Nonfactorization in hadronic two-body Cabibbo-favored decays of D^0 and D^+

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With the inclusion of nonfactorized amplitudes in a scheme with $N_c = 3$, we studied Cabibbo-favored decays of D^0 and D^+ into two-body hadronic states involving two isospins in the final state. We have shown that it is possible to understand the measured branching ratios and determine the sizes and signs of nonfactorized amplitudes required.

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I. INTRODUCTION

In recent past there has been a growing interest [1–7] in exploring the role played by nonfactorized terms in the hadronic decays of charmed and bottom mesons. References [1] and [2] have endeavored to calculate the nonfactorized contribution to two-body hadronic decays of the B meson. These calculations lend support to the $N_c \to \infty$ rule in two-body hadronic B decays. Experimentally, however, the evidence in support [8] of the $N_c \to \infty$ rule which appeared to be there in the earlier B-decay data has since weakened [9] and the sign of the phenomenological parameter a_2 appears to be positive [9], contrary to the prediction of the $N_c \to \infty$ rule.

More recently, the view that the phenomenological parameters a_1 and a_2 are effective and process dependent has been pursued further [3–7]. The effective a_1 and a_2, evaluated with $N_c = 3$, depend on the nonfactorized contribution. In particular, it was shown in Ref. [5] how the conundrum of the failure [10] of all popular models to explain the longitudinal polarization fraction in $B^0 \to \psi \bar{K}^{*0}$ could be resolved in a scheme that uses $N_c = 3$ but allows a small nonfactorized amplitude. This idea was carried over to the charm sector in Ref. [6] where it was shown that with $N_c = 3$ allowing nonfactorized terms somewhat larger than in B decays (by nonfactorized terms "large" or "small" we mean: in relation to factorized terms), one could understand data in $D_s^+ \to \phi\pi^+$, $\phi\rho^+$ and $\phi l^+ \nu_l$ decays. The introduction and description of nonfactorized terms is purely phenomenological in Refs. [5, 6] as is also the case in [3, 4, 7]. No attempt is made to calculate the nonfactorized terms but, rather, the emphasis is to glean some systematic behavior of these terms so that more can be learned about them in future.

With this objective, we have studied those hadronic two-body Cabibbo-favored decays of D^0, D^+ mesons that involve two isospins in the final state in $N_c = 3$ scheme. These decays are $D \to \bar{K}\pi$, $\bar{K}^*\pi$, $\bar{K}\rho$, $\bar{K}a_1$, and $\bar{K}^*\rho$. By fitting data, we have calculated the size and the sign of the nonfactorized term in each decay. Annihilation terms, wherever permitted, have been neglected in D^0 decays because of the smallness of a_2 $(= C_2 + \frac{C_1}{N_c})$ for $N_c = 3$ and other reasons as argued in the text. We have included final-state interaction phases in all decays except $\bar{K}^*\rho$ for reasons we elaborate later. However, we have neglected inelastic final state interactions because of the ignorance of the rescattering parameters to be used in such an analysis.

For decays involving a single Lorentz scalar structure, such as $D \to \bar{K}\pi$, $\bar{K}^*\pi$, $\bar{K}\rho$, and $\bar{K}a_1$, one can extract effective a_1 and a_2 which we show to be process dependent. We also argue that color-suppressed decays are more likely to reveal presence or otherwise of nonfactorized effects.

This paper is organized as follows: Sec. II contains the conventions and definitions used throughout. We discuss the decays $D \to \bar{K}\pi$ in Sec. III, $D \to \bar{K}^*\pi$, $\bar{K}\rho$, $\bar{K}a_1$ in Sec. IV, and $D \to \bar{K}^*\rho$ in Sec. V. The results are discussed in Sec. VI.

II. DEFINITIONS

The effective Hamiltonian for Cabibbo-favored hadronic charm decays is given by

$$H_w = \tilde{G}_F \left\{ C_1 (\bar{u}d)(\bar{s}c) + C_2 (\bar{u}c)(\bar{s}d) \right\} , \quad (1)$$

where $\tilde{G}_F = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^*$ and $(\bar{u}d)$, etc., represent color-singlet $(V-A)$ Dirac currents. C_1 and C_2 are the Wilson coefficients for which we adopt the values

$$C_1 = 1.26 \pm 0.04 , \qquad C_2 = -0.51 \pm 0.05 . \quad (2)$$

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The central values of C_1 and C_2 are taken from Ref. [8] and the errors are ours.

Fierz transforming the product of two Dirac currents of Eq. (1) in N_c-color-space, we get

$$(\bar{u}c)(\bar{s}d) = \frac{1}{N_c}(\bar{u}d)(\bar{s}c) + \frac{1}{2}\sum_{a=1}^{8}(\bar{u}\lambda^a d)(\bar{s}\lambda^a c) \quad (3)$$

and an analogous relation for $(\bar{u}d)(\bar{s}c)$, where λ^a are the Gell-Mann matrices. Using Eq. (3) and its analogue, we reduce the effective Hamiltonian of Eq. (1) to the forms

$$H_w^{\rm CF} = \tilde{G}_F\left\{a_1(\bar{u}d)(\bar{s}c) + C_2 H_w^8\right\} \quad (4)$$

and

$$H_w^{\rm CS} = \tilde{G}_F\left\{a_2(\bar{u}c)(\bar{s}d) + C_1 \tilde{H}_w^8\right\} \quad (5)$$

to describe color-favored (CF) and color-suppressed (CS) decays, respectively. The matrix elements of the first terms in (4) and (5) are expected to be dominated by factorized contributions; any nonfactorized part arising from them is parametrized as detailed in the text. The second terms $H_w^{(8)}[\equiv \frac{1}{2}\sum(\bar{u}\lambda^a d)(\bar{s}\lambda^a c)]$ and $\tilde{H}_w^{(8)}[\equiv \frac{1}{2}\sum(\bar{u}\lambda^a c)(\bar{s}\lambda^a d)]$ involving color-octet currents generate nonfactorized contributions. We have defined here, for $N_c = 3$,

$$a_1 = C_1 + \frac{C_2}{3} = 1.09 \pm 0.04\,,$$
$$a_2 = C_2 + \frac{C_1}{3} = -0.09 \pm 0.05\,. \quad (6)$$

It should be obvious from (4) and (5) that nonfactorized effects are more likely to manifest themselves in color-suppressed decays than in color-favored decays because of the fact that C_1 is much larger than a_2 in magnitude.

Further, in calculating the factorized amplitudes, we use the following matrix elements [8, 11] for the weak vector (j_μ^V) and axial vector (j_μ^A) currents between the vacuum and the pseudoscalar (P), vector (V), and axial (A) vector states

$$\langle V(p,\varepsilon)|j_\mu^V|0\rangle = \varepsilon_\mu^* m_V f_V\,,$$
$$\langle A(p,\varepsilon)|j_\mu^A|0\rangle = \varepsilon_\mu^* m_A f_A\,,$$
$$\langle P(p)|j_\mu^A|0\rangle = -if_P p_\mu\,, \quad (7)$$

and the form factors for the transition of a pseudoscalar meson (M) to pseudoscalar (P) and vector (V) mesons,

$$\langle P(p')|j_\mu^V|M(p)\rangle = \left\{(p+p')_\mu - \frac{m_M^2 - m_P^2}{q^2}q_\mu\right\}F_1^{MP}(q^2) + \frac{m_M^2 - m_P^2}{q^2}q_\mu F_0^{MP}(q^2)\,, \quad (8)$$

$$\langle V(p',\varepsilon)|j_\mu^V - j_\mu^A|M(p)\rangle = i\left\{(m_M + m_V)\varepsilon_\mu^* A_1^{MV}(q^2) - \frac{\varepsilon^*\cdot q}{m_M + m_V}(p+p')_\mu A_2^{MV}(q^2) - 2m_V\frac{\varepsilon^*\cdot q}{q^2}q_\mu\left[A_3^{MV}(q^2) - A_0^{MV}(q^2)\right]\right\} + \frac{2}{m_M + m_V}\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho p'^\sigma V^{MV}(q^2)\,, \quad (9)$$

where $q_\mu = (p-p')_\mu$. In addition, the following constraint applies at all q^2:

$$2m_V A_3^{MV}(q^2) = (m_M + m_V)A_1^{MV}(q^2) - (m_M - m_V)A_2^{MV}(q^2)\,. \quad (10)$$

The following relations are also needed to cancel the poles at $q^2 = 0$:

$$F_0^{MP}(0) = F_1^{MP}(0)\,, \qquad A_0^{MV}(0) = A_3^{MV}(0). \quad (11)$$

For an axial vector meson, A, we define, analogously to (9),

$$\langle A(p',\varepsilon)|j_\mu^V - j_\mu^A|M(p)\rangle = -i\left\{(m_M + m_A)\varepsilon_\mu^* V_1^{MA}(q^2) - \frac{\varepsilon^*\cdot q}{m_M + m_A}(p+p')_\mu V_2^{MA}(q^2) - 2m_A\frac{\varepsilon^*\cdot q}{q^2}q_\mu\left[V_3^{MA}(q^2) - V_0^{MA}(q^2)\right]\right\} - \frac{2}{m_M + m_A}\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho p'^\sigma A^{MA}(q^2)\,, \quad (12)$$

with the same conditions (10) and (11) imposed on $V_i^{MA}(q^2)$.

The branching ratio for $M \to P_1P_2$, where P_1 and P_2 are pseudoscalar mesons, is given by

$$B(M \to P_1P_2) = \tau_M\frac{|\vec{p}|}{8\pi m_M^2}|A(M \to P_1P_2)|^2\,, \quad (13)$$

and that for $M \to V_1V_2$, where V_1 and V_2 are vector mesons, is written as

$$B(M \to V_1 V_2) = \tau_M \frac{|\vec{p}|}{8\pi m_M^2} \sum_\lambda |A(M \to V_1 V_2)_{\lambda\lambda}|^2 \ , \quad (14)$$

where $|\vec{p}|$ is the magnitude of final-state three-momentum in M-rest frame, τ_M is the life time of M, and $A(M \to P_1 P_2)$, etc. are the decay amplitudes. The branching ratio formula for $M \to PV$ decay is the same as (14) with a sum over polarizations of V.

In the following, we list some of the parameters we have used throughout this paper:

$$f_\pi = 130.7 \text{ MeV}, \quad f_K = 159.8 \text{ MeV},$$
$$f_\rho = 212.0 \text{ MeV}, \quad f_{K^*} = 221.0 \text{ MeV},$$
$$f_{a_1} = 212.0 \text{ MeV},$$
$$V_{cs} = 0.975 \ , \quad V_{ud} = 0.975. \quad (15)$$

III. $D \to P_1 P_2$

$D^0 \to K^- \pi^+$, $\bar{K}^0 \pi^0$, and $D^+ \to \bar{K}^0 \pi^+$. To illustrate our method we write, using Eq. (4) for the effective Hamiltonian, the decay amplitude of $D^0 \to K^- \pi^+$ as

$$A(D^0 \to K^-\pi^+) = \tilde{G}_F\{a_1\langle K^-\pi^+|(\bar{s}c)(\bar{u}d)|D^0\rangle + C_2\langle K^-\pi^+|H_w^{(8)}|D^0\rangle\} \ . \quad (16)$$

We write the first term as a sum of a factorized and a nonfactorized part,

$$\begin{aligned}\left\langle K^-\pi^+\middle|(\bar{s}c)(\bar{u}d)\middle|D^0\right\rangle &= \left\langle \pi^+\middle|(\bar{u}d)\middle|0\right\rangle \left\langle K^-\middle|(\bar{s}c)\middle|D^0\right\rangle + \left\langle \pi^+K^-\middle|(\bar{s}c)(\bar{u}d)\middle|D^0\right\rangle^{\text{nf}} \\ &= -if_\pi(m_D^2 - m_K^2)[F_0^{DK}\left(m_\pi^2\right) + F_0^{(1)nf}] \ , \end{aligned} \quad (17)$$

where we have defined the nonfactorized matrix element of the product of the color-singlet currents $(\bar{s}c)(\bar{u}d)$ as

$$\left\langle K^-\pi^+|(\bar{s}c)(\bar{u}d)|D^0\right\rangle^{\text{nf}} \equiv -if_\pi(m_D^2 - m_K^2)F_0^{(1)\text{nf}} \ . \quad (18)$$

For the second term in (16), we write

$$\left\langle K^-\pi^+\middle|H_w^{(8)}\middle|D^0\right\rangle \equiv -if_\pi(m_D^2 - m_K^2)F_0^{(8)\text{nf}} \ . \quad (19)$$

Both $F_0^{(1)\text{nf}}$ and $F_0^{(8)\text{nf}}$ (as also all nonfactorized contributions to follow) are functions of the Mandelstam variables, $s = m_D^2$, $t = m_K^2$, and $u = m_\pi^2$. We have chosen to suppress these variables in writing the last three and all ensuing equations. The decay amplitude of Eq. (16) is then written in the form

$$A(D^0 \to K^-\pi^+) = -i\tilde{G}_F(a_1^{\text{eff}})_{K\pi} f_\pi (m_D^2 - m_K^2) F_0^{DK}(m_\pi^2) \ , \quad (20)$$

where

$$\left(a_1^{\text{eff}}\right)_{K\pi} = a_1 \left(1 + \frac{F_0^{(1)\text{nf}}}{F_0^{DK}(m_\pi^2)} + \frac{C_2}{a_1}\frac{F_0^{(8)\text{nf}}}{F_0^{DK}(m_\pi^2)}\right) \ . \quad (21)$$

This defines a process-dependent effective a_1. We shall see that it is possible to do so for all decays involving a single Lorentz scalar structure. We notice also that as the coefficient C_2/a_1 (≈ -0.47) is smaller than unity, the effect of the nonfactorized amplitude arising from $H_w^{(8)}$ is suppressed relative to the factorized amplitude in color-favored decays. For the same reason, the nonfactorized term proportional to $F_0^{(1)\text{nf}}$ could compete favorably with $F_0^{(8)\text{nf}}$.

The decay amplitude for the color-suppressed decay $D^0 \to \bar{K}^0\pi^0$ by following an analogous procedure is given by

$$A(D^0 \to \bar{K}^0\pi^0) = -i\frac{\tilde{G}_F}{\sqrt{2}}\left(a_2^{\text{eff}}\right)_{K\pi} f_K \left(m_D^2 - m_\pi^2\right) \times F_0^{D\pi}(m_K^2) \ , \quad (22)$$

where

$$\left(a_2^{\text{eff}}\right)_{K\pi} = a_2 \left(1 + \frac{\tilde{F}_0^{(1)\text{nf}}}{F_0^{D\pi}(m_K^2)} + \frac{C_1}{a_2}\frac{\tilde{F}_0^{(8)\text{nf}}}{F_0^{D\pi}(m_K^2)}\right) \ . \quad (23)$$

In writing (22) we have used

$$\begin{aligned}\left\langle \bar{K}^0\pi^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle &= \left\langle \bar{K}^0|(\bar{s}d)|0\right\rangle \left\langle \pi^0|(\bar{u}c)|D^0\right\rangle + \left\langle \bar{K}^0\pi^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle^{\text{nf}} \\ &= -i\frac{f_K}{\sqrt{2}}\left(m_D^2 - m_\pi^2\right)\left[F_0^{D\pi}\left(m_K^2\right) + \tilde{F}_0^{(1)\text{nf}}\right]\end{aligned} \quad (24)$$

and

$$\left\langle \bar{K}^0\pi^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle^{\text{nf}} \equiv -i\frac{f_K}{\sqrt{2}}\left(m_D^2 - m_\pi^2\right)\tilde{F}_0^{(1)\text{nf}} \ ,$$

$$\langle \bar{K}^0\pi^0|\tilde{H}_w^{(8)}|D^0\rangle \equiv -i\frac{f_K}{\sqrt{2}}\left(m_D^2 - m_\pi^2\right)\tilde{F}_0^{(8)\text{nf}} \ . \quad (25)$$

We note that in (24) there would also be a contribution from an annihilation term which in factorized form is proportional to $\left\langle \bar{K}^0\pi^0|(\bar{s}d)|0\right\rangle \left\langle 0|(\bar{u}c)|D^0\right\rangle$. However, such a term is proportional to [12] $\left(m_K^2 - m_\pi^2\right) F_0^{\pi K}\left(m_D^2\right)$. Because of the fact that $\left(m_K^2 - m_\pi^2\right) \ll \left(m_D^2 - m_\pi^2\right)$, it has been neglected.

Now, as $\frac{C_1}{a_2}$ in Eq. (23) is large (≈ -14), the nonfactorized contribution $\tilde{F}_0^{(8)\rm nf}$, arising from $\tilde{H}_w^{(8)}$, is greatly enhanced in comparison to $\tilde{F}_0^{(1)\rm nf}$. Nevertheless, it is not possible to separate the individual contributions from $\tilde{F}_0^{(8)\rm nf}$ and $\tilde{F}_0^{(1)\rm nf}$.

The amplitude for $D^+ \to \bar{K}^0\pi^+$ decay is obtained from Eqs. (20) and (22) via the isospin sum rule

$$A(D^+ \to \bar{K}^0\pi^+) = A(D^0 \to K^-\pi^+) + \sqrt{2}A(D^0 \to \bar{K}^0\pi^0) . \qquad (26)$$

In terms of isospin amplitudes $A_{1/2}$ and $A_{3/2}$ and the final-state interaction (FSI) phases,

$$\begin{aligned} A(D^0 \to K^-\pi^+) &= \frac{1}{\sqrt{3}}(A_{3/2}\exp(i\delta_{3/2}) + \sqrt{2}A_{1/2}\exp(i\delta_{1/2})), \\ A(D^0 \to \bar{K}^0\pi^0) &= \frac{1}{\sqrt{3}}(\sqrt{2}A_{3/2}\exp(i\delta_{3/2}) - A_{1/2}\exp(i\delta_{1/2})), \\ A(D^+ \to \bar{K}^0\pi^+) &= \sqrt{3}A_{3/2}\exp(i\delta_{3/2}). \end{aligned} \qquad (27)$$

The relative phase is known [13] to be

$$\delta^{\bar{K}\pi} \equiv \delta_{1/2}^{\bar{K}\pi} - \delta_{3/2}^{\bar{K}\pi} = (86 \pm 8)^0 , \qquad (28)$$

with the relative sign of $A_{1/2}$ and $A_{3/2}$ positive. The determination of relative phase of (28) in [13] used the 1994 listing [14] of the branching ratios. It differs somewhat from an earlier [15] determination. There would be another solution where the relative sign of $A_{1/2}$ and $A_{3/2}$ is odd and $\delta \to (\pi - \delta)$ [16].

We determine $A_{1/2}$ and $A_{3/2}$ by equating Eqs. (20) and (22) to Eq. (27) with the phases $\delta_{1/2}$ and $\delta_{3/2}$ set equal to zero, and then reinstate the phases to calculate the branching ratios from Eq. (13). This procedure is equivalent to assuming that the effect of FSI in this mode is simply to rotate the isospin amplitudes without affecting their magnitudes. For the form factors we have used the following normalizations at $q^2 = 0$:

$$\begin{aligned} F_0^{DK}(0) &= 0.77 \pm 0.04 \qquad [17], \\ F_0^{D\pi}(0) &= 0.83 \pm 0.08 \qquad [13,18,19]. \end{aligned} \qquad (29)$$

We extrapolated $F_0^{DK}(q^2)$ and $F_0^{D\pi}(q^2)$ as monopoles with 0^+ pole masses of 2.01 and 2.47 GeV, respectively as in [11]. As these form factors are needed at a relatively small q^2 ($= m_\pi^2$ or m_K^2), the results are not very sensitive to the manner of extrapolation.

The results are summarized below. We first determined $A_{1/2}$ and $A_{3/2}$ from (20), (22), and (27) with $\delta_{1/2}^{K\pi}$ and $\delta_{3/2}^{K\pi}$ set equal to zero. Next, we searched for the allowed ranges of $(a_1^{\rm eff})_{K\pi}$, $(a_2^{\rm eff})_{K\pi}$ that fit the branching ratio data [14] as $\delta_{1/2}^{K\pi} - \delta_{3/2}^{K\pi}$ was allowed to vary in the range indicated in (28) for $A_{3/2}/A_{1/2} > 0$. The resulting ranges were

$$1.11 \le \left(a_1^{\rm eff}\right)_{K\pi} \le 1.17 , \qquad -0.46 \le \left(a_2^{\rm eff}\right)_{K\pi} \le -0.39. \qquad (30)$$

Equivalently, if we define the two parameters

$$\begin{aligned} \chi_{K\pi} &\equiv \frac{F_0^{(8)\rm nf}}{F_0^{DK}(m_\pi^2)} + \frac{a_1}{C_2}\frac{F_0^{(1)\rm nf}}{F_0^{DK}(m_\pi^2)}, \\ \xi_{K\pi} &\equiv \frac{\tilde{F}_0^{(8)\rm nf}}{F_0^{D\pi}(m_K^2)} + \frac{a_2}{C_1}\frac{\tilde{F}_0^{(1)\rm nf}}{F_0^{D\pi}(m_K^2)}, \end{aligned} \qquad (31)$$

such that

$$\left(a_1^{\rm eff}\right)_{K\pi} = a_1\left(1 + \frac{C_2}{a_1}\chi_{K\pi}\right)$$

and (32)

$$\left(a_2^{\rm eff}\right)_{K\pi} = a_2\left(1 + \frac{C_1}{a_2}\xi_{K\pi}\right) ,$$

the allowed ranges of these two parameters were

$$-0.22 \le \chi_{K\pi} \le 0 , \qquad -0.32 \le \xi_{K\pi} \le -0.21. \qquad (33)$$

In calculating the ranges of $\chi_{K\pi}$ and $\xi_{K\pi}$, we have employed the stated errors in C_1 and C_2, Eq. (2).

There is another allowed solution where $\frac{A_{3/2}}{A_{1/2}}$ is negative and $\delta^{\bar{K}\pi}$ is replaced by $(\pi - \delta^{\bar{K}\pi})$ [16]. This solution requires

$$0.76 \le \left(a_1^{\rm eff}\right)_{K\pi} \le 0.84$$

and (34)

$$-0.92 \le \left(a_2^{\rm eff}\right)_{K\pi} \le -0.87.$$

However, this solution requires $\chi_{K\pi}$ and $\xi_{K\pi}$, which are measures of nonfactorization effects, to be larger and of opposite signs. That is,

$$0.41 \le \chi_{K\pi} \le 0.76 \qquad \text{and} \qquad -0.71 \le \xi_{K\pi} \le -0.58. \qquad (35)$$

Of the two solutions, the solution shown in (30) and (33), which yields $\frac{A_{3/2}}{A_{1/2}} > 0$ has $(a_1^{\rm eff})_{K\pi}$ and $(a_2^{\rm eff})_{K\pi}$ closer to the values that have been in vogue over the past decade and it also requires a smaller nonfactorized contribution. In principle, there is a sign ambiguity in $a_1^{\rm eff}$ and $a_2^{\rm eff}$: One could reverse the signs of both $a_1^{\rm eff}$ and $a_2^{\rm eff}$ which only serves to change the sign of the decay amplitudes. We think that this is a very unlikely solution because a sign reversal of $a_1^{\rm eff}$ can only be accomplished at the expense of a very large nonfactorized term [> (3–4) times the factorized term] in class I [11] decays.

We shall return to a discussion of our numerical estimates of $\chi_{K\pi}$ and $\xi_{K\pi}$ [equivalently, $(a_1^{\rm eff})_{K\pi}$ and $(a_2^{\rm eff})_{K\pi}$] in Sec. VI.

IV. $D \to P_1 V_1$

A. $D^0 \to K^{*-}\pi^+, \bar{K}^{*0}\pi^0$ and $D^+ \to \bar{K}^{*0}\pi^+$

Using the definitions introduced in Sec. II and the method of calculation detailed for $D \to \bar{K}\pi$ decays, the amplitudes for the decays $D^0 \to K^*\pi$ are given by

$$A(D^0 \to K^{*-}\pi^+) = 2\tilde{G}_F f_\pi m_{K^*} A_0^{DK^*}(m_\pi^2)(\varepsilon^* \cdot p_D) \times \left(a_1^{\rm eff}\right)_{K^*\pi} ,$$
$$A(D^0 \to \bar{K}^{*0}\pi^0) = \sqrt{2}\tilde{G}_F f_{K^*} m_{K^*} F_1^{D\pi}\left(m_{K^*}^2\right)(\varepsilon^* \cdot p_D) \times \left(a_2^{\rm eff}\right)_{K^*\pi} ,$$
$$A(D^+ \to \bar{K}^{*0}\pi^+) = A(D^0 \to K^{*-}\pi^+) + \sqrt{2}A(D^0 \to \bar{K}^{*0}\pi^0) , \quad (36)$$

where

$$\left(a_1^{\rm eff}\right)_{K^*\pi} = a_1\left(1 + \frac{A_0^{(1){\rm nf}}}{A_0^{DK^*}(m_\pi^2)} + \frac{C_2}{a_1}\frac{A_0^{(8){\rm nf}}}{A_0^{DK^*}(m_\pi^2)}\right) ,$$
$$\left(a_2^{\rm eff}\right)_{K^*\pi} = a_2\left(1 + \frac{\tilde{F}_1^{(1){\rm nf}}}{F_1^{D\pi}(m_{K^*}^2)} + \frac{C_1}{a_2}\frac{\tilde{F}_1^{(8){\rm nf}}}{F_1^{D\pi}(m_{K^*}^2)}\right) . \quad (37)$$

In (36) and (37), in addition to (8) and (9), we have used the definitions

$$\langle K^{*-}\pi^+|(\bar{s}c)(\bar{u}d)|D^0\rangle^{\rm nf} \equiv 2\tilde{G}_F f_\pi m_{K^*} A_0^{(1){\rm nf}}(\varepsilon^* \cdot p_D) ,$$
$$\langle K^{*-}\pi^+|H_w^{(8)}|D^0\rangle \equiv 2\tilde{G}_F f_\pi m_{K^*} A_0^{(8){\rm nf}}(\varepsilon^* \cdot p_D) ,$$
$$\langle \bar{K}^{*0}\pi^0|(\bar{u}c)(\bar{s}d)|D^0\rangle^{\rm nf} \equiv \sqrt{2}\tilde{G}_F f_{K^*} m_{K^*} \tilde{F}_1^{(1){\rm nf}} \times(\varepsilon^* \cdot p_D) ,$$

and

$$\langle \bar{K}^{*0}\pi^0|\tilde{H}_w^{(8)}|D^0\rangle \equiv \sqrt{2}\tilde{G}_F f_{K^*} m_{K^*} \tilde{F}_1^{(8){\rm nf}}(\varepsilon^* \cdot p_D) . \quad (38)$$

It is known [13] from an analysis of 1994 listed data [14] that FSI phases in this decay are large, $\delta^{K^*\pi} \equiv \delta_{1/2}^{K^*\pi} - \delta_{3/2}^{K^*\pi} = (103 \pm 17)^0$ for $\frac{A_{3/2}}{A_{1/2}} > 0$. To take the FSI phases into account we follow a procedure similar to that for $D \to K\pi$ decays; we calculate the isospin amplitudes by equating the amplitudes in (36) to those in (27) with phases set equal to zero. Having so determined $A_{1/2}$ and $A_{3/2}$, we reinstate the phases. For the form factors we have used the following normalizations at $q^2 = 0$:

$$A_0^{DK^*}(0) = 0.70 \pm 0.09 \quad [13,17],$$
$$F_1^{D\pi}(0) = 0.83 \pm 0.08 \quad [13,18]. \quad (39)$$

We used the normalization of the form factors given in (39) and considered monopole [referred to as Bauer-Stech-Wirbel I (BSWI) hereafter) as well as dipole (referred to as BSWII hereafter) forms for the q^2 extrapolation of the form factors $A_0^{DK^*}(q^2)$ and $F_1^{D\pi}(q^2)$ with pole masses 2.11 and 1.87 GeV, respectively. Allowing $\delta^{K^*\pi}$ to vary in the range $(103 \pm 17)^0$, we determined the allowed ranges of $\left(a_1^{\rm eff}\right)_{K^*\pi}$ and $\left(a_2^{\rm eff}\right)_{K^*\pi}$ for $\frac{A_{3/2}}{A_{1/2}} > 0$ and $\frac{A_{3/2}}{A_{1/2}} < 0$ that fit the branching ratio data [14]. The results were

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$1.74 \le \left(a_1^{\rm eff}\right)_{K^*\pi} \le 1.96 ,$$
$$-0.53 \le \left(a_2^{\rm eff}\right)_{K^*\pi} \le -0.43 \quad {\rm BSWI} ,$$
$$1.73 \le \left(a_1^{\rm eff}\right)_{K^*\pi} \le 1.95 ,$$
$$-0.43 \le \left(a_2^{\rm eff}\right)_{K^*\pi} \le -0.34 \quad {\rm BSWII} . \quad (40)$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$1.29 \le \left(a_1^{\rm eff}\right)_{K^*\pi} \le 1.51 ,$$
$$-0.90 \le \left(a_2^{\rm eff}\right)_{K^*\pi} \le -0.81 \quad {\rm BSWI} ,$$
$$1.28 \le \left(a_1^{\rm eff}\right)_{K^*\pi} \le 1.52 ,$$
$$-0.71 \le \left(a_2^{\rm eff}\right)_{K^*\pi} \le -0.63 \quad {\rm BSWII} . \quad (41)$$

From (37) the parameters $\chi_{K^*\pi}$ and $\xi_{K^*\pi}$, analogous to $\chi_{K\pi}$ and $\xi_{K\pi}$ defined through (31) and (32), were estimated to be

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$-1.91 \le \chi_{K^*\pi} \le -1.14 ,$$
$$-0.38 \le \xi_{K^*\pi} \le -0.25 \quad {\rm BSWI} ,$$
$$-1.89 \le \chi_{K^*\pi} \le -1.13 ,$$
$$-0.30 \le \xi_{K^*\pi} \le -0.18 \quad {\rm BSWII} , \quad (42)$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$-0.93 \le \chi_{K^*\pi} \le -0.34 ,$$
$$-0.68 \le \xi_{K^*\pi} \le -0.53 \quad {\rm BSWI} ,$$
$$-0.96 \le \chi_{K^*\pi} \le -0.34 ,$$
$$-0.52 \le \xi_{K^*\pi} \le -0.40 \quad {\rm BSWII} . \quad (43)$$

Simply by fitting the three branching ratios for $(D^0, D^+) \to \bar{K}^*\pi$, it is not possible to favor one solution or the other. Here, the solutions obtained with the constraint $\frac{A_{3/2}}{A_{1/2}} > 0$ require large nonfactorization contributions to $(a_1^{\rm eff})_{K^*\pi}$ as evidenced by $\chi_{K^*\pi}$ in (42). On the contrary, the solutions corresponding to $\frac{A_{3/2}}{A_{1/2}} < 0$ [with $\delta^{K^*\pi} \to (\pi - \delta^{K^*\pi})$] require larger nonfactorization contributions to $(a_2^{\rm eff})_{K^*\pi}$ as seen by comparing $\xi_{K^*\pi}$'s of (43) with those of (42). A discussion of these results is given in Sec. VI.

B. $D^0 \to K^-\rho^+$, $\bar{K}^0\rho^0$ and $D^+ \to \bar{K}^0\rho^+$

We write, using the definitions given in Sec. II, the amplitudes for the decays $D^0 \to K\rho$ as

$$A(D^0 \to K^-\rho^+)$$
$$= 2\tilde{G}_F f_\rho m_\rho(\varepsilon^* \cdot p_D) F_1^{DK}\left(m_\rho^2\right)\left(a_1^{\rm eff}\right)_{K\rho} ,$$
$$A(D^0 \to \bar{K}^0\rho^0)$$
$$= \sqrt{2}\tilde{G}_F f_K m_\rho(\varepsilon^* \cdot p_D) A_0^{D\rho}\left(m_K^2\right)\left(a_2^{\rm eff}\right)_{K\rho} ,$$

and

$$A(D^+ \to \bar{K}^0\rho^+) = A(D^0 \to K^-\rho^+) + \sqrt{2}A(D^0 \to \bar{K}^0\rho^0) , \quad (44)$$

where

$$\left(a_1^{\rm eff}\right)_{K\rho} = a_1\left(1 + \frac{F_1^{(1)\rm nf}}{F_1^{DK}(m_\rho^2)} + \frac{C_2}{a_1}\frac{F_1^{(8)\rm nf}}{F_1^{DK}(m_\rho^2)}\right) ,$$
$$\left(a_2^{\rm eff}\right)_{K\rho} = a_2\left(1 + \frac{\tilde{A}_0^{(1)\rm nf}}{A_0^{D\rho}(m_K^2)} + \frac{C_1}{a_2}\frac{\tilde{A}_0^{(8)\rm nf}}{A_0^{D\rho}(m_K^2)}\right) . \quad (45)$$

We have also used, in addition to (8) and (9), the definitions

$$\left\langle K^-\rho^+|(\bar{s}c)(\bar{u}d)|D^0\right\rangle^{\rm nf} \equiv 2\tilde{G}_F f_\rho m_\rho F_1^{(1)\rm nf}(\varepsilon^* \cdot p_D) ,$$
$$\langle K^-\rho^+|H_w^{(8)}|D^0\rangle \equiv 2\tilde{G}_F f_\rho m_\rho F_1^{(8)\rm nf}(\varepsilon^* \cdot p_D) ,$$
$$\left\langle \bar{K}^0\rho^0|(\bar{u}c)(\bar{s}d)|D^0\right\rangle^{\rm nf} \equiv \sqrt{2}\tilde{G}_F f_K m_\rho \tilde{A}_0^{(1)\rm nf}(\varepsilon^* \cdot p_D) ,$$
$$\langle \bar{K}^0\rho^0|\tilde{H}_w^{(8)}|D^0\rangle \equiv \sqrt{2}\tilde{G}_F f_K m_\rho \tilde{A}_0^{(8)\rm nf}(\varepsilon^* \cdot p_D) . \quad (46)$$

Fits [13, 15] to $D \to \bar{K}\rho$ data admit a solution with $\frac{A_{3/2}}{A_{1/2}} > 0$ and a relative FSI phase $\delta^{K\rho} \equiv \delta_{1/2}^{K\rho} - \delta_{3/2}^{K\rho} = (0 \pm 30)^0$. We use $F_1^{DK}(0)$ from Eq. (29) and, for want of better information, the BSW [11] value of $A_0^{D\rho}(0) = 0.67$. In this decay also we have considered both monopole (BSWI) and dipole (BSWII) extrapolations of the form factors $F_1^{DK}(q^2)$ and $A_0^{D\rho}(q^2)$ with 1^- pole at 2.11 GeV and 0^- pole at 1.87 GeV, respectively [11]. To search for the allowed ranges of $(a_1^{\rm eff})_{K\rho}$ and $(a_2^{\rm eff})_{K\rho}$, we followed the same procedure as that outlined in the analysis of $D \to \bar{K}\pi$ and $\bar{K}^*\pi$ decays by varying $\delta^{K\rho}$ in the domain $(0 \pm 30)^0$ and searching for allowed values of $a_1^{\rm eff}$ and $a_2^{\rm eff}$ that fit the data [14]. We found the following allowed ranges for $(a_1^{\rm eff})_{K\rho}$ and $(a_2^{\rm eff})_{K\rho}$.

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$1.17 \le (a_1^{\rm eff})_{K\rho} \le 1.32 ,$$
$$-1.00 \le (a_2^{\rm eff})_{K\rho} \le -0.75 \qquad \text{(BSWI)} ,$$
$$1.01 \le (a_1^{\rm eff})_{K\rho} \le 1.15$$
$$-0.92 \le (a_2^{\rm eff})_{K\rho} \le -0.69 \qquad \text{(BSWII)} . \quad (47)$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$0.71 \le (a_1^{\rm eff})_{K\rho} \le 0.85,$$
$$-2.53 \le (a_2^{\rm eff})_{K\rho} \le -2.19 \qquad \text{(BSWI)},$$
$$0.62 \le (a_1^{\rm eff})_{K\rho} \le 0.74 ,$$
$$-2.35 \le (a_2^{\rm eff})_{K\rho} \le -2.04 , \qquad \text{(BSWII)} . \quad (48)$$

These ranges translate into the following limits on $\chi_{K\rho}$ and $\xi_{K\rho}$ defined in analogy with $\chi_{K\pi}$ and $\xi_{K\pi}$ of (31).

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$-0.52 \le \chi_{K\rho} \le -0.11 ,$$
$$-0.75 \le \xi_{K\rho} \le -0.50 , \qquad \text{(BSWI)},$$
$$-0.15 \le \chi_{K\rho} \le 0.20 ,$$
$$-0.69 \le \xi_{K\rho} \le -0.46 , \qquad \text{(BSWII)} . \quad (49)$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$-0.41 \le \chi_{K\rho} \le -0.85 ,$$
$$-2.00 \le \xi_{K\rho} \le -1.60 , \qquad \text{(BSWI)},$$
$$0.61 \le \chi_{K\rho} \le 1.04 ,$$
$$-1.86 \le \xi_{K\rho} \le -1.50 , \qquad \text{(BSWII)} . \quad (50)$$

Note that the solutions with $\frac{A_{3/2}}{A_{1/2}} < 0$ require much larger nonfactorized contributions. A discussion of $\chi_{K\rho}$ and $\xi_{K\rho}$ is given in the last section.

C. $D^0 \to K^- a_1^+$, $\bar{K}^0 a_1^0$ and $D^+ \to \bar{K}^0 a_1^+$

We write, using definitions given in Sec. II, decay amplitudes for $D \to K a_1$ as

$$A(D^0 \to K^- a_1^+) = 2\tilde{G}_F f_{a_1} m_{a_1}(\varepsilon^* \cdot p_D) \times F_1^{DK}(m_{a_1}^2)(a_1^{\rm eff})_{Ka_1} ,$$
$$A(D^0 \to \bar{K}^0 a_1^0) = \sqrt{2}\tilde{G}_F f_K m_{a_1}(\varepsilon^* \cdot p_D) C_1 \tilde{V}_0^{\rm nf} ,$$
$$A(D^+ \to \bar{K}^0 a_1^+) = A(D^0 \to K^- a_1^+) + \sqrt{2}A(D^0 \to \bar{K}^0 a_1^0) , \quad (51)$$

where

$$\tilde{V}_0^{\rm nf} = \tilde{V}_0^{(8)\rm nf} + \frac{a_2}{C_1}\tilde{V}_0^{(1)\rm nf} ,$$
$$\left(a_1^{\rm eff}\right)_{Ka_1} = a_1\left(1 + \frac{F_1^{(1)\rm nf}}{F_1^{DK}(m_{a_1}^2)} + \frac{C_2}{a_1}\frac{F_1^{(8)\rm nf}}{F_1^{DK}(m_{a_1}^2)}\right) . \quad (52)$$

In deriving (51), in addition to (8) and (12), we have used the definitions

$$\langle K^- a_1^+ |(\bar{s}c)(\bar{u}d)|D^0\rangle^{\rm nf} \equiv 2\tilde{G}_F f_{a_1} m_{a_1}(\varepsilon^*\cdot p_D)F^{(1)\rm nf}\,,$$
$$\langle K^- a_1^+ |H_w^{(8)}|D^0\rangle^{\rm nf} \equiv 2\tilde{G}_F f_{a_1} m_{a_1}(\varepsilon^*\cdot p_D)F^{(8)\rm nf}\,,$$
$$\langle \bar{K}^0 a_1^0 |(\bar{u}c)(\bar{s}d)|D^0\rangle^{\rm nf} \equiv \sqrt{2}\tilde{G}_F f_K m_{a_1}(\varepsilon^*\cdot p_D)\tilde{V}_0^{(1)\rm nf}\,,$$
$$\langle \bar{K}^0 a_1^0 |\tilde{H}_w^{(8)}|D^0\rangle^{\rm nf} \equiv \sqrt{2}\tilde{G}_F f_K m_{a_1}(\varepsilon^*\cdot p_D)\tilde{V}_0^{(8)\rm nf}\,. \quad (53)$$

In the decay amplitude for $D^0 \to \bar{K}^0 a_1^0$ we have retained only the nonfactorized contributions arising from the product of color-singlet currents and $\tilde{H}_w^{(8)}$. The reason being that the factorized amplitude cannot be calculated in the BSW scheme, $a_1(1260)$ being a $^3P_1$ state, unlike for K^* which is a $^3S_1$ state, BSW procedure does not define the null-plane wave function for L =1 quark-antiquark pairs. However, the relevant form factor $V_0^{Da_1}(q^2)$ [see Eq. (12)] can be calculated in the model proposed by Isgur, Scora, Grinstein, and Wise [20] where it can be shown that it vanishes at the zero-recoil point. This does not imply that it vanishes everywhere but as it also comes multiplied by the rather small coefficient $a_2(\approx -0.09)$, we have neglected the factorized amplitude all together.

We use $F_1^{DK}(0)$ from Eq. (29) and both monopole (BSWI) and dipole (BSWII) forms for q^2 extrapolation of the form factor $F_1^{DK}(q^2)$ with 1^- pole at 2.11 GeV.

We allowed the isospin phase $\delta^{Ka_1} \equiv \delta_{1/2}^{Ka_1} - \delta_{3/2}^{Ka_1}$ to vary in the domain $(0 \pm 37)^0$ [15] and searched for the allowed ranges of $(a_1^{\rm eff})_{Ka_1}$ and $\tilde{V}_0^{\rm nf}$ that fitted the branching ratio data [14]. This case differs from those discussed thus far. Whereas in the earlier cases, the color-suppressed amplitude depended on the product of $a_2^{\rm eff}$, which was unknown, and a form factor which is treated as known, the color-suppressed amplitude here depends on the product of C_1, which is known and $\tilde{V}_0^{\rm nf}$, which is unknown. Hence, we varied $(a_1^{\rm eff})_{Ka_1}$ and $\tilde{V}_0^{\rm nf}$ to fit the branching ratios for $D^0 \to K^- a_1^+$ and $D^+ \to \bar{K}^0 a_1^+$ [14] and the upper limit $B(D^0 \to \bar{K}^0 a_1^0) \le 1.9\%$ [14, 15]. It is worth bearing in mind that the range of δ^{Ka_1} determined by Mark III Collaboration [15] uses $B(D^0 \to \bar{K}^0 a_1^0) = (0.4\pm0.4\pm0.9)\%$ in doing their amplitude analysis even though they only have an upper limit of $< 1.9\%$ at 90% C.L. for this branching ratio. Thus their determination of δ^{Ka_1} and the ratio of the isospin amplitudes have to be understood with this caveat.

We obtain the following solutions.

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$2.28 \le (a_1^{\rm eff})_{Ka_1} \le 2.75\,,\quad -1.65 \le \tilde{V}_0^{\rm nf} \le -0.69 \quad \text{(BSWI)},$$

$$1.51 \le (a_1^{\rm eff})_{Ka_1} \le 1.81\,,\quad -1.61 \le \tilde{V}_0^{\rm nf} \le -0.69 \quad \text{(BSWII)}. \quad (54)$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$1.44 \le (a_1^{\rm eff})_{Ka_1} \le 1.80\,,\quad -4.09 \le \tilde{V}_0^{\rm nf} \le -3.39 \quad \text{(BSWI)},$$

$$0.75 \le (a_1^{\rm eff})_{Ka_1} \le 1.19\,,\quad -4.09 \le \tilde{V}_0^{\rm nf} \le -3.39 \quad \text{(BSWII)} \quad (55)$$

The corresponding ranges for χ_{Ka_1} are the following.

$\frac{A_{3/2}}{A_{1/2}} > 0$:

$$-3.61 \le \chi_{Ka_1} \le -2.11 \quad \text{(BSWI)},$$
$$-1.56 \le \chi_{Ka_1} \le -0.75 \quad \text{(BSWII)}. \quad (56)$$

$\frac{A_{3/2}}{A_{1/2}} < 0$:

$$-1.54 \le \chi_{Ka_1} \le -0.10 \quad \text{(BSWI)},$$
$$-0.22 \le \chi_{Ka_1} \le 0.74 \quad \text{(BSWII)}. \quad (57)$$

From the above it is seen that for $\frac{A_{3/2}}{A_{1/2}} > 0$ one needs large nonfactorized contributions to $(a_1^{\rm eff})_{Ka_1}$ and a relatively small $\tilde{V}_0^{\rm nf}$, while for $\frac{A_{3/2}}{A_{1/2}} < 0$ solutions can be found with very little nonfactorized contribution to $(a_1^{\rm eff})_{Ka_1}$ but a much larger $\tilde{V}_0^{\rm nf}$. Not unexpectedly, there is a considerable dependence on the manner of extrapolation of the form factors because of the large mass of a_1.

V. $D \to V_1 V_2$

$D^0 \to K^{*-}\rho^+$, $\bar{K}^{*0}\rho^0$ and $D^+ \to \bar{K}^{*0}\rho^+$. Using the definitions given in Sec. II, one can write the decay amplitudes for $D^0 \to K^{*-}\rho^+$, $\bar{K}^{*0}\rho^0$ and $D^+ \to \bar{K}^{*0}\rho^+$ as,

$$A(D^0 \to K^{*-}\rho^+) = \tilde{G}_F m_\rho f_\rho \Bigg\{ (m_D + m_{K^*})\varepsilon_{K^*}\cdot\varepsilon_\rho [a_1 A_1^{DK^*}(m_\rho^2) + a_1 A_1^{(1)\rm nf} + C_2 A_1^{(8)\rm nf}]$$
$$- \frac{\varepsilon_{K^*}\cdot(p_D - p_{K^*})\varepsilon_\rho\cdot(p_D + p_{K^*})}{m_D + m_{K^*}}[a_1 A_2^{DK^*}(m_\rho^2) + a_1 A_2^{(1)\rm nf} + C_2 A_2^{(8)\rm nf}]$$
$$+ \frac{2i}{m_D + m_{K^*}}\varepsilon_{\mu\nu\sigma\delta}\varepsilon_\rho^\mu \varepsilon_{K^*}^\nu p_{K^*}^\sigma p_D^\delta [a_1 V^{DK^*}(m_\rho^2) + a_1 V^{(1)\rm nf} + C_2 V^{(8)\rm nf}] \Bigg\}\,,$$

$$A(D^0 \to \bar{K}^{*0}\rho^0) = \frac{1}{\sqrt{2}}\tilde{G}_F m_{K^*} f_{K^*}\Bigg\{(m_D + m_\rho)\varepsilon_{K^*}\cdot\varepsilon_\rho[a_2 A_1^{D\rho}(m_{K^*}^2) + a_2\tilde{A}_1^{(1)\mathrm{nf}} + C_1\tilde{A}_1^{(8)\mathrm{nf}}]$$
$$-\frac{\varepsilon_\rho\cdot(p_D - p_\rho)\varepsilon_{K^*}\cdot(p_D + p_\rho)}{m_D + m_\rho}[a_2 A_2^{D\rho}(m_{K^*}^2) + a_2\tilde{A}_2^{(1)\mathrm{nf}} + C_1\tilde{A}_2^{(8)\mathrm{nf}}]$$
$$+\frac{2i}{m_D + m_\rho}\varepsilon_{\mu\nu\sigma\delta}\varepsilon_{K^*}^{\mu}\varepsilon_\rho^{\nu}p_\rho^{\sigma}p_D^{\delta}[a_2 V^{D\rho}(m_{K^*}^2) + a_2\tilde{V}^{(1)\mathrm{nf}} + C_1\tilde{V}^{(8)\mathrm{nf}}]\Bigg\}\,,$$

and

$$A(D^+ \to \bar{K}^{*0}\rho^+) = A(D^0 \to K^{*-}\rho^+) + \sqrt{2}A(D^0 \to \bar{K}^{*0}\rho^0)\,, \tag{58}$$

where the quantities in the color-favored decay $D^0 \to K^{*-}\rho^+$ with super index 1 (e.g., $A_1^{(1)\mathrm{nf}}$) arise from the nonfactorized contribution to the matrix elements of the color-singlet currents $(\bar{s}c)(\bar{u}d)$; those with super index 8 (e.g., $A_1^{(8)\mathrm{nf}}$) arise from $H_w^{(8)}$ made up of color-octet currents; the tilded quantities refer to the color-suppressed decay $D^0 \to \bar{K}^{*0}\rho^0$ (e.g., $\tilde{A}_1^{(8)\mathrm{nf}}$ arises from $\tilde{H}_w^{(8)}$).

The decay rate can be calculated using (14). For the form factors we use the following normalizations with errors where available:

$$\begin{aligned} &A_1^{DK^*} = 0.61 \pm 0.05\,, \quad &&A_2^{DK^*} = 0.45 \pm 0.09\,, \quad &&V^{DK^*} = 1.16 \pm 0.16 \quad &&[17]\,,\\ &A_1^{D\rho} = 0.78\,, &&A_2^{D\rho} = 0.92\,, &&V^{D\rho} = 1.23 &&[11]\,, \end{aligned} \tag{59}$$

and extrapolate them to relevant q^2 with monopole forms with pole masses [11] 2.53 GeV for $A_1^{DK^*}$ and $A_2^{DK^*}$, 2.11 GeV for V^{DK^*}, 2.42 GeV for $A_1^{D\rho}$ and $A_2^{D\rho}$, and 2.01 GeV for $V^{D\rho}$. We tried putting nonfactorized contribution in A_1-, A_2-, or V- like terms and combinations thereof. We found that nonfactorization is needed in A_1-like terms to fit all three branching ratios $B(D^0 \to K^{*-}\rho^+)$, $B(D^0 \to \bar{K}^{*0}\rho^0)$, and $B(D^+ \to \bar{K}^{*0}\rho^+)$. It was possible, for example, to fit $B(D^0 \to K^{*-}\rho^+)$ and $B(D^0 \to \bar{K}^{*0}\rho^0)$ with nonfactorized contributions to A_2-like terms alone; however, such a scheme failed for $B(D^+ \to K^{*0}\rho^+)$.

Thus, putting nonfactorized effects only in the A_1-like terms and defining

$$\chi_{K^*\rho} = \frac{A_1^{(8)\mathrm{nf}}}{A_1^{DK^*}(m_\rho^2)} + \frac{a_1}{C_2}\frac{A_1^{(1)\mathrm{nf}}}{A_1^{DK^*}(m_\rho^2)}\,,$$
$$\xi_{K^*\rho} = \frac{\tilde{A}_1^{(8)\mathrm{nf}}}{A_1^{D\rho}(m_{K^*}^2)} + \frac{a_2}{C_1}\frac{\tilde{A}_1^{(1)\mathrm{nf}}}{A_1^{D\rho}(m_{K^*}^2)}\,, \tag{60}$$

we found that the three branching ratios $B(D^0 \to K^{*-}\rho^+)$, $B(D^0 \to \bar{K}^{*0}\rho^0)$, and $B(D^+ \to \bar{K}^{*0}\rho^+)$ [14] could be understood for $\chi_{K^*\rho}$ and $\xi_{K^*\rho}$ lying in the ranges

$$0.02 \le \chi_{K^*\rho} \le 0.80\,, \qquad -0.31 \le \xi_{K^*\rho} \le -0.24\,. \tag{61}$$

A further comment is in order: The problem of introducing FSI phases in this decay is a complicated one. There are three partial waves, S, P and D, and for each of them there are three helicity configurations. One, therefore, does not expect a single relative isospin phase $\delta_{1/2}^{K^*\rho} - \delta_{3/2}^{K^*\rho}$ (as seems to be the case in the fit to the data in Ref. [15]) to be applicable to the problem. It would be more appropriate to use a different relative isospin phase for each helicity amplitude. For this reason, we have chosen to work with zero FSI phases rather than use the phase determination of Ref. [15].

VI. SUMMARY AND CONCLUSIONS

We have carried out an analysis of those Cabibbo-favored two-body hadronic decays of D^0 and D^+ which involve two isospins in the final state in a formalism that uses $N_c = 3$ and includes nonfactorized amplitudes. These decays are: $D \to \bar{K}\pi$, $\bar{K}^*\pi$, $\bar{K}\rho$, $\bar{K}a_1$, and $\bar{K}^*\rho$. We have included the measured FSI phases in all but the $\bar{K}^*\rho$ decays but only in so far as they rotate the isospin amplitudes without affecting their magnitudes. We have ignored annihilation terms and inelastic FSI. The rationale for the former in $D \to \bar{K}\pi$ decays is that these terms are proportional to $a_2(m_K^2 - m_\pi^2)$ while the terms that are kept are proportional to $a_1(m_D^2 - m_K^2)$ or $a_2(m_D^2 - m_\pi^2)$.

Justifying the neglect of annihilation terms in $D \to K^*\pi$ or $\bar{K}\rho$ is harder as they involve the divergence of the axial vector current. If, however, the annihilation form factors $A_0^{K^*\pi}(m_D^2)$ and $A_0^{\rho\bar{K}}(m_D^2)$ would be much smaller than the form factors $A_0^{DK^*}(m_\pi^2)$ or $A_0^{D\rho}(m_K^2)$, the annihilation term would be smaller than the terms retained. The neglect of the inelastic FSI is largely because of ignorance of the parameters to be used in implementing a believable calculation.

Despite the statement above regarding the size of the annihilation terms, perhaps it is fair to say that at our present level of understanding of nonfactorization effects, we do not fully understand their role.

From the data, one only determines $(a_1)^{\rm eff}$ and $(a_2)^{\rm eff}$ which, as we and others [7] have shown, are process dependent. The next question is: What effects contribute to $(a_1)^{\rm eff}$ and $(a_2)^{\rm eff}$ in a scheme that uses $N_c = 3$? We have tacitly assumed [3–7] that these effects arise from the following sources: the nonfactorized matrix elements of $H_w^{(8)} = \frac{1}{2}\sum_a(\bar{s}\lambda^a c)(\bar{u}\lambda^a d)$, $\tilde{H}_w^{(8)} = \frac{1}{2}\sum_a(\bar{s}\lambda^a d)(\bar{u}\lambda^a c)$, and parts of the effective Hamiltonians made up of color-singlet currents $(\bar{s}c)(\bar{u}d)$ and $(\bar{u}c)(\bar{s}d)$. With these assumptions, we have extracted the relative size of the nonfactorized contribution in each specific channel. We now turn to a detailed discussion of specific decays.

From $D \to \bar{K}\pi$ decays we have determined the values of $(a_1^{\rm eff})_{\bar{K}\pi}$ and $(a_2^{\rm eff})_{\bar{K}\pi}$. We found that if we allowed the ratios of the isospin amplitudes $\frac{A_{3/2}}{A_{1/2}}$ to be positive and chose $\delta_{\bar{K}\pi}$ to be $(86 \pm 8)^0$, then $(a_1^{\rm eff})_{\bar{K}\pi}$ and $(a_2^{\rm eff})_{\bar{K}\pi}$ had the values given in (30). However, as the branching ratios remain invariant under simultaneous change of sign of the ratio $\frac{A_{3/2}}{A_{1/2}}$ and $\delta_{K\pi} \to (\pi - \delta_{K\pi})$, there is another solution given by (34). The values of the parameters $\chi_{K\pi}$ and $\xi_{K\pi}$, which are measures of the nonfactorized contributions, were extracted and are shown in (33) and (35) for the two sets of solutions of $(a_1^{\rm eff})_{K\pi}$ and $(a_2^{\rm eff})_{K\pi}$. The latter set of solutions require much larger nonfactorized contributions to both $(a_1^{\rm eff})_{K\pi}$ and $(a_2^{\rm eff})_{K\pi}$.

In our formalism it is not possible to separate the contribution of $F_0^{(1){\rm nf}}$ from that of $F_0^{(8){\rm nf}}$. One might also be tempted to assume that $F_0^{(8){\rm nf}} = \tilde{F}_0^{(8){\rm nf}}$; however, such an assumption would be flawed since $H_w^{(8)}$ and $\tilde{H}_w^{(8)}$ are related by V-spin symmetry (s $\Longleftrightarrow$ u), but under the same transformation $|D^0\rangle \to |D_s^+\rangle$ and $|K^-\pi^+> \to |K^+\bar{K}^0\rangle$. Thus V-spin symmetry leads to

$$\langle K^-\pi^+|H_w^{(8)}|D^0\rangle = \langle K^+\bar{K}^0|\tilde{H}_w^{(8)}|D_s^+\rangle \quad (62)$$

and not to a relation between $F^{(8){\rm nf}}$ and $\tilde{F}^{(8){\rm nf}}$. We also emphasize that the nonfactorized contribution in the color-suppressed decay $D^0 \to \bar{K}^0\pi^0$ is enhanced relative to the factorized term by a factor of $C_1/a_2(\approx -14)$ which is not the case in the color-favored decay $D^0 \to K^-\pi^+$. Thus, the color-suppressed processes are more likely to reveal the presence of nonfactorized contributions than those revealed by color-favored processes. Further, in the color-favored decay the nonfactorized amplitude arising from the color-singlet currents $(\bar{s}c)(\bar{u}d)$ (called $F_0^{(1){\rm nf}}$ here) could be just as important as the one from $H_w^{(8)}$ (called $F_0^{(8){\rm nf}}$ here).

In $D \to K^*\pi$ decay, we again found two sets of solutions for $(a_1^{\rm eff})_{K^*\pi}$ and $(a_2^{\rm eff})_{K^*\pi}$ when we allowed $\delta_{K^*\pi}$ to vary in the range $(103 \pm 17)^0$. These solutions are shown in (40) for $\frac{A_{3/2}}{A_{1/2}} > 0$ and in (41) $\frac{A_{3/2}}{A_{1/2}} < 0$.

In the color-suppressed decay $D^0 \to \bar{K}^0\rho^0$, for the case $\frac{A_{3/2}}{A_{1/2}} > 0$, we find large nonfactorized contributions: $-0.75 \le \xi_{K\rho} \le -0.50$ for monopole form factors and $-0.69 \le \xi_{K\rho} \le -0.46$ for dipole form factors. They result in $-1.00 \le (a_2^{\rm eff})_{K\rho} \le -0.75$ and $-0.92 \le (a_2^{\rm eff})_{K\rho} \le -0.69$, respectively. The nonfactorized contribution to the color-favored decay $D^0 \to K^-\rho^+$ appears to be small leading to: $1.17 \le (a_1^{\rm eff})_{K\rho} \le 1.32$ for monopole form factors and $1.01 \le (a_1^{\rm eff})_{K\rho} \le 1.15$ for dipole form factors. A solution was also found for $\frac{A_{3/2}}{A_{1/2}} < 0$, shown in Eq. (48). However, this solution requires rather large nonfactorized contributions in color-suppressed $D^0 \to \bar{K}^0\rho^0$ decay as seen from Eq. (49). In all the above, we used $\delta_{1/2}^{K\rho} - \delta_{3/2}^{K\rho} = (0 \pm 30)$.

The decays $D \to \bar{K}a_1$ have long posed a problem for the factorization model. Inclusion of nonfactorized amplitudes allows us to understand the branching ratios involved. Our picture suggests that the color-suppressed decay $D^0 \to \bar{K}^0a_1^0$ proceeds almost entirely through a nonfactorized amplitude, parametrized by $\tilde{V}_0^{\rm nf}$ in Eq. (51), whose size we limit by the experimental upper limit on $B(D^0 \to \bar{K}^0a_1^0)$. We are then able to understand the measured branching ratios $B(D^0 \to K^-a_1^+)$ and $B(D^+ \to \bar{K}^0a_1^+)$ provided that: For $\frac{A_{3/2}}{A_{1/2}} > 0$, $(a_1^{\rm eff})_{Ka_1}$ and $\tilde{V}_0^{\rm nf}$ are given by Eq. (54) for monopole and dipole extrapolations of the form factor. For $\frac{A_{3/2}}{A_{1/2}} < 0$, the allowed values of $(a_1^{\rm eff})_{Ka_1}$ and $\tilde{V}_0^{\rm nf}$ are those shown in Eq. (55). Note that the case $\frac{A_{3/2}}{A_{1/2}} < 0$ requires a smaller nonfactorized contribution to $(a_1^{\rm eff})_{Ka_1}$ but a larger one to $\tilde{V}_0^{\rm nf}$ than those for the case $\frac{A_{3/2}}{A_{1/2}} > 0$. We allowed $(\delta_{1/2}^{Ka_1} - \delta_{3/2}^{Ka_1}) = (0 \pm 37)^0$ [15].

For the decays $D \to K^*\rho$ (and, in general, for any $P \to VV$ decay), one cannot define $(a_1^{\rm eff})$ and $(a_2^{\rm eff})$ as the decay amplitude involves three independent Lorentz-scalar structures and it is not possible to factor out an effective a_1 and a_2. We tried to fit data on the three branching ratios $B(D^0 \to K^{*-}\rho^+)$, $B(D^0 \to \bar{K}^{*0}\rho^0)$, and $B(D^+ \to \bar{K}^{*0}\rho^+)$ by assuming nonfactorized contributions to A_1-, A_2-, and V-like terms in the decay amplitude but had success in reproducing data to one standard deviation only if nonfactorized effects were included in A_1-like terms. Thus, retaining the nonfactorized effects only in A_1-like terms, we find significant nonfactorized effects in the color-suppressed decay $D^0 \to \bar{K}^{*0}\rho^0$, characterized by the parameter $\xi_{K^*\rho}$ of Eq. (61): $-0.31 \le \xi_{K^*\rho} \le -0.24$. The analogous parameter $\chi_{K^*\rho}$, Eq. (61), which is a measure of nonfactorized contribution to the color-favored decay $D^0 \to K^{*-}\rho^+$, has the opposite sign, and could, in principle, be very small: $0.02 \le \chi_{K^*\rho} \le 0.80$. We do not believe that we have said the last word on the problem of $D \to \bar{K}^*\rho$ decays. Fitting the branching ratio data for $D^+ \to \bar{K}^{*0}\rho^+$, $D^0 \to K^{*-}\rho^+$, and $D^0 \to \bar{K}^{*0}\rho^0$, along with the separation of the branching ratios into longitudinal and transverse states of polarization, is a nontrivial task [21].

We take this opportunity to remedy the use [3, 7] of imprecise language, for which we are partly to blame [5], which attributes S waves entirely to A_1-like terms, P waves to V-like terms and D waves entirely to A_2-like terms. In fact, while V-like terms give rise to P-wave final states only, A_1- and A_2-like terms give rise to both

S and D waves [22].

We conclude by saying that one can understand D decays in a picture with $N_c = 3$ but only with the inclusion of nonfactorized amplitudes. This picture results in process-dependent effective a_1 and a_2 (except for $D \to VV$ decays), which ought to be complex as are all the nonfactorized amplitudes. We have not included the inelastic final-state interaction effects which would further complicate the analysis. The effort here was to parametrize the nonfactorized amplitudes and determine their sizes. The understanding of any systematics that emerge is yet to come.

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