Chiral perturbation theory for $\tau\to\rho\pi\nu_\tau$, $\tau\to K^*\pi\nu_\tau$, and $\tau\to\omega\pi\nu_\tau$

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(Received 23 October 1995)

We use heavy vector meson $SU(2)_L\times SU(2)_R$ chiral perturbation theory to predict differential decay distributions for $\tau\to\rho\pi\nu_\tau$ and $\tau\to K^*\ \pi\nu_\tau$ in the kinematic region where $p_V\cdot p_\pi/m_V$ (here $V=\rho$ or K^*) is much smaller than the chiral symmetry-breaking scale. We also predict the rate for $\tau\to\omega\pi\nu_\tau$ in this region (now $V=\omega$). Comparing our prediction with experimental data, we determine one of the coupling constants in the heavy vector meson chiral Lagrangian.

PACS number(s): 13.35.Dx, 12.39.Fe

I. INTRODUCTION

Chiral perturbation theory provides a systematic method for describing the interactions of hadrons at low momentum. It applies not only to strong interactions of the pseudo Goldstone bosons, π, K, and η, with themselves (e.g., $\pi\pi$ scattering), but also to the interactions of the pseudo Goldstone bosons with heavy matter fields like nucleons [1] and hadrons containing a heavy charm or bottom quark [2].

Recently, chiral perturbation theory has been applied to describe strong interactions of the lowest lying vector mesons ρ, K^*, ω, and ϕ with the pseudo Goldstone bosons [3]. The vector mesons were treated as heavy and an effective Lagrangian based on the SU $(3)_L\times SU(3)_R$ chiral symmetry was given for couplings between the vector mesons and the pseudo Goldstone bosons. At leading order in the derivative expansion, the chiral Lagrangian has two coupling constants g_1 and g_2 that are related in the large N_c (i.e., number of colors) limit [4]. While it is known from the value of the octet singlet mixing angle and the smallness of the $\phi\to\rho\pi$ amplitude that the $N_c\to\infty$ relation, $g_1=2g_2/\sqrt{3}$, is a reasonable approximation, the value of g_2 has not been determined.

In this paper, we use heavy vector meson chiral perturbation theory to study the decays $\tau\to\rho\pi\nu_\tau$ and $\tau\to K^*\pi\nu_\tau$ in the kinematic regime where the pion is "soft" in the vector meson's rest frame. At the present time, there is little experimental information that bears on the applicability of chiral perturbation theory for vector meson interactions. These τ decays provide an interesting way to test whether low orders in the momentum expansion yield a good approximation. We also predict the differential decay rate for $\tau\to\omega\pi\nu_\tau$, in the kinematic regime where the pion is soft in the ω rest frame. In heavy vector meson chiral perturbation theory, this decay amplitude is dominated by a ρ pole and is proportional to g_2^2. Comparing with experimental data [5], we find that $g_2\simeq0.6$. An important aspect of this work is that we will only use chiral $SU(2)_L\times SU(2)_R$ symmetry and consequently do not treat the strange quark mass as small.

The decays $\tau\to\rho\pi\nu_\tau$, $\tau\to K^*\ \pi\nu_\tau$, and $\tau\to\omega\pi\nu_\tau$ result in final hadronic states that contain three and four pseudo Goldstone bosons. The amplitude for the vector and axial currents to produce pseudo Goldstone bosons is determined by ordinary chiral perturbation theory [6] but only in a limited kinematic region where their invariant mass is small compared with the chiral symmetry breaking scale. The situation is similar for heavy vector meson chiral perturbation theory. It partially constrains the multi pseudo Goldstone boson amplitudes in a small (but different) part of the available phase space. This paper is meant to illustrate the usefulness of heavy vector meson chiral perturbation theory for τ decay. Since the ρ and K^* widths are not negligible, a more complete calculation that includes vector meson decay and interference between different vector meson amplitudes that give the same three pseudo Goldstone boson final hadronic state may be necessary for a detailed comparison with experiment in these cases.

For the τ decays $\tau\to\rho\nu_\tau$, $\tau\to K^*\nu_\tau$, $\tau\to\rho\pi\nu_\tau$, $\tau\to K^*\pi\nu_\tau$, and $\tau\to\omega\pi\nu_\tau$, we need matrix elements of the left-handed currents $\bar{d}\gamma_\mu(1-\gamma_5)u$ and $\bar{s}\gamma_\mu(1-\gamma_5)u$ between the vacuum and a vector meson or a vector meson and a low momentum pion. In the next section, we derive the hadron level operators that represent these currents in chiral perturbation theory. Section III contains expressions for the $\tau\to\rho\pi\nu_\tau$, $\tau\to K^*\pi\nu_\tau$, and $\tau\to\omega\pi\nu_\tau$ differential decay rates. Concluding remarks are made in Sec. IV.

II. CHIRAL PERTURBATION THEORY FOR VECTOR MESONS

An effective Lagrangian based on $SU(2)_L\times SU(2)_R$ chiral symmetry that describes the interactions of ρ and K^* vector mesons with pions can be derived in the standard way. The pions are incorporated into a 2×2 special unitary matrix

$$\Sigma=\exp(2i\Pi/f), \tag{1}$$

where

$$\Pi=\begin{bmatrix}\pi^0/\sqrt{2} & \pi^+\\ \pi^- & -\pi^0/\sqrt{2}\end{bmatrix}. \tag{2}$$

Under chiral $SU(2)_L\times SU(2)_R$, $\Sigma\to L\Sigma R^\dagger$, where $L\in SU(2)_L$ and $R\in SU(2)_R$. At leading order in chiral perturbation theory, f can be identified with the pion decay constant $f_\pi\simeq132$ MeV. For describing the interactions of the pions with other fields it is convenient to introduce

$$\xi = \exp\left(\frac{i\Pi}{f}\right) = \sqrt{\Sigma}. \tag{3}$$

Under chiral $SU(2)_L \times SU(2)_R$,

$$\xi \to L\xi U^\dagger = U\xi R^\dagger, \tag{4}$$

where U is a complicated function of L, R, and the pion fields Π. However, in the special case of transformations where $L=R=V$ in the unbroken $SU(2)_V$ vector subgroup, $U=V$.

The ρ fields are introduced as a 2×2 matrix,

$$R_\mu = \begin{bmatrix} \rho_\mu^0/\sqrt{2} & \rho_\mu^+ \\ \rho_\mu^- & -\rho_\mu^0/\sqrt{2} \end{bmatrix}, \tag{5}$$

and the K^*, $\bar{K}^*$ fields as doublets:

$$K_\mu^* = \begin{bmatrix} K_\mu^{*+} \\ K_\mu^{*0} \end{bmatrix}, \quad \bar{K}_\mu^* = \begin{bmatrix} K_\mu^{*-} \\ \bar{K}_\mu^{*0} \end{bmatrix}. \tag{6}$$

Under chiral $SU(2)_L \times SU(2)_R$,

$$R_\mu \to U R_\mu U^\dagger, \quad K_\mu^* \to U K_\mu^*, \quad \bar{K}_\mu^* \to U^* \bar{K}_\mu^*. \tag{7}$$

The doublets K_μ^* and $\bar{K}_\mu^*$ are related by charge conjugation which acts on the fields as

$$C R_\mu C^{-1} = -R_\mu^T, \quad C K_\mu^* C^{-1} = -\bar{K}_\mu^*, \quad C\xi C^{-1} = \xi^T. \tag{8}$$

We construct an effective Lagrangian for strong transitions of the form $V \to V'X$, where V and V' are vector mesons and X is either the vacuum or one or more soft pions. The vector meson fields are treated as heavy with fixed four velocity v^μ, $v^2=1$, satisfying the constraint $v\cdot R = v\cdot K^* = v\cdot \bar{K}^* = 0$. The chiral Lagrange density has the general structure

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm mass} - \frac{i}{2}\mathcal{L}_{\rm width}. \tag{9}$$

The interaction terms are

$$\begin{aligned} \mathcal{L}_{\rm int} = {} & i g_2^{(\rho)} \, {\rm Tr}(\{R_\mu^\dagger, R_\nu\} A_\lambda) v_\sigma \epsilon^{\mu\nu\lambda\sigma} \\ & + i g_2^{(K^*)} \bar{K}_\mu^{*\dagger} A_\lambda^T \bar{K}_\nu^* v_\sigma \epsilon^{\mu\nu\lambda\sigma} \\ & + i g_2^{(K^*)} K_\mu^{*\dagger} A_\lambda K_\nu^* v_\sigma \epsilon^{\mu\nu\lambda\sigma}, \end{aligned} \tag{10}$$

where

$$A_\lambda = \frac{i}{2}(\xi\partial_\lambda \xi^\dagger - \xi^\dagger \partial_\lambda \xi). \tag{11}$$

Comparing with the Lagrange density in Eq. (11) of Ref. [3], we find that in the case of $SU(3)_L \times SU(3)_R$ symmetry $g_2^{(K*)} = g_2$, at leading order in $SU(3)_L \times SU(3)_R$ chiral perturbation theory. Note that for the vector mesons $\rho_\mu^{-\dagger} \neq \rho_\mu^+$, etc. In heavy vector meson chiral perturbation theory, ρ_μ^+ destroys a ρ^+, but it does not create the corresponding antiparticle. The field $\rho_\mu^{-\dagger}$ creates a ρ^-.

The kinetic terms are

$$\begin{aligned} \mathcal{L}_{\rm kin} = {} & -i\,{\rm Tr} R_\mu^\dagger v\cdot\partial R^\mu - i\,{\rm Tr} R_\mu^\dagger [v\cdot V, R^\mu] - i K_\mu^{*\dagger} v\cdot\partial K^{*\mu} \\ & - i K_\mu^{*\dagger} v\cdot V K^{*\mu} - i \bar{K}_\mu^{*\dagger} v\cdot \partial \bar{K}^{*\mu} + i \bar{K}_\mu^{*\dagger} v\cdot V^T \bar{K}^{*\mu}, \end{aligned} \tag{12}$$

where

$$V_\nu = \frac{1}{2}(\xi\partial_\nu \xi^\dagger + \xi^\dagger \partial_\nu \xi). \tag{13}$$

The mass terms are

$$\begin{aligned} \mathcal{L}_{\rm mass} = {} & \lambda_2^{(\rho)} \, {\rm Tr}(\{R_\mu^\dagger, R^\mu\} M_\xi) + \lambda_2^{(K^*)} K_\mu^{*\dagger} M_\xi K^{*\mu} + \lambda_2^{(K^*)} \bar{K}_\mu^{*\dagger} M_\xi^T \bar{K}^{*\mu} + \sigma_8^{(\rho)} \, {\rm Tr}(M_\xi) {\rm Tr}(R_\mu^\dagger R^\mu) + \sigma_8^{(K^*)} \, {\rm Tr}(M_\xi) K_\mu^{*\dagger} K^{*\mu} \\ & + \sigma_8^{(K^*)} \, {\rm Tr}(M_\xi) \bar{K}_\mu^{*\dagger} \bar{K}^{*\mu}. \end{aligned} \tag{14}$$

In Eq. (14)

$$M_\xi = \frac{1}{2}(\xi M \xi + \xi^\dagger M \xi^\dagger), \tag{15}$$

where $M = {\rm diag}(m_u, m_d)$ is the 2×2 quark mass matrix. At leading order in $SU(3)_L \times SU(3)_R$ chiral perturbation theory, the couplings in Eq. (14) are related to those in Ref. [3] by

$$\lambda_2^{(\rho)} = \lambda_2^{(K^*)} = \lambda_2 \quad \text{and} \quad \sigma_8^{(\rho)} = \sigma_8^{(K^*)} = \sigma_8. \tag{16}$$

The ρ and K^* are not stable. In heavy vector meson chiral perturbation theory, their widths appear as anti-Hermitian terms in the Lagrange density (9). Since the ρ and K^* widths vanish in the large N_c (i.e., number of colors) limit and are comparable with the pion mass, we treat the widths as of order one derivative [the mass terms in (14) go like two derivatives and are less important in chiral perturbation theory than the terms in $\mathcal{L}_{\rm kin}$, $\mathcal{L}_{\rm int}$, and $\mathcal{L}_{\rm width}$]. The width terms are

$$\mathcal{L}_{\rm width} = \Gamma^{(\rho)} \, {\rm Tr}\, R_\mu^\dagger R^\mu + \Gamma^{(K^*)} K_\mu^{*\dagger} K^{*\mu} + \Gamma^{(K^*)} \bar{K}_\mu^{*\dagger} \bar{K}^{*\mu}. \tag{17}$$

In the SU(3) limit $\Gamma^{(\rho)} = \Gamma^{(K^*)}$, however, the physical values of the widths $\Gamma^{(\rho)} = 151$ MeV and $\Gamma^{(K^*)} = 50$ MeV are far from this situation. In heavy vector meson chiral perturbation theory, the vector meson propagator is

$$\frac{-i(g^{\mu\nu}-v^\mu v^\nu)}{v\cdot k+i\Gamma/2}, \tag{18}$$

where Γ is the corresponding width. Note that we are treating the vector meson widths differently than Ref. [3]. In Ref. [3], chiral $SU(3)_L\times SU(3)_R$ was used and since the vector meson widths are small compared with the kaon mass they were treated as of the order of a light quark mass or, equivalently, two derivatives. Hence, in Ref. [3], the widths could be neglected in the propagator at leading order in chiral perturbation theory.

At the quark level, the effective Hamiltonian density for weak semileptonic τ decay is

$$H_W=\frac{G_F}{\sqrt{2}}\,V_{ud}\bar{\nu}_\tau\gamma_\mu(1-\gamma_5)\tau\bar{d}\gamma^\mu(1-\gamma_5)u$$
$$+\frac{G_F}{\sqrt{2}}\,V_{us}\bar{\nu}_\tau\gamma_\mu(1-\gamma_5)\tau\bar{s}\gamma^\mu(1-\gamma_5)u, \tag{19}$$

where G_F is the Fermi constant and V_{ud} and V_{us} are elements of the Cabibbo-Kobayashi-Maskawa matrix where, experimentally, $|V_{ud}|\simeq 1$ and $|V_{us}|\simeq 0.22$. At leading order in chiral perturbation theory, we need to represent the currents $\bar{d}\gamma^\mu(1-\gamma_5)u$ and $\bar{s}\gamma^\mu(1-\gamma_5)u$ by operators involving the hadron fields that transform respectively as $(3_L,1_R)$ and $(2_L,1_R)$ under chiral $SU(2)_L\times SU(2)_R$ and contain the least number of derivatives or insertions of the light quark mass matrix. These operators are

$$\bar{d}\gamma_\mu(1-\gamma_5)u=\frac{f_\rho}{\sqrt{2}m_\rho}\,\mathrm{Tr}\,R_\mu^\dagger\xi^\dagger\begin{pmatrix}0&0\\1&0\end{pmatrix}\xi \tag{20a}$$

and

$$\bar{s}\gamma_\mu(1-\gamma_5)u=\frac{f_{K^*}}{\sqrt{2}m_{K^*}}\,\bar{K}_\mu^{*\dagger}\xi^T\begin{pmatrix}1\\0\end{pmatrix}. \tag{20b}$$

The coefficients are fixed in terms of the vector meson decay constants f_ρ and f_{K^*} by the matrix elements $\langle K^{*-}|\bar{s}\gamma_\mu(1-\gamma_5)u|0\rangle$ and $\langle\rho^-|\bar{d}\gamma_\mu(1-\gamma_5)u|0\rangle$, which are equal to $f_{K^*}\epsilon_\mu^*$ and $f_\rho\epsilon_\mu^*$, respectively, and follow from Eqs. (20a) and (20b) by setting ξ equal to unity. (Note that because of the parity invariance of the strong interactions the axial currents do not contribute to these matrix elements.)

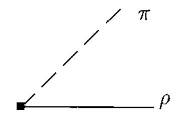

FIG. 1. Feynman diagram representing the matrix element of the left-handed current from the vacuum to $\rho\pi$. In this case, only the axial current contributes.

In the large N_c limit, couplings involving the ω are related to those involving the ρ. They can be derived from the Lagrange densities in Eqs. (10), (12), (14) and the expression for the current in Eq. (20a) by replacing the isospin triplet matrix R_μ by the quartet matrix

$$Q_\mu=\begin{bmatrix}\rho_\mu^0/\sqrt{2}+\omega_\mu/\sqrt{2} & \rho_\mu^+\\ \rho_\mu^- & -\rho_\mu^0/\sqrt{2}+\omega_\mu/\sqrt{2}\end{bmatrix}. \tag{21}$$

However, the effect of the ω width cannot be included by replacing R_μ in Eq. (17) with Q_μ. Since the widths vanish in the large N_c limit, a separate term $\Gamma^{(\omega)}\omega_\mu^\dagger\omega^\mu$ must be added to Eq. (17). Experimentally, $\Gamma^{(\omega)}=8.4$ MeV. Note that for the ρ alone the term proportional to $g_2^{(3)}$ in Eq. (10) vanishes by G parity. Since our predictions do not rely on the couplings in Eq. (14) they do not depend on the large N_c limit.

III. DIFFERENTIAL DECAY RATES

The amplitude for $\tau\to\rho\pi\nu_\tau$ follows from the Feynman diagram for the vacuum to $\rho\pi$ matrix element of the current shown in Fig. 1. Note that there is no pole diagram since the Lagrange density (10) has no $\rho\rho\pi$ coupling. The invariant matrix element is

$$\mathscr{M}(\tau\to\rho^0\pi^-\nu_\tau)=\frac{G_F V_{ud}f_\rho}{f_\pi}\,\bar{u}_\nu\gamma^\mu\epsilon_\mu^*(\rho)(1-\gamma_5)u_\tau, \tag{22}$$

where $u_{\nu,\tau}$ are four component spinors for the neutrino and τ.

It is convenient to express the differential decay distribution in terms of the $\rho\pi$ mass $s=(p_\rho+p_\pi)^2$ and the angle θ between the ρ direction and the τ direction in the $\rho-\pi$ center of mass frame. Then the differential decay rate is

$$\frac{d\Gamma(\tau\to\rho^0\pi^-\nu_\tau)}{ds\,d\cos\theta}=\frac{G_F^2|V_{ud}|^2 f_\rho^2 m_\tau}{2^7 f_\pi^2\pi^3}\left(1-\frac{s}{m_\tau^2}\right)\frac{\sqrt{(s-m_\rho^2+m_\pi^2)^2-4m_\pi^2 s}}{4s^2}\,[A(s)+B(s)\cos\theta+C(s)\cos^2\theta], \tag{23}$$

where the dimensionless functions $A(s)$, $B(s)$, and $C(s)$ are

$$A(s)=\frac{1}{8s^2m_\rho^2}\left(1-\frac{s}{m_\tau^2}\right)[(s+m_\rho^2-m_\pi^2)^2(s+m_\tau^2)+4s^2m_\rho^2], \tag{24a}$$

$$B(s)=-\frac{m_\tau^2}{4s^2m_\rho^2}\left(1-\frac{s}{m_\tau^2}\right)(s+m_\rho^2-m_\pi^2)\sqrt{(s-m_\rho^2+m_\pi^2)^2-4m_\pi^2 s}, \tag{24b}$$

and

$$C(s)=\frac{m_\tau^2}{8s^2m_\rho^2}\left(1-\frac{s}{m_\tau^2}\right)^2[(s-m_\rho^2+m_\pi^2)^2-4m_\pi^2 s]. \qquad (24c)$$

The differential decay rate is the same for $\rho^-\pi^0$ mode. Our expression for the invariant matrix element in Eq. (22) was derived using heavy vector meson chiral perturbation theory, which is an expansion in m_π/m_p and $v\cdot p_\pi/m_\rho$. In A, B, and C, terms suppressed by powers of these quantities should be neglected. To focus on the kinematic region where chiral perturbation theory is valid, it is convenient to change from the variable s to the dimensionless variable $x=v\cdot p_\pi/m_\pi$, using

$$s=m_\rho^2+m_\pi^2+2m_\pi m_\rho x. \qquad (25)$$

Then expanding in (m_π/m_ρ),

$$A(x)\simeq\frac{1}{2}\left(1-\frac{m_\rho^2}{m_\tau^2}\right)\left[2+\frac{m_\tau^2}{m_\rho^2}\right], \qquad (26a)$$

$$B(x)\simeq-\left(\frac{m_\pi}{m_\rho}\right)\left(\frac{m_\tau^2}{m_\rho^2}-1\right)\sqrt{x^2-1}, \qquad (26b)$$

and

$$C(x)\simeq\frac{1}{2}\left(\frac{m_\pi^2}{m_\rho^2}\right)\left(\frac{m_\tau^2}{m_\rho^2}\right)\left(1-\frac{m_\rho^2}{m_\tau^2}\right)^2(x^2-1). \qquad (26c)$$

Hence, B and C are negligible compared with A and our expression for the differential decay rate becomes

$$\frac{d\Gamma(\tau\to\rho^0\pi^-\nu_\tau)}{dx\,d\cos\theta}=\frac{G_F^2|V_{ud}|^2f_\rho^2m_\tau m_\pi^2}{2^7f_\pi^2\pi^3}\sqrt{x^2-1}\left(1-\frac{m_\rho^2}{m_\tau^2}\right)^2\times\left[2+\frac{m_\tau^2}{m_\rho^2}\right]. \qquad (27)$$

Normalizing to the $\tau\to\rho^-\nu_\tau$ width gives the simple expression

$$\frac{1}{\Gamma(\tau\to\rho^-\nu_\tau)}\frac{d\Gamma(\tau\to\rho^0\pi^-\nu_\tau)}{dx}=\left(\frac{m_\pi}{f_\pi}\right)^2\frac{\sqrt{x^2-1}}{4\pi^2}. \qquad (28)$$

It seems reasonable that lowest order chiral perturbation theory will be a useful approximation in the region $x\in[1,2]$. Integrating x over this region gives a $\tau\to\rho^0\pi^-\nu_\tau$ width that is 0.03 times the $\tau\to\rho^-\nu_\tau$ width.

The amplitude for $\tau\to K^*\pi\nu_\tau$ follows from the Feynman diagrams for the vacuum to $K^*\pi$ matrix element of the left-handed current shown in Fig. 2. In this case, there is a pole contribution proportional to the $K^*K^*\pi$ coupling $g_2^{(K^*)}$. The resulting invariant matrix element is

$$\mathscr{M}(\tau\to\bar{K}^{*0}\pi^-\nu_\tau)=\frac{G_FV_{us}f_{K^*}}{\sqrt{2}\,f_\pi}\bar{u}_\nu\gamma_\mu(1-\gamma_5)u_\tau\left[\epsilon^{*\mu}(K^*)+\frac{ig_2^{(K^*)}\epsilon^{\nu\mu\beta\sigma}}{(v\cdot p_\pi+i\Gamma^{(K^*)}/2)}p_{\pi\beta}v_\sigma\epsilon_\nu^*(K^*)\right]. \qquad (29)$$

The term proportional to $g_2^{(K^*)}$ arises from the pole diagram and it corresponds to the p-wave $K^*\pi$ amplitude. In the nonrelativistic constituent quark model [7] $g_2^{(K^*)}=1$. Following the same procedure as for the $\tau\to\rho\pi\nu_\tau$ case, we arrive at the differential decay rate

$$\frac{d\Gamma(\tau\to\bar{K}^{*0}\pi^-\nu_\tau)}{dx\,d\cos\theta}=\frac{G_F^2|V_{us}|^2f_{K^*}^2m_\pi^2m_\tau}{2^8f_\pi^2\pi^3}\sqrt{x^2-1}\left(1-\frac{m_{K^*}^2}{m_\tau^2}\right)^2\left\{\left(\frac{m_\tau^2}{m_{K^*}^2}+2\right)+\frac{g_2^{(K^*)2}}{x^2(1+\gamma^2)}\left(\frac{m_\tau^2}{m_{K^*}^2}+1\right)(x^2-1)+\frac{4g_2^{(K^*)}}{x(1+\gamma^2)}\sqrt{x^2-1}\cos\theta-\frac{{g_2^{(K^*)}}^2}{x^2(1+\gamma^2)}(x^2-1)\left(\frac{m_\tau^2}{m_{K^*}^2}-1\right)\cos^2\theta\right\}. \qquad (30)$$

In Eq. (30),

$$\gamma=\Gamma^{(K^*)}/(2xm_\pi). \qquad (31)$$

In this case, $s=m_{K^*}^2+m_\pi^2+2m_\pi m_{K^*}x$. The rate for $\tau\to K^{*-}\pi^0\nu_\tau$ is one-half the rate for $\tau\to\bar{K}^{*0}\pi^-\nu_\tau$.

Normalizing to the $\tau\to K^{*-}\nu_\tau$ decay width and integrating over $x\in[1,2]$, Eq. (30) gives

$$\frac{1}{\Gamma(\tau\to K^{*-}\nu_\tau)}\int_1^2 dx\,\frac{d\Gamma(\tau\to\bar{K}^{*0}\pi^-\nu_\tau)}{dx\,d\cos\theta}\simeq7.5\times10^{-3}[(1+0.48{g_2^{(K^*)}}^2)+0.51g_2^{(K^*)}\cos\theta-0.28{g_2^{(K^*)}}^2\cos^2\theta]. \qquad (32)$$

The shape of the $\bar{K}^{*0}\pi^-\nu_\tau$ decay distribution in cos θ depends on the value of $g_2^{(K^*)}$ and it may be possible at a τ-charm or B factory to determine this coupling from a study of $\tau\to K^*\pi\nu_\tau$ decay. In the SU(3) and large N_c limits $g_2^{(K^*)}=g_2^{(\rho)}$ and in what follows we discuss how to determine $g_2^{(\rho)}$.

Using heavy vector meson chiral perturbation theory the amplitude for $\tau\to\omega\pi\nu_\tau$ follows from the Feynman diagram for vacuum to $\omega\pi$ matrix element of the left-handed current in Fig. 3. In this case, there is only a pole graph and the invariant matrix element is

$$\mathscr{M}(\tau\to\omega\pi^-\nu_\tau)=\frac{G_F V_{ud} f_\rho}{f_\pi}\,\bar{u}_\nu\gamma_\mu(1-\gamma_5)u_\tau\left[\frac{ig_2^{(\rho)}}{(v\cdot p_\pi+i\Gamma^{(\rho)}/2)}\,\epsilon^{\nu\mu\beta\sigma}p_{\pi\beta}v_\sigma\epsilon_\nu^*(\omega)\right]. \tag{33}$$

Here, the difference between the ρ and ω masses is neglected as is appropriate in the large N_c limit. The resulting differential decay rate is

$$\frac{d\Gamma(\tau\to\omega\pi^-\nu_\tau)}{dx d\cos\theta}=\frac{G_F^2|V_{ud}|^2 f_\rho^2 m_\pi^2 m_\tau}{2^7 f_\pi^2\pi^3}(x^2-1)^{3/2}\left(1-\frac{m_\omega^2}{m_\tau^2}\right)^2\frac{g_2^{(\rho)^2}}{x^2(1+\gamma^2)}\left[\left(\frac{m_\tau^2}{m_\omega^2}+1\right)-\left(\frac{m_\tau^2}{m_\omega^2}-1\right)\cos^2\theta\right] \tag{34}$$

where now

$$\gamma=\Gamma^{(\rho)}/(2xm_\pi), \tag{35}$$

and $s=m_\omega^2+m_\pi^2+2m_\omega m_\pi x$. Integrating over cos θ and dividing by the rate for $\tau\to\rho\nu_\tau$ give (again we neglect the difference between the ρ and ω masses) the simple expression

$$\frac{1}{\Gamma(\tau\to\rho^-\nu_\tau)}\frac{d\Gamma(\tau\to\omega\pi^-\nu_\tau)}{dx}=\left(\frac{m_\pi}{f_\pi}\right)^2\frac{(x^2-1)^{3/2}g_2^{(\rho)^2}}{6\pi^2x^2(1+\gamma^2)}. \tag{36}$$

Reference [5] plots the differential decay rate as a function of the $\omega\pi$ invariant mass [see Fig. 3(b)]. The first bin corresponds to $x\leqslant 1.7$. Integrating the $\tau\to\omega\pi\nu_\tau$ differential decay rate over $x\in[1, 1.7]$ and comparing with the experimental rate in this region [8] give $|g_2^{(\rho)}|\simeq 0.57$. If both the first and second bins are included the region corresponds to $x\in[1, 2.7]$ and integrating over this region gives $|g_2^{(\rho)}|\simeq 0.65$. It is not likely that lowest order chiral perturbation theory will be a good approximation for values of x greater than this.

IV. CONCLUDING REMARKS

In this paper, we have studied the decay modes $\tau\to\rho\pi\nu_\tau$, $\tau\to K^*\pi\nu_\tau$, and $\tau\to\omega\pi\nu_\tau$, using heavy vector meson chiral perturbation theory. Equations (27), (30), and (34) are our main results. Our predictions are valid in the kinematic region where the pion is soft in the vector meson rest frame. For these modes, vector meson decay results in three or four pseudo Goldstone boson hadronic final states, and heavy vector meson chiral perturbation theory restricts these amplitudes in a small part of phase space. This is similar to applications of ordinary chiral perturbation theory which are valid in a different small kinematic region.

Modes similar to those discussed in this paper, such as $\tau\to\rho K\nu_\tau$, can also be studied, using chiral perturbation theory. They will be related to those we considered in chiral $SU(3)_L\times SU(3)_R$. Using chiral $SU(3)_L\times SU(3)_R$, the left-handed current $J^A_{L\lambda}=\bar{q}T^A\gamma\lambda(1-\gamma_5)q$ is represented by

$$J^A_{L\lambda}=\frac{f_V}{\sqrt{2m_V}}\,\mathrm{Tr}(O^\dagger_\lambda\xi^\dagger T^A\xi), \tag{37}$$

where O_λ is the 3×3 octet matrix of vector meson fields.

We found a branching ratio for $\tau\to\rho^0\pi^-\nu_\tau$ in the region where the hadronic mass satisfies $m_{\rho\pi}<1022$ MeV, of 0.69%, and a branching ratio for $\tau\to\bar{K}^{*0}\pi^-\nu_\tau$ in the region $m_{K^*\pi}<1151$ MeV, of $(0.02+0.008g_2^{(K^*)^2})$%. It may be possible to study $\tau\to K^*\pi\nu_\tau$ decay in the kinematic region where chiral perturbation theory is valid at a τ-charm or B factory [9].

In τ decay, the $\rho\pi$ final hadronic states get a significant contribution from the $a_1(1260)$ resonance which has a large width of around 400 MeV, while $K^*\pi$ final states get contri-

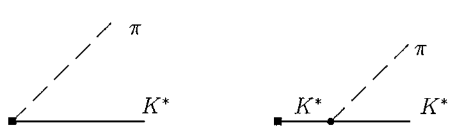

FIG. 2. Feynman diagrams representing the matrix element of the left-handed current from the vacuum to $K^*\pi$. For the first diagram, the axial vector current contributes while for the second pole diagram, the vector current contributes.

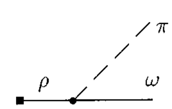

FIG. 3. Feynman diagram representing the matrix element of the left-handed current from the vacuum to $\omega\pi$. In this case, only the vector current contributes.

butions from the $K_1(1270)$, $K_1(1400)$, and $K^*(1410)$ which have widths of 90 MeV, 174 MeV, and 227 MeV, respectively. Since in our formulation of chiral perturbation theory these heavier resonances are integrated out, one can take the view that the "tails" of their contributions are constrained by our results. Note that the $K_1(1270)$ has a branching ratio of only 16% to $K^*\pi$.

The narrow width of the ω makes $\tau\to\omega\pi\nu_\tau$ easier to study experimentally than $\tau\to\rho\pi\nu_\tau$. Using the heavy vector meson chiral perturbation theory, we predicted the differential decay rate for $\tau\to\omega\pi\nu_\tau$ in the kinematic region where the pion is soft in the ω rest frame. Comparing with experimental data, we find that the $\rho\omega\pi$ coupling $|g_2^{(\rho)}|\simeq 0.6$. $\tau\to\omega\pi\nu_\tau$ decay proceeds via the vector part of the weak current and the rate for this decay is related by isospin to the $e^+e^-\to\omega\pi^0$ cross section. Experimental data [10] on $e^+e^-\to\omega\pi^0$ lead to a comparable value for $g_2^{(\rho)}$.

Our predictions for τ decay amplitudes get corrections suppressed by just $\sim v\cdot p_\pi/(1\text{ GeV})$ from operators with one derivative (e.g., $\mathrm{Tr}O_\lambda^\dagger v\cdot A\xi^\dagger T^A\xi$) that occur in the left-handed current. This is different from pseudo Goldstone boson self-interactions where corrections to leading order results are suppressed by $p^2/(1\text{ GeV}^2)$, where p is a typical momentum. Hence, even in the region $1<v\cdot p_\pi/m_\pi<2$, we expect sizable corrections to our results. This is particularly true for the $\rho\pi$ case where this region overlaps with a significant part of the a_1 Breit-Wigner distribution.

We have applied heavy vector meson chiral perturbation theory to τ decay and used data on $\tau\to\omega\pi\nu_\tau$ to determine the magnitude of the coupling g_2 in the chiral Lagrangian. The value we extract, $|g_2|\simeq 0.6$, is not too far from the prediction, $g_2=0.75$, of the chiral quark model [11]. The value of g_2 is relevant for other processes of experimental interest. For example, heavy vector meson chiral perturbation theory can be used to predict differential decay rates for $D\to K^*\pi e^+\nu_e$ in the kinematic region where both $p_D\cdot p_\pi/m_D$ and $p_{K^*}\cdot p_\pi/m_{K^*}$ are small compared with the chiral symmetry breaking scale. Here, one combines chiral perturbation theory for hadrons containing a heavy quark [2] with heavy vector meson chiral perturbation theory.

ACKNOWLEDGMENTS

We thank A. Weinstein, J. Urheim, A. Manohar, and A. Leibovich for useful discussions. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-92-ER40701.

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[8] In Fig. 3(b) of Ref. [5] the function v is plotted. Using Eq. (34) our prediction for the spectral function v as a function of $x=(m_{\omega\pi}^2-m_\omega^2-m_\pi^2)/2m_\pi m_\omega$ is

$$v(x)=\frac{1}{6\pi}\left(\frac{m_\pi}{m_\omega}\right)\left(\frac{f_\rho}{f_\pi}\right)^2\frac{1}{m_\omega^2}\frac{(x^2-1)^{3/2}}{x^2(1+\gamma^2)}\,g_2^{(\rho)^2}.$$

Experimentally, the ρ decay constant is $f_\rho\simeq(407\text{ MeV})^2$. In Fig. 3(b), in the first bin, $0.9\text{ GeV}<m_{\omega\pi}<1.0\text{ GeV}$, $v=(0.0029\pm0.0004)$, and in the second bin $1\text{ GeV}<m_{\omega\pi}<1.1\text{ GeV}$, $v=(0.0156\pm0.0018)$. These errors are only statistical.

[9] For a recent discussion of data on $K\pi\pi$ hadronic final states, see CLEO Collaboration, J. G. Smith, in *Tau 94*, Proceedings of the Third Workshop on Tau Lepton Physics, Montreux, Switzerland, edited by L. Rolandi [Nucl. Phys. B (Proc. Suppl.) **40**, 351 (1995)]; CLEO Collaboration, M. Battle *et al.*, Phys. Rev. Lett. **73**, 1079 (1994). There, it is reported that $B(\tau\to\bar{K}^0\pi^-\pi^0\nu_\tau)=(0.39\pm0.06\pm0.06)\%$; $B(\tau\to K^-\pi^0\pi^0\nu_\tau)=(0.14\pm0.1\pm0.03)\%$.

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