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Systematic gauge-invariant approach to heavy quarkonium decays

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We present a method which, starting directly from QCD, permits a systematic gauge-invariant expansion to be made for all hard processes involving quarkonia in powers of the quark relative velocity, a small natural parameter for heavy quark systems. Our treatment automatically introduces soft gluons in the expansion. Corrections arising from the incorporation of gauge symmetry turn out to be important for decay and fragmentation processes involving $Q\bar{Q}$ systems. The contribution of soft gluons is shown to be of higher order in v and so is neglected for calculations done up to and including $O(v^2)$.

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INTRODUCTION

The principles of quantum chromodynamics (QCD) were applied almost 20 years ago to the bound states of heavy quarks, such as $c\bar{c}$ and $b\bar{b}$. These are possibly the simplest strongly bound systems that exist. The large mass of the heavy quark sets a mass scale large enough so that perturbative QCD, together with a nonrelativistic potential model description of the bound state, provides a good starting point to describe the decay and formation of quarkonia. However, quantitative predictions of the simple quarkonium model, even supplemented by radiative corrections, sometimes fail badly.[1] Over the years, hundreds of papers have been written to rectify some of the failures. Nevertheless one still does not have a complete solution to this important problem of nonperturbative QCD.

Our investigation into this venerable subject was prompted by the observation that the fundamental symmetry to which QCD owes its origin, gauge symmetry, is manifestly violated by the naive quarkonium model. This is not hard to see: under a local gauge transformation $q(\vec{x},t)\to U(\vec{x},t)q(\vec{x},t)$ the state normally used to describe quarkonia,

$$\int d^3x_1 d^3x_2 f(\vec{x}_1-\vec{x}_2)\bar{q}(\vec{x}_1,t)\Gamma q(\vec{x}_2,t)|0\rangle, \qquad (1)$$

does not remain invariant. In the above equation Γ is a space-time-independent matrix in spin, color, and flavor indices and $f(\vec{x})$ is the relative wave function. A gauge-invariant state can be constructed, however, by inserting a path-ordered gauge link operator between quark operators. This amounts to including arbitrary numbers of soft gluons for transporting color between quarks. Building on this idea, in a previous publication [2] we had proposed a manifestly gauge-invariant effect field theory describing the interaction between heavy quarks, gluons, and quarkonium. The corrections accruing from the incorporation of gauge symmetry turned out to be substantial for decay and fragmentation processes, as well as radiative transitions, and this indicated the importance of a correct treatment. However, the relation of the effective theory to QCD was not transparent and it was not clear how the theory could be systematically extended to higher orders.

In this paper we have developed a method which starts directly from QCD and which does allow for a systematic treatment of all high momentum transfer processes involving quarkonia such as inclusive decays, production, and fragmentation. The natural expansion parameter is the quark relative velocity which, for a heavy quark system, is small. All of the nonperturbative physics turns out to reside in a small number of matrix elements of gauge-invariant operators which are identified from symmetry considerations. The method used in this paper was inspired by the Feynman diagram treatment of deeply inelastic scattering as originally developed by Ellis, Furmanski, and Petronzio [3], and recently further expanded upon by Jaffe and Ji [4].

While this work was nearing completion, we received a preprint authored by Bodwin, Braaten, and Lepage [5] which presents a comprehensive QCD analysis of hard processes involving quarkonia. Their analysis is based upon a nonrelativistic formulation of QCD. We share similar conclusions although these two approaches are quite different; with suitable identification of parameters the results are identical. Perhaps an advantage of our method is its relative simplicity and its closer relation to the more familiar relativistic QCD. On the other hand, the work of Bodwin *et al.* [5] has wider scope because it is ultimately aimed at also generating the static properties of quarkonia through lattice calculations. We see the two approaches as complementary to each other.

FORMALISM

Our goal is to arrive at a systematically improvable, gauge-invariant, description of all hard processes involving a

[1]For a recent review of quarkonium phenomenology see, for example, Schuler [1].

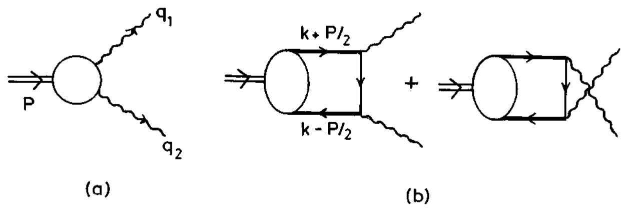

FIG. 1. (a) Heavy quarkonium decay into two photons. (b) The lowest order diagrams contributing to $Q\bar{Q}\to 2\gamma$.

$Q\bar{Q}$ system. By way of introduction, consider the decay of a positive $\mathscr{C}$ parity state into 2 photons [Fig. 1(a)] and the simplest Feynman graphs [Fig. 1(b)] which contribute to it. Decay widths of specific hadrons were computed from the leading approximation to these graphs long ago, and references may be found in Schuler [1]. For our purposes, it is useful to write the zero-gluon amplitude in Fig. 1(b) as

$$T_0^{\mu\nu}=\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}M(k)h^{\mu\nu}(k). \tag{2}$$

$M(k)$ is the usual, but obviously non-gauge-invariant, Bethe-Salpeter (BS) amplitude:

$$M(k)=\int d^4x\; e^{ik\cdot x}\langle 0|T[\bar{\psi}(-x/2)\psi(x/2)]|P\rangle. \tag{3}$$

The tensor $h^{\mu\nu}(k)$ is the amplitude for two quarks, not necessarily on their mass-shells, to annihilate into 2 photons. To leading order in α_s this is

$$h^{\mu\nu}(k)=-ie^2[\gamma^\nu S_F(k+K)\gamma^\mu+\gamma^\mu S_F(k-K)\gamma^\nu]. \tag{4}$$

In Eqs. (2)–(4), x^μ is the relative distance between quarks, $k^\mu=\frac{1}{2}(p_1-p_2)^\mu$ is the quark relative momentum, $K^\mu=\frac{1}{2}(q_2-q_1)^\mu$ is the photon relative momentum and S_F is the free fermion propagator. We shall refer to $M(k)$ and $h^{\mu\nu}$ as "soft" and "hard" parts, respectively, in the following.

Now consider the fact that a large momentum of $O(m)$, where m is the heavy quark mass, flows through the single propagator in Fig. 1(b) but that, on the other hand, the soft part has typically quark momenta much less than m. This suggests that we expand the hard part in powers of k^α:

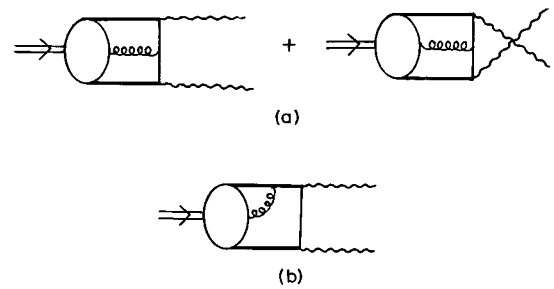

FIG. 2. (a) Single gluon diagrams. (b) Interacting quark field diagram which is properly included in Fig. 1(b).

$$h^{\mu\nu}(k)=\sum \frac{1}{n!}k^{\alpha_1}\cdots k^{\alpha_n}U^{\mu\nu}_{\alpha_1\cdots\alpha_n}, \tag{5}$$

where

$$U^{\mu\nu}_{\alpha_1\cdots\alpha_n}=\frac{\partial}{\partial k^{\alpha_1}}\cdots\frac{\partial}{\partial k^{\alpha_n}}h^{\mu\nu}|_{k=0}. \tag{6}$$

Inserting Eq. (5) into Eq. (2) and integrating by parts gives, for the amplitude,

$$T_0^{\mu\nu}(k)=\mathrm{Tr}\sum_n \mathscr{M}^{\alpha_1\cdots\alpha_n}U^{\mu\nu}_{\alpha_1\cdots\alpha_n}, \tag{7}$$

where

$$\mathscr{M}^{\alpha_1\cdots\alpha_n}=\frac{1}{n!}\langle 0|\bar{\psi}i\overleftrightarrow{\partial}^{\alpha_1}\cdots i\overleftrightarrow{\partial}^{\alpha_n}\psi|P\rangle. \tag{8}$$

The matrix elements in Eq. (8) have derivatives $\overleftrightarrow{\partial}^\alpha=\frac{1}{2}(\vec{\partial}^\alpha\overleftarrow{\partial}^\alpha)$ evaluated at zero relative quark separation. As we shall see later, in a nonrelativistic model the $n=0$ matrix element is proportional to the wave function at the origin, and so on. However, we do not need to appeal to any particular model at this point.

Next consider the single gluon diagram in Fig. 2. The corresponding amplitude is

$$T_1^{\mu\nu}=\int \frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\,\mathrm{Tr}M_\rho(k,k')H^{\mu\nu\rho}(k,k'). \tag{9}$$

The "soft" part $M_\rho(k,k')$ is a generalized BS amplitude,

$$M_\rho(k,k')=\int d^4x\; d^4z\; e^{ik\cdot x}e^{ik'\cdot z}\langle 0|T[\bar{\psi}(-x/2)A_\rho(z)\psi(x/2)]|P\rangle, \tag{10}$$

and $A_\rho\equiv\frac{1}{2}\lambda^a A^a_\rho$ is the gluon field matrix. The "hard" part $H^{\mu\nu\rho}$ is the annihilation amplitude for $\bar{Q}Qg\to\gamma\gamma$. To leading order this is

$$H^{\mu\nu\rho}(k,k')=-ie^2g[\gamma^\nu S_F(k+\tfrac{1}{2}k'+K)\gamma^\rho S_F(k-\tfrac{1}{2}k'+K)\gamma^\mu]+(\mu\leftrightarrow\nu). \tag{11}$$

Excluded are the diagrams such as in Fig. 2(b). These are properly included in Fig. 1(b) since the lines emerging from the right correspond to interacting fields. Expanding the hard part,

$$H^{\mu\nu\rho}(k,k')=\sum \frac{1}{n!l!}k^{\alpha_1}\cdots k^{\alpha_n}k'^{\beta_1}\cdots k'^{\beta_l}V^{\mu\nu\rho}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_l}, \tag{12}$$

where

$$V^{\mu\nu\rho}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_l}=\frac{\partial}{\partial k^{\alpha_1}}\cdots\frac{\partial}{\partial k^{\alpha_n}}\frac{\partial}{\partial k'^{\beta_1}}\cdots\frac{\partial}{\partial k'^{\beta_l}}H^{\mu\nu\rho}|_{k=k'=0}. \tag{13}$$

An integration by parts on k, k' yields an alternate form for the amplitude $T_1^{\mu\nu}$:

$$T_1^{\mu\nu}(k)=\mathrm{Tr}\sum_{nl} M_\rho^{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_l}V^{\mu\nu\rho}_{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_l}, \tag{14}$$

where

$$M_\rho^{\alpha_1\cdots\alpha_n,\beta_1\cdots\beta_l}=\frac{1}{n!l!}\langle 0|\bar{\psi}i\overleftrightarrow{\partial}^{\alpha_1}\cdots i\overleftrightarrow{\partial}^{\alpha_n}\psi i\partial^{\beta_1}\cdots i\partial^{\beta_l}A_\rho|P\rangle. \tag{15}$$

The derivatives $i\overleftrightarrow{\partial}^{\alpha}$ act only upon the quark operators.

Finally consider the two-gluon contribution to the amplitude shown in Fig. 3. The amplitude corresponding to Fig. 3(a) is

$$T_{2a}^{\mu\nu}=\int\frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}\mathrm{Tr}M_{\rho'\rho''}(k,k',k'')H_a^{\mu\nu\rho'\rho''}(k,k',k''), \tag{16}$$

where

$$M_{\rho'\rho''}(k,k',k'')=\int d^4x\ d^4x'\ d^4x''\ e^{i(k\cdot x+k'\cdot x'+k''\cdot x'')}\langle 0|\bar{\psi}(-x/2)A_{\rho'}(x')A_{\rho''}(x'')\psi(x/2)|P\rangle. \tag{17}$$

The product of color matrix fields is appropriately symmetrized in the above because of Bose symmetry. The hard part $H_a^{\mu\nu\rho'\rho''}$ is, at lowest order,

$$H_a^{\mu\nu\rho'\rho''}(k,k',k'')=-ie^2g^2[\gamma^\nu S_F(k+\tfrac{1}{2}k''+K)\gamma^{\rho''}S_F(k+k'-\tfrac{1}{2}k''+K)\gamma^{\rho'}S_F(k-\tfrac{1}{2}k''+K)\gamma^\mu+\text{ crossed}]. \tag{18}$$

This may be expanded as before about $k=k'=k''=0$.

We now make the observation that the simple Ward identity

$$\frac{\partial}{\partial p^\alpha}S_F(p)=-S_F(p)\gamma_\alpha S_F(p), \tag{19}$$

leads to a number of useful relations. In particular,

$$V^{\mu\nu\rho}=-gU^{\mu\nu\rho} \tag{20}$$

allows us to combine the $n=1$ term in Eq. (7) and the $n=l=0$ term in Eq. (14) into a gauge-invariant sum:

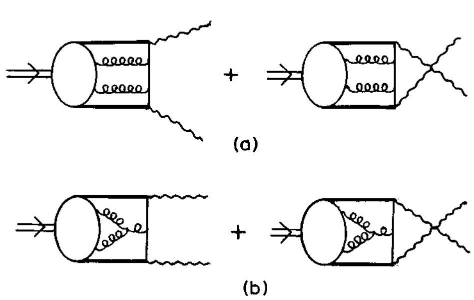

FIG. 3. (a) Two gluon diagrams. (b) Three gluon vertex diagrams.

$$\mathscr{M}^\alpha U^{\mu\nu}_\alpha+M^\alpha V^{\mu\nu}_\alpha=\langle 0|\bar{\psi}i\overleftrightarrow{D}^\alpha\psi|P\rangle U^{\mu\nu}_\alpha. \tag{21}$$

Similarly, the leading order term in the hard two-gluon amplitude is just the second-order term in the zero-gluon amplitude:

$$H^{\mu\nu\rho'\rho''}(0)=g^2U^{\mu\nu\rho'\rho''}. \tag{22}$$

Collecting together appropriate terms leads to another gauge-invariant matrix element:

$$\mathscr{M}^{\alpha\alpha'}U^{\mu\nu}_{\alpha\alpha'}+M^\alpha_\rho V^{\mu\nu\rho}_\alpha+M_{\rho'\rho''}H_a^{\mu\nu\rho'\rho''}(0)$$
$$=\tfrac{1}{2}\langle 0|\bar{\psi}i\overleftrightarrow{D}_{\rho'}i\overleftrightarrow{D}_{\rho''}\psi|P\rangle U^{\mu\nu\rho'\rho''}. \tag{23}$$

Here $M_{\rho'\rho''}$ is just the leading order term from Eq. (17). It is the matrix element with al fields at the same space-time point:

$$M_{\rho'\rho''}=\langle 0|\bar{\psi}A_{\rho'}A_{\rho''}\psi|P\rangle. \tag{24}$$

Next, look at the $n=0$ $l=1$ term in Eq. (14):

$$\langle 0|\bar{\psi}\psi i\partial_\beta A_\rho|P\rangle\partial'^\beta H^{\mu\nu\rho}=\frac{i}{2}\langle 0|\bar{\psi}\psi(\partial_\beta A_\rho$$
$$-\partial_\rho A_\beta)|P\rangle\partial'^\beta H^{\mu\nu\rho}. \tag{25}$$

The last step made use of $\partial'^\beta H^{\mu\nu\rho}=-\partial'^\rho H^{\mu\nu\beta}$, with all derivatives evaluated at $k=k'=0$. The quantity inside the angular brackets is the Abelian field strength tensor; the non-

Abelian part, $ig[A_\beta,A_\rho]$, can be shown after some effort to arise from the amplitude in Fig. 3(b).

We now collect results together and summarize. The contribution from the 0, 1, 2 gluon diagrams in Figs. 1–3 have been expanded in powers of relative momentum, added together, and terms suitably rearranged. The total amplitude is, neglecting higher powers of momentum,

$$(T_0+T_1+T_2)^{\mu\nu}=\mathrm{Tr}\Bigg[\langle 0|\bar{\psi}\psi|P\rangle h^{\mu\nu}+\langle 0|\bar{\psi}i\vec{D}_\alpha\psi|P\rangle\partial^\alpha h^{\mu\nu}+\langle 0|\bar{\psi}i\vec{D}_\alpha i\vec{D}_\beta\psi|P\rangle\frac{1}{2}\partial^\alpha\partial^\beta h^{\mu\nu}+\langle 0|\bar{\psi}F_{\alpha\beta}\psi|P\rangle\frac{i}{2}\partial'^\alpha H^{\mu\nu\beta}\Bigg]. \tag{26}$$

The hard amplitude $h^{\mu\nu}(k)$, $H^{\mu\nu\alpha}(k,k')$, and their derivatives are all evaluated at $k=k'=0$. We see that each term in the above is a product of a gauge invariant matrix element characteristic of the decaying hadron and a simple, calculable, hard part. In the following, hadrons of specific J^{PC} will be considered and the relative order of importance of the terms in Eq. (26) will be explicated. Radiative corrections, which are note included in the lowest order amplitudes h and H, will be considered separately.

MATRIX ELEMENTS

In the previous section the amplitude for a $\mathscr{C}=+$ quarkonium state to decay into 2 photons was expressed in terms of matrix elements of leading gauge-invariant operators. Further progress requires we specify the angular momentum and parity: we take $J=0$, $\mathscr{P}=-$(η_c and η_b mesons) for now, leaving other mesons for later analysis. From Lorentz invariance, and invariance under charge conjugation and parity, the only nonzero matrix elements are

$$\langle 0|\bar{\psi}\psi|0^{-+}\rangle=a_1M^2\gamma_5+a_2M\not{P}\gamma_5, \tag{27}$$

$$\langle 0|\bar{\psi}i\vec{D}^\mu\psi|0^{-+}\rangle=ibM^2\sigma^{\mu\nu}\gamma_5P_\nu, \tag{28}$$

$$\langle 0|\bar{\psi}F^{\mu\nu}\psi|0^{-+}\rangle=cM^3\epsilon^{\mu\nu\alpha\beta}\gamma_\alpha P_\beta, \tag{29}$$

$$\langle 0|\bar{\psi}i\vec{D}^\mu i\vec{D}^\nu\psi|0^{-+}\rangle=M^2[d_1M^2g^{\mu\nu}+d_2P^\mu P^\nu]\gamma_5+M^3\left[e_1g^{\mu\nu}P^\alpha+e_2\frac{P^\alpha P^\mu P^\nu}{M^2}+e_3(g^{\alpha\mu}P^\nu+g^{\alpha\nu}P^\mu)\right]\gamma_\alpha\gamma_5. \tag{30}$$

For brevity, color has not been explicitly indicated in Eqs. (27)–(30). It is clear that in Eq. (27) the two quarks must be in a color singlet and so, regarded as matrix in color space, only the unit operator appears on the right-hand side. However, when one gluon appears, as in Eqs.(28) and (29), the quarks may be in either singlet or octet states and the corresponding constants $b^{(1)}\cdots b^{(8)}$ then appear on the right-hand side (RHS). In Eq. (30), the two gluons can combine into a color singlet or octet, and those in turn can combine with the quark singlet and octet, respectively, to give an overall singlet. Clearly this leads to a large number of constants which must be known in order to describe corrections to 0^{-+} decay and if our approach is to have any practical utility, this number must be curtailed according to some well-defined principle.

To progress beyond this point, it will be necessary to specialize our hitherto general discussion and select a particular gauge. The Coulomb gauge is natural for this problem, as shown by vast experience with positronium states. We shall not repeat here the arguments of Lepage *et al.* [6] who, using the QCD equations of motion in the Coulomb gauge, make the estimates,

$$\partial^0\sim mv^2,\quad gA^0\sim mv^2,\quad \vec{\partial}\sim mv,\quad g\vec{A}\sim mv^2,$$
$$g\vec{E}\sim m^2v^3,\quad g\vec{B}\sim m^2v^4. \tag{31}$$

Here v is the relative velocity of quarks, the small parameter in the theory. The estimates (31) allow us to see that explicit gluons will not enter in the leading order corrections to the naive quarkonium model. Therefore, working to $O(v^2)$, one may effectively replace the covariant derivatives in Eqs. (27)–(30) with ordinary derivatives, and ignore $\vec{E}$ and $\vec{B}$.

We next observe that tracing Eq. (30) with $\gamma_5\gamma_\mu$ or $\gamma_5\gamma_\nu$, and using the equation of motion $i\not{D}\psi=m\psi$, yields the constraint

$$e_1+e_2+5e_3=0. \tag{32}$$

Working in the rest frame of the meson $P^\mu=(M,\vec{0})$ and putting $\mu=0$ $\nu=i$ yields $e_3=O(v^3)$. Hence $e_1=-e_2+O(v^3)$. Setting $\mu=\nu=0$ yields $d_2=-d_1+O(v^3)$. This leaves us with having to deal with a_1,a_2,b,d_1, and e_1—five independent parameters at the $O(v^2)$ level.

Further progress demands that we specialize a step further and specify a model framework for the 0^{-+} quarkonium state. We shall assume, in common with many other authors, that the Bethe-Salpeter equation with an instantaneous kernel does provide an adequate description. This has been conveniently reviewed by Keung and Muzinich [7] and we adopt their notation. The momentum space BS amplitude $\chi(p)$ satisfies the homogeneous equation

$$\chi(p)=iG_0(P,p)\int\frac{d^4p'}{(2\pi)^4}K(P,p,p')\chi(p'), \tag{33}$$

which, after making the instantaneous approximation $K(P,p,p')=V(\vec{p},\vec{p}\,')$ and reduction to the nonrelativistic limit yields [7]

$$\chi(p)=\frac{M^{1/2}(M-2E)(E+m-\vec{p}\cdot\vec{\gamma})\gamma_5(1-\gamma_0)(E+m-\vec{p}\cdot\vec{\gamma})\phi(|\vec{p}|)}{4E(E+m)\left(p^0+\frac{M}{2}-E+i\epsilon\right)\left(p^0-\frac{M}{2}+E-i\epsilon\right)}. \quad (34)$$

The scalar wave function $\phi(|\vec{p}|)$ is normalized to unity:

$$\int \frac{d^3p}{(2\pi)^3}|\phi(|\vec{p}|)|^2=1, \quad (35)$$

and

$$E=\sqrt{\vec{p}^2+m^2}. \quad (36)$$

Fourier transforming $\chi(p)$ to position space yields $\langle 0|\bar{\psi}(-x/2)\psi(x/2)|P\rangle$ from which, by tracing with appropriate gamma matrices, the coefficients a_1, a_2, b, d_1, and e_1 can be extracted. So finally, to $O(v^2)$, one has a rather simple result:

$$\langle 0|\bar{\psi}\psi|0^{-+}\rangle=\frac{1}{2}M^{1/2}\phi(0)\left(1+\frac{\not{P}}{M}\right)\gamma_5 + M^{-1/2}\frac{\vec{\nabla}^2\phi(0)}{M^2}\not{P}\gamma_5, \quad (37)$$

$$\langle 0|\bar{\psi}iD^\mu\psi|0^{-+}\rangle=\frac{1}{3}M^{1/2}\frac{\vec{\nabla}^2\phi(0)}{M^2}i\sigma^{\mu\nu}\gamma_5P_\nu, \quad (38)$$

$$\langle 0|\bar{\psi}F^{\mu\nu}\psi|0^{-+}\rangle=0, \quad (39)$$

$$\langle 0|\bar{\psi}iD^\mu iD^\nu\psi|0^{-+}\rangle=\frac{1}{6}M^{5/2}\frac{\vec{\nabla}^2\phi(0)}{M^2}\left(g^{\mu\nu}-\frac{P^\mu P^\nu}{M^2}\right)\times\left(1+\frac{\not{P}}{M}\right)\gamma_5. \quad (40)$$

Equations (37)–(40) express hadronic matrix elements, up to $O(v^2)$, in terms of two basic parameters: $\phi(0)$ and $\vec{\nabla}^2\phi(0)$. These may be obtained for any given phenomenological potential from a nonlocal Schrödinger-type equation [7].

DECAY RATES

The decay rate for $0^{-+}\to 2\gamma$ may be directly computed from Eqs. (26) and (37)–(40). The calculation is facilitated by the observation that, from invariance under time reversal, the crossed diagrams in Figs. 1–3 exactly double the uncrossed ones. The result is

$$(T_0+T_1+T_2)^{\mu\nu}=\frac{4\sqrt{3}}{M^{3/2}(\frac{1}{4}M^2+m^2)}\left(\phi(0)+\frac{8}{3}\frac{\vec{\nabla}^2\phi(0)}{M^2}\right)\times\epsilon^{\mu\nu\rho\lambda}q_{1\rho}q_{2\lambda}. \quad (41)$$

The factor of $\sqrt{3}$ comes from the sum over colors. The quark mass m differs from $M/2$ because of strong binding:

$$\epsilon_B=2m-M. \quad (42)$$

ϵ_B/M is of $O(v^2)$ from virial theorem, and thus of the same order of magnitude as ∇^2/M^2. From Eq. (41) it is simple to get the decay rate (excluding radiative corrections)

$$\Gamma_{0^{-+}\to 2\gamma}=\Gamma_0+\Gamma_B+\Gamma_C+\Gamma_R. \quad (43)$$

In Eq. (43), Γ_0 is the conventional result

$$\Gamma_0=\frac{12\alpha_e^2e_Q^4}{M^2}R^2(0), \quad (44)$$

where e_Q is the quark charge and $R(0)=\phi(0)\sqrt{4\pi}$. Γ_B is the correction coming from $m\neq M/2$,

$$\Gamma_B=-2\frac{\epsilon_B}{M}\Gamma_0, \quad (45)$$

and Γ_C is the term coming from differentiating the quark propagator once, and then twice:

$$\Gamma_C=\frac{16}{3M^2}\frac{\nabla^2R(0)}{R(0)}\Gamma_0. \quad (46)$$

Lowest order radiative corrections to $0^{-+}\to 2\gamma$ were calculated by Barbieri *et al.* [8] a long time ago. These are $O(v^2)$ also:

$$\Gamma_R=\frac{\alpha_s}{\pi}\left(\frac{\pi^2-20}{3}\right)\Gamma_0. \quad (47)$$

For decay into 2 gluons, the only difference in Eqs. (44)–(46) is from the color factor, but the 3 gluon vertex changes the form of the radiative correction:

$$\Gamma_{0^{-+}\to 2g}=\frac{2\alpha_s^2}{9\alpha_e^2e_Q^4}(\Gamma_0+\Gamma_B+\Gamma_C+\Gamma_R'), \quad (48)$$

where

$$\Gamma_R'=\left[\beta_0\ln\frac{\mu}{m}+\frac{159}{6}-\frac{31}{24}\pi^2-11\ \ln 2 + n_f\left(\frac{2}{3}\ln 2-\frac{8}{9}\right)\right]\frac{\alpha_s}{\pi}\Gamma_0. \quad (49)$$

The radiative corrections to the decay into gluons involves both the renormalization scale μ and the renormalization scheme; for a discussion of this point see Kwong *et al.* [9].

1^{--} DECAY

The formalism developed for two photon decay can be used quite trivially to calculate the important decay $1^{--}\to\gamma^*\to l^+l^-$. The "hard part" is the single, momentum independent vertex, $h^\mu=-ie\gamma^\mu$. There are therefore no corrections from expanding the hard part, and the amplitude analogous to Eq. (26) is simply

$$T^{\mu}=\mathrm{Tr}\langle 0|\bar{\psi}\psi|P,\epsilon\rangle h^{\mu}, \tag{50}$$

where ϵ^{μ} is the vector meson polarization vector. Going to the Coulomb gauge, and reducing the BS equation, yields the amplitude analogous to Eq. (34) with the simple replacement $\gamma_5\to\not{\epsilon}$. Using $\mathscr{C}$ and $\mathscr{P}$ invariance of the matrix element in Eq. (1), we find that, to $O(v^2)$,

$$\langle 0|\bar{\psi}\psi|P,\epsilon\rangle=\frac{1}{2}M^{1/2}\left(1+\frac{\nabla^2}{M^2}\right)\phi(0)\left(1+\frac{\not{P}}{M}\right)\not{\epsilon} - \frac{1}{2}M^{1/2}\frac{\nabla^2\phi(0)}{3M^2}\left(1-\frac{\not{P}}{M}\right)\not{\epsilon}. \tag{51}$$

This yields, for the decay to leptons,

$$\Gamma_{1^{--}\to l^+l^-}=\Gamma_{\mathrm{VW}}+\Gamma_{\mathrm{rad}}+\Gamma_{\mathrm{cor}}. \tag{52}$$

Γ_{VW} is the usual Van Royen–Weisskopf [10] formula[2]

$$\Gamma_{\mathrm{VW}}=\frac{4\alpha_e^2e_Q^2}{M^2}R^2(0), \tag{53}$$

Γ_{rad} is the radiative correction calculated some time ago by Celmaster [11],

$$\Gamma_{\mathrm{rad}}=-\frac{16}{3\pi}\alpha_s\Gamma_{\mathrm{VW}}, \tag{54}$$

and Γ_{cor} is the correction term which comes from Eqs. (50) and (51):

$$\Gamma_{\mathrm{cor}}=\frac{4}{3M^2}\frac{\nabla^2R(0)}{R(0)}\Gamma_{\mathrm{VW}}. \tag{55}$$

Although we have used the same symbol $R(r)$ for the radial wave function of the 1^{--} and 0^{-+} states, these wave functions are in principle different. We shall return to this point later.

COMPARISON

In two important previous works, $O(v^2)$ corrections to 0^{-+} and 1^{--} quarkonium decays have been evaluated. the first approach by Keung and Muzinich [7] starts from the BS equation with an instantaneous kernel. Subsequently a nonrelativistic reduction is made, followed by an expansion of the lowest order amplitude about the mass-shell value of the relative momentum $\vec{p}^{\,2}=(M/2)^2-m^2=-m\epsilon_B$. The relevant results of their work are in Table I. Their treatment does not satisfactorily resolve the issue of QCD gauge invariance of decay rates, although they do raise this question.

The second approach is that of Bodwin *et al.* [5] which builds systematically upon the rigorous formulation of nonrelativistic QCD (NRQCD) by Lepage and coworkers [6].

TABLE I. A comparison of the $O(v^2)$ correction factor, excluding radiative corrections, which multiply the zeroth order formulas for the electromagnetic decay of quarkonium states. M is the hadron mass, $\epsilon_B=2m-M$ is the binding energy, and both R and ∇^2R are evaluated at $r=0$. Note that all six entries become identical upon making the identification $\epsilon_B/M=2\nabla^2R/M^2R$.

	$0^{-+}\to 2g$	$1^{--}\to l^+l^-$
Keung *et al.* [7]	$1+\frac{2}{3}\frac{\epsilon_B}{M}$	$1+\frac{2}{3}\frac{\epsilon_B}{M}$
Bodwin *et al.* [5]	$1-\frac{2\epsilon_B}{M}+\frac{16}{3M^2}\frac{\nabla^2R}{R}$	$1-\frac{2\epsilon_B}{M}+\frac{16}{3M^2}\frac{\nabla^2R}{R}$
This work	$1-\frac{2\epsilon_B}{M}+\frac{16}{3M^2}\frac{\nabla^2R}{R}$	$1+\frac{4}{3M^2}\frac{\nabla^2R}{R}$

These authors introduce an ultraviolet cutoff Λ of $O(m)$ and then construct a NRQCD Lagrangian by successively adding new local interaction with 2-component fermion spinors. To account for annihilation into photons, higher dimensional terms involving 4 fermion operators are introduced into the Lagrangian, and their coefficients are computed in a power series in α_s by putting the annihilating quarks on mass-shell. The annihilation process, which cannot be got directly from NRQCD, is taken into account via the optical theorem which relates annihilation rates to the imaginary parts of $\bar{Q}Q\to\bar{Q}Q$ scattering amplitudes. Bodwin *et al.* [5] express their results (see Table I) in terms of nonrelativistic wave functions, their derivatives and the quark mass m. They do not use the meson mass M. However, to enable a comparison, we have expressed their results in terms of M using $\epsilon_B=2m-M$ after expanding to first order in ϵ_B/M. Note also that this definition of ϵ_B is opposite in sign to that of Keung and Muzinich [7].

The third approach is that of this paper. For completeness we summarize this too: the decay amplitude is given by the sum of all distinct Feynman diagrams leading from the initial quarkonium state to the final state. Each diagram is put into the form of a (multiple) loop integral with a kernel which is a product of a hard part and a soft part. The hard part is treated with perturbative QCD, and the soft part is analyzed into its different components with the use of Lorentz, $\mathscr{C}$, and $\mathscr{P}$ symmetries. Use of the QCD equations of motion enables separation of these components according to their importance of v. As the last step, a specific commitment to dynamics is made and the BS equation is used to express the components in the form of wave functions.

The first comment regarding the results summarized in Table I is that all six entries collapse into a single one, $1+(4/3)(\nabla^2R/M^2R)$, upon making the identification[3] $\epsilon_B/M=2\nabla^2R/M^2R$. It is interesting to note that his condition is precisely that which follows for a potential $V(\vec{r})$

[2]The lepton mass correction is simply included by multiplying Eq. (53) by $\sqrt{1-(m_l^2/M^2)}[1+(2m_l^2/M^2)]$.

[3]The relation between the binding energy and ∇^2R/R is explained briefly as a renormalization condition in Labelle *et al.* [12] in the NRQED approach [see their Eqs. (11) and (12)]. However, in our case there is no principle which *a priori* constraints ϵ_B to bear a fixed relation to ∇^2R/R, and therefore both will be considered adjustable parameters.

which has $V(0)=0$. The Schrödinger equation for this potential at $\vec{r}=0$ is

$$-\frac{2}{M}\nabla^2 R=-\epsilon_B R. \tag{56}$$

However, it is well known that at small r the potential is Coulomb-like, $V(r)\sim 1/r$. In this case the entries in Table I are not identical for arbitrary choices of ϵ_B or equivalently, the quark mass m. Furthermore, $\nabla^2 R$ is apparently singular at the origin $\nabla^2 R(r)\sim MR(r)/r$. As is clear from the uncertainty principle, the local kinetic energy becomes very large at short distances and the expansion in powers of v breaks down. This difficulty may be circumvented by imagining that annihilation takes place in a diffused region of size $O(1/m)$, i.e., that R and $\nabla^2 R$ are quantities renormalized at this scale. In any case, they are simply parameters which serve instead of the parameters in Eqs. (27)–(30).

In order to estimate the correction factors for charmonium, we used the following values of the independent parameters:

$$\alpha_s=0.19,$$

$$m=1.43 \text{ GeV},$$

$$\frac{\nabla^2 R}{R}=-0.7 \text{ GeV}^2.$$

With this particular choice of parameters and using Eqs. (43), (48), and (52), the decay rates are calculated to be

$$\Gamma(J/\psi\to e^+e^-)=5.61 \text{ keV},$$

$$\Gamma(\eta_c\to\text{hadrons})=9.99 \text{ MeV},$$

$$\Gamma(\eta_c\to 2\gamma)=6.48 \text{ keV}. \tag{57}$$

These values agree reasonably well with the experimentally measured decay widths which are [13]

$$\Gamma(J/\psi\to e^+e^-)=5.36\pm 0.28 \text{ keV},$$

$$\Gamma(\eta_c\to\text{hadrons})=10.3\pm 3.6 \text{ MeV},$$

$$\Gamma(\eta_c\to 2\gamma)=8.1\pm 2.0 \text{ keV}. \tag{58}$$

In evaluating expressions (57), the radiative corrections are calculated at the renormalization point $\mu=m$ [9]. The wave functions of J/ψ and η_c at the origin differ from each other to $O(v^2)$. This difference is neglected in taking the ratio $\nabla^2 R/R$. Their values are

$$|R_{J/\psi}|^2=0.978 \text{ GeV}^3,$$

$$|R_{\eta_c}|^2=0.936 \text{ GeV}^3.$$

One remarks concerns the value of α_s used above, which differs from the value deduced from deep inelastic scattering, $\alpha_s(m_c)\approx 0.3$. The reason is the following: the value of the parameter $\nabla^2 R/R$ depends upon the value of α_s chosen and, for smaller values of α_s, this is negative. The corresponding values of the wave functions of J/ψ and η_c at the origin should differ from each other by $O(v^2)$ by the assumptions used in this paper. However, for larger values of $\alpha_s, \nabla^2 R/R$ becomes positive and correspondingly the difference between the wave functions becomes rather large. For example, for $\alpha_s=0.24$, we have, $\nabla^2 R/R=2.8$ GeV^2, with $|R_{J/\psi}|^2=0.582$ GeV^3 and $|R_{\eta_c}|^2=0.194$ GeV^3. The difficulty in using large values of α_s has also been noted by Consoli and Field [13], and suggests that $O(\alpha_s^2)$ radiative corrections to charmonium decays may well be significant.

In conclusion, we have investigated higher order corrections to the decay of 0^{-+} and 1^{--} heavy quarkonia and shown how these corrections can be systematically incorporated in terms of various bound state matrix elements of gauge-invariant quark and gluon operators. Investigation of $\mathscr{P}=+, \mathscr{C}=+$ states, which correspond to P waves in the NR limit, is in progress. We are also currently calculating, using the framework developed in this paper, the more complicated case of the decay of negative $\mathscr{C}$ parity quarkonium into 3 gluons or photons. This will enable a more detailed comparison of theory vs experiment.

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