CP noninvariance effects induced by quantum gravity in *B* systems

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The effects of a new CP-violating source originated from quantum gravity are studied systematically for the B system. A comparison with the K system is presented. It is found that in terms of the parameter values obtained in the K system the observational prospect of such effects in the B system is not optimistic even at the B factory.

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I. INTRODUCTION

It is interesting to explore possible sources of CP violation. It is believed [1] that in the standard theory of Kobayashi and Maskawa [2] all CP-violating effects reside in the quark-Higgs-boson Yukawa couplings, whereas if the Higgs sector contains several scalar fields, an additional CP-violating phase may appear in the Higgs self-couplings [3]; thus, all CP-violation sources are associated with the Higgs boson sector.

In the regular theory, the neutral meson systems, K, B, etc., can be described by quantum mechanics; their evolutions are determined by the Hamiltonian. By studying black hole physics, Hawking [4] proposed that, due to quantum gravity effects, quantum-mechanical evolution should be modified. The new phenomenological term induced by Hawking's action should be taken into account for the evolution of a quantum system. Namely, the quantum-mechanics violation causes CP and even CPT violation. In the most general form, the evolution of the density matrix can be expressed [6,5]

$$\frac{d}{dt}\rho_B^A = H\!\!\!/_{BC}^{AD}\rho_D^C, \tag{1}$$

where the linear operator $H\!\!\!/$ contains an additional term to the usual quantum-mechanical piece, so

$$H\!\!\!/_{BC}^{AD}\rho_D^C = i[\rho,H]_B^A + \delta H\!\!\!/_{BC}^{AD}\rho_D^C \tag{2}$$

and

$$\delta H\!\!\!/\rho = h^{0j}\rho^j 1 + h^{j0}\rho^0\sigma^j + h^{ij}\sigma^i\rho^j, \tag{3}$$

where ρ is the density matrix of the quantum system, H is the regular Hamiltonian, and h is a parameter set. Considering that the probability must be conserved and the entropy should not decrease at any time, $h^{0j}=h^{j0}=0$.

The extra term causes a pure state to evolve into a mixed state, but not the inverse. This leads to a new evolution equation for the density matrix components [5] as

$$\frac{d}{d\tau}\rho^0 = -\Gamma^0\rho^0 - \Gamma^i\rho^i,$$
$$\frac{d}{d\tau}\rho^i = 2\epsilon^{ijk}M^j\rho^k - \Gamma^i\rho^0 + h^{ij}\rho^j, \tag{4}$$

where $H=M-(i/2)\Gamma$ is a 2×2 matrix and determined by the standard model and Γ^0, Γ^i, M^0, M^i are described in the basis of the Pauli matrices $\sigma^0\equiv1$ and σ^i.

Huet and Peskin (HP) and Ellis, Hagelin, Nanopoulos, and Srednicki (EHNS) studied the possible effects of $\delta H\!\!\!/\rho$ in the K system [5,6] and paramterized h in a 3×3 matrix form as

$$h = 2\begin{pmatrix} 0 & 0 & 0 \\ 0 & -\alpha & -\beta \\ 0 & -\beta & -\gamma \end{pmatrix}. \tag{5}$$

It is noted that this new additional term is Hermitian negative definite, that requires α and γ to be positive and an inequality $\alpha\gamma>\beta^2$ must be satisfied. h should be phenomenologically relevant to the CP-violation observables (see below). More explicitly, for the K system, the first row and column in Eq. (5) are set to zero because of the assumption of strangeness conservation. Here we simply generalize this form to the B system where the b flavor corresponds to the s flavor in the K system. Because of close analogue of K and B systems for mixing effects and decay mechanisms, this simple generalization may be reasonable; namely the arguments given in [6,5] for the K system can apply to the B system.

The solution of ρ can be obtained as

$$\rho(t) = A_H\rho_H e^{-\Gamma_H t} + A_L\rho_L e^{-\Gamma_L t} + A_I\rho_I e^{-(\bar\Gamma+\alpha-\gamma)t}e^{-i\Delta mt} + A_{\bar I}\rho_{\bar I}e^{-(\bar\Gamma+\alpha-\gamma)t}e^{+i\Delta mt}, \tag{6}$$

where all notation is taken from [6,5] and $\Gamma_{H(L)}$ denotes the total widths of the heavy and light eigenstates of the B system, which will be clarified in a better way in the next section. Comparing with the original expression of ρ where α, β, and γ do not appear, it is noted that the new effects cause

$\bar{\Gamma}=(\Gamma_H+\Gamma_L)/2$ to shift by $\alpha-\gamma$ and change the eigenmodes of the ρ_H, ρ_L, ρ_I, and ρ_I^-. We will discuss the results in the third section.

In the K system, by comparing expression (6) where the subscripts H and L for the B system should be changed to S (short) and L (long) with data, the parameters are obtained

$$\gamma=(0.1\pm3)\times10^{-22}\ \text{GeV},$$

$$\beta=(1\pm23)\times10^{-20}\ \text{GeV}\quad([6]), \tag{7}$$

and

$$\gamma=(-0.2\pm2.2)\times10^{-21}\ \text{GeV},$$

$$\beta=(0.32\pm0.29)\times10^{-18}\ \text{GeV}\quad([5]). \tag{8}$$

α cannot be determined by the CP-violation data of the K system.[1] In [5,6], there are only mild constraints for α. In our calculations, considering the positiveness of α and γ and the inequality, we set the parameter values as $\alpha=5.8\times10^{-18}$ GeV, $\beta=0.12\times10^{-18}$ GeV, and $\gamma=2.5\times10^{-21}$ GeV.

Because of large uncertainties of the parameters, one can only estimate the order of magnitude and expect to fix them through future precision measurements on the CP and CPT violation in the ϕ factory which is discussed by Huet and Peskin [5] in some detail.

Obviously, it is natural to apply the same scenario to the B systems. The reason is twofold: first, the B system is very similar to the K system, especially, both of them have large mixing effects and are good subject to quantum mechanics; second because B is heavy, the contributions from the long distance effects associated with the intermediate resonances can be less important and negligible [7]. In fact the suggestion of detecting direct and indirect CPT violation at a B factory has been raised by some authors [8].

Moreover, according to [6,5], the parameters of h (α, β, and γ) for the K system are of order $M_K^2/M_{\rm Pl}\sim10^{-19}$ GeV, where $M_{\rm Pl}$ is the Planck mass. The reason is that the effects are induced by the quantum gravity, so the physics is at or near the Planck scale and optimistically, only one Planck mass suppression exists. Thus one might expect that in the B system, β and γ can be 2 orders of magnitude larger. It would be helpful for observation.

Motivated by the idea, we study the effect of the quantum-gravity induced additional term $\delta\not{H}\rho$ in the B system. Thus we investigate its effect on χ_d ($\bar{\chi}_d$), etc., in the semileptonic decays and then on

$$A_B=\frac{\Gamma(B^0\to2\pi)-\Gamma(\bar{B}^0\to2\pi)}{\Gamma(B^0\to2\pi)+\Gamma(\bar{B}^0\to2\pi)}$$

for nonleptonic decays.

In the next section, we give the necessary formulas, but for saving space, we refer the readers to the original references for details. In Sec. III, we present our numerical results. The last section is devoted to the discussions and conclusions.

II. FORMULATION

The B system has been carefully studied by many authors [9]. The $B^0-\bar{B}^0$ mixing is large because the top quark is heavy. Since the intermediate resonance contributions can be neglected in the B system, the box diagram fully determines M_{12} and Γ_{12}.

Recently, the top quark mass has been measured by the Collider Detector at Fermilab (CDF) and DØ Collaborations at Fermilab [10,11]:

$$m_t=176\pm8(\text{stat})\pm10(\text{syst})\ \text{GeV}/c^2\quad(\text{CDF}),$$

$$m_t=199^{+19}_{-21}(\text{stat})\pm22(\text{syst})\ \text{GeV}/c^2\quad(\text{D}\emptyset).$$

The values are measured at the m_t mass shell, and may run to a larger value at the M_B energy scale, the running depends on $\Lambda_{\overline{\rm MS}}$ and is not very certain so far. Following Buras [12], we take $m_t\sim176-200$ GeV. Since the purpose of this work is to test the new CP-violation effects induced by $\delta\not{H}\rho$ compared to that of the box diagram, considering large uncertainties of the parameters in the K system, the small deviation caused by the varied m_t value is not important.

(i) Solution (6) is obtained by following the procedure given in [5]. Explicitly, Eq. (4) can be rewritten as

$$\frac{d}{dt}\begin{pmatrix}\rho_1\\ \rho_2\\ \rho\\ \bar{\rho}\end{pmatrix}=\left[-\bar{\Gamma}+\begin{pmatrix}-\Delta\Gamma/2 & 0 & +i\epsilon^*d^* & -i\epsilon d\\ 0 & \Delta\Gamma/2 & +i\epsilon d & -i\epsilon d\\ -i\epsilon^*d^* & -i\epsilon d & +i\Delta m & 0\\ +i\epsilon d & +i\epsilon^*d^* & 0 & -i\Delta m\end{pmatrix}\right]\begin{pmatrix}\rho_1\\ \rho_2\\ \rho\\ \bar{\rho}\end{pmatrix}, \tag{9}$$

[1] In the previous version of the paper of Huet and Peskin, the parameters β and γ were different from that listed at Eq. (7), the original one set was $\gamma=(-1.1\pm3.6)\times10^{-21}$ GeV and $\beta=(0.12\pm0.44)\times10^{-18}$ GeV, to make the α value not too large we choose a reasonable set of α, β, and γ given in the text. In fact, at the present stage, with the large uncertainties, especially, for the B system, only an estimation of order of magnitude is feasible, therefore selecting a reasonable set of the parameters which are within the possible ranges would be sufficient, of course, adjusting the parameters, one may obtain larger or smaller CP-violation results, but cannot deviate by orders. So in this work we only constrain ourselves with this set of parameters.

where $d=\Delta m+i\Delta\Gamma/2$ and since we do not consider the CPT violation, so ignore the difference between ϵ_H and ϵ_L in the expression. Since the $\epsilon\ll 1$ assumption does not exist for the B system, the corresponding approximation adopted in [5] does not apply. Therefore, instead, we directly diagonalize the matrix and obtain the eigenstates of the ρ vector which appear in Eq. (6).

(ii) The mixing effects and the semileptonic decays. Generally, one defines

$$|B_L\rangle=\frac{1}{\sqrt{|p|^2+|q|^2}}(p|B^0\rangle+q|\bar{B}^0\rangle), \tag{10}$$

$$|B_H\rangle=\frac{1}{\sqrt{|p|^2+|q|^2}}(p|B^0\rangle-q|\bar{B}^0\rangle), \tag{11}$$

where we use the phase convention of $CP|B^0\rangle=|\bar{B}^0\rangle$, the superscripts H and L denote "heavy" and "light" eigenstates of $H=M-(i/2)\Gamma$, and $\Delta m\equiv M_{B_H^0}-M_{B_L^0}$.

Diagonalizing the Hamiltonian $H=M-(i/2)\Gamma$, one achieves [13]

$$\frac{p}{q}=\sqrt{\frac{M_{12}^*-i\Gamma_{12}^*/2}{M_{12}-i\Gamma_{12}/2}}, \tag{12}$$

$$\Delta m=2\ \ \mathrm{Re}\Delta, \tag{13}$$

$$\Delta\Gamma=-4\ \ \mathrm{Im}\Delta \tag{14}$$

where $\Delta=[(M_{12}-i\Gamma_{12}/2)(M_{12}^*-i\Gamma_{12}^*/2)]^{1/2}$.

One of the authors [14] pointed out that as m_t is very large, the formulas for M_{12} and Γ_{12} given in the earlier literatures should be modified, the explicit expressions for M_{12} and Γ_{12} are very complicated and are presented in [14].

It is noted that M_{12} and Γ_{12} are fully determined by the box diagram and so are Δm and Γ. However, some measurable quantities are associated with the evolution of the quantum system, so closely related to $\delta H\!\!\!/\rho$ via Eq. (6), such as r, $\chi_d(\chi_s)$, and A_B, etc.

By definition,

$$\xi=\frac{\Gamma(ll)+\Gamma(\overline{ll})}{\Gamma(l\bar{l})+\Gamma(\bar{l}l)} \tag{15}$$

and

$$r=\frac{\Gamma(B^0\to l^+X\ \ \mathrm{via}\ \ \bar{B}^0)}{\Gamma(B^0\to l^\pm X)}\ , \tag{16}$$

$$\chi=\frac{r}{1+r}=P(B^0\to\bar{B}^0), \tag{17}$$

$$a=\frac{r-\bar{r}}{r+\bar{r}}. \tag{18}$$

It is noted that $\xi=2r/(1+r^2)$ if the $B^0\bar{B}^0$ pair is produced incoherently, in contrast, $\xi=r$ if it is produced coherently [for example, at $\Upsilon(4s)$] [15]. Calculating the hadronic matrix elements of the exclusive semileptonic or nonleptonic decay width needs more knowledge on the nonperturbative QCD. With the PCAC (partial conservation of axial-vector current) theorem, one can simplify

$$\langle l^-\nu D^+|H_W|B^0\rangle\sim j_\mu^l f_+^{B^0D}(p_B+p_D)^\mu,$$

where j_μ^l is the lepton current $\bar{\nu}\gamma_\mu(1-\gamma_5)e$ and the form factor $f_+^{B^0D}$ is not calculable in the framework of perturbative QCD. Fortunately, to evaluate r and a, the troublesome factor can be canceled out at the ratios.

It is easy to obtain

$$r=\frac{\int_0^\infty\{\mathrm{tr}[\rho_{B^0}(\tau)O_{l^+}]\}d\tau}{\int_0^\infty\{\mathrm{tr}[\rho_{B^0}(\tau)O_{l^-}]\}d\tau}, \tag{19}$$

where the density matrix is the expression in Eq. (6) and the operator matrix O_{l^+} reads

$$O_{l^+}=|B_i\rangle\langle B_i|H_W|l^+\nu X\rangle\langle l^+\nu X|H_W|B_j\rangle\langle B_j|, \tag{20}$$

where the subscripts i and j refer to B^0 and $\bar{B}^0$, but in the calculations, one needs to project them into the basis of $|B_{1,2}\rangle=(1/\sqrt{2})(|B^0\rangle\pm|\bar{B}^0\rangle)$, while the O_{l^-} is similar. The other quantities can be easily obtained.

(iii) The CP violation in the nonleptonic decay processes. The nonleptonic decay processes concern direct CP violation resulted by the CP noninvariant interactions. Following the literature, we study

$$A_B=\frac{N(B^0\to\pi^+\pi^-)-N(\bar{B}^0\to\pi^+\pi^-)}{N(B^0\to\pi^+\pi^-)+N(\bar{B}^0\to\pi^+\pi^-)}. \tag{21}$$

A_B is an integrated value over time, so related to the evolution of the quantum system and thus depends on the quantum-gravity induced $\delta H\!\!\!/\rho$:

$$A_B=\frac{\int_0^\infty d\tau\ \mathrm{tr}\{[\rho_{B^0}(\tau)-\rho_{\bar{B}^0}(\tau)]O_{\pi^+\pi^-}\}}{\int_0^\infty d\tau\ \mathrm{tr}\{[\rho_{B^0}(\tau)+\rho_{\bar{B}^0}(\tau)]O_{\pi^+\pi^-}\}}. \tag{22}$$

The operator matrix $O_{\pi^+\pi^-}$ is defined as

$$(O_{\pi^+\pi^-})_{ij}=|B_i\rangle\langle B_i|H_W|\pi^+\pi^-\rangle\langle\pi^+\pi^-|H_W|B_j\rangle\langle B_j|, \tag{23}$$

where the operator matrix $O_{\pi^+\pi^-}$ has the same notation as $O_{l^\pm}$.

Generally, the direct CP violation emerges via an interference between two (or more) channels whose weak and strong phases are all different; namely, a CP asymmetry is proportional to $\sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)$ where δ_1, δ_2 are the strong phase shifts and ϕ_1, ϕ_2 are the weak phases due to the Kobayashi-Maskawa (KM) phase for the two channels. In the $K\to 2\pi$ case, 1 and 2 correspond to $|\pi\pi(I=0)\rangle$ and $|\pi\pi(I=2)\rangle$ channels.

In $B\to 2\pi$, the main contribution comes from the interference between the tree diagram and the one-loop penguin diagram [16]:

$$A(B^0\to f)=T\ \exp[i(\phi_T+\alpha_T)] + P\ \exp[i(\phi_P+\theta_P+\alpha_P)], \tag{24}$$

$$A(\bar{B}^0\to f)=\bar{T}\ \exp[i(-\phi_T+\alpha_T)] + \bar{P}\ \exp[i(-\phi_P+\theta_P+\alpha_P)] \tag{25}$$

where ϕ_i are the weak phases, α_i are the final state interaction induced phase shifts and θ_P is the loop-induced phase. In general, $\alpha_T=\alpha_P$, θ_P is determined by the absorptive and dispersive parts of the penguin diagram. In fact, only the timelike penguin diagram can have an absorptive part and contributes to θ_P; however, recent research [17] pointed out that a spacelike penguin diagram also affects the CP asymmetry by modifying the dispersive or absorptive parts of the timelike penguin amplitudes. For the time being, in this work we ignore the contribution from the spacelike penguin diagram because it would make the calculation very complicated, but does not influence the qualitative conclusion.

Defining

$$\gamma=\frac{P}{T},\quad \bar{\gamma}=\frac{\bar{P}}{\bar{T}},$$

one has [16]

$$\gamma\approx c_\gamma\left|I_{ut}+\frac{v_c^*}{v_u^*}I_{ct}\right|,\quad \bar{\gamma}\approx c_\gamma\left|I_{ut}+\frac{v_c}{v_u}I_{ct}\right|$$

where

$$c_\gamma=\frac{\alpha_s}{2\pi}\left(1+\frac{2M_\pi^2}{(m_d+m_u)(m_b-m_u)}\right).$$

$v_i=V_{ib}V_{id}^*$ and $I_{ut}\equiv I_u-I_t$, $I_{ct}\equiv I_c-I_t$ are very complicated functions of $y_i=m_i^2/M_W^2$ which are given explicitly in [16]. Substituting all the information into Eq. (23), it would be straightforward to achieve the operator matrix $O_{\pi^+\pi^-}$.

In the next section, we will present our numerical results for the quantities.

III. NUMERICAL RESULTS

Except for the Cabibbo angle θ_C, all KM entries are not well determined, and the phase δ is not either. They disperse in a wide range $|V_{ub}|\sim 0.002$–0.005, $|V_{td}|\sim 0.004$–0.015, and $\sin\theta_C=0.22$ [18]. In the B system, x_d is relatively well measured (even with a small portion of x_s), so we take reasonable values for $\sin\theta_2$ and $\sin\theta_3$ which guarantee $x_d=\Delta m/\Gamma$ to remain within the experimentally allowed region of 0.71 ± 0.06 [18]. Since $m_t\gg m_c$, M_{12} mainly is determined by $V_{td}V_{tb}^*$, which is only sensitive to θ_2, thus in our later computations we adopt $|V_{td}|=0.0075$ and $|V_{ub}|=0.005$. Furthermore, unlike in the K case, for $B^0-\bar{B}^0$ mixing, $|\Gamma_{12}|\ll|M_{12}|$, because $|M_{12}|$ is related to the large m_t, while Γ_{12} only to much smaller m_c and m_u. Therefore,

$$\frac{q}{p}\approx\sqrt{\frac{M_{12}^*}{M_{12}}},$$

by the KM parametrization, the phase $\sim 2\delta$. If one writes $q/p=(1-\epsilon)/(1+\epsilon)$ in the traditional way, approximately $\epsilon\simeq -i\ \tan\delta$, thus at $\delta=\pi/2$ there is a discontinuity in sign, but it does not change the physics. Even so, we try to avoid the vicinity of $\pi/2$ and restrict the δ value at 0.2 – 0.73 and 2.4 – 2.9 (getting rid of the region $\pi/2\pm7\pi/30$). It is noted that if m_t were small, the situation would be different, since the above approximation would not be valid.

As aforementioned, the parameters β and γ are proportional to $M_{\rm meson}^2/M_{\rm Pl}\sim10^{-19}$ GeV for K mesons, since $M_B\sim10M_K$ approximately, one can expect β and γ to be 2 orders of magnitude larger than that obtained for the K system. In our numerical calculations, we take two sets of β and γ, the first one is that obtained by Huet and Peskin [5], and for the second set, we enlarge the first one by 100 times.

ξ and r concern the mixing parameters which are experimentally measurable, but do not refer to the CP violation. The calculated $\chi_d=r/(1+r)=0.168$, compared to the data 0.156 ± 0.024 and our result shows $\xi\approx2r/(1+r^2)$. In the standard way, by varying the δ value, we obtain $\xi_0\sim0.388$, while $(\xi_1-\xi_0)/\xi_0\sim10^{-6}-10^{-5}$, $(\xi_2-\xi_0)/\xi_0\sim10^{-4}-10^{-3}$, and $r_0\sim0.2$, $(r_1-r_0)/r_0\sim10^{-6}-10^{-5}$, $(r_2-r_0)/r_0\sim10^{-4}-10^{-3}$, where the subscripts 0, 1, and 2 denote the cases without gravity, α, β, γ being the values listed in last section and with α, β, γ being 100 times larger than the first set, respectively. Indeed by an estimate of the order of magnitude,

$$M_B^2/m_{\rm Pl}\Delta M\sim10^{-5},$$

which indicates $\Delta\xi/\xi$ and $\Delta r/r$ to be of such orders and much smaller than unity, so may not be very meaningful for experiments.

But for the CP-violation parameters a and A_B defined above an estimate of the CP-violation effects originated from quantum gravity may somehow make some sense. Below we tabulate the dependence of the CP violation on the phase δ, the parameters α, β, and γ which characterize the quantum-gravity effects.

In Table I, we will present a and A_B corresponding to the CP violation in semileptonic and nonleptonic decay, respectively. Their definitions were given in the last section.

IV. DISCUSSIONS AND CONCLUSIONS

Hawking's generalization of quantum mechanics which encompasses gravity allows the evolution of pure states into mixed states, so it is a model violating quantum mechanics. CP violations may occur at the evolution processes of a quantum system. EHNS [6] and HP [5] investigated possible effects of such gravity-induced term $\delta\not{H}\rho$ on the CP violation in the K system. Comparing with the data of the K system, they obtained corresponding parameters β and γ. It is noted that the parameters obtained by EHNS and HP all have very large uncertainties, or in fact they are still consistent with zero.

It is natural to extend their work to the B system which is another good place for observing CP violation besides the K mesons. We adopt the parameters β and γ given in [5] and another set with them being enlarged by 2 orders of magnitude according to the aforementioned discussion: α, β,

TABLE I. The CP asymmetry parameters for in semileptonic and nonleptonic decays.

	a			A_B		
δ	a_0	$\frac{a_1-a_0}{a_0}$	$\frac{a_2-a_0}{a_0}$	$(A_B)_0$	$\frac{(A_B)_1-(A_B)_0}{(A_B)_0}$	$\frac{(A_B)_2-(A_B)_0}{(A_B)_0}$
0.209	5.53×10^{-4}	-5.60×10^{-5}	-5.59×10^{-3}	0.145	6.06×10^{-6}	6.04×10^{-4}
0.314	8.21×10^{-4}	-3.21×10^{-5}	-3.20×10^{-3}	0.184	1.80×10^{-6}	1.76×10^{-4}
0.419	1.08×10^{-3}	-1.59×10^{-5}	-1.58×10^{-3}	0.222	-1.82×10^{-6}	-1.89×10^{-4}
0.524	1.33×10^{-3}	-3.64×10^{-6}	-3.62×10^{-4}	0.258	-4.61×10^{-6}	-4.68×10^{-4}
0.628	1.56×10^{-3}	5.57×10^{-6}	5.54×10^{-4}	0.291	-6.40×10^{-6}	-6.47×10^{-4}
0.730	1.77×10^{-3}	1.19×10^{-5}	1.19×10^{-3}	0.322	-7.18×10^{-6}	-7.23×10^{-4}
2.40	1.77×10^{-3}	-2.30×10^{-5}	-2.32×10^{-3}	0.304	-4.44×10^{-6}	-4.42×10^{-4}
2.51	1.56×10^{-3}	-3.00×10^{-5}	-3.00×10^{-3}	0.257	-6.75×10^{-6}	-6.74×10^{-4}
2.62	1.32×10^{-3}	-3.32×10^{-5}	-3.32×10^{-3}	0.207	-8.63×10^{-6}	-8.63×10^{-4}
2.72	1.08×10^{-3}	-3.11×10^{-5}	-3.11×10^{-3}	0.156	-9.50×10^{-6}	-9.50×10^{-4}
2.83	8.19×10^{-4}	-2.07×10^{-5}	-2.07×10^{-3}	0.105	-8.39×10^{-6}	-8.40×10^{-4}
2.93	5.51×10^{-4}	5.08×10^{-6}	-5.08×10^{-4}	0.053	-1.82×10^{-6}	-1.82×10^{-4}

$\gamma\sim M_M^2/M_{\rm Pl}$. Since for the K system there is only a mild bound for α, for example, in [5] the authors gave a distribution $Q(\pi^+\pi^-;\pi^+\pi^-;\tau)$ which is less sensitive to α and its leading α effect is of order of $\alpha|\eta_{+-}|^2$, so very small. Therefore we take the α value according to the inequality $\alpha\gamma>\beta^2$. Then we study the effects of $\delta\not{H}\rho$ on the mixing χ_d, ξ, and CP violation of the semileptonic and nonleptonic decays.

Since the quantum-gravity effects enter this game through the evolution of the density matrix of the system, we can analyze the changes caused by theses effects and find similarities and differences between the K and B systems.

The time dependence of the density matrix is given in Eq. (6), one can notice that there are two sorts of changes, the first is the shift of the average lifetime $\bar{\Gamma}$ to $\bar{\Gamma}+\alpha-\gamma$ and the second is the variation of the eigenmodes ρ_i. It is noticed from our numerical results that the changes of the matrix elements of ρ_i are of the order of α, β, and γ. For the K system, $\bar{\Gamma}=(\Gamma_S+\Gamma_L)/2\sim3.7\times10^{-15}$ GeV, while for the B system, $\bar{\Gamma}=(\Gamma_H+\Gamma_L)/2\sim2.2\times10^{-13}$ GeV, 2 orders larger than that for K, so if we adopt the first set of α,β,γ, the relative changes in B would be smaller than in K, whereas, with the second set of the parameters which is 100 times larger than the first one, the relative changes are of the same order as for the K system. For the density matrix, the situation is similar, however, ϵ is measured in the K system which is as small as 10^{-3}, but for B, there are no data yet and the calculated value of ϵ is not so small [9]. Since all corrections to ρ_i for the K case appear at the terms proportional to $|\epsilon|^2$ [6,5], it seems that the direct effects in the B system is much smaller than that in the K system. But this is not true, since the small ϵ approximation does not hold for the B system, there is no simple analytic solution for Eq. (9) as for the K system, and it should be solved numerically. The numerical results show that the changes caused by the quantum-gravity effects are similar to that for the K system as long as the second set of parameters is employed.

Thus one can see that the K system and B system have a very similar response to the effects of the quantum gravity.

Our numerical results show that if the parameters β and γ obtained in the K meson are valid, the additional phenomenological term $\delta\not{H}\rho$ can only result in a change of about 10^{-6} for the CP violation from that caused by the regular mechanisms, for example, the box and penguin diagrams, etc. It is impossible to observe it in any available or in near future experiments. However, if the parameters can be 2 orders of magnitude larger than in the K system, the additional CP-violation effect can reach 10^{-3}–10^{-4} of the regular ones. Namely the expected observable CP violation may be (1 ± 10^{-3}) times of that resulted by the standard mechanisms. Of course, it is still a small effect. In addition, even though the contribution of the long distance effects (intermediate resonances) is much smaller than that in the K system, it may still be as large as the effect of gravity. It would be hard to distinguish between them in experiments.

Moreover, experimental difficulties for measuring such effects caused by the phenomenological term in the B system are greater than in the K system, because the time scales for the neutral B system are 2 orders shorter than for the K system and the measurement background is larger.

Even in the proposed B-meson factory, it is not easy to observe so small effects and identify them from other sources, such as the long distance effects. Therefore our conclusion is that the prospect for observing the effects of gravity on the CP violation in the B system is pessimistic, even not completely impossible.

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