SU(6) symmetric value of one. This appears to be an inescapable result of our fits and is, of course, important in

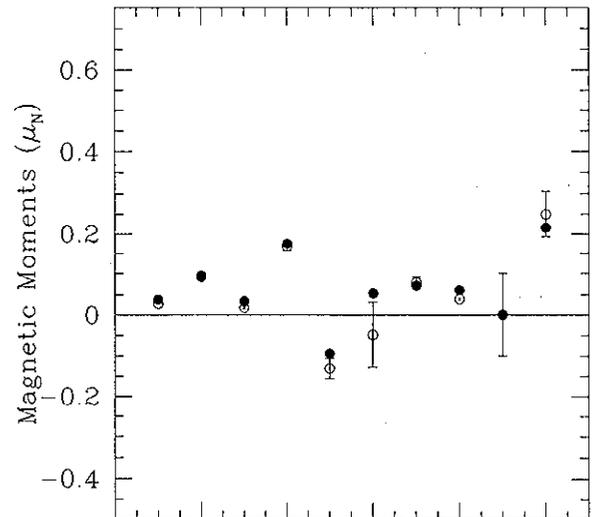


FIG. 4. Deviation of the magnetic moments from the best SU(3)-symmetric fit plus the chiral loop diagram of Fig. 1. The deviations should be compared with those in Fig. 3.

the analysis of spin-dependent deep inelastic scattering. For the axial vector couplings we also found  $c_2 \approx -b/2$  which suggests that the strange quark is heavy enough that its spin-dependent interactions are rather strongly suppressed. Beyond that, however, the fit to the axial couplings, while good and seemingly stable, is rather unsatisfying from a theoretical point of view. In particular, we do not know why  $c_4$  and especially  $c_1$  are so small. Further progress on the axial vector couplings probably awaits better data, which is likely to be years away, or a better understanding of large  $N_c$  baryon dynamics, which might produce a simple reason for the smallness of  $c_1$ .

*Note added.* This paper differs from the previous version in that we have included recent measurements of  $g_A/g_V$  in neutron  $\beta$  decay [29] and of the  $\Omega^-$  magnetic moment [30]. The small change in the value of  $g_A$  does not significantly affect the fit. The new measurement of the  $\Omega^-$  magnetic moment is in good agreement with the best fit value given in the preprint, and supports the pattern of symmetry breaking discussed in Sec. IV.

We have used  $g_A$  in  $\Xi \rightarrow \Lambda$  decay from the asymmetry measurement alone. The conclusions are essentially unchanged if one uses the average of the asymmetry and rate determinations of  $g_A$ . For example, the central values for  $F$ ,  $D$ , and  $3F - D$  are 0.39, 0.87, and 0.30 instead of 0.39, 0.88, and 0.28, respectively.

#### ACKNOWLEDGMENTS

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#### APPENDIX: HIGHER ORDER OPERATORS

The point of this appendix is to show that the truncation scheme is consistent in the sense that matrix elements of the operators that have been dropped are always smaller, for large  $N_c$ , than the corresponding matrix elements of at least one of the operators that has been retained. This has already been done for the singlet and octet operators — the corresponding analysis for the  $10 \oplus \overline{10}$  and  $27$  follows. Up to terms which simply renormalize the symmetrical couplings and irrelevant constants, the operators that we keep are

$$\begin{aligned} O_1 &= \left\{ G^{ia}, \frac{N_s}{N_c} \right\}, \\ O_2 &= \left\{ T^a, \frac{J_s^i}{N_c} \right\}, \end{aligned} \quad (A1)$$

where the scaling with  $N_c$  has been made explicit, and we drop the operators

$$\begin{aligned} O_3 &= \frac{J^i}{N_c} \left\{ T^a, \frac{N_s}{N_c} \right\}, \\ O_4 &= \left\{ \frac{J^k G^{ka}}{N_c}, \frac{J_s^i}{N_c} \right\}, \\ O_5 &= \left\{ \frac{G^{ia}}{N_c}, \frac{J^k J_s^k}{N_c} \right\}. \end{aligned} \quad (A2)$$

First consider the operators  $O_1$ ,  $O_4$ , and  $O_5$ , all of which contain the basic operator  $G^{ia}$ . Since  $J_s^2 = (N_s/2)(N_s/2+1)$ ,  $J_s^i$  cannot have matrix elements larger than  $[(N_s/2)(N_s/2+1)]^{1/2}$  and it is clear that for low spin states any matrix element of  $O_4$  or  $O_5$  is of order  $N_c^{-1}$  times the corresponding matrix element of  $O_1$ . Dropping  $O_4$  and  $O_5$  is therefore justified by the fact that these operators are always small compared to  $O_1$ . Now consider the matrix elements of  $O_2$  and  $O_3$ , both of which contain the basic operator  $T^a$ . The comparison of these operators is simplified by noting that  $O_3$  has matrix elements only between states of the same spin and that the matrix element of  $J_s^i$  between states of equal spin is equal to the matrix element of  $J^i J^k J_s^k / J^2$ . Therefore the statement that  $O_3$  is small compared to  $O_2$  is equivalent to the statement that, for low spin states,  $\{T^a, N_s/N_c\}$  is small compared to  $\{T^a, J_s^k J^k / J^2\}$ . We will explicitly carry out the calculation for the experimentally interesting case of spin-1/2 — other cases can be studied in a similar way. The structure of the baryon multiplet is such that (i) the isospin of a baryon is equal to the total angular momentum (spin) of the up and down quarks and (ii) the total angular momentum (spin) of the strange quarks is  $N_s/2$ . It follows that (i) there are no spin-1/2 baryons with  $N_s > (N_c + 1)/2$  and (ii) a spin-1/2 baryon must have isospin  $(N_s - 1)/2$  or  $(N_s + 1)/2$  where either possibility is allowed for  $1 \leq N_s \leq (N_c - 1)/2$  and only the smaller isospin is allowed when  $N_s = (N_c + 1)/2$ . The case  $N_s = 0$  also is exceptional, with only the larger isospin allowed,  $I = 1/2$ . A straightforward calculation then shows that  $J^k J_s^k / J^2$  is equal to  $-N_s/3$  for the higher isospin and  $(N_s + 2)/3$  for the lower isospin. Note that, unlike the matrix elements of  $N_s$ , the matrix elements of  $J_s^k J^k / J^2$  can have different signs giving rise to the possibility of cancellations between the two terms in  $\{T^a, J_s^k J^k / J^2\}$ . If the SU(3) index  $a$  is 1, 2, 3, or 8, then neither  $O_2$  nor  $O_3$  can change the isospin of a state, so there can be no cancellation for any  $N_s$  and the matrix elements of  $O_3$  are down by  $N_c^{-1}$  relative to the matrix elements of  $O_2$ . However, in the case of strangeness changing operators, a cancellation can occur. There are two types of strangeness changing matrix elements, those with  $\Delta N_s = 2\Delta I$  where the state with the largest number of strange quarks also has the larger isospin and those with  $\Delta N_s = -2\Delta I$  where the state with the largest number of strange quarks has the smaller isospin. For the  $\Delta N_s = 2\Delta I$  transitions, the two terms in  $O_2$  have the same sign and, independent of the value of  $N_s$ , the matrix element of  $O_3$  is always smaller than that of  $O_2$  by a factor of  $1/N_c$ . However, for the  $\Delta N_s = -2\Delta I$  transitions there is a cancellation and, for large  $N_s$ , the matrix element of  $O_3$  is smaller than that of  $O_2$  by a factor of order  $N_s/N_c$ , i.e., a factor which is of order unity when  $N_s \sim N_c$ . However,

for these transitions the matrix elements of  $O_1$  are large compared to the matrix elements of either  $O_2$  or  $O_3$ . To see this, we note that the matrix element of  $G^{ia}$  between states of the same spin is equal to the matrix element of  $J^i J^k G^{ka}/J^2$  and use the identity

$$J^k G^{ka} = \frac{1}{2} d^{abc} T^b T^c + \frac{1}{4} \left( \frac{N_c}{3} + 1 \right) T^a, \quad (\text{A3})$$

which is a linear combination of two of the (0,adj) SU(6) identities listed in Ref. [1]. Taking matrix elements of this identity and working out the Clebsch-Gordan coefficients yields

$$J^k G^{ka}/J^2 = \frac{2}{3} \left( T^a + \frac{1}{2} \{N_s, T^a\} \right) \quad (\text{A4})$$

for  $\Delta N_s = -2\Delta I$  transitions, where this relation is understood to be valid only for matrix elements of strangeness changing operators between spin-1/2 baryons. It follows immediately that for  $N_s \sim N_c$  the strangeness changing matrix elements of  $O_3$  are order  $N_c^{-2}$  times the corresponding matrix elements of  $O_1$ . The conclusion of these rather lengthy calculations is that in all cases matrix elements of  $O_3$ ,  $O_4$ , and  $O_5$  are smaller than a corresponding matrix element of  $O_1$  or  $O_2$  by at least a factor of  $N_c^{-1}$ .

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