Spacelike penguin diagram effects in $B \to PP$ decays

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The spacelike penguin diagram contributions to branching ratios and CP asymmetries in charmless decays of B to two pseudoscalar mesons are studied using the next-to-leading order low energy effective Hamiltonian. Both the gluonic penguin and the electroweak penguin diagrams are considered. We find that the effects are significant.

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I. INTRODUCTION

Penguin diagrams play an important role in charmless B decays and direct CP violation [1,2]. But only timelike penguin diagrams were considered in the literature because they can provide the necessary different strong phases for CP violation by different loop effects of the internal u and c quarks [1]. The contribution of spacelike penguin diagrams is usually neglected assuming form factor suppression. This assumption for neglecting spacelike penguin diagram effects is used not only for gluonic penguin but also for electroweak penguin diagrams [3]. But it does not lie on a solid ground because the spacelike penguin amplitudes can be remarkably enhanced by the hadronic matrix elements involving $(V-A)(V+A)$ or $(S+P)(S-P)$ currents [4]. Although spacelike penguin diagrams alone can only provide an overall CP-conserving phase due to final state interaction, it affects CP asymmetry by modifying the dispersive or absorptive parts of timelike penguin amplitudes, or by interference with tree diagrams. Obviously, it affects branching fractions too. In our recent paper [5], we illustrated the spacelike penguin diagram effects in CP asymmetries for the exclusive B decays $B_u^- \to \bar{K}^0\pi^-$ and K^0K^- using leading order Hamiltonian. In contrast to the naive expectation, the spacelike penguin diagram effects on CP asymmetries are found to be significant. In this paper we study spacelike penguin diagram effects in B to two pseudoscalar decays systematically. We concentrate on the charmless B decays because penguin diagrams play an important role in these decays. We use the next-to-leading order low energy effective Hamiltonian in order to consider both gluonic and electroweak penguin diagrams. We can see later that the contribution of the electroweak penguin diagram is not negligible. This article is organized as following: In Sec. II, we present the next-to-leading order effective Hamiltonian and the computation method. Section III is devoted to the numerical results and corresponding discussions.

II. EFFECTIVE HAMILTONIAN AND FACTORIZATION APPROXIMATION

Following Ref. [6], the next-to-leading order low energy effective Hamiltonian describing $\Delta B=-1$, $\Delta C=\Delta U=0$ transitions is given at the renormalization scale $\mu=O(m_b)$ as

$$\mathcal{H}_{\rm eff}(\Delta B=-1)=\frac{G_F}{\sqrt{2}}\left[\sum_{q=u,c}v_q\left\{Q_1^qC_1(\mu)+Q_2^qC_2(\mu)+\sum_{k=3}^{10}Q_kC_k(\mu)\right\}\right], \quad (1)$$

where the Wilson coefficient functions $C_i(\mu)$ ($i=1,\ldots,10$) are calculated in the renormalization group improved perturbation theory and include the leading and next-to-leading order QCD corrections and the leading order corrections in α. The Cabibbo-Kobayashi-Maskawa (CKM) factors v_q are defined as

$$v_q=\begin{cases}V_{qd}^*V_{qb} & \text{for } b\to d \text{ transitions,}\\ V_{qs}^*V_{qb} & \text{for } b\to s \text{ transitions.}\end{cases} \quad (2)$$

Here, we make use of the Wolfenstein parametrization [7] in which the CKM matrix can be written in terms of the four parameters λ, A, ρ, and η in the following form:

$$V=\left|\begin{array}{ccc}1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3A(\rho-i\eta)\\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2A\\ \lambda^3A(1-\rho-i\eta) & -\lambda^2A & 1\end{array}\right|. \quad (3)$$

The preferred values of the CKM parameters are $\lambda=0.22$, $A=0.8$, $\eta=0.34$, and $\rho=-0.12$, which are obtained from the fit to the experimental data [8]. The operators Q_1^u, Q_2^u, $Q_3,\ldots,Q_{10}$ are given as the following forms:

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$$
\begin{aligned}
Q_1^u &= (\bar{q}_\alpha u_\beta)_{V-A}(\bar{u}_\beta b_\alpha)_{V-A}, & Q_2^u &= (\bar{q}u)_{V-A}(\bar{u}b)_{V-A},\\
Q_3 &= (\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V-A}, & Q_4 &= (\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A},\\
Q_5 &= (\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V+A}, & Q_6 &= (\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A},\\
Q_7 &= \tfrac{3}{2}(\bar{q}b)_{V-A}\sum_{q'}e_{q'}(\bar{q}'q')_{V+A}, & Q_8 &= \tfrac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A},\\
Q_9 &= \tfrac{3}{2}(\bar{q}b)_{V-A}\sum_{q'}e_{q'}(\bar{q}'q')_{V-A}, & Q_{10} &= \tfrac{3}{2}(\bar{q}_\alpha q_\beta)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A},
\end{aligned}
\tag{4}
$$

where Q_1^u and Q_2^u are the current-current operators, and the current-current operators Q_1^c and Q_2^c are obtained from Q_1^u and Q_2^u through the substitution of $u \to c$. $Q_3, \ldots, Q_6$ are the QCD penguin diagram operators, whereas $Q_7, \ldots, Q_{10}$ are the electroweak penguin diagram operators. The quark $q = d$ or s is for $b \to d$ or s transitions, respectively; q' is running over the quark flavors being active at the scale $\mu = O(m_b)$ ($q' \in \{u, d, c, s, b\}$); $e_{q'}$ are the corresponding quark charges; the indices α, β are $SU(3)_c$ color indices; $(V \pm A)$ refer to $\gamma_\mu(1 \pm \gamma_5)$. It should be noted that the Hamiltonian (1) can be viewed as the generalization of the leading logarithmic Hamiltonians presented in [9,10].

Beyond the leading logarithmic approximation, the Wilson coefficient functions $C_i(\mu)$ depend both on the form of the operator basis (4) and on the renormalization scheme. Here, we use the renormalization scheme independent Wilson coefficient functions [11]:

$$
\bar{\mathbf{C}}(\mu) = \left[\hat{1} + \frac{\alpha_s(\mu)}{4\pi}\hat{r}_s^T + \frac{\alpha(\mu)}{4\pi}\hat{r}_e^T\right]\cdot \mathbf{C}(\mu), \tag{5}
$$

where $\hat{r}_s$ and $\hat{r}_e$ are obtained from one-loop matching conditions. Now, taking the QCD and electroweak one-loop level matrix elements of the operators Q_i (Q_i=Q_1^u, Q_2^u, $Q_3, \ldots, Q_{10}$) into account through

$$
\langle \mathbf{Q}^T(\mu)\rangle = \langle \mathbf{Q}^T\rangle_0 \cdot \left[\hat{1} + \frac{\alpha_s(\mu)}{4\pi}\hat{m}_s^T(\mu) + \frac{\alpha_{em}}{4\pi}\hat{m}_e^T(\mu)\right], \tag{6}
$$

which define matrices $\hat{m}_s(\mu)$ and $\hat{m}_e(\mu)$. In Eqs. (5) and (6), $\mathbf{C}(\mu)$, $\bar{\mathbf{C}}(\mu)$, and $\mathbf{Q}$ are all column vectors, where the vector $\mathbf{Q}$ are given by the operator basis Q_i, and $\langle\mathbf{Q}\rangle_0$ denote the tree level matrix elements of these operators. Combining Eqs. (5) and (6), we obtain

$$
\begin{aligned}
\langle \mathbf{Q}^T(\mu)\cdot\mathbf{C}(\mu)\rangle &= \langle \mathbf{Q}^T\rangle_0 \cdot \left[\hat{1} + \frac{\alpha_s(\mu)}{4\pi}\left[\hat{m}_s(\mu) - \hat{r}_s\right]^T + \frac{\alpha(\mu)}{4\pi}\left[\hat{m}_e(\mu) - \hat{r}_e\right]^T\right]\cdot\bar{\mathbf{C}}(\mu)\\
&\equiv \langle \mathbf{Q}^T\rangle_0 \cdot \mathbf{C}'(\mu),
\end{aligned}
\tag{7}
$$

where $\mathbf{C}'(\mu)$ are defined as

$$
\begin{aligned}
C_1' &= \overline{C}_1, & C_2' &= \overline{C}_2,\\
C_3' &= \overline{C}_3 - P_s/3, & C_4' &= \overline{C}_4 + P_s,\\
C_5' &= \overline{C}_5 - P_s/3, & C_6' &= \overline{C}_6 + P_s,\\
C_7' &= \overline{C}_7 + P_e, & C_8' &= \overline{C}_8,\\
C_9' &= \overline{C}_9 + P_e, & C_{10}' &= \overline{C}_{10},
\end{aligned}
\tag{8}
$$

where $P_{s,e}$ are given by

$$
\begin{aligned}
P_s &= \frac{\alpha_s}{8\pi}\overline{C}_2(\mu)\left[\frac{10}{9} - G(m_q, q^2, \mu)\right],\\
P_e &= \frac{\alpha_{em}}{9\pi}\left[3\overline{C}_1 + \overline{C}_2(\mu)\right]\left[\frac{10}{9} - G(m_q, q^2, \mu)\right],\\
G(m, q^2, \mu) &= -4\int_0^1 dx\, x(1-x)\ln\left[\frac{m^2 - x(1-x)q^2}{\mu^2}\right],
\end{aligned}
\tag{9}
$$

here $q = u,\ c$. For a numerical calculation, we take $m_u = 0.005$ GeV, $m_c = 1.35$ GeV, and q^2 denotes the momentum transfer squared of the virtual gluons, photons, and Z^0 appearing in the QCD and electroweak penguin matrix elements. For the details of this calculation, see Refs. [12,13].

The renormalization scheme independent Wilson coefficient functions $\bar{C}_i(\mu)$ at the scale $\mu = O(m_b)$ are obtained by first calculating the Wilson coefficients at $\mu = O(m_W)$ and then using the renormalization group equation to evolve them down to $O(m_b)$. We use, in our analysis, $\alpha_s(m_Z) = 0.118$, $\alpha(m_Z) = 1/128$ [14], and $m_t = 174$ GeV [15], and the numerical values of the renormalization scheme independent Wilson coefficients $\overline{C}_i(\mu)$ at $\mu = O(m_b)$ [13]

$$
\begin{aligned}
\bar{c}_1 &= -0.313, \quad \bar{c}_2 = 1.150, \quad \bar{c}_3 = 0.017,\\
\bar{c}_4 &= -0.037, \quad \bar{c}_5 = 0.010, \quad \bar{c}_6 = -0.046,\\
\bar{c}_7 &= -0.001\alpha_{em}, \quad \bar{c}_8 = 0.049\alpha_{em},\\
\bar{c}_9 &= -1.321\alpha_{em}, \quad \bar{c}_{10} = 0.267\alpha_{em}.
\end{aligned}
\tag{10}
$$

With the help of Eq. (7), the two-body decay amplitude $\langle PP'|H_{\text{eff}}(\Delta B = -1)|B\rangle$ can be expressed as linear combinations of $\langle PP'|Q_i|B\rangle_0$. The hadronic matrix elements $\langle PP'|Q_i|B\rangle_0$ are evaluated using the factorization approximation [16]. It should be noted that this approach has already been used in the literature to an-

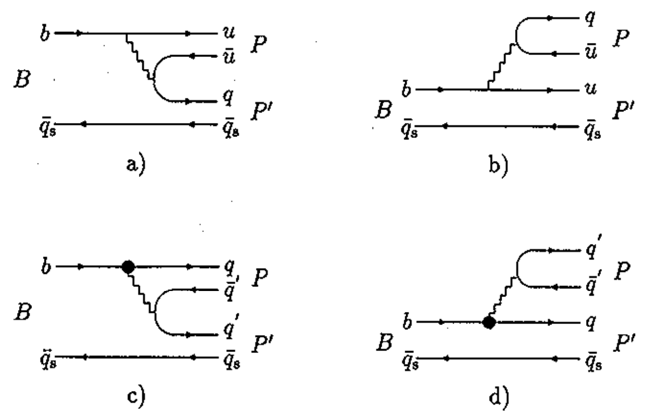

FIG. 1. Quark diagrams for a B meson decaying into two light pseudoscalar mesons P and P' through the tree process $b\to u(\bar{u}q)$: (a) the internal W-emission diagram and (b) the external W-emission diagram. The timelike penguin diagram process $b\to q(q'\bar{q}')$: (c) the timelike pure penguin diagram and (d) the timelike hairpin diagram. The subscripts "s" denote "spectator." The dot stands for the contraction of the W loop.

alyze the QCD or electroweak timelike penguin diagram contributions [12]. However, we go further in this paper by including the spacelike penguin diagrams. As in [5,12], we also neglect W-annihilation or W-exchange diagram contributions in our present analysis which are commonly assumed to be form factor suppressed.

Using the vacuum-saturation approximation, the decay amplitude $\langle PP'|H_{\rm eff}|B\rangle$ can be factorized into a product of two current matrix elements $\langle P|J^\mu|0\rangle$ and $\langle P'|J'_\mu|B\rangle$ for the tree and timelike penguin diagrams (Fig. 1), or the product of $\langle pp'|J^\mu|0\rangle$ and $\langle 0|J'_\mu|B\rangle$ for the spacelike

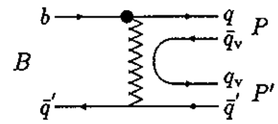

FIG. 2. Quark diagrams for a B meson decaying into two light pseudoscalar mesons P and P' through the spacelike penguin process $(b\bar{q}')\to(q\bar{q}')$. The subscripts "$v$" denote "vacuum." The dot stands for the contraction of the W loop.

penguin diagrams (Fig. 2). In this work, the hadronic matrix elements are calculated in the Bauer, Stech, and Wirbel (BSW) method [16]. We define

$$\begin{aligned} M^{pp'}_{q_1q_2q_3} &= \langle P|(\bar{q}_1q_2)_{V-A}|0\rangle\langle P'|(\bar{b}q_3)_{V-A}|B\rangle,\\ &\overset{\text{or}}{\cong} \langle P|(\bar{q}_1q_2)_{V-A}|0\rangle\langle P'|(\bar{q}_3b)_{V-A}|B\rangle \end{aligned} \tag{11}$$

and

$$\begin{aligned} S^{pp'}_{q_1q_2q_3} &= \langle PP'|(\bar{q}_1q_2)_{V-A}|0\rangle\langle 0|(\bar{b}q_3)_{V-A}|B\rangle,\\ &\overset{\text{or}}{\cong} \langle PP'|(\bar{q}_1q_2)_{V-A}|0\rangle\langle 0|(\bar{q}_3b)_{V-A}|B\rangle, \end{aligned} \tag{12}$$

where $M^{pp'}_{q_1q_2q_3}$ denotes the hadronic matrix element in tree and timelike penguin diagram case, while $S^{pp'}_{q_1q_2q_3}$ denotes the spacelike penguin case. When the $(V-A)(V+A)$ current are transformed into $(S+P)(S-P)$ and further into $(V-A)(V-A)$ ones using equation of motion for the timelike and spacelike penguin diagram amplitudes, there appear the terms which are proportional to

$$\frac{2m_X^2}{(m_q+m_{q'})(m_b-m_{q'})} \quad\text{and}\quad \frac{2m_B^2}{(m_q-m_{q'})(m_b+m_{q'})},$$

respectively. If $q=q'$ as in the decay modes:

$$\begin{aligned} &\bar{B}^0_d\to\pi^-\pi^+,\pi^0\pi^0,\pi^0\eta,\pi^0\eta',\eta\eta,\eta\eta',\eta'\eta',K^0\bar{K}^0 \ \ (\text{for } b\to d \text{ transitions}),\\ &\bar{B}^0_s\to K^-K^+,\bar{K}^0K^0,\pi^0\eta,\pi^0\eta',\eta\eta,\eta\eta',\eta'\eta' \ \ (\text{for } b\to s \text{ transitions}), \end{aligned}$$

the denominator of the factor

$$\frac{2m_B^2}{(m_q-m_{q'})(m_b+m_{q'})}$$

is zero. So, we cannot use the equation of motion to compute the amplitudes of these decays. We have to compute the matrix elements of $(S+P)(S-P)$ operators directly. We shall discuss it elsewhere.

As an example of how to factorize the decay amplitudes into the product of hadronic matrix elements, we give the result of $\langle\pi^-\pi^0|H_{\rm eff}|B_u^-\rangle$ in the following,

$$\begin{aligned} \langle\pi^-\pi^0|H_{\rm eff}|B_u^-\rangle = \frac{G_F}{\sqrt{2}}\sum_{q=u,c} v_q \Bigg[&\left(a_1\delta_{uq}+a_3-\frac{2M^2_{\pi^-}}{(m_d+m_u)(m_u-m_b)}(a_5+a_7)+a_9\right)M^{\pi^-\pi^0}_{duu}\\ &+\left(a_2\delta_{uq}-a_3+\frac{M^2_{\pi^0}}{m_d(m_d-m_b)}(a_5-a_7/2)-\frac{3}{2}a_8+\frac{3}{2}a_{10}+\frac{1}{2}a_9\right)M^{\pi^0\pi^-}_{uud}\\ &+\left(a_3+\frac{2M_B^2}{(m_u+m_b)(m_d-m_u)}(a_5+a_7)+a_9\right)\left(S^{\pi^-\pi^0}_{duu}+S^{\pi^0\pi^-}_{duu}\right)\Bigg], \end{aligned} \tag{13}$$

where the term $(S^{\pi^-\pi^0}_{duu}+S^{\pi^0\pi^-}_{duu})$ is the contribution obtained from two spacelike penguin diagrams, and the quark masses are taken as $m_d=0.01$ GeV, $m_u=0.005$ GeV, $m_s=0.175$ GeV, and $m_b=5.0$ GeV. a_k is defined as

$$a_{2i-1}\equiv\frac{C'_{2i-1}}{3}+C'_{2i},$$

$$a_{2i}\equiv C'_{2i-1}+\frac{C'_{2i}}{3}\ \ (i=1,2,3,4,5).$$

The general expression for the one-body pseudoscalar matrix element of the axial vector is

$$\langle 0|V_\mu - A_\mu|P(q)\rangle = if_P q_\mu, \tag{14}$$

where q represents the momentum of the pseudoscalar meson, and f_P is the decay constant. The two-body pseudoscalar-pseudoscalar matrix element of the vector current is [4,17]

$$\langle P_2(q_2)|V_\mu - A_\mu|P_1(q_1)\rangle = f_+(q_-^2)q_{+\mu} + f_-(q_-^2)q_{-\mu}, \tag{15}$$

where $q_\pm = q_1 \pm q_2$, and the form factor $f_\pm$ is given by the monopole parametrization

$$f_+(q_-^2) \simeq \frac{f_+(0)}{1-q^2/m_{\rm pole}^2}, \tag{16a}$$

$$f_-(q_-^2) = -\frac{m_1-m_2}{m_1+m_2} f_+(q_-^2). \tag{16b}$$

With equation (14)–(16), we obtain

$$\begin{aligned} M_{duu}^{\pi^-\pi^0} &= -\frac{i}{\sqrt{2}} f_{\pi^-} f_+^{B_u^-\pi^0}(M_{\pi^-}^2)\frac{M_{B_u^-}-M_{\pi^0}}{M_{B_u^-}+M_{\pi^0}} \\ &\quad\times\left[(M_{B_u^-}+M_{\pi^0})^2 - M_{\pi^-}^2\right], \\ M_{uud}^{\pi^0\pi^-} &= -if_{\pi^0}^{\bar u u} f_+^{B_u^-\pi^-}(M_{\pi^-}^2)\frac{M_{B_u^-}-M_{\pi^-}}{M_{B_u^-}+M_{\pi^-}} \\ &\quad\times\left[(M_{B_u^-}+M_{\pi^-})^2 - M_{\pi^0}^2\right], \end{aligned} \tag{17}$$

and

$$\begin{aligned} S_{duu}^{\pi^-\pi^0} &= \frac{i}{\sqrt{2}} f_{B_u^-} f_+^a(M_{B_u^-}^2)\frac{M_{\pi^-}-M_{\pi^0}}{M_{\pi^-}+M_{\pi^0}} \\ &\quad\times\left[(M_{\pi^-}+M_{\pi^0})^2 - M_{B_u^-}^2\right], \\ S_{duu}^{\pi^0\pi^-} &= -\frac{i}{\sqrt{2}} f_{B_u^-} f_+^a(M_{B_u^-}^2)\frac{M_{\pi^0}-M_{\pi^-}}{M_{\pi^-}+M_{\pi^0}} \\ &\quad\times\left[(M_{\pi^-}+M_{\pi^0})^2 - M_{B_u^-}^2\right], \end{aligned} \tag{18}$$

where the factors $1/\sqrt{2}$, $-1/\sqrt{2}$ come from the constituent of $\pi^0 = (1/\sqrt{2})(\bar u u - \bar d d)$.

In order to give numerical results, we need to know the form factors. For the decay form factors such as $f_+^{B\pi}(M^2)$, etc., we can use BSW [16] method to calculate them. For the annihilation form factor $f_+^a(Q^2)$, we do not have a reliable method to compute it. But at $Q^2 = M_B^2$, one is far from the $K\pi$, $\pi\pi$, ηK resonance region. So, for the charmless B decays, because of the large energy release, we can use the form factor in its asymptotic form. For charmless B to two pseudoscalars decays, the asymptotic form factor predicted by QCD [18] should be a resonable approximation. So we take $f_+^a(Q^2) = i16\pi\alpha_s f_B^2/Q^2$. Now we are in a position to give the numerical results.

III. RESULTS AND DISCUSSIONS

The decay width of a B meson at rest decaying into two pseudoscalars is

$$\Gamma(B\to PP') = \frac{1}{8\pi}|\langle PP'|H_{\rm eff}|B\rangle|^2\frac{|p|}{M_B^2}, \tag{19}$$

where

$$|p| = \frac{\{[M_B^2-(M_P+M_{P'})^2][M_B^2-(M_P-M_{P'})^2]\}^{1/2}}{2M_B} \tag{20}$$

is the momentum of the pseudoscalar meson P or P'. The corresponding branching ratios ($B_{\rm BR}$) are given by

$$B_{\rm BR}(B\to PP') = \frac{\Gamma(B\to PP')}{\Gamma_{\rm tot}}. \tag{21}$$

In our numerical calculation, we take [14] $\Gamma_{\rm tot}^{B_u^-} = 4.27\times 10^{-13}$ GeV, $\Gamma_{\rm tot}^{B_d^0} = 4.39\times10^{-13}$ GeV, and $\Gamma_{\rm tot}^{B_s^0} = 4.91\times 10^{-13}$ GeV.

In order to obtain the CP violating parameter, the B meson decay amplitude can be generally expressed as

$$\begin{aligned} \langle PP'|H_{\rm eff}|B\rangle &= \frac{G_F}{\sqrt{2}}\sum_{q=u,c} v_q\Big(C_1'\langle Q_1^q\rangle + C_2'\langle Q_2^q\rangle \\ &\quad + \sum_{k=3}^{10} C_k'\langle Q_k\rangle\Big) \\ &\equiv \frac{G_F}{\sqrt{2}}\sum_{q=u,c} v_q(T_q+F_q). \end{aligned} \tag{22}$$

where T_q and F_q denote the tree and Penguin diagram amplitude, respectively. With the help of Eq. (22), one can get the CP violating asymmetry parameter

$$\begin{aligned} \mathcal{A}_{CP} &= \frac{\Gamma(B\to PP') - \Gamma(\bar B\to \bar P\bar P')}{\Gamma(B\to PP') + \Gamma(\bar B\to \bar P\bar P')} \\ &= \frac{2\,{\rm Im}(v_u v_c^*){\rm Im}[(T_c+F_c)/(T_u+F_u)]}{v_u^2 + v_c^2[(T_c+F_c)/(T_u+F_u)]^2 + 2\,{\rm Re}[(T_c+F_c)/(T_u+F_u)]}. \end{aligned} \tag{23}$$

Since the branching ratios and CP asymmetries depend crucially on the parameter q^2 describing the momentum squared of the exchanged virtual particles appearing in the penguin matrix elements of Figs. 1 and 2, we should consider it in detail. Here, we use the same simple picture for two-body decays illustrated in Figs. 1(c) and 2 as the one in Ref. [5]. With the simple physical picture presented in Ref. [5], the average value of the momentum squared $\langle q^2\rangle$ of the exchanged virtual particles can be given by

$$\langle q^2 \rangle = m_b^2 + m_q^2 - 2 m_b E_q \ , \tag{24}$$

where E_q is determined from

$$E_q + \sqrt{E_q^2 - m_q^2 + m_{q'}^2} + \sqrt{4E_q^2 - 4m_q^2 + m_{q'}^2} = m_b \tag{25a}$$

for the timelike penguin diagram channels; or from

$$E_q + \sqrt{E_q^2 - m_q^2 + m_{q'}^2} = m_b + m_{q'} \tag{25b}$$

for the spacelike penguin diagram channels. When we factorize $\langle Q_k \rangle_0$ of hairpin diagrams illustrated in Fig. 1(d), we find $\langle Q_3 \rangle_0 = -\langle Q_5 \rangle_0$, $\langle Q_4 \rangle_0 = -\langle Q_6 \rangle_0$, and $\langle Q_7 \rangle_0 = -\langle Q_9 \rangle_0$, and hence the factor in Eq. (6):

$$\left\{ \frac{\alpha_s}{8\pi} \left[-\frac{1}{N_c} \langle Q_3 \rangle_0 + \langle Q_4 \rangle_0 - \frac{1}{N_c} \langle Q_5 \rangle_0 + \langle Q_6 \rangle_0 \right] \bar{C}_2(\mu) \right.$$
$$\left. + \frac{\alpha}{3\pi} \left[\langle Q_7 \rangle_0 + \langle Q_9 \rangle_0 \right] \left[\bar{C}_1(\mu) + \frac{1}{N_c} \bar{C}_2(\mu) \right] \right\} \tag{26}$$

vanishes because of the cancellation. So, we do not need to consider the hairpin diagrams.

The numerical results of the spacelike penguin diagram contributions to the branching ratios and CP violating asymmetries are given in Table I. In the meantime, we also calculate the branching ratios and CP violating asymmetries with only the tree and timelike penguin diagram contributions for comparison. We also present the results with only tree and gluonic penguin diagram contributions. All the parameters such as meson decay constants and form factors needed in our calculation are taken as $f_{\pi^\pm} = 0.13$ GeV, $f_K = 0.160$ GeV [14], $f_{\pi^0}^{\bar{u}u} = -f_{\pi^0}^{\bar{d}d} = f_{\pi^\pm}/\sqrt{2}$, $f_\eta^{\bar{u}u} = f_\eta^{\bar{d}d} = 0.092$, $f_\eta^{\bar{s}s} = -0.105$, $f_{\eta'}^{\bar{u}u} = f_{\eta'}^{\bar{d}d} = 0.049$, $f_{\eta'}^{\bar{s}s} = 0.12$ [4], $f_D = 0.23$, $f_{D_s} = 0.281$ [19], $f_B = 1.5 \times f_{\pi^\pm}$ [20], $f_{B_s} = 0.206$ [21], and $f_+^{B_u^- \pi^-}(0) = 0.29$, $f_+^{B_u^- K^-}(0) = 0.32$ [22], $f_+^{B_u^- \eta_{au}}(0) = 0.307$, $f_+^{B_u^- \eta'_{au}}(0) = 0.254$, $f_+^{BD}(0) = 0.690$ [16], $f_+^{B_s^- \eta_{ss}}(0) = 0.335$, $f_+^{B_s^- \eta'_{ss}}(0) = 0.282$ [23], $f_+^{B_s D_s}(0) = 0.648$ [24].

From Table I we can see the following features:

(i) For most of the charmless decays, penguin diagram contributions are important.

(ii) For $B_u^- \to \pi^- \eta$, $K^- \pi^0$, $K^- \eta$, $K^- \eta'$, $\bar{B}_d^0 \to \bar{K}^0 \pi^0$, $\bar{K}^0 \eta$, $\bar{K}^0 \eta'$, and $B_s^0 \to \pi^0 K^0$, the contribution of the electroweak penguin diagrams are not negligible.

(iii) The spacelike penguin diagram effects in $B_u^- \to \pi^- \pi^0$ are amazingly large. The correction to the branch-

TABLE I. The branching ratios and the CP asymmetries, where the "only tree" means the branching ratios with only tree diagram contribution, "T-like" denotes the timelike penguin contributions, the "S-like" denotes the spacelike penguin contributions, "QCD" means QCD penguin and tree diagrams contributions, and "QCD+EW" denotes full tree, QCD, and EW (electroweak) penguin contributions.

Decay mode		$\mathcal{B}_{\rm BR}$				$\mathcal{A}_{cp}$			
	Only tree	Tree+T-like		Tree+T-like+S-like		Tree+T-like		Tree+T-like+S-like	
		QCD	QCD+EW	QCD	QCD+EW	QCD	QCD+EW	QCD	QCD+EW
$B_u^- \to \pi^- \pi^0$	3.54×10^{-6}	3.46×10^{-6}	3.27×10^{-6}	1.55×10^{-6}	1.34×10^{-6}	0.69%	0.77%	−67.4%	−70.4%
$B_u^- \to \pi^- \eta$	2.62×10^{-6}	4.39×10^{-6}	4.55×10^{-6}	4.39×10^{-6}	4.55×10^{-6}	35.1%	33.5%	35.1%	33.5%
$B_u^- \to \pi^- \eta'$	8.3×10^{-7}	6.98×10^{-6}	6.92×10^{-6}	6.98×10^{-6}	5.92×10^{-6}	16.0%	16.1%	16.0%	16.1%
$B_u^- \to K^0 K^-$	0	4.97×10^{-7}	4.88×10^{-7}	4.52×10^{-7}	4.44×10^{-7}	2.47%	2.49%	−2.21%	−2.24%
$B_u^- \to K^- \pi^0$	2.66×10^{-7}	3.52×10^{-6}	5.06×10^{-6}	6.37×10^{-6}	7.88×10^{-6}	−8.73%	−6.27%	23.3%	17.9%
$B_u^- \to K^- \eta$	1.97×10^{-7}	1.90×10^{-7}	2.29×10^{-7}	5.17×10^{-8}	9.07×10^{-8}	6.03%	4.29%	−76.6%	−65.3%
$B_u^- \to K^- \eta'$	6.29×10^{-8}	9.97×10^{-6}	9.26×10^{-6}	9.60×10^{-6}	9.07×10^{-6}	−3.35%	−3.53%	−1.09%	−1.18%
$B_u^- \to \bar{K}^0 \pi^-$	0	5.91×10^{-6}	5.80×10^{-6}	8.48×10^{-6}	8.32×10^{-6}	−0.18%	−0.18%	0.89%	0.90%
$\bar{B}_d^0 \to K^- \pi^+$	4.60×10^{-7}	6.75×10^{-6}	6.97×10^{-6}	1.21×10^{-5}	1.24×10^{-5}	−8.38%	−8.2%	22.2%	21.8%
$\bar{B}_d^0 \to \bar{K}^0 \pi^0$	8.25×10^{-10}	2.85×10^{-6}	1.81×10^{-6}	4.22×10^{-6}	3.18×10^{-6}	0.45%	0.75%	−0.76%	−1.19%
$\bar{B}_d^0 \to \bar{K}^0 \eta$	8.20×10^{-10}	4.93×10^{-8}	2.35×10^{-9}	7.42×10^{-8}	2.69×10^{-8}	1.44%	47.0%	−11.5%	−27.6%
$\bar{B}_d^0 \to \bar{K}^0 \eta'$	2.28×10^{-10}	9.26×10^{-6}	8.63×10^{-6}	8.88×10^{-6}	8.25×10^{-6}	−0.43%	−0.46%	−0.06%	−0.070%
$\bar{B}_s^0 \to \pi^- K^+$	4.97×10^{-6}	4.36×10^{-6}	4.34×10^{-6}	2.46×10^{-6}	2.38×10^{-6}	8.34%	8.41%	−44.1%	−44.4%
$\bar{B}_s^0 \to \pi^0 K^0$	1.13×10^{-8}	1.18×10^{-7}	7.06×10^{-8}	3.36×10^{-7}	2.94×10^{-7}	−6.05%	−10.2%	18.5%	23.2%
$\bar{B}_s^0 \to \eta K^0$	1.09×10^{-8}	1.45×10^{-6}	1.59×10^{-6}	1.50×10^{-6}	1.64×10^{-6}	5.27%	4.87%	6.44%	5.96%
$\bar{B}_s^0 \to \eta' K^0$	3.13×10^{-9}	8.33×10^{-6}	8.23×10^{-6}	8.14×10^{-6}	8.05×10^{-6}	3.09%	3.11%	2.41%	2.43%

ing ratio is more than 100%, while to the CP asymmetry is about 2 order of magnitude, actually, $A_{CP} \sim 0.77\%$ with only the timelike penguin diagram, but $A_{CP} \sim -70.4\%$ when including the spacelike penguin diagram. This is surprising. The reason is the following:

Using the amplitude definition in Eq. (22), for $B_u^- \to \pi^-\pi^0$, the tree amplitude is

$$T_u \sim -0.83i. \tag{27}$$

The timelike penguin diagram amplitudes are

$$\begin{aligned} F_u^t &\sim \ 0.0013 + 0.0176i, \\ F_c^t &\sim \ 0.0011 + 0.0181i, \end{aligned} \tag{28}$$

while the spacelike penguin diagram amplitudes are

$$\begin{aligned} F_u^s &\sim \ -0.1952, \\ F_c^s &\sim \ -0.1373 \,. \end{aligned} \tag{29}$$

Although the tree amplitude still dominates, the spacelike penguin diagram amplitudes are an order of magnitude larger than the timelike ones. This is because the enhancement factor in Eq. (13) $2M_B^2/[(m_u+m_b)(m_d-m_u)]$ for the spacelike penguin diagram is 3 orders larger than that of the timelike ones. It should be noted that the interference of the spacelike penguin diagram amplitude with the tree amplitude can already induce CP asymmetry. That is why the CP asymmetry changes so drastically.

For $B_u^- \to K^-\pi^0$, $K^-\eta$, $K^-\eta'$, $\bar{K}^0\pi^-$, $\bar{B}_d^0 \to K^-\pi^+$, $\bar{K}^0\eta$,$\bar{K}^0\eta'$, and $\bar{B}_s^0 \to \pi^-K^+$, π^0K^0, the spacelike penguin diagram contributions are also dominant.

In $B_u^- \to \pi^-\eta$, $\pi^-\eta'$, the spacelike penguin diagram contribution is zero. The reason is that there are two spacelike penguin diagrams in each channel and the contributions of the two diagrams exactly cancel each other.

In general, we can conclude that the spacelike penguin diagram effects are not negligible in most of the charmless two-pseudoscalar decays of the B mesons. The spacelike penguin diagrams can affect not only CP asymmetries, but also decay branching ratios. Of course, there are many uncertainties, such as the true phases and magnitudes of the annihilation form factors, the true value of the momentum squared carried by the virtual gluons, photons, and Z bosons, etc. Further investigations are definitely needed.

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